Journal of Advanced Research in Applied Sciences and Engineering Technology

Journal homepage:
https://semarakilmu.com.my/journals/index.php/applied_sciences_eng_tech/index
ISSN: 2462-1943

Implementing Reliability Analysis of a Soil Slope using First-Order Second Moment Method, Monte-Carlo Simulation Method and Subset Simulation Method in a MS-Excel Spreadsheet

Saurav Shekhar Kar[1,*], Anupama Arunkumar Athawale[2], Avijit Burman[3], Lal Bahadur Roy[3]

[1] Universidade Federal de São Carlos, 13565-905, São Paulo, Brazil
[2] AISSMS IOIT, Pune, 411001, Maharashtra, India
[3] Department of Civil Engineering, National Institute of Technology, Patna, 800005, Bihar, India

ARTICLE INFO

Article history:
Received 3 January 2023
Received in revised form 7 September 2023
Accepted 10 November 2023
Available online 25 December 2023

Keywords:
Spatial variation of soil; β; P_f; FOSM; MCS; SS

ABSTRACT

The spatial variation in the soil properties affects the analysis and design of geotechnical structures such as slope stability analysis. This paper presents the reliability analysis of a soil slope using three different methods namely, first-order second moment (FOSM) method, Monte-Carlo simulation (MCS) method and Subset simulation (SS) method. The analysis has been carried out by considering the spatial variation of the soil properties of the slope. The correlation function and the correlation distance (λ) for the uncertain parameters are modelled using random field theory developed by Vanmarcke. The factor of safety (FOS) of the slope is determined using Ordinary method of slices. The reliability based probabilistic slope stability analysis has been carried out in a MS-Excel spreadsheet to determine the probability of failure (P_f) and reliability index (β) of the slope. The spreadsheet mainly consists of three parts i.e., deterministic analysis, uncertainty analysis and uncertainty propagation. The study shows that the SS method has shown better performance as compared to FOSM method and MCS method especially at low failure probability levels. Also, SS method ensures the generation of samples in the failure region which is not always possible in MCS method.

1. Introduction

The soil characteristics changes from one point to another point along both the horizontal as well as vertical directions. These variations are termed as the inherent spatial variability of soil. Baecher and Christian [1] studied that these inherent spatial variabilities of soil are independent in nature and thus cannot be minimized. Phoon and Kulhawy [2,3] found out that the soil properties with same elevation but different locations are nearly equal and the soil properties along the horizontal direction are more correlated to each other as compared to the vertical direction. Lump [4] proposed that the spatial variability of soil along the vertical direction if soil properties are normally distributed,

* *Corresponding author.*
E-mail address: kar.sauravshekhar2008@gmail.com

https://doi.org/10.37934/araset.36.1.176188

can be modelled using the mean and standard deviation of the soil properties. But, the correlations between the soil properties at different location were not taken into consideration. Similarly, if the soil properties are log-normally distributed, then the spatial variability of soil along the vertical direction can be modelled using the mean and standard deviation of the logarithmic parameters soil properties [5]. Vanmarcke [6,7] modelled the variation in the soil properties with depth along the different locations by developing the correlation between the soil properties in terms of correlation function and correlation distance (λ). This concept is also known as random field theory. The correlation distance is defined as the significant distance up to which the soil property shows a strong correlation among each other. The correlation between the soil properties at different locations are characterized by a correlation function. Phoon and Kulhawy [2,3] suggested the typical value of correlation distance in vertical direction in the range of 0.8 – 6.1 m and average value of 2.5 m.

The uncertainties present in the soil such as inherent spatial variability affects the analysis and design of geotechnical structures [8]. The probabilistic method of analysis can help to account these uncertainties in slope stability analysis [9]. In probabilistic slope stability analysis, the safety of the slope is measured in terms of reliability index (β) or probability of failure (P_f) of the slope. The probability of failure of the slope is defined as the probability of having factor of safety (FOS) value less than one. The term reliability index and probability of failure are correlated to each other and given as Eq. (1).

$$P_f = 1 - \Phi(\beta) = \Phi(-\beta) \quad (1)$$

in which, Φ is the standard normal cumulative distribution function. As per U.S. army corps of engineers [10], the values of β varies from 5 to 1, which corresponds to P_f of 3×10^{-3} to 0.16. For a better performance of a geotechnical structure, the value of β should be at least equal to 3 which corresponds to P_f of 0.001 i.e., one out of thousand times, the structure will fail. The reliability index (β) and probability of failure (P_f) of the slope can be estimated by several methods such as first-order second moment (FOSM) method [11-17], Monte-Carlo simulation (MCS) method [14-22] and Subset simulation (SS) method [21-25].

FOSM method is a very simple approach for performing the reliability based probabilistic analysis of slope. The reliability index (β) of the slope is calculated by FOSM as Eq. (2).

$$\beta = \frac{\mu_{FOS} - 1}{\sigma_{FOS}} \quad (2)$$

in which μ_{FOS} is mean value of the factor of safety, σ_{FOS} is standard deviation of the factor of safety. The probability of failure (P_f) of slope is calculated using Eq. (1).

MCS method is a computational process of continuously calculating a mathematical operator to obtain a numerical result. The mathematical operator contains the random samples of the uncertain parameter having specified probability distributions [26]. The samples obtained from the MCS method is analogous to the samples observed from a physical experiment. In MCS method for slope stability, the mathematical operator is used to calculate the FOS of the slope and the failure occurs when FOS < 1. To expect a desired performance in P_f, the number of samples to be generated is equal to $10/P_f$ [18]. That means to obtain a failure probability level of 0.001, a total 10,000 samples of FOS should be generated. The probability of failure (P_f) and its corresponding reliability index (β) of the slope using MCS method is calculated as Eq. (3) and Eq. (4) respectively.

$$P_f = \frac{number\ of samples\ having\ F.O.S < 1\ (i.e., failure\ samples)}{total\ number\ of\ samples} \quad (3)$$

$$\beta = \Phi^{-1}(1 - P_f) \quad (4)$$

SS method is an advanced MCS method developed by Au *et al.,* [21], Wang *et al.,* [22], Au and Wang [23] and Au and Beck [24] to improve the efficiency and resolution of MCS method especially at low failure probability levels. This method uses Markov Chain Monte-Carlo Simulation [27,28] method to generate the samples based on Bayes' conditional probability theorem. This method uses the concept that the small failure probability event can be expressed in terms of several intermediate failure events having larger conditional probability. The conditional samples of these intermediate failure events are generated using Markov Chains until the target failure region is obtained. Let $Y = FOS$ be the output for slope stability problem and the probability that Y is lesser than a threshold value x is of interest, (i.e., $P(Y = FOS < x)$) and $x = x_n < x_{n-1} <\ldots< x_2 < x_1$be increasing order of n intermediate threshold value. Let $F_i = Y < x_i, i = 1, 2, \ldots, n$ be the intermediate events. The probability of failure can be written as Eq. (5).

