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Unsteady Three-Dimensional Free Convection Flow Near the Stagnation Point Over a General Curved Isothermal Surface in a Nanofluid

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Norshaza Atika Saidin[1], Mohd Ariff Admon[2], Khairy Zaimi[1,*]

[1] Institute of Engineering Mathematics, Universiti Malaysia Perlis, Pauh Putra Campus, 02600 Arau, Perlis, Malaysia
[2] Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310 Johor Bahru, Johor, Malaysia

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ABSTRACT

This study deals with an unsteady three-dimensional free convection flow near the stagnation point region over a general curved isothermal surface placed in a nanofluid. Nanofluids are great scientific interest because these new thermal transport phenomena surpass the fundamental limits of conventional macroscopic theories of suspensions. Since the heat and mass transfer are very extensive in the industry, the unsteady three-dimensional body near stagnation point can give a significant impact on the heat transfer process. The main objective of the present study is to investigate the effects of some governing parameters on the skin friction coefficients, local Nusselt and local sheerwood numbers as well as related profiles of unsteady free convection in a nanofluid. The momentum equations in x- and y-directions, energy balance equation, and nanoparticle concentration equation are reduced to a set of four fully-coupled nonlinear differential equations under appropriate similarity transformations. The well-known technique Keller-box method is used numerically for different values of governing parameters entering these equations. Further, the present results have been compared with the previous published results for a particular case and the comparisons are found to be in good agreement. The skin friction, local Nusselt number and Sherwood number is increases with an increase in curvature parameter. Rising values of the Lewis number and Brownