

# Analysis of MHD Effects on Porous Flat Plate and Curved Circular Plate with Couple Stress Fluid

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ARTICLE INFO	ABSTRACT
Article history: Received 26 March 2024 Received in revised form 25 July 2024 Accepted 9 August 2024 Available online 30 August 2024	The purpose of this article is to examine the effect of MHD on curved circular and porous flat plates lubricated with couple stress fluid. It is assumed that the flow in the porous region follows Darcy's law. A generalized form of Reynolds's equation is derived on the basis of Stokes Theory of couple stress fluids in the presence of applied external magnetic field. The non-dimensional form of modified Reynolds's equation is solved and the closed form expressions are obtained for the fluid film pressure, load carrying capacity and squares flim time. It is found that the offset of applied magnetic field and
<i>Keywords:</i> Couple stress; squeeze film; magnetohydrodynamics; flat plate and circular plate	couple stress lubricants are to increase the squeeze film pressure, load carrying capacity and to lengthen the squeezing time. Also, the effect of permeability parameter is to decrease the pressure, load-carrying capacity and squeeze-film time as compared to the non-porous case.

### 1. Introduction

Magnetohydrodynamics (MHD) is the study of the effects of electromagnetic phenomena on conducting fluids. The magnetic field plays an important role in the investigation and control of tribological characteristics. MHD bearings with conducting fluids have superior thermal and electric conductivity when compared to ordinary bearings. Magnetic and electric fields improve the load carrying efficiency of bearings lubricated with liquid metal, and these applications encouraged the development of MHD lubricants. Many researchers have carried out theoretical and experimental research into the effect of magnetic fields on the flow of conducting lubricants. Kuzma [1] observed that using a magnetic field improved the bearing's capacity to support loads when he investigated the MHD journal bearing. The flow between parallel plates was examined by Maki *et al.*, [2]. Many studies have been conducted in the presence of a magnetic field. Several researchers, including Kuzma [1], Lin [3], Mouda *et al.*, [4] have investigated, changes in the properties of bearings

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lubricated by electrically conduction fluid [5-7]. They noticed that as the magnetic field increased, it also increased the capacity of supporting load.

In the classical hydrodynamic lubrication theory, the lubricant is treated as a Newtonian fluid in all of these experiments. This prognosis is not particularly precise in terms of applications to engineering problems. The researcher developed the most ideal lubricant, known as non-Newtonian fluid, by adding certain polymers to Newtonian fluid. Numerous studies examined the effects of couple stress lubricants on the functioning of bearings employing Stokes theory [8]. The fluid couple stress in journal bearings was looked at. Short journal bearings with couple stress fluid were presented by Naduvinamani *et al.*, [9]. Das [10] and Guha [11] carried out research on dynamics slider journal bearings and revealed that the impact of couple stress lubricant boosts load support and squeezing time.

Many researchers have investigated MHD couple stress lubrication theory, for example, secant slider and plane slider bearings were studied by Hanumagowda *et al.*, [12] and Hanumagowda [13]. Composite slider bearings by Kashinath and Hanumagowda [14], different types of finite plates by Fathima *et al.*, [15,16], and parabolic and plane slider bearings by Naduvinamani *et al.*, [17]. Alkasasbeh [18] analysed the study on magnetic field and casson hybrid Nano fluid over vertical stretching sheet, from this study it is cleared that the use of mangetic field and casson hybrid fluid will enhance the temperature profile and declines the Nusselt number and velocity profile. And Peristalsis of Fractional Second Grade Fluid in the Presence of Electro Osmotic Phenomenon with Heat and Mass Transfer was studied by Channakote *et al.*, [19]. Numerical solution of MHD Reynolds equation for squeeze film lubrication between two parallel surfaces by Kudenatti *et al.*, [20], MHD couple stress on Triangular plates was analyzed Kashinath [21], Pivoted curved slider bearings and porous sine slider bearings with MHD couple stress described by Hanumagowda *et al.*, [22], and Ayyappa *et al.*, [23,24]. biradar *et al.*, [25] presented sine curved slider bearings and curved circular plate and flat plates by Hiremath *et al.*, [26], non-Newtonian couple stress lubrication of MHD elastic journal bearings by Kardoudi [27].

These investigations have shown that, when compared to an equivalent non-magnetic environment, the impact of magnetic field and couple stress is for boosting load bearing capacity and delaying approach time.

Porous media are employed in many disciplines of applied research and engineering, including filtration, mechanics, and geosciences, biophysics, and materials science. When a porous bearing with oil impregnated into the porous material is installed, the bearing no longer requires a separate oil supply. Morgan [28] were the first to establish a lubrication mechanism for porous metal bearings. Rectangular Shaped Plates Wu [29] Analyzed the results reveal that increasing the permeability limitation of porous face and inverse cube of dimensionless film thickness has a negative effect on pressure, load support, and squeeze film time. This implies that the permeability parameter, as well as the thickness of the film, controls the amount of the porous effect.

Porous inclined slider bearing by Verma *et al.*, [30] after he came to know inclined slider bearings have more prominent than the viscous porous case. Porous-rough circular stepped plates and rectangular plates with MHD couple stress were introduced and reported on by Naduvinamani *et al.*, [31,32]. Also, Kudenatti *et al.*, [33] made the analysis on porous-rough rectangular plates, from this study we can able to know the pressure and load bearing capacity decrease as values of permeability parameter increases, whereas squeeze film characteristics are significantly increases when roughness parameter value increases along with Hartmann number.

Until now, no investigation has been done on the effect of couple stress lubricant on curved circular plate and porous flat plate under existence of an MHD. In this article, we made a theoretical investigation of MHD effects on curved circular and porous flat plates with couple stress

fluid. The current work is compared with Hiremath *et al.,* [26] and numerical analysis is done in Table 1.

## Table 1

Numerical Analysis of Squeeze film attributes between *P*,  $W^*$  and  $T^*$  between Hiremath *et al.*, [26] and present analysis by varying Magnetic field ( $M_0$ ), couple stress ( $I^*$ ) and Porous Parameter ( $\psi$ ) with fixed = 0.8,  $h_m^* = 0.8, r = 0.6, a = 0.6, m = 0.6, \delta = 0.01, \phi = 0.2$ 

		Hiremath <i>et al.,</i> [26]		Present Analysis			
				$\psi = 0$		$\psi = 0.01$	
	Mo	<i>I</i> <sup>*</sup> = 0	<i>I</i> <sup>*</sup> = 0.2	/* = 0	/ <sup>*</sup> = 0.2	<i>I</i> <sup>*</sup> = 0	<i>I</i> <sup>*</sup> = 0.2
Р	0	6.67249	8.58686	6.67249	8.58686	3.93646	4.53261
	2	7.63177	9.56433	7.63177	9.56433	7.03275	8.64185
	4	10.4896	12.4909	10.4896	12.4909	10.166	12.0348
$W^*$	0	4.00349	5.15212	4.00349	5.15212	2.36187	2.71957
	2	4.57906	5.7386	4.57906	5.7386	4.21965	5.18511
	4	6.29374	7.49453	6.29374	7.49453	6.09962	7.22088
Τ*	0	1.6014	2.06085	1.6014	2.06085	0.94475	1.08783
	2	1.83163	2.29544	1.83163	2.29544	1.68786	2.07405
	4	2.5175	2.99781	2.5175	2.99781	2.43985	2.88835

## 2. Methodology

Geometrical representation of curved circular plate and porous flat plate with couple stress fluid under magnetic field as portrayed in Figure 1. Lower plate is porous and has thickness  $\delta$ . Movement of the fluid in to the porous material is controlled by Darcy's law. Lower and upper plates are separated by film thickness *h*.



Fig. 1. Configuration geometry of the problem

Film shape *h* is taken to be an exponential type as

$$h = h_m \exp(-\alpha r^2) \qquad \qquad 0 \le r \le a \tag{1}$$

Here,  $\alpha$  is curvature parameter and  $h_m$  is minimal film thickness.

It is supposed that the fluid film is thin, the body couples and the body forces are insignificant, in view of these suppositions, the theory of hydrodynamic lubrication applied to thin films, and the equation of continuity and MHD governing equation of motion in polar coordinate are

$$\mu \frac{\partial^2 u}{\partial z^2} - \eta \frac{\partial^4 u}{\partial z^4} - \sigma B_0^2 u = \frac{\partial p}{\partial r}$$
(2)

$$\frac{\partial p}{\partial z} = 0 \tag{3}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0 \tag{4}$$

and for porous region

$$\frac{1}{r}\frac{\partial(ru^*)}{\partial r} + \frac{\partial w^*}{\partial z} = 0$$
(5)

$$u^* = \frac{-k}{\mu \left[1 - \phi + \frac{kM_0^2}{mh_m^2}\right]} \frac{\partial p^*}{\partial r}$$
(6)

$$w^* = \frac{-k}{\mu [1 - \phi]} \frac{\partial p^*}{\partial z}$$
(7)

where, u stand for velocity component, P represent pressure of the film,  $\mu$  is dynamic viscosity of the lubricant,  $\sigma$  is the electrical conductivity of the lubricant and  $B_0$  is the applied magnetic field to the bearing in the z-direction. Also, velocity components  $u^*$  and  $w^*$  in porous matrix is deliberated with modified version of Darcy's law.

