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An Optimal Control Problem of SECR Corruption Dynamic Model

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ABSTRACT

In this paper, the SECR model of corruption dynamics was proposed and will be analyzed, including non-negativity and boundedness of the model, corruption cases-free equilibrium point, and local stability. In order to reduce the number of corruption cases, the SECR model is extended with two controls strategies, i.e, u_1 : preventive campaign for corruption by the KPK institution, and u_2 : giving punishment for corrupted individual. Furthermore, it is investigated how control strategies should be applied to prevent corruption cases optimally. The model will be simulated in three scenarios, i.e. with full control u_1 and u_2 simultaneously, only u_1 control, and only u_2 control. Then, each of these simulations results will be compared with the results of the simulation model without control. The optimal control problem will be solved using Pontryagin's Minimum Principle and Runge Kutta order 4th method for numerical simulation with Matlab. The simulation results show that the first scenario give the optimal solution in reducing the number of exposed and corrupted subpopulations with minimum controls effort and short time period. Based on these results, it can be concluded that in order to prevent and reduce the number of corruption cases, the control strategy giving punishment for the corrupted individuals must be also followed by prevention campaign corruption in society by the KPK, where both these control strategies must be carried out together to give the optimal results.

1. Introduction

Corruption is defined as an illegal act involving the abuse of public or office trust for personal gain. According to Indonesian Law Number 31 of 1999 concerning the eradication of corruption crimes, what is included in the criminal act of corruption is anyone who is categorized as violating the law, committing acts of enriching themselves, benefiting themselves or another person or a corporation, abusing authority or opportunity or facilities available to them or that can harm state finances or the state economy because of their position. Corruption is a serious problem in some countries, especially developing countries, including Indonesia. The phenomenon of corruption is a very interesting study so that several related studies have emerged on corruption. The dynamics of

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corruption can be expressed in a mathematical model, and the model can be analyzed to determine the behavior of the system.

Several studies that discuss corruption models include in 2005, Blackburn *et al.,* construct how corruption can harm economic development and low-level development, which in turn can encourage greater corruption [1]. Caulkins *et al.,* in 2013 developed a corruption model to determine the level of corruption in the bureaucracy by taking into account the state variables of honest employees and corrupt employees [2]. Malafeyev *et al.,* made a formulation of corruption dynamics using lattice model which is similar to the three-dimensional Ising model [3]. In 2018, the SCJH model that describe the transmission process of corruption was proposed by Obsu *et al.,* [4]. Furthermore, researchers formulated a corruption dynamics model with an optimal control approach in order to minimize the number of corruption cases. The simple model formulation of corruption dynamics with optimal control approach have been done in the previous studies [5-9].

The number of corruption cases in Indonesia is high and continues to increase every year. Therefore, the government must have several strategies to eradicate corruption case. In an effort to overcome the problem of corruption in Indonesia, "Komisi Pemberantasan Korupsi" (KPK) was formed which was given the mandate to eradicate corruption in a professional, intensive, and sustainable manner. The spread of corruption in society can be described as the spread of infectious diseases, but controlling corruption is more difficult than controlling infectious diseases [5]. Therefore, it is very important to apply an optimal control model to reduce the corruption cases. The main objective of optimal control is to determine control signals that will cause a process to satisfy some physical constraints and at the same time extremize a chosen performance criterion [10,11]. The application of optimal control in various problem areas can be seen in the previous papers by several authors [12-20].

Based on these problems, this paper will propose a new SECR model to describe the dynamics of corruption. Furthermore, in an effort to reduce the number of corruption cases, the model will be developed with the addition of two control strategies, namely preventive campaign for corruption by KPK institution, and giving punishment for corrupted individual. Then, the optimal strategies will be investigated to prevent and reduce the number of corruption cases. To solve these problems, the optimal control is applied and will be analyzed using Pontryagin's Minimum Principle. The optimal control model will be solved numerically using Runge Kutta order 4^{th} method to find the optimal strategy to prevent and reduce the number of corruption cases. The results of this study are expected to be used as a reference and consideration for the government to determine policies in eradicating corruption cases in Indonesia.

2. Mathematical Modeling of Corruption Dynamic

In this paper, the SECR model of corruption dynamics that is constructed from four subpopulations is introduced. The total population N(t) is divided into four compartments, namely susceptible S(t) are individuals who are susceptible to doing corruption; exposed E(t) are individuals who have the intention to do corruption but do not perform it; corrupted C(t) are individuals performing corruption, and recovered R(t) are individuals who have stopped doing corruption. The SECR model is given as system in Eq. (1) and the compartment diagram is shown in Figure 1.

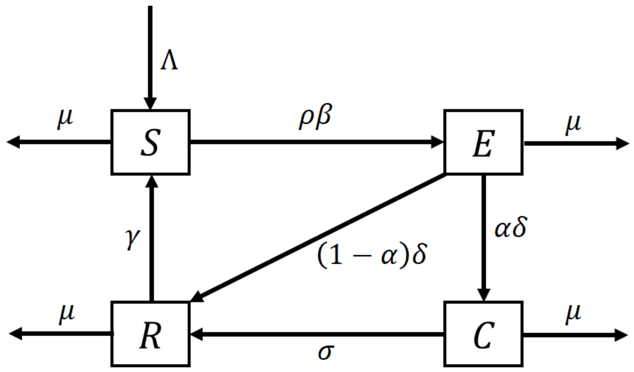


Fig. 1. The compartment diagram of SECR model

$$\frac{dS}{dt} = \Lambda - \rho\beta SC + \gamma R - \mu S$$

$$\frac{dE}{dt} = \rho\beta SC - \delta E - \mu E \tag{1}$$

$$\frac{dC}{dt} = \alpha\delta\mathrm{E} - \sigma C - \mu C$$

$$\frac{dR}{dt} = \sigma C + (1-\alpha)\delta E - \gamma R - \mu R$$

The parameters are:
S(t) : subpopulation of susceptible
E(t) : subpopulation of exposed
C(t) : subpopulation of corrupted
R(t) : subpopulation of recovered
Λ : number of natural birth and migration
μ : rate of natural death
β : contact rate between corrupted and susceptible individuals
ρ : probability of susceptible individuals become exposed
α : proportion of exposed individuals become corrupted
δ : rate of exposed individuals become corrupted
σ : rate of corrupted individuals become recovered
γ : rate of recovered individuals become susceptible

The number of susceptible individuals in the subpopulations increase with birth and migration Λ, also by recovered individuals who become susceptible again with rate γ (γR). In addition, the number of susceptible subpopulations will decrease because there are susceptible individuals interacting with corrupted individuals with the rate β and corruption transmission probability per contact ρ become exposed $(\rho\beta SC)$ and also by natural death with rate μ (μS). Then, the number of exposed individuals in the subpopulation increase by susceptible individuals that become exposed $(\rho\beta SC)$, and they will decrease because these exposed individuals become corrupted with rate δ (δE) and also by natural death μE. The number of corrupted individuals increase by exposed individual who become corrupted with rate δ and proportion α $(\alpha\delta E)$. On the other hand, corrupted individuals decrease because there are corrupted individuals who recovered with rate σ (σC), and natural death (μC). The presence of corrupted individuals that recovered (σC) causes an increase in recovered individuals in the subpopulation. Also, the number of recovered individuals increase by exposed individuals that becomes recovered $(1-\alpha)\delta E$. Lastly, the number of recovered decrease because there are recovered individuals that becomes susceptible (γR), and by natural death (μR).

3. Model Analysis

3.1 Non-Negativity of the Model

The SECR model in Eq. (1) with initial conditions S(0), E(0), C(0), R(0)≥0 will have a non-negative solution for all t≥0. To prove it, S(t) was obtained from

$$\frac{dS}{dt} = \Lambda - \rho\beta SC + \gamma R - \mu S$$

$$\frac{dS}{dt} \geq -(\rho\beta C + \mu)S \tag{2}$$

$$\frac{1}{S}dS \geq -(\rho\beta C + \mu)\, dt$$

By solving equation above, the solution is

$$S(t) \geq S(0)e^{-(\rho\beta C_0+\mu)t} \geq 0 \tag{3}$$

Obtain E(t) from

$$\frac{dE}{dt} = \rho\beta SC - \delta E - \mu E$$

$$\frac{dE}{dt} \geq -(\delta + \mu)E \tag{4}$$

$$\frac{1}{E}dE \geq -(\delta + \mu)dt$$

Solving equation above, we obtain that

$$E(t) \geq E(0)e^{-(\delta+\mu)t} \geq 0 \tag{5}$$

By doing similar steps to obtain C(t) and R(t), the solutions for system in Eq. (1) given as follow

$$S(t), E(t), C(t), R(t) \geq 0 \tag{6}$$

3.2 Boundedness of the Model

The SECR model in Eq. (1) with non-negative initial conditions will have bounded solutions for all $t \geq 0$. To prove it, N(t) is total population given by

$$N(t) = S(t) + E(t) + C(t) + R(t)$$

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dE}{dt} + \frac{dC}{dt} + \frac{dR}{dt} \tag{7}$$