$$P_f = P(Y < x) = P(Y < x_1)P(Y < x_2|Y < x_1) \times \cdots \times P(Y < x_m|Y < x_{m-1}) \quad (5)$$

The threshold intermediate values $(i.e.\ x_n, \ldots, x_2, x_1)$ are generated such that the sample estimates of $P(Y < x_1)$ and $\{P(Y < x_i|Y < x_{i-1}), i = 1, 2, \ldots, n\}$ corresponds to a conditional probability value equals to 0.10 [21-24]. The number of samples generated by SS depends on four parameters i.e., number of runs (R), number of samples generated per level (N), Conditional probability (P_o) and highest SS level (m). The total number of samples generated by SS per run is equal to $N + mN(1 - P_o)$.

This paper presents reliability based probabilistic analysis of a soil slope using FOSM method, MCS method and SS method. The four cases based on inherent spatial variation of soil properties of the slope have been considered in this study. The value of correlation distance (λ) among the uncertain soil parameters along the vertical direction is taken as 2.0 m. The FOS of the soil slope is determined using limit equilibrium based ordinary method of slices [29]. The analysis has been developed and executed in a MS-Excel spreadsheet, which mainly consists of three parts i.e., deterministic model generation, uncertainty model generation and uncertainty propagation [15-17]. It has been seen that SS method has shown better performance as compared to FOSM method and MCS method especially at low failure probability levels. Also, the number of samples generated by SS method has significantly reduced as compared to the MCS method. Moreover, SS method guarantee the sample generation in the failure region which is not always possible in MCS method.

2. Problem Statement

A soil slope considered by Malkawi *et al.,* [30] has been studied to assess its reliability. The cross section of the slope is shown in Figure 1.

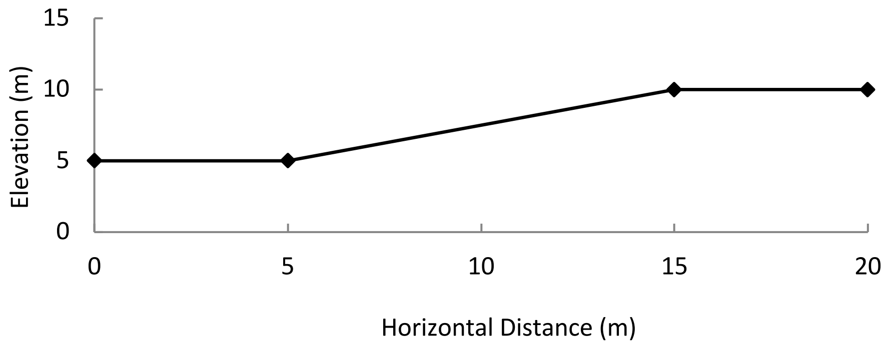


Fig. 1. Cross-section of the slope

The soil properties of slope and their coefficient of variation (c.o.v) taken in this study are shown in Table 1. The cohesion (c) and friction angle (ϕ) of the soil is considered as an uncertain parameter and the unit weight of soil (γ) is taken as constant. The hard stratum is present at 10 m below the top of the soil slope.