Velocity components boundary conditions are

(i) At lower plate z = 0

$$u = 0, \quad \frac{\partial^2 u}{\partial z^2} = 0 \tag{8}$$

w = 0

(9)

# (ii) At upper plate z = h

$$u = 0 , \quad \frac{\partial^2 u}{\partial z^2} = 0 \tag{10}$$

$$w = V = \frac{-\partial h}{\partial t}$$
(11)

Solution of Eq. (2) utilising boundary conditions (8) and (10) is

$$u = \left\{ \left(g_1 - g_2\right) - 1 \right\} \frac{h_m^2}{\mu M_0^2} \frac{\partial p}{\partial r}$$
(12)

Here,  $l = (\eta / \mu)^{1/2}$  is couple stress parameter and  $M_0 = B_0 h_m (\sigma / \mu)^{1/2}$  is Hartmann number.

$$g_1 = g_{11}, \ g_2 = g_{12}$$
 for  $4M_0^2 l^2 / h_m^2 < 1$  (13)

$$g_1 = g_{21}, \ g_2 = g_{22}$$
 for  $4M_0^2 l^2 / h_m^2 = 1$  (14)

$$g_1 = g_{31}, \ g_2 = g_{32}$$
 for  $4M_0^2 l^2 / h_m^2 > 1$  (15)

Related equations in (13), (14) and (15) are specified in equations below.

$$g_{11} = \frac{A^2}{\left(A^2 - B^2\right)} \frac{\cosh\left\{B(2z - h)/2l\right\}}{\cosh\left(Bh/2l\right)}$$

$$g_{12} = \frac{B^2}{\left(A^2 - B^2\right)} \frac{\cosh\left\{A(2z - h)/2l\right\}}{\cosh\left(Ah/2l\right)}$$

$$A = \left\{\frac{1 + \left(1 - 4M^2l^2/h_m^2\right)^{\frac{1}{2}}}{2}\right\}^{\frac{1}{2}} \qquad B = \left\{\frac{1 - \left(1 - 4M^2l^2/h_m^2\right)^{\frac{1}{2}}}{2}\right\}^{\frac{1}{2}}$$

$$g_{21} = \frac{2\cosh\left\{(z - h)/\sqrt{2l}\right\} + 2\cosh\left(z/\sqrt{2l}\right)}{2\left\{\cosh\left(h/\sqrt{2l}\right) + 1\right\}}$$

$$g_{22} = \frac{\left(z/\sqrt{2l}\right)\sinh\left\{(z - h)/\sqrt{2l}\right\} + \left\{(z - h)/\sqrt{2l}\right\}\sinh\left(z/\sqrt{2l}\right)}{(z/\sqrt{2l})}$$

$$2\left\{\cosh\left(h/\sqrt{2l}\right)+1\right\}$$

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$$g_{31} = \frac{\cos B_2 z \cosh A_2 (z - h) + \cosh A_2 z \cosh B_2 (z - h)}{\cosh A_2 h + \cosh B_2 h}$$

$$g_{32} = \frac{\cot\theta\left\{\sinh A_2 z \sinh B_2 \left(z - h\right) + \sin B_2 z \sinh A_2 \left(z - h\right)\right\}}{\cosh A_2 h + \cosh B_2 h}$$

$$A_2 = \sqrt{M/lh_m} \cos(\theta/2) \qquad B_2 = \sqrt{M/lh_m} \sin(\theta/2) \qquad \theta = \tan^{-1}\left(\sqrt{4l^2 M^2/h_m^2 - 1}\right)$$

Performing an integrating of Eq. (4) over the film thickness and utilising boundary conditions (9) and (11) we get

$$\frac{1}{r\mu}\frac{\partial}{\partial r}\left\{rS(h,l,M_0)\frac{\partial p}{\partial r}\right\} = w_h - w_0$$
(16)

Here,  $w_h = 0$  because upper plate is non-porous.

Velocity component in z-direction is incessant at interface amongst lower plate and film so that

$$w_{0} = -\left[\frac{dh}{dt} + \frac{k}{\mu[1-\phi]} \left(\frac{\partial p^{*}}{\partial z}\right)_{z=0}\right]$$
(17)

Substituting (17) in (16) we get modified form Reynolds equation in form

$$\frac{1}{r\mu}\frac{\partial}{\partial r}\left\{rS(h,l,M_0)\frac{\partial p}{\partial r}\right\} = \frac{dh}{dt} + \frac{k}{\mu\left[1-\phi\right]}\left(\frac{\partial p^*}{\partial z}\right)_{z=0}$$
(18)

Where,

$$S(h,l,M_{0}) = \begin{cases} \frac{h_{m}^{2}}{M_{0}^{2}} \left\{ \frac{2l}{(A^{2} - B^{2})} \left( \frac{B^{2}}{A} \tanh \frac{Ah}{2l} - \frac{A^{2}}{B} \tanh \frac{Bh}{2l} \right) + h \right\}, & \text{for } M_{0}^{2}l^{2}/h_{m}^{2} < 1 \\ \frac{h_{m}^{2}}{M_{0}^{2}} \left\{ \frac{h}{2} \sec h^{2} \left( \frac{h}{2\sqrt{2l}} \right) - 3\sqrt{2l} \tanh \left( \frac{h}{2\sqrt{2l}} \right) + h \right\}, & \text{for } M_{0}^{2}l^{2}/h_{m}^{2} = 1 \\ \frac{h_{m}^{2}}{M_{0}^{2}} \left\{ \frac{2lh_{m}}{M} \left( \frac{(A_{2}\cot\theta - B_{2})\sin B_{2}h - (B_{2}\cot\theta + A_{2})\sin A_{2}h}{\cos B_{2}h + \cosh A_{2}h} \right) + h \right\}, & \text{for } M_{0}^{2}l^{2}/h_{m}^{2} > 1 \end{cases}$$

Fluid pressure into porous region satisfies equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p^*}{\partial r}\right) + \left[\frac{D}{1-\phi}\right]\frac{\partial p^*}{\partial z^*} = 0$$
(19)

Where 
$$D = \left[1 - \phi + \frac{kM_0^2}{mh_m^2}\right]$$

Let's doing an integration to Eq. (19) with respect to z over  $\delta$  and utilising boundary conditions of solid  $\frac{\partial p^*}{\partial r} = 0$  at  $z = -\delta$  [Morgan-Cameron Approximation] we get

$$\left[\frac{\partial p^*}{\partial z}\right]_{z=0} = -\left[\frac{1-\phi}{D}\right]_{-\delta}^0 \frac{1}{r} \frac{\partial}{\partial r} \left(r\frac{\partial p^*}{\partial r}\right) dz$$

With assumption of porous layer thickness  $\delta$  is comparatively smaller and utilising pressure boundary condition  $p = p^*$  at z = 0 we get

$$\left[\frac{\partial p^*}{\partial z}\right]_{z=0} = -\delta \left[\frac{1-\phi}{D}\right] \frac{1}{r} \frac{\partial}{\partial r} \left(r\frac{\partial p}{\partial r}\right)$$
(20)

Utilising Eq. (20) into Eq. (18) we attain Reynolds equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{\left(S(h,l,M_0) + \frac{\delta k}{D}\right)r\frac{\partial p}{\partial r}\right\} = V\mu$$
(21)

Considering the dimensionless quantities,

$$r^{*} = \frac{r}{a} , h^{*} = \frac{h}{h_{m}}, l^{*} = \frac{2l}{h_{m}}, P = -\frac{h_{1}^{3}p}{\mu a^{2}V}, \delta^{*} = \frac{\delta}{h_{m}}, \Psi = \frac{k\delta}{h_{m}^{3}}, D_{1} = \left[1 - \phi + \frac{\psi M_{0}^{2}}{m\delta^{*}}\right]$$

Using above quantities in Eq. (21) modified dimensionless Reynolds equation is attained as

$$\frac{1}{r^*} \frac{\partial}{\partial r^*} \left\{ G(h^*, l^*, M_0, \psi) r^* \frac{\partial P}{\partial r^*} \right\} = -1$$
(22)