$\frac{dN}{dt} = \Lambda - \mu(S + E + C + R) \Longrightarrow \frac{dN}{dt} = \Lambda - \mu N \Longrightarrow \frac{dN}{dt} + \mu N = \Lambda$

The solution of equation above obtained using integral factor $v = e^{\mu t}$ as follows

$$N(t)e^{\mu t} = \int \Lambda e^{\mu t}\, dt + c \Longrightarrow N(t) = \frac{\Lambda}{\mu} + \frac{c}{e^{\mu t}} \tag{8}$$

Thus, while $t \longrightarrow \infty$, we find that

$$N(t) = \frac{\Lambda}{\mu} \tag{9}$$

Which implies that

$$0 \leq N(t) \leq \frac{\Lambda}{\mu} \tag{10}$$

3.3 Corruption Cases-Free Equilibrium Point

The corruption cases-free equilibrium point was obtained by setting model system in Eq. (1) to be zero as follow

$$\frac{dS}{dt} = \frac{dE}{dt} = \frac{dC}{dt} = \frac{dR}{dt} = 0 \tag{11}$$

There are no corruption cases, which means the number of corrupted individual is zero, $C = 0$. By setting the third equation from Eq. (1) to be zero, we find that

$$\frac{dC}{dt} = 0 \Longrightarrow \alpha\delta E = 0 \tag{12}$$

Because $\alpha\delta \neq 0$, then $E = 0$.
From the fourth equation of Eq. (1)

$$\frac{dR}{dt} = 0 \Longrightarrow -(\gamma + \mu)R = 0 \tag{13}$$

Assume that $\gamma + \mu \neq 0$, then $R = 0$.
Finally, by setting the first equation of Eq. (1) to zero, we get

$$\Lambda - \mu S = 0 \Longrightarrow S = \frac{\Lambda}{\mu} \tag{14}$$

Therefore, the corruption cases-free equilibrium point is

$$P_0 = (S^*, E^*, C^*, R^*) = \left(\frac{\Lambda}{\mu}, 0,0,0\right) \tag{15}$$

3.4 Local Stabilily of Corruption Cases-Free Equilibrium

The Jacobian matrix of Eq. (1) was obtained as follows

$$J = \begin{bmatrix} -\rho\beta C^* - \mu & 0 & -\rho\beta S^* & \gamma \\ 0 & -(\delta+\mu) & \rho\beta S^* & 0 \\ 0 & \alpha\delta & -(\sigma+\mu) & 0 \\ 0 & (1-\alpha)\delta & \sigma & -(\gamma+\mu) \end{bmatrix} \tag{16}$$

Then the Jacobian matrix evaluated at corruption cases-free equilibrium point P_0 becomes

$$J(P_0) = \begin{bmatrix} -\mu & 0 & -\rho\beta\Lambda\mu^{-1} & \gamma \\ 0 & -(\delta+\mu) & \rho\beta\Lambda\mu^{-1} & 0 \\ 0 & \alpha\delta & -(\sigma+\mu) & 0 \\ 0 & (1-\alpha)\delta & \sigma & -(\gamma+\mu) \end{bmatrix} \tag{17}$$

Eigen values are obtained from $|\lambda I - J(P_0)| = 0$ as follow

$$\begin{vmatrix} \lambda+\mu & 0 & \rho\beta\Lambda\mu^{-1} & -\gamma \\ 0 & \lambda+(\delta+\mu) & -\rho\beta\Lambda\mu^{-1} & 0 \\ 0 & -\alpha\delta & \lambda+(\sigma+\mu) & 0 \\ 0 & -(1-\alpha)\delta & -\sigma & \lambda+(\gamma+\mu) \end{vmatrix} = 0 \tag{18}$$

The characteristic polynomial of equation above becomes

$$(\lambda+\mu)(\lambda+[\gamma+\mu])\left(\lambda^2+[\delta+\sigma+2\mu]\lambda+[\delta+\mu][\sigma+\mu]-\frac{\alpha\delta\rho\beta\Lambda}{\mu}\right)=0 \tag{19}$$