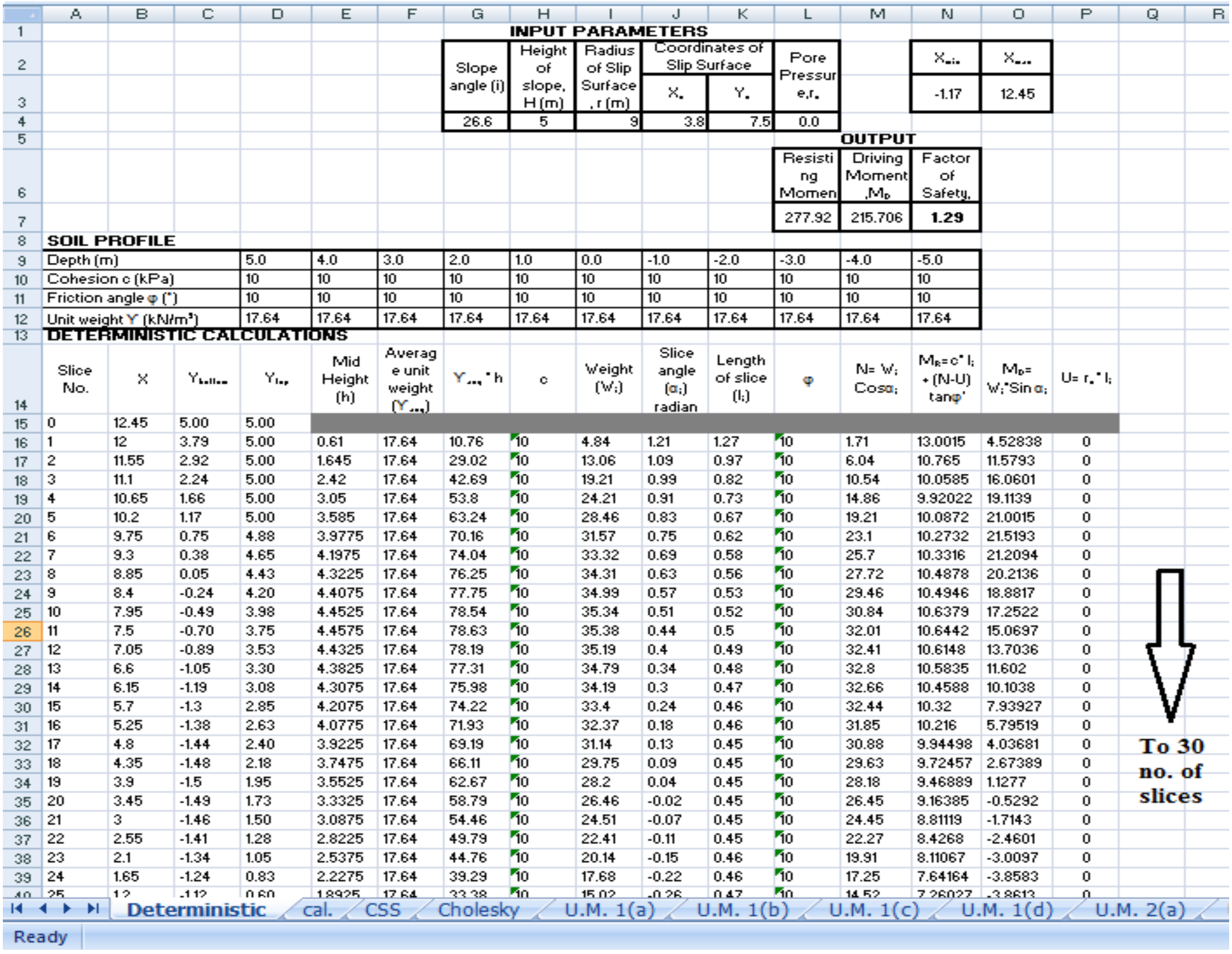
Table 1
Soil properties and their c.o.v

Soil parameters	Mean value	Coefficient of variation, c.o.v. (v)			
		Case 1	Case 2	Case 3	Case 4
Cohesion, c	10.0 kN/m^2	0.10	0.20	0.30	0.40
Angle of internal friction, ϕ	10.0°	0.05	0.10	0.15	0.20
Unit weight, γ	17.64 kN/m^3	----	----	----	----

3. Methodology

The reliability analysis has been performed in a MS-Excel spreadsheet with the help of Visual Basic for Application (VBA) functions. The spreadsheet mainly consists of three parts i.e., deterministic model generation, uncertainty model generation and uncertainty propagation model. The Deterministic model is developed to calculate the FOS value of the slope using ordinary method of slices. In order to determine the FOS, the system parameters such as slope geometry, soil profile, soil characteristics, slip surface geometry etc. should be properly defined in the deterministic model. The deterministic model developed for the slope stability problem is shown in Figure 2. VBA codes have been run for calculating the FOS values for different combinations of centre coordinates (x, y) and radius of slip surface and then identifying the minimum value as the FOS of the slope, which corresponds to the critical slip surface having centre coordinate(x_c, y_c) and radius(r). For slope stability problem, the FOS is obtained as 1.29 having critical slip surface coordinate (3.8, 7.5) and radius of 9.0 m.

An uncertainty model shown in Figure 3 is developed to generate the random samples of uncertain soil parameters based on their probability distribution. The 10 m soil slope is assumed to be divided into 10 layers of 1.0 m thickness. Total eleven independent and identically distributed random variables are required at 11 depths. Let $\bar{c} = [c_1(d_1), c_2(d_2), \dots, c_{11}(d_{11})]^T$ be vector of$c_i$at depth$d_i$and $\bar{\phi} = [\phi_1(d_1), \phi_2(d_2), \dots, \phi_{11}(d_{11})]^T$ be vector of ϕ_i at depth d_i, where $i = 1,2, \dots, 11$.



INPUT PARAMETERS

Slope angle (i)	Height of slope, H (m)	Radius of Slip Surface, r (m)	Coordinates of Slip Surface X_c	Coordinates of Slip Surface Y_c	Pore Pressure, r_u	X_{min}	X_{max}
26.6	5	9	3.8	7.5	0.0	-1.17	12.45

OUTPUT

Resisting Moment	Driving Moment, M_D	Factor of Safety,
277.92	215.706	**1.29**

SOIL PROFILE

Depth (m)	5.0	4.0	3.0	2.0	1.0	0.0	-1.0	-2.0	-3.0	-4.0	-5.0
Cohesion c (kPa)	10	10	10	10	10	10	10	10	10	10	10
Friction angle φ (°)	10	10	10	10	10	10	10	10	10	10	10
Unit weight γ (kN/m³)	17.64	17.64	17.64	17.64	17.64	17.64	17.64	17.64	17.64	17.64	17.64

DETERMINISTIC CALCULATIONS

Slice No.	X	Y_{bottom}	Y_{top}	Mid Height (h)	Average unit weight (γ_{avg})	γ_{avg}*h	c	Weight (W_i)	Slice angle (α_i) radian	Length of slice (l_i)	φ	$N = W_i \cos\alpha_i$	$M_R = c^* l_i + (N-U) \tan\varphi'$	$M_D = W_i^* \sin\alpha_i$	$U = r_u^* l_i$
0	12.45	5.00	5.00												
1	12	3.79	5.00	0.61	17.64	10.76	10	4.84	1.21	1.27	10	1.71	13.0015	4.52838	0
2	11.55	2.92	5.00	1.645	17.64	29.02	10	13.06	1.09	0.97	10	6.04	10.765	11.5793	0
3	11.1	2.24	5.00	2.42	17.64	42.69	10	19.21	0.99	0.82	10	10.54	10.0585	16.0601	0
4	10.65	1.66	5.00	3.05	17.64	53.8	10	24.21	0.91	0.73	10	14.86	9.92022	19.1139	0
5	10.2	1.17	5.00	3.585	17.64	63.24	10	28.46	0.83	0.67	10	19.21	10.0872	21.0015	0
6	9.75	0.75	4.88	3.9775	17.64	70.16	10	31.57	0.75	0.62	10	23.1	10.2732	21.5193	0
7	9.3	0.38	4.65	4.1975	17.64	74.04	10	33.32	0.69	0.58	10	25.7	10.3316	21.2094	0
8	8.85	0.05	4.43	4.3225	17.64	76.25	10	34.31	0.63	0.56	10	27.72	10.4878	20.2136	0
9	8.4	-0.24	4.20	4.4075	17.64	77.75	10	34.99	0.57	0.53	10	29.46	10.4946	18.8817	0
10	7.95	-0.49	3.98	4.4525	17.64	78.54	10	35.34	0.51	0.52	10	30.84	10.6379	17.2522	0
11	7.5	-0.70	3.75	4.4575	17.64	78.63	10	35.38	0.44	0.5	10	32.01	10.6442	15.0697	0
12	7.05	-0.89	3.53	4.4325	17.64	78.19	10	35.19	0.4	0.49	10	32.41	10.6148	13.7036	0
13	6.6	-1.05	3.30	4.3825	17.64	77.31	10	34.79	0.34	0.48	10	32.8	10.5835	11.602	0
14	6.15	-1.19	3.08	4.3075	17.64	75.98	10	34.19	0.3	0.47	10	32.66	10.4588	10.1038	0
15	5.7	-1.3	2.85	4.2075	17.64	74.22	10	33.4	0.24	0.46	10	32.44	10.32	7.93927	0
16	5.25	-1.38	2.63	4.0775	17.64	71.93	10	32.37	0.18	0.46	10	31.85	10.216	5.79519	0
17	4.8	-1.44	2.40	3.9225	17.64	69.19	10	31.14	0.13	0.45	10	30.88	9.94498	4.03681	0
18	4.35	-1.48	2.18	3.7475	17.64	66.11	10	29.75	0.09	0.45	10	29.63	9.72457	2.67389	0
19	3.9	-1.5	1.95	3.5525	17.64	62.67	10	28.2	0.04	0.45	10	28.18	9.46889	1.1277	0
20	3.45	-1.49	1.73	3.3325	17.64	58.79	10	26.46	-0.02	0.45	10	26.45	9.16385	-0.5292	0
21	3	-1.46	1.50	3.0875	17.64	54.46	10	24.51	-0.07	0.45	10	24.45	8.81119	-1.7143	0
22	2.55	-1.41	1.28	2.8225	17.64	49.79	10	22.41	-0.11	0.45	10	22.27	8.4268	-2.4601	0
23	2.1	-1.34	1.05	2.5375	17.64	44.76	10	20.14	-0.15	0.46	10	19.91	8.11067	-3.0097	0
24	1.65	-1.24	0.83	2.2275	17.64	39.29	10	17.68	-0.22	0.46	10	17.25	7.64164	-3.8583	0
25	1.2	-1.12	0.60	1.8925	17.64	33.38	10	15.02	-0.26	0.47	10	14.52	7.26027	-3.8613	0