Here,  $G(h^*, l^*, M_0, \psi) = \left(S^*(h^*, l^*, M_0) + \frac{\psi}{D_1}\right)$ 

$$S^{*}(h^{*}, l^{*}, M_{0}) = \begin{cases} \frac{1}{M_{0}^{2}} \left\{ \frac{l^{*}}{(A^{*^{2}} - B^{*^{2}})} \left( \frac{B^{*^{2}}}{A^{*}} \tanh \frac{A^{*}h^{*}}{l^{*}} - \frac{A^{*^{2}}}{B^{*}} \tanh \frac{B^{*}h^{*}}{l^{*}} \right) + h^{*} \right\}, \quad \text{for } M_{0}^{2}l^{*2} < 1 \\ \frac{1}{M_{0}^{2}} \left\{ \frac{h^{*}}{2} \sec h^{2} \left( \frac{h^{*}}{\sqrt{2l^{*}}} \right) - \frac{3l^{*}}{\sqrt{2}} \tanh \left( \frac{h^{*}}{\sqrt{2l^{*}}} \right) + h^{*} \right\}, \quad \text{for } M_{0}^{2}l^{*2} = 1 \\ \frac{1}{M_{0}^{2}} \left\{ \left( \frac{l^{*}(A^{*}_{2} \cot \theta^{*} - B^{*}_{2}) \sin B^{*}_{2}h^{*} - l^{*}(B^{*}_{2} \cot \theta^{*} + A^{*}_{2}) \sinh A^{*}_{2}h^{*}}{M(\cos B^{*}_{2}h^{*} + \cosh A^{*}_{2}h^{*})} \right) + h^{*} \right\}, \quad \text{for } M_{0}^{2}l^{*2} > 1 \end{cases}$$
where,  $A^{*} = \left( \frac{1 + \left(1 - l^{*}M_{0}^{2}\right)^{\frac{1}{2}}}{2} \right)^{\frac{1}{2}} \text{ and } B^{*} = \left( \frac{1 - \left(1 - l^{*}M_{0}^{2}\right)^{\frac{1}{2}}}{2} \right)^{\frac{1}{2}}$ 

Integrating Eq. (22) twice with respect to  $r^*$  and utilising boundary conditions as obtained nondimensional mean squeeze film pressure.

$$\frac{\partial p}{\partial r^*} = 0 \quad \text{When} \quad r^* = 0 \tag{24}$$

$$P = 0$$
 When  $r^* = 1$  (25)

We get

$$P = -\frac{1}{2} \int_{1}^{r^{*}} \frac{r^{*}}{G(h^{*}, l^{*}, M_{0}, \psi)} dr^{*}$$
(26)

Load carrying capacity can be mentioned

$$W = \int_{0}^{a} P dr$$
 (27)

Performing the integration of the Eq. (21) w.r. to  $r^*$  with the limit 0 to 1 from this we get nondimensional load carrying capacity  $W^*$  as

$$W^* = -\frac{1}{2} \int_0^1 \left\{ \int_1^{r^*} \frac{r^*}{G(h^*, l^*, M_0, \psi)} dr^* \right\} r^* dr^*$$
(28)

The expression for time height relation into dimensionless form is attained from Eq. (28) as

$$T^{*} = -\frac{1}{2} \int_{h_{1}^{*}}^{h_{2}^{*}} \left[ \int_{0}^{r} \left\{ \int_{1}^{r^{*}} \frac{r^{*}}{G(h^{*}, l^{*}, M_{0}, \psi)} dr^{*} \right\} r^{*} dr^{*} \right] dh^{*}$$
<sup>(29)</sup>

The above derived three parameters likes pressure, load carrying capacity and squeeze film time are all in non-dimensional form it cannot be integrated directly by theoretically but can be integrated numerically using Simpson's 1/3<sup>rd</sup> rule.

# 3. Results

This study presents impact of MHD on porous flat plate with curved circular plate with dimensionless couple stress parameter, using MHD thin film lubrication and Stokes microcontinuum theory as the theoretical foundations. Parameter  $M_0$  is Hartmann number, couple stress

 $l^*\left(=\frac{2l}{h}\right)$  here  $l=\sqrt{\eta/\mu}$  rises because of smaller polar additives in lubricant. Dimension of  $\left(\frac{\eta}{\mu}\right)$  is of length square and this length is considered as chain length of polar additives into lubricant. Therefore, following range of non-dimensional characteristics are employed in the current analysis permeability parameter ( $\psi = 0 \sqcup 0.01$ ), couple stress parameter ( $l^* = 0 \amalg 0.6$ ), curvature parameter ( $\beta = -0.25 \sqcup 0.25$ ) and Hartmann number ( $M_0 = 0 \square 6$ ).