The corruption cases-free equilibrium point of system in Eq. (1) is locally asymptotically stable if

$$\delta+\sigma+2\mu>0 \tag{20}$$

and

$$\mu(\delta+\mu)(\sigma+\mu)>\alpha\delta\rho\beta\Lambda \tag{21}$$

4. Extended Model with Optimal Control Strategies

In order to prevent and reduce the number of corruption cases, the SECR model is extended with two controls strategies, i.e., u_1: preventive campaign for corruption by KPK institution, and u_2: giving punishment for corrupted individual. The SECR model with controls strategies is given as system in Eq. (22)

$$\begin{aligned} \frac{dS}{dt} &= \Lambda - \rho\beta SC + \gamma R - (\mu+u_1)S \\ \frac{dE}{dt} &= \rho\beta SC - (\delta+\mu+u_1)E \\ \frac{dC}{dt} &= \alpha\delta\mathrm{E} - (\sigma+\mu+u_2)C \end{aligned} \tag{22}$$

$$\frac{dR}{dt} = (\sigma + u_2)C + \big((1-\alpha)\delta + u_1\big)E - (\gamma + \mu)R + u_1 S$$

The main objective in this paper is to minimize the number of exposed subpopulation and corrupted subpopulation using two control strategies. Control variables u_1 and u_2 will minimize optimal control of model subject to objective function that defined below

$$J = \int_0^{t_f} \left(a_1 E + a_2 C + \frac{1}{2}(b_1 u_1^2 + b_2 u_2^2) \right) dt \tag{23}$$

t_f : final time
a_1 : weighting parameter of exposed subpopulation
a_2 : weighting parameter of corrupted subpopulation
b_1 : weighting coefficient of u_1 control
b_2 : weighting coefficient of u_2 control

Find the optimal control of (u_1^*, u_2^*) that must satisfy

$$J(u_1^*, u_2^*) = \min\{J(u_1,u_2) | (u_1,u_2) \in \boldsymbol{u}\} \tag{24}$$

subject to system in Eq. (22) and appropriate initial condition given at t = 0, where the control set is defined as

$$J = \left\{(u_1,u_2) \middle| u_i \text{ } is\ Lebesgue\ measurable, 0 \le u_i(t) \le 1, t \in [0, t_f]\ for\ i = 1,2 \right\} \tag{25}$$

is closed set.

The Hamiltonian function $\mathcal{H}$ for the system in Eq. (22) was defined to minimize the objective function, where λ_i ; $i = 1,2,3,4$ are the adjoint variables.

$$\begin{aligned}\mathcal{H} &= \left(a_1 E + a_2 C + \frac{1}{2}(b_1 u_1^2 + b_2 u_2^2) \right) + \lambda_1(\Lambda - \rho\beta SC + \gamma R - (\mu + u_1)S) \\ &+ \lambda_2(\rho\beta SC - (\delta + \mu + u_1)E) + \lambda_3(\alpha\delta E - (\sigma + \mu + u_2)C) \\ &+ \lambda_4\left((\sigma + u_2)C + \big((1-\alpha)\delta + u_1\big)E - (\gamma + \mu)R + u_1 S\right)\end{aligned} \tag{26}$$

For $t \in [0, t_f]$, the optimal control u_1 and u_2 can be solved from the optimality condition $\frac{\partial \mathcal{H}}{\partial \boldsymbol{u}} = 0$

$$\frac{\partial \mathcal{H}}{\partial u_1} = b_1 u_1 - \lambda_1 S - \lambda_2 E + \lambda_4(S + E) = 0 \Longrightarrow u_1 = \frac{(\lambda_1 - \lambda_4)S + (\lambda_2 - \lambda_4)E}{b_1}$$

$$\frac{\partial \mathcal{H}}{\partial u_2} = b_2 u_2 - \lambda_3 C + \lambda_4 C = 0 \Longrightarrow u_2 = \frac{(\lambda_3 - \lambda_4)C}{b_2} \tag{27}$$

For the boundary condition of the controls $\boldsymbol{u}$, we can obtain the optimal control of u_1^* and u_2^* below

$$u_1^* = \max\left\{0, \min\left(1, \frac{(\lambda_1 - \lambda_4)S + (\lambda_2 - \lambda_4)E}{b_1}\right)\right\}$$

$$u_2^* = \max\left\{0, \min\left(1, \frac{(\lambda_3-\lambda_4)C}{b_2}\right)\right\} \tag{28}$$