To 30 no. of slices

Deterministic | cal. | CSS | Cholesky | U.M. 1(a) | U.M. 1(b) | U.M. 1(c) | U.M. 1(d) | U.M. 2(a)

Ready

Fig. 2. Deterministic model for slope stability problem

The inherent spatial variation in c and $\phi$of soil along the vertical depth is modelled by 1-D random field theory and are log-normally distributed having an exponential correlation function. Let$c(d)$ and $\phi(d)$be the cohesion and friction angle of soil at depth(d), then the correlation between $ln[c_i(d_i)]$ and $ln[c_j(d_j)]$and between $ln[\phi_i(d_i)]$ and $ln[\phi_j(d_j)]$at depth d_i and $d_j$is given as Eq. (6).

$$\bar{R} \ = \ R_{ij} = e^{\left(-\frac{2|d_i-d_j|}{\lambda}\right)} \tag{6}$$

in which $\bar{R}$ is the correlation function between the uncertain soil parameters and λ is correlation distance between the uncertain soil parameters. When the c and ϕ of soil are log-normally distributed, then in space domain as per Cao *et al.,* [14], it can be represented as Eq. (7) and Eq. (8) respectively.

$$ln(c) = U\bar{l} + C\bar{L}\,\bar{Z} \tag{7}$$

$$ln(\phi) = M\bar{l} + D\bar{L}\,\bar{Z} \tag{8}$$

where U and C are the mean and standard deviation of $ln[c(d)]$, M and D are the mean and standard deviation of $ln[\phi(d)]$, $\bar{l}$ is a column vector with 11 components all equal to one, $\bar{Z}$ is 11 dimensional standard normal vector, $\bar{L}$ is a 11×11 dimensional lower triangular matrix obtained by Cholesky decomposition of correlation function $\bar{R}$ such that $\bar{R} = \bar{L}\bar{L}^T$. The generation of $\bar{L}$ matrix is obtained using MATLAB code.

Figure 3 shows the uncertainty model generated for coefficient of variation (c.o.v.) of cohesion (v_c) and c.o.v. of friction angle (v_φ) equals to 0.10 and 0.05 respectively. Similarly, different uncertainty models are generated for the different c.o.v. values of cohesion and friction angle. For this slope stability problem, four different uncertainty models are generated for four cases. The value of c (cells D18:N18 of Figure 3) and ϕ (cells D24:N24 of Figure 3) generated are random and these values are copied to the values of c (cells D10:N10 in Figure 2) and ϕ (cells D11:N11 in Figure 2) in deterministic model by entering formulas in the cells, which links both the model. Now, the values of FOS which will be generated in the deterministic model will be random (Cell N7 in Figure 2). At this stage, by pressing the F9 key in Excel can generate the random values of FOS and one can generate the required sample size. In place of constantly pressing the F9 key, a VBA macro code has been written and run in MS-Excel to calculate the n number of random samples of FOS at a time. For the slope stability problem, a total of 5000 samples of the FOS are generated and reliability analysis of the soil slope has been performed using FOSM method and MCS method.

Uncertainity Modelling Worksheet

Parameters

Mean of c (kPa)	10	U =	2.29761	mean of Ln(c)	Mean of ϕ (degree)	10	M =	2.30134	mean of Ln(ϕ)
Std. of c (kPa)	1	C =	0.09975	Std. of Ln(c)	Std. of ϕ (degree)	0.5	D =	0.04997	Std. of Ln(ϕ)
Cov of c	0.1	λ =	2 m	correlation length (assumed value)	Cov of ϕ	0.05	λ =	2 m	correlation length (assumed value)

Random Sample generation

Vertical depth (m)	5	4	3	2	1	0	-1	-2	-3	-4	-5
Uniform I.I.D.	0.48008	0.70336	0.44242	0.97579	0.04427	0.94817	0.27861	0.30709	0.9435	0.91405	0.87824
Std. Normal I.I.D., Z	-0.05	0.5341	-0.1448	1.9737	-1.7031	1.6274	-0.587	-0.5041	1.5849	1.3661	1.1662
PDF of Std. Normal I.I.D.	0.3984	0.3459	0.3948	0.0569	0.0936	0.1061	0.3358	0.3513	0.1136	0.1569	0.2021

Generation of Lognormal random field for c by cholesky transformation

L*Z	-0.05	0.47821	0.04132	1.85031	-0.9027	1.18086	-0.1112	-0.5098	1.28605	1.74349	1.72549
Ln(c) = U+C*L*Z	2.29262	2.34531	2.30173	2.48218	2.20756	2.4154	2.28652	2.24676	2.4259	2.47152	2.46973
c (kPa)	9.90087	10.4365	9.99146	11.9673	9.0935	11.1943	9.84061	9.457	11.3124	11.8405	11.8193

Generation of Lognormal random field for ϕ by cholesky transformation

L*Z	-0.05	0.47821	0.04132	1.85031	-0.9027	1.18086	-0.1112	-0.5098	1.28605	1.74349	1.72549
Ln(ϕ) = M+D*L*Z	2.29884	2.32523	2.3034	2.39379	2.25623	2.36034	2.29578	2.27586	2.3656	2.38846	2.38756
ϕ (degree)	9.9626	10.2291	10.0082	10.955	9.54701	10.5946	9.93218	9.73631	10.6504	10.8967	10.8869