# 3.1 Pressure Distribution

Figure 2 depicts disparity of dimensionless pressure P with fixed values  $r^*$  different values of the permeability parameter  $\psi$  and Hartmann number  $M_0$  with  $l^* = 0.2$ ,  $\beta = 0.5$ , m = 0.6,  $\delta = 0.01$  and  $\phi = 0.2$ . When a magnetic field is present in porous media, non-dimensional pressure is found to be higher than in non-magnetic situation. Furthermore, P is more significant in solid case as compared to the porous medium. Variation of non-dimensional pressure P with  $r^*$  for different values  $\psi$  and  $l^*$  under fixed values porosity  $M_0 = 3$ ,  $\beta = 0.5$ , m = 0.6,  $\delta = 0.01$  and  $\phi = 0.2$  is displayed in Figure 3. It found as non-dimensional pressure P increases significantly as increasing values of  $l^*$  until it reaches maximum there after falls down gradually. Figure 4 Represents P with  $r^*$  for distinctive values  $\psi$  and  $\beta$  with  $M_0 = 3$ ,  $l^* = 0.2$ , m = 0.6,  $\delta = 0.01$  and  $\phi = 0.2$ . It's seen as non-dimensional pressure P increase significantly as values increase for curvature parameter  $\beta$  until reaches maximum there after 5 describes 3D pictures in that the higher in pressure and also decrements noticed due to rise of magnetic field and porous parameter.



**Fig. 2.** Variation of non-dimensional pressure P versus  $r^*$  with different values of  $\Psi$  and  $M_0$  with  $l^* = 0.2$ ,  $\beta = 0.5$ , m = 0.6,  $\delta = 0.01$  and  $\phi = 0.2$ 



**Fig. 3.** Variation of non-dimensional pressure P versus  $r^*$  with different values of  $\Psi$  and  $l^*$  with  $M_0$  = 3,  $\beta$  = 0.5, m = 0.6,  $\delta$  = 0.01 and  $\phi$  = 0.2



**Fig. 4.** Variation of non-dimensional pressure *P* versus  $r^*$  with different values of  $\Psi$  and  $\beta$  with  $M_0 = 3$ ,  $l^* = 0.2$ , m = 0.6,  $\delta = 0.01$  and  $\phi = 0.2$ 



Fig. 5. Combined effect of  $\psi$  and  $M_0$  on dimensionless load P

#### 3.2 Load Carrying Capacity

Figure 6 depicts deviation of  $W^*$  with  $\beta$  for diverse values of  $\psi$  and Hartmann number  $M_0$  with  $l^* = 0.2, m = 0.6, \delta = 0.01$  and  $\phi = 0.2$ . It's seen as impact of  $M_0$  is increasing  $W^*$  comparing with nonmagnetic case ( $M_0 \rightarrow 0$ ). By reducing the flow of the lubricant with the help of a magnetic field, significant pressure is created, leading to an increase in the bearings load capacity. Figure 7 describes the  $W^*$  with  $\beta$  for different  $\psi$  and  $l^*$  with  $M_0 = 3$ . As  $l^*$  is increased, it is found that load bearing capacity is enhanced in comparison to the Newtonian situation. With an increase in pressure inside the fluid zone, couple stress fluid improves the region load-bearing capability. Figure 8 is 3D graph of load  $W^*$  versus different values of  $M_0$  and  $\psi$ , this 3D pictures describes the increase and falling down of load support due to impacts of  $M_0$  and  $\psi$ .