Obtain state equation $\dot{\boldsymbol{x}} = \frac{\partial \mathcal{H}}{\partial \boldsymbol{\lambda}}$:

$$\frac{dS}{dt} = \frac{\partial \mathcal{H}}{\partial \lambda_1} = \Lambda - \rho\beta SC + \gamma R - (\mu + u_1)S$$

$$\frac{dE}{dt} = \frac{\partial \mathcal{H}}{\partial \lambda_2} = \rho\beta SC - (\delta + \mu + u_1)E \tag{29}$$

$$\frac{dC}{dt} = \frac{\partial \mathcal{H}}{\partial \lambda_3} = \alpha\delta\mathrm{E} - (\sigma + \mu + u_2)C$$

$$\frac{dR}{dt} = \frac{\partial \mathcal{H}}{\partial \lambda_4} = (\sigma + u_2)C + \big((1-\alpha)\delta + u_1\big)E - (\gamma + \mu)R + u_1 S$$

Obtain costate equation $\dot{\boldsymbol{\lambda}} = -\frac{\partial \mathcal{H}}{\partial \boldsymbol{x}}$:

$$\frac{d\lambda_1}{dt} = -\frac{\partial \mathcal{H}}{\partial S} = \lambda_1(\rho\beta C + \mu + u_1) - \lambda_2\rho\beta C - \lambda_4 u_1$$

$$\frac{d\lambda_2}{dt} = -\frac{\partial \mathcal{H}}{\partial E} = -a_1 + \lambda_2(\delta + \mu + u_1) - \lambda_3\alpha\delta - \lambda_4\big((1-\alpha)\delta + u_1\big) \tag{30}$$

$$\frac{d\lambda_3}{dt} = -\frac{\partial \mathcal{H}}{\partial C} = -a_2 + \lambda_1\rho\beta S - \lambda_2\rho\beta S + \lambda_3(\sigma + \mu + u_2) - \lambda_4(\sigma + u_2)$$

$$\frac{d\lambda_4}{dt} = -\frac{\partial \mathcal{H}}{\partial R} = -\lambda_1\gamma + \lambda_4(\gamma + \mu)$$

Given initial condition $x(t_0) = x_0$ and fixed-final time and free-final state system, substitute $\delta t_f = 0$ and $\delta x_f \neq 0$ into final condition

$$\left[\mathcal{H} + \frac{\partial S}{\partial t}\right]_{t_f} \delta t_f + \left[\frac{\partial S}{\partial x} - \lambda\right]_{t_f} \delta x_f = 0$$

$$\left[\frac{\partial S}{\partial x} - \lambda\right]_{t_f} = 0 \tag{31}$$

$$\lambda\big(t_f\big) = \left(\frac{\partial S}{\partial x}\right)_{t_f}$$

$$\lambda\big(t_f\big) = 0$$

The optimal system is obtained by substitute optimal control $\boldsymbol{u}^*$ into the system of state and costate equations, so the optimal system is obtained as follow

$$\frac{dS}{dt} = \Lambda - \rho\beta SC + \gamma R - (\mu + u_1^*)S$$

$$\frac{dE}{dt} = \rho\beta SC - (\delta + \mu + u_1^*)E$$

$$\frac{dC}{dt} = \alpha\delta E - (\sigma + \mu + u_2^*)C$$

$$\frac{dR}{dt} = (\sigma + u_2^*)C + \big((1-\alpha)\delta + u_1^*\big)E - (\gamma + \mu)R + u_1^* S$$

$$\frac{d\lambda_1}{dt} = \lambda_1(\rho\beta C + \mu + u_1^*) - \lambda_2\rho\beta C - \lambda_4 u_1^*$$

$$\frac{d\lambda_2}{dt} = -a_1 + \lambda_2(\delta + \mu + u_1^*) - \lambda_3\alpha\delta - \lambda_4\big((1-\alpha)\delta + u_1^*\big) \tag{32}$$

$$\frac{d\lambda_3}{dt} = -a_2 + \lambda_1\rho\beta S - \lambda_2\rho\beta S + \lambda_3(\sigma + \mu + u_2^*) - \lambda_4(\sigma + u_2^*)$$

$$\frac{d\lambda_4}{dt} = -\lambda_1\gamma + \lambda_4(\gamma + \mu)$$

$$u_1^* = \max\left\{0, \min\left(1, \frac{(\lambda_1-\lambda_4)S + (\lambda_2-\lambda_4)E}{b_1}\right)\right\}$$

$$u_2^* = \max\left\{0, \min\left(1, \frac{(\lambda_3-\lambda_4)C}{b_2}\right)\right\}$$