Lower Traingular Matrix, L

	5	4	3	2	1	0	-1	-2	-3	-4	-5
5	1	0	0	0	0	0	0	0	0	0	0
4	0.3679	0.9298	0	0	0	0	0	0	0	0	0
3	0.1353	0.3421	0.9298	0	0	0	0	0	0	0	0
2	0.0498	0.1258	0.3421	0.9298	0	0	0	0	0	0	0
1	0.0183	0.0463	0.1258	0.3421	0.9298	0	0	0	0	0	0
0	0.0067	0.0170	0.0463	0.1258	0.3421	0.9298	0	0	0	0	0
-1	0.0025	0.0062	0.0170	0.0463	0.1258	0.3421	0.9298	0	0	0	0
-2	0.0009	0.0023	0.0062	0.0170	0.0463	0.1258	0.3421	0.9298	0	0	0
-3	0.0003	0.0008	0.0023	0.0062	0.0170	0.0463	0.1258	0.3421	0.9298	0	0
-4	0.0001	0.0003	0.0008	0.0023	0.0062	0.0170	0.0463	0.1258	0.3421	0.9298	0
-5	0.00005	0.00009	0.0003	0.0008	0.0023	0.0062	0.0170	0.0463	0.1258	0.3421	0.9298

Deterministic / cal. / CSS / Cholesky / U.M. 1(a) / U.M. 1(b) / U.M. 1(c) / U.M. 1(d) / U.M. 2(a) / U.M. 2(b) / U.M. 2(c) / U.M. 2(d) / U.M. 3(a) / U.M. 3(b)

Ready

Fig. 3. Uncertainty model generated for v_c =0.10 and v_φ = 0.05 (case 1)

After linking the above two models, the uncertainty propagation is carried out using subset simulation (SS) method as an Excel Add-In called Uncertainty Propagation using Subset Simulation (UPSS). After each simulation run, the UPSS gives the plot for driving variable ($P(Y = FOS < x)$)

versus threshold value$x$ and based on the information, histogram or probability of failure (P_f) and its corresponding reliability index (β) of the slope can be estimated. For the slope stability problem, SS method is performed using following parameters i.e., number of samples per level, N = 500, P_0 = 0.1, number of simulation level, m = 2 and number of runs, R = 1. The total number of samples generated is equal to 500 + 2 (1 – 0.1) 500 = 1400 (i.e., 500 samples in level 0, 450 samples in level 1 and 450 samples in level 2).

4. Results and Discussion

Total 5000 samples of FOS have been generated using MCS method for four different cases. The FOS histogram obtained from MCS samples have been shown in Figure 4. The detailed analysis of the FOS histogram is presented in Table 2.

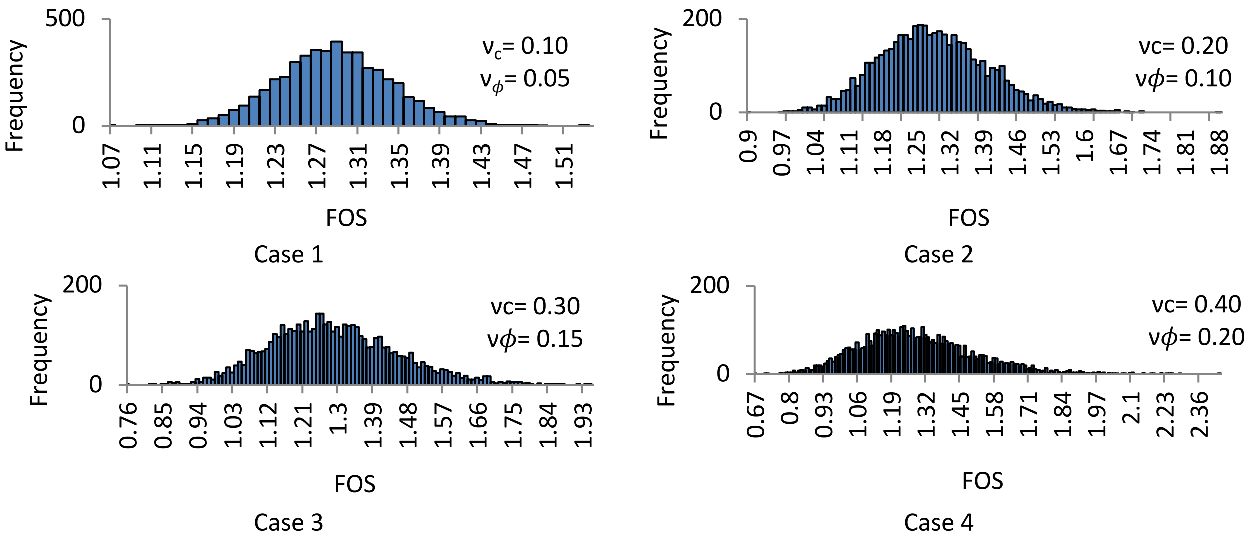


Fig. 4. FOS histogram obtained from MCS method

Table 2 shows the probability of failure (P_f) and reliability index (β) of the soil slope obtained for four different cases using MCS method. For example, (say case 2), 7 samples out of 5000 samples have failed i.e., 7 samples have FOS value less than one. The probability of failure (P_f) of the slope is calculated as 7/5000 = 0.14 % which corresponds to the reliability index (β) equals to ϕ^{-1}(0.14%) = 2.99. Similarly, P_f and β have been calculated for other cases as shown in Table 2. It has been observed that with the increase in the c.o.v. values of soil parameters (c and ϕ), the probability of failure of the slope increases and thus, the slope becomes less reliable.

Table 2
Result obtained from MCS method

Method	Total number of samples generated	Coefficient of variation (v)			Failure samples (FOS < 1)	Probability of failure (P_f) %	Reliability Index (β)
		Cohesion (v_c)	Angle of internal friction (v_ϕ)	Cases			
MCS	5000	0.10	0.05	Case 1	0	-	-
		0.20	0.10	Case 2	7	0.14	2.99
		0.30	0.15	Case 3	118	2.36	1.98
		0.40	0.20	Case 4	357	7.14	1.46

The FOSM method has been applied on 5000 MCS samples of FOS. The mean and standard deviation of FOS samples for four different cases has been shown in Table 3. For example, (say case 3), the mean and standard deviation of 5000 samples of FOS is obtained as 1.291 and 0.165 respectively. The reliability index (β) is equals to $(1.291 - 1)/0.165 = 1.76$ and its corresponding probability of failure (P_f) of the slope is equals to $\Phi(-1.76) = 3.92$ %. The same procedure has been adopted for calculating the β and P_f for other cases.