**Fig. 6.** Variation of non-dimensional pressure  $W^*$  versus  $r^*$  with different values of  $\Psi$  and  $M_0$  with  $l^* = 0.2$ , m = 0.6,  $\delta = 0.01$  and  $\phi = 0.2$ 



**Fig. 7.** Variation of non-dimensional load capacity  $W^*$  versus  $\beta$  with different values of  $\Psi$  and  $l^*$  with  $M_0$  = 3, m = 0.6,  $\delta$  = 0.01 and  $\phi$  = 0.2



**Fig. 8.** Combined effect of  $M_0$  and  $\psi$  on dimensionless load  $W^*$ 

## 3.3 Non-Dimensional Squeeze Film Time

Essential aspects of squeeze film are squeeze film time, its time needed to reduce film thickness  $h_m^*$  non-dimensional  $T^*$  with  $h_m^*$  for different values of  $\psi$  and Hartmann number  $M_0$  with  $l^*$ =0.2,  $\beta$  =0.5, m =0.6,  $\delta$  =0.01 and  $\phi$  =0.2 is presented in Figure 9. By comparison to the non-magnetic condition, the squeeze film time is found to be increased when a magnetic field is applied. Magnetic bearing's principal role is to rise and sustain a rotating shaft without any physical touch. Increasing the strength of the magnetic field can improve levitation force, allowing for greater load carrying capability. This can be advantageous in applications requiring larger loads to be supported. Figure 10 Depicts squeeze film  $T^*$  with  $h_m^*$  for different  $\psi$  and  $l^*$  with  $M_0 = 3, \beta = 0.5, m = 0.6, \delta =$ 0.01 and  $\phi$  = 0.2. Noted the squeezing time is higher in case of in non-Newtonian situation than the Newtonian. And also came to know the response time is more efficient in the absence of porous case i.e.  $(\psi = 0)$ , response squeeze film time is longer when the couple stress parameter is included. Figure 11 Depicts  $T^*$  with  $h_m^*$  for different values of  $\psi$  and curvature parameter  $\beta$  with  $M_0$ = 3,  $l^* = 0.2$ , m = 0.6,  $\delta = 0.01$  and  $\phi = 0.2$ . It's clear that  $T^*$  upsurges for increasing values of  $\beta$ . Figure 12 is 3D pictures of squeezing time along minimum thickness for distinctive values magnetic field and permeability parameter from this graph it revealed that there is a rise in squeezing time due to impacts of  $M_{_0}$  and squeezing time reduces under existence of  $\psi\,$  .



**Fig. 9.** Variation of non-dimensional squeeze film time  $T^*$  versus  $h_m^*$  with different values of  $\Psi$  and  $M_0$  with  $l^*$  = 0.2,  $\beta$  = 0.5, m = 0.6,  $\delta$  = 0.01 and  $\phi$  = 0.2



**Fig. 10.** Variation of non-dimensional squeeze film time  $T^*$  versus  $h_m^*$  with different values of  $\Psi$  and  $l^*$  with  $M_0 = 3$ ,  $\beta = 0.5$ , m = 0.6,  $\delta = 0.01$  and  $\phi = 0.2$ 



**Fig. 11.** Variation of non-dimensional squeeze film time  $T^*$  versus  $h_m^*$  with different values of  $\Psi$  and  $\beta$  with  $M_0 = 3$ ,  $l^* = 0.2$ , m = 0.6,  $\delta = 0.01$  and  $\phi = 0.2$ 



Fig. 12. Combined effect of  $M_{_0}$  and  $\psi$  on  $T^{^*}$ 

# 4. Conclusions

As per Stokes micro continuum theory for couple stress fluid, we investigated effect of a magnetic field upon porous flat plate and curved circular plates using couple stress fluid as lubricant. An equation of kind used in MHD is generated. Following inferences may be made from theoretical findings presented in this article.

- (i) It is noticed the impact of magnetic field and non-Newtonian fluid lubricants is enhances the squeeze film attributes when compared to non-magnetic and Newtonian situations.
- (ii) Impact of porous parameter provides the reduction of pressure, load support and squeezing time than the absence porous medium.
- (iii) Increasing the curvature parameter causes the dimensionless pressure to rise until it reaches a maximum, after which it falls and the squeeze film duration grows.

Overall, pressure, load bearing capacity, and delay in the squeeze film duration were all enhanced by application of a magnetic field with couple stress fluids as lubricant in porous flat plate and curved circular plates.

# Justification

The present work is compared with Hiremath *et al.*, [26] and numerical comparison is analysed in Table 1. When Permeability Parameter ( $\psi = 0$ ) the current analysis is reduced to Hiremath *et al.*, [26] and excellent agreement of result was observed in absence of porous parameters. The performance of our framework significantly improves as these parameters vary according to the variation effects brought on by the MHD couple stress. The results of this research are anticipated to assist lubrication engineers in selecting the proper parameters for a given magnetic field to extend the bearings life.

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