$$\lambda_i(t_f) = 0; \quad i = 1,2,3,4$$

$$S(0) = S_0\ ;\ E(0) = E_0\ ;\ C(0) = C_0\ ;\ R(0) = R_0$$

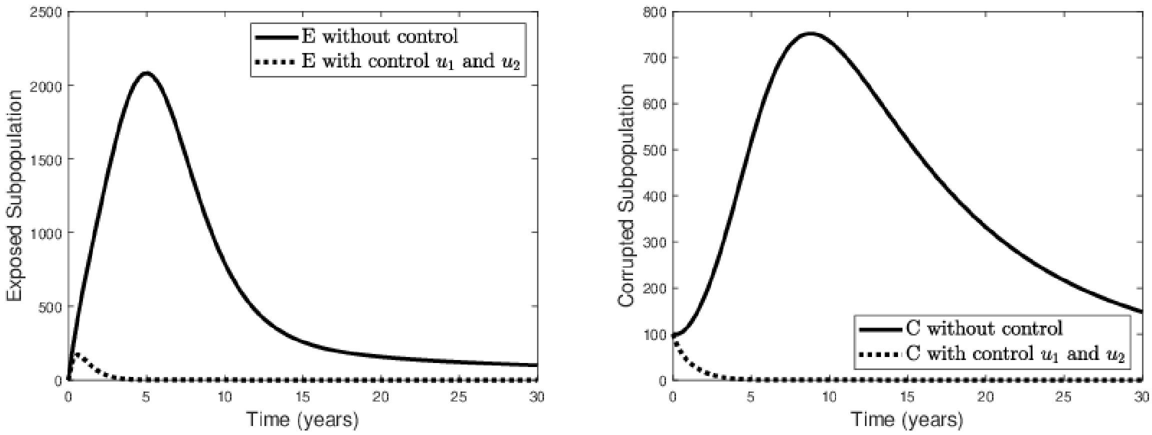
5. Numerical Simulation and Discussion

The dynamics system in Eq. (32) was solved numerically by using the forward-backward Runge-Kutta order 4th method, for detail see the book by Lenhart *et al.*, [21]. The numerical simulations were carried out using Matlab and the initial condition are setting as follow : S_0 = 10000 , E_0 = 0 , C_0 = 100 , and R_0 = 0. The initial condition for susceptible (S_0) is taken based on the average government employee in Indonesia who is susceptible to doing corruption. Meanwhile, the initial condition of corrupted (C_0) based on the average corruption cases that occur in Indonesia in recent 10 years. The value of parameter used in the simulation is shown in Table 1, where all the parameters are per year. For this set of parameters, the system in Eq. (1) has the corruption cases-free equilibrium point and it is locally asymptotically stable. Furthermore, the effect of applying each of the proposed controls on the SECR model was obtained. Therefore, the model will be simulated in three scenarios, i.e. full control u_1 and u_2 simultaneously, only u_1 control, and only u_2 control. Then, each of these simulations results will be compared with the results of the simulation model without control.

Table 1
The value of parameter

Parameter	Value	Source
Λ	85	Assumed
μ	0.125	Assumed
β	0.0234	[6]
ρ	0.036	[6]
α	0.3	[6]
δ	0.2	[6]
σ	0.007	[6]
γ	0.01	Assumed

Figure 2 shows the simulation result for first scenario, where two control strategies u_1 and u_2 were applied simultaneously, and it can be seed the comparison number of each subpopulation between system without control and system with two strategies control. The number of exposed subpopulations before the implementation of the control strategies increased very quickly and reached the largest number in the 5th to 6th years, with about 2081 individuals, and after that decreased, but slowly. For the number of corrupted subpopulation before controlled with initial condition C_0=100 it increased until its largest number in the 9th year, with about 753 individuals. Meanwhile, the simulation results show that the implementation of the two control strategies was successful in reducing the number of exposed and corrupted subpopulations, as shown in Figure 2(a) and Figure 2(b), respectively. In which for the exposed and corrupted subpopulation decreases and reaches zero at the end of the 6th year.



(a) Exposed (b) Corrupted

Fig. 2. Dynamics of exposed and corrupted subpopulation with u_1 and u_2 controls

The controls effort of u_1 and u_2 that are given into the system are shown in Figure 3. For u_1 control, the effort that must be given to the model in the first 6 years reaches the maximum limit, and after that it can slowly decrease until it reaches the minimum limit at the end of the 25th year. Meanwhile, for u_2 control, the effort that must be given to the model in the first 7.5 years reaches the maximum limit, and after that it can slowly decrease until it reaches the minimum limit at the end of the 28th year. This shows that the u_2 control effort that must be given to the system is relatively smaller with a shorter time period than the u_1 control.