Table 3
Result obtained from FOSM method

Method	Total number of samples generated	Coefficient of variation (v)			Mean of the samples	Standard deviation of the samples	Reliability Index (β)	Probability of failure (P_f) %
		Cohesion (v_c)	Angle of internal friction (v_ϕ)	Cases				
FOSM	5000	0.10	0.05	Case 1	1.289	0.056	5.16	1.23E-05
		0.20	0.10	Case 2	1.288	0.114	2.53	0.57
		0.30	0.15	Case 3	1.291	0.165	1.76	3.92
		0.40	0.20	Case 4	1.288	0.221	1.30	9.68

Figure 5 shows the reliability index obtained using FOSM method for different sample size ranging from 50 to 5000 for four cases. It can be seen that with the increase in the sample size, the value of reliability index is almost constant.

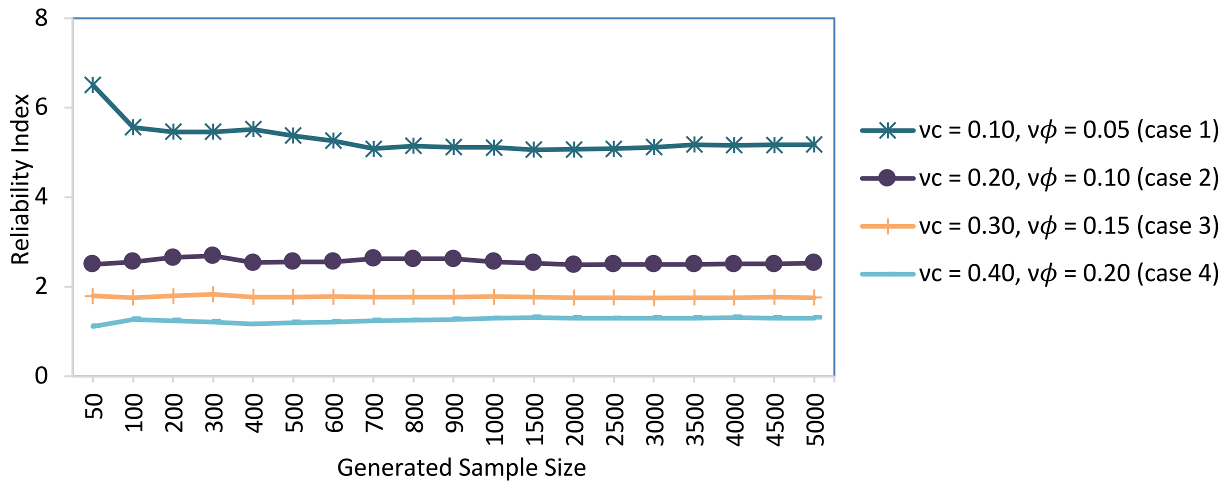


Fig. 5. Reliability index for different sample size using FOSM method

The FOS histogram obtained from three different levels of SS method for four cases are shown in Figure 6. The number of samples generated in level 0 is equal to 500, level 1 is equal to 450 and level 2 is equal to 450. It can be seen from Figure 6 that with the increase in simulation level, the samples are shifting towards the failure region. The detailed analysis of the histogram obtained from SS method is presented in Table 4.