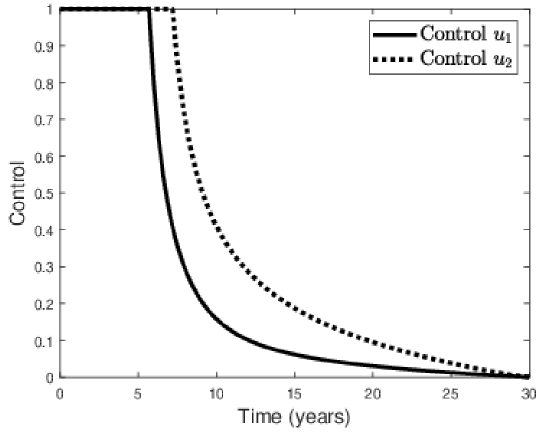
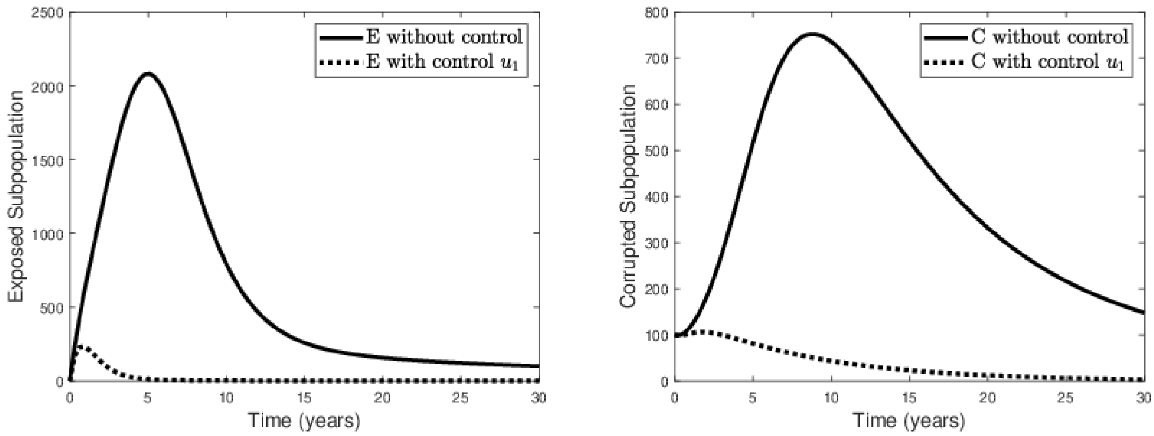


Fig. 3. The effort of u_1 and u_2 controls

Figure 4 show the simulation result for the model with u_1 control, and we can see the comparison number of each subpopulation between system without control and system with u_1 control. The simulation results show that the implementation of u_1 control was successful in reducing the number of exposed and corrupted subpopulations, as shown in Figure 4(a) and Figure 4(b), respectively, but the results were still not good enough when compared to the first control scenario (applying u_1 and u_2 controls simultaneously). As the exposed subpopulation decreases and reaches zero at the end of the 23th year. Meanwhile, the corrupted subpopulation decreases and at the end of the 30th year, the number of corrupted subpopulations remains only 4 individuals. The controls effort of u_1 that are given into the system are shown in Figure 5. The effort that must be given to the model in the first 27 years reaches the maximum limit, and after that it can slowly decrease until it reaches the minimum limit at the end of the 30th year. This shows that applying only u_1 control into system causes the amount of control effort that must be given to be large and over a longer period of time.