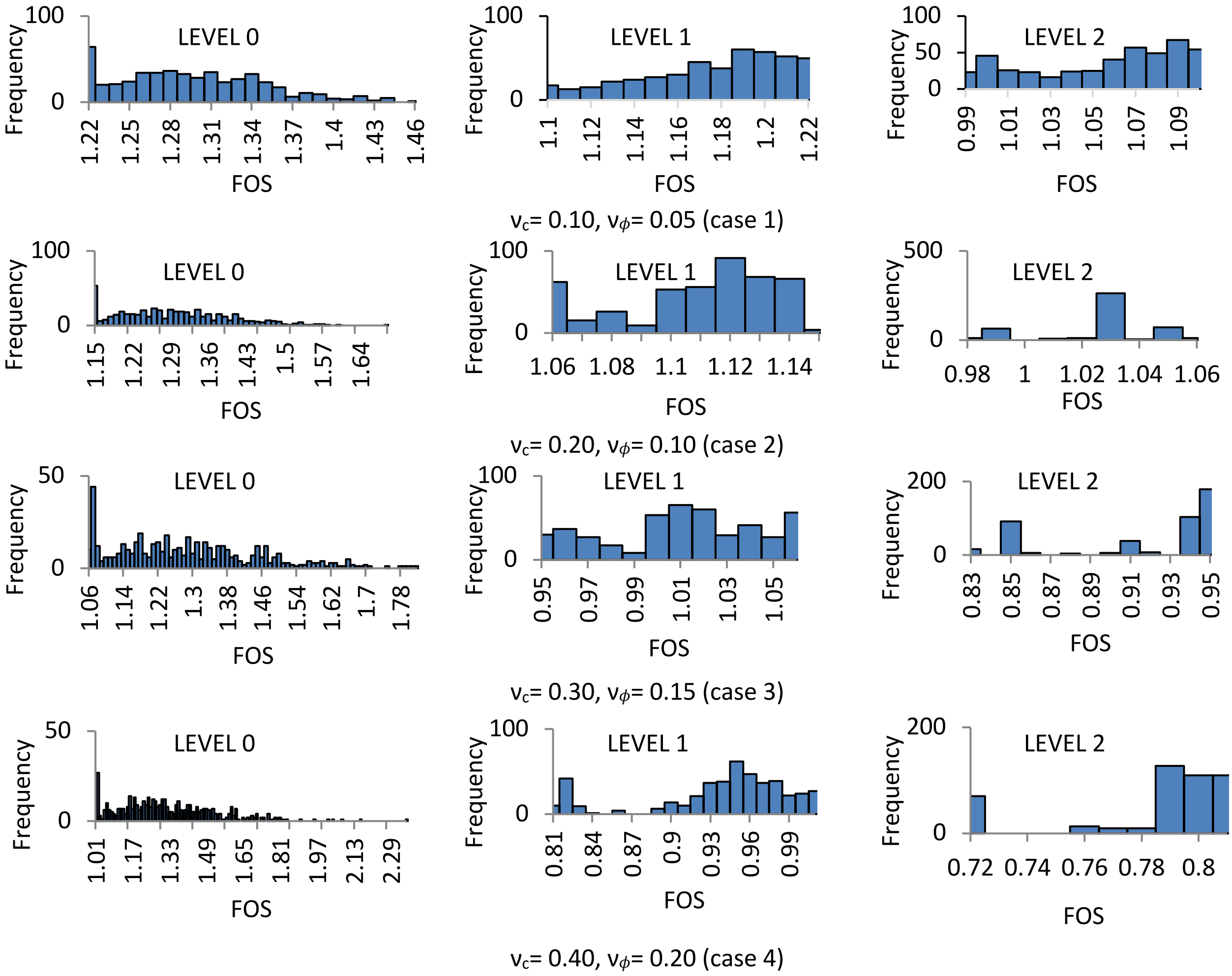


Fig. 6. FOS histogram obtained from SS method

Table 4 shows the result obtained using SS method for four different cases. For example, (say case 2), 0 samples out of 500 samples have failed (i.e., FOS < 1) in level 0 of SS, 0 samples out of 450 samples failed (i.e., FOS < 1) in level 1 of SS and 76 samples out of 450 samples have failed (i.e., FOS < 1) in level 2 of SS. The probability of failure (P_f) is calculated as 0.1 × 0.1 × 76/450 = 0.17 % and its corresponding reliability index (β) is found out to be ϕ^{-1} (0.17%) = 2.93. As compared to the MCS method, the occurrence of failure events increases significantly in SS method.

Table 4
Result obtained from SS method

Method	Total number of samples generated	Coefficient of variation (v)			Number of samples having FOS < 1			Probability of failure (P_f) %	Reliability Index (β)
		Cohesion (v_c)	Angle of internal friction (v_ϕ)	Cases	Level 0	Level 1	Level 2		
SS	1400	0.10	0.05	Case 1	0	0	23	0.05	3.29
		0.20	0.10	Case 2	0	0	76	0.17	2.93
		0.30	0.15	Case 3	0	119	450	2.64	1.94
		0.40	0.20	Case 4	0	399	450	8.87	1.35

Table 5 shows the comparison of the results obtained for four different cases from FOSM method, MCS method and SS method of reliability analysis. The reliability index of the slope varies from 5.16 to 1.30 (FOSM method), 2.99 to 1.46 (MCS method) and 3.29 to 1.35 (SS method). It can be observed that in case of rare events (i.e., probability of failure is very low), the SS method generates the samples in failure region which is not always possible with MCS method (i.e., Case 1). Also, it can be seen that MCS method underestimates the probability of failure and thus, gives higher value of reliability. This situation is not ideal for a geotechnical structure especially the heavy ones like dams, bridges etc. Moreover, the SS method has generated less samples as compared to MCS method and shown a better performance in terms of generating failure samples.

Table 5
Comparison of results obtained from different reliability methods

Coefficient of variation (v)			FOSM Method		MCS Method		SS Method	
Cohesion (v_c)	Angle of internal friction (v_ϕ)	Cases	Probability of failure (P_f) %	Reliability Index (β)	Probability of failure (P_f) %	Reliability Index (β)	Probability of failure (P_f) %	Reliability Index (β)
0.10	0.05	Case 1	1.23E-05	5.16	-	-	0.05	3.29
0.20	0.10	Case 2	0.57	2.53	0.14	2.99	0.17	2.93
0.30	0.15	Case 3	3.92	1.76	2.36	1.98	2.64	1.94
0.40	0.20	Case 4	9.68	1.30	7.14	1.46	8.87	1.35

5. Conclusion

This paper presents reliability analysis of a soil slope by considering various uncertainties in soil parameters. The analysis has been performed using FOSM method, MCS method and SS method. The whole procedure for determining the reliability of slope has been carried out in a MS-Excel worksheet. The deterministic model of the slope is developed based on the limit equilibrium method and further has been extended to a probabilistic method of analysis by considering the inherent spatial variation in soil parameters. Based on the above results and discussion, the following conclusions can be made:

i. The combination of uncertainty model using random field theory and deterministic model using limit equilibrium method can provide a better understanding to consider the failure mechanism caused due to inherent spatial variation of soil parameters.
ii. The probability of failure of the slope increases significantly with increase in the c.o.v. value of cohesion and friction angle of the soil and subsequently, the reliability of the slope decreases.
iii. The FOSM method directly calculates the reliability index of the slope using the samples generated by MCS method. This method does not provide any information regarding the failure samples.
iv. MCS method is very simple and efficient approach for performing the reliability analysis but this method may not generate the samples in the failure region in rare events (e.g., Table 5, case 1).
v. For such rare events, a very large number of samples (>5000) are required to be generated in MCS method for getting the desired failure probability level.
vi. The SS method makes sure that the samples are generated in the failure region with lesser number of samples (1400). The results also indicate that the SS method has performed better as compared to MCS method and FOSM method and is more reliable.

Acknowledgement
The authors would like to acknowledge Prof. Siu-Kui Au, University of Liverpool, UK; Prof. Yu Wang, City University of Hong Kong, China and Prof. Zijun Cao, Wuhan University, China for providing the MS-Excel Add-In UPSS module which has been used in the study.

References
[1] Baecher, Gregory B., and John T. Christian. *Reliability and statistics in geotechnical engineering*. John Wiley & Sons, 2005.
[2] Phoon, Kok-Kwang, and Fred H. Kulhawy. "Evaluation of geotechnical property variability." *Canadian Geotechnical Journal* 36, no. 4 (1999): 625-639. https://doi.org/10.1139/t99-039
[3] Phoon, Kok-Kwang, and Fred H. Kulhawy. "Characterization of geotechnical variability." *Canadian geotechnical journal* 36, no. 4 (1999): 612-624. https://doi.org/10.1139/t99-038
[4] Lumb, Peter. "The variability of natural soils." *Canadian geotechnical journal* 3, no. 2 (1966): 74-97. https://doi.org/10.1139/t66-009
[5] Ang, Alfredo H-S., and Wilson H. Tang. "Probability concepts in engineering planning and design, vol. 2: Decision, risk, and reliability." *JOHN WILEY & SONS, INC., 605 THIRD AVE., NEW YORK, NY 10158, USA, 1984, 608* (1984).
[6] Vanmarcke, Erik H. "Reliability of earth slopes." *Journal of the Geotechnical Engineering Division* 103, no. 11 (1977): 1247-1265. https://doi.org/10.1061/AJGEB6.0000518
[7] Vanmarcke, Erik H. "Probabilistic modeling of soil profiles." *Journal of the geotechnical engineering division* 103, no. 11 (1977): 1227-1246. https://doi.org/10.1061/AJGEB6.0000517