(a) Exposed

(b) Corrupted

Fig. 4. Dynamics of exposed and corrupted subpopulation with u_1 control

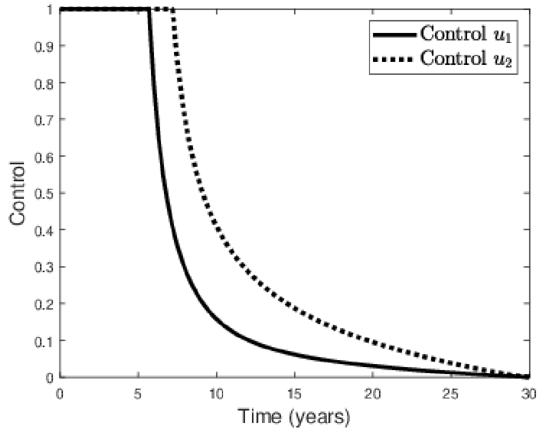
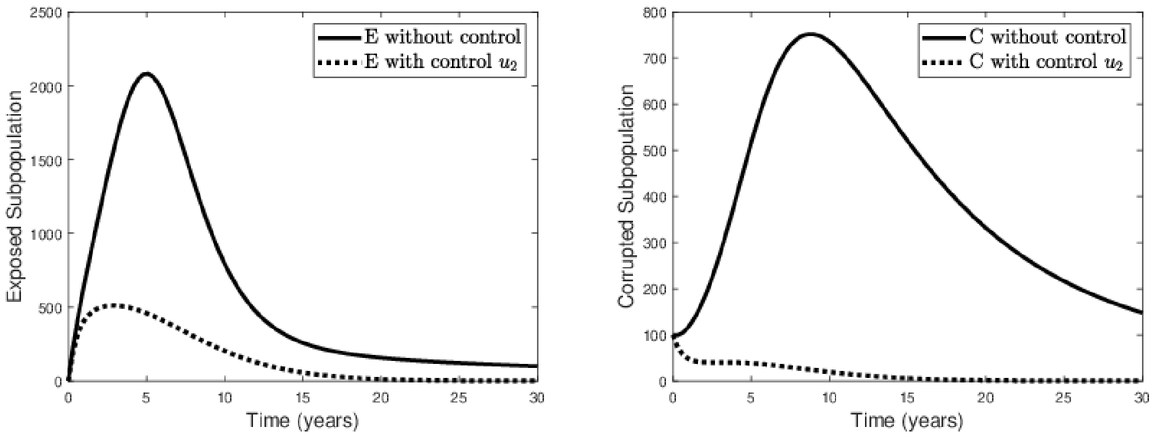


Fig. 5. The effort of u_1 control

Figure 6 show the simulation result for the model with u_2 control, and we can see the comparison number of each subpopulation between system without control and system with u_2 control. The simulation results show that the implementation of u_2 control was successful in reducing the number of exposed and corrupted subpopulations, as shown in Figure 6(a) and Figure 6(b), respectively, but the results were still not good enough when compared to the first control scenario. While for the exposed subpopulation decreases and reaches zero at the end of the 30th year, the corrupted subpopulation decreased and reached zero at the end of the 24th year. The controls effort of u_2 that are given into the system is shown in Figure 7. The effort that must be given to the model in the first 28 years reaches the maximum limit, and after that it can slowly decrease until it reaches the minimum limit at the end of the 30th year. This shows that applying only u_2 control into system causes the amount of control effort that must be given to be large and over a longer period of time. The comparison of optimal performance index (minimum control effort) for each control scenario can be seen in Table 2, where by applying two control strategies simultaneously, it give the optimal solution with minimum control effort.



(a) Exposed (b) Corrupted

Fig. 6. Dynamics of exposed and corrupted subpopulation with u_2 control

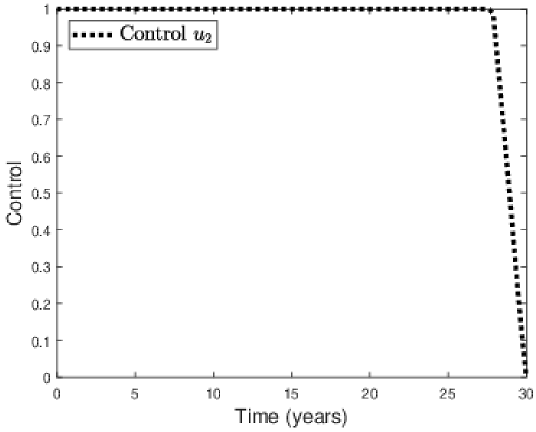


Fig. 7. The effort of u_2 control

Table 2
Performance index for the minimum control effort

Control scenario	u_1	u_2	J (million rupiah)
u_1 and u_2	0.1303	0.3522	202.61
u_1	1	0	844.7
u_2	0	1	2562.02

6. Conclusions

In this paper, the SECR mathematical model based on four compartments which describes the corruption dynamics was successfully developed. The analysis of the model shows that the system has non-negative solutions with a bounded domain and corruption cases-free equilibrium point which was local asymptotically stable. The simulation results of optimal control problem show that the scenario which applied two control strategies u_1 and u_2 simultaneously, produced the most effective results in reducing the number of exposed and corrupted subpopulations. Where the number of exposed and corrupted subpopulation decreased and reached zero in short time period. This strategy gives the optimal solution with minimum control effort. Meanwhile, the implementation of only one type control caused a decrease in the number of exposed and corrupted subpopulations, which took a long time with large control efforts. From these results, it can be concluded that in order to prevent and reduce the number of corruption cases, the control strategy in the form of giving punishment for the corrupted individuals must be also followed by prevention campaign corruption in society by the KPK, where both these control strategies must be carried out together to give the optimal results.

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