[8] Kalantari, A. R., A. Johari, M. Zandpour, and M. Kalantari. "Effect of spatial variability of soil properties and geostatistical conditional simulation on reliability characteristics and critical slip surfaces of soil slopes." *Transportation Geotechnics* 39 (2023): 100933. https://doi.org/10.1016/j.trgeo.2023.100933
[9] Ray, Rahul, Shiva Shankar Choudhary, Lal Bahadur Roy, Mosbeh R. Kaloop, Pijush Samui, Pradeep U. Kurup, Jungkyu Ahn, and Jong Wan Hu. "Reliability analysis of reinforced soil slope stability using GA-ANFIS, RFC, and GMDH soft computing techniques." *Case Studies in Construction Materials* 18 (2023): e01898. https://doi.org/10.1016/j.cscm.2023.e01898
[10] USACE. "Risk-based analysis in geotechnical engineering for support of planning studies, engineering and design." (1997): 20314-21100.
[11] Ahmed, Ashraf, and Abdul-Hamid Soubra. "Subset simulation and its application to a spatially random soil." In *Geo-Risk 2011: Risk Assessment and Management*, pp. 209-216. 2011. https://doi.org/10.1061/41183(418)12
[12] Kumar, Manoj, Manav Mittal, and Pijush Samui. "Performance assessment of genetic programming (GP) and minimax probability machine regression (MPMR) for prediction of seismic ultrasonic attenuation." *Earthquake science* 26 (2013): 147-150. https://doi.org/10.1007/s11589-013-0018-z
[13] Budhu, Muniram. *Soil mechanics fundamentals*. John Wiley & Sons, 2015.
[14] Cao, Zijun, Yu Wang, and Dianqing Li. *Probabilistic approaches for geotechnical site characterization and slope stability analysis*. Springer Berlin Heidelberg, 2017. https://doi.org/10.1007/978-3-662-52914-0
[15] Kar, Saurav Shekhar, and Lal Bahadur Roy. "Reliability analysis of a finite slope considering the effects of soil uncertainty." *International Journal of Performability Engineering* 17, no. 5 (2021): 473. https://doi.org/10.23940/ijpe.21.05.p7.473483
[16] Kar, Saurav Shekhar, and Lal Bahadur Roy. "A comparative study on reliability analysis of cohesive soil slope using subset simulation and other methods." *U. Porto Journal of Engineering* 8, no. 2 (2022): 135-155. https://doi.org/10.24840/2183-6493_008.002_0011
[17] Kar, Saurav Shekhar, and Lal Bahadur Roy. "Probabilistic based reliability slope stability analysis using FOSM, FORM, and MCS." *Engineering, Technology & Applied Science Research* 12, no. 2 (2022): 8236-8240. https://doi.org/10.48084/etasr.4689
[18] Robert, Christian P., George Casella, and George Casella. *Monte Carlo statistical methods*. Vol. 2. New York: Springer, 1999. https://doi.org/10.1007/978-1-4757-3071-5
[19] Jiang, Shui-Hua, Dian-Qing Li, Zi-Jun Cao, Chuang-Bing Zhou, and Kok-Kwang Phoon. "Efficient system reliability analysis of slope stability in spatially variable soils using Monte Carlo simulation." *Journal of Geotechnical and Geoenvironmental Engineering* 141, no. 2 (2015): 04014096. https://doi.org/10.1061/(ASCE)GT.1943-5606.0001227
[20] Rubinstein, Reuven Y., and Dirk P. Kroese. *Simulation and the Monte Carlo method*. John Wiley & Sons, 2016. https://doi.org/10.1002/9781118631980
[21] Au, Siu Kui, Z. J. Cao, and Yu Wang. "Implementing advanced Monte Carlo simulation under spreadsheet environment." *Structural Safety* 32, no. 5 (2010): 281-292. https://doi.org/10.1016/j.strusafe.2010.03.004
[22] Wang, Yu, Zijun Cao, and Siu-Kui Au. "Practical reliability analysis of slope stability by advanced Monte Carlo simulations in a spreadsheet." *Canadian Geotechnical Journal* 48, no. 1 (2011): 162-172. https://doi.org/10.1139/T10-044
[23] Au, Siu-Kui, and Yu Wang. *Engineering risk assessment with subset simulation*. John Wiley & Sons, 2014. https://doi.org/10.1002/9781118398050
[24] Au, Siu-Kui, and James L. Beck. "Estimation of small failure probabilities in high dimensions by subset simulation." *Probabilistic engineering mechanics* 16, no. 4 (2001): 263-277. https://doi.org/10.1016/S0266-8920(01)00019-4
[25] Li, Dian-Qing, Te Xiao, Zi-Jun Cao, Chuang-Bing Zhou, and Li-Min Zhang. "Enhancement of random finite element method in reliability analysis and risk assessment of soil slopes using Subset Simulation." *Landslides* 13 (2016): 293-303. https://doi.org/10.1007/s10346-015-0569-2
[26] Zheng, Zhou, Yanlong Li, Lifeng Wen, Ye Zhang, and Ting Wang. "Reliability analysis of an embankment dam slope based on an ellipsoid model and PSO-ELM." In *Structures*, vol. 55, pp. 2419-2432. Elsevier, 2023. https://doi.org/10.1016/j.istruc.2023.06.125
[27] Hastings, W. Keith. "Monte Carlo sampling methods using Markov chains and their applications." (1970): 97-109. https://doi.org/10.1093/biomet/57.1.97
[28] Wang, Mao-Xin, Xiao-Song Tang, Dian-Qing Li, and Xiao-Hui Qi. "Subset simulation for efficient slope reliability analysis involving copula-based cross-correlated random fields." *Computers and Geotechnics* 118 (2020): 103326. https://doi.org/10.1016/j.compgeo.2019.103326
[29] Fellenius, Wolmar. "Calculation of the stability of earth dams." In *Proc. of the second congress on large dams*, vol. 4, pp. 445-463. 1936.

[30] Malkawi, Abdallah I. Husein, Waleed F. Hassan, and Fayez A. Abdulla. "Uncertainty and reliability analysis applied to slope stability." *Structural safety* 22, no. 2 (2000): 161-187. https://doi.org/10.1016/S0167-4730(00)00006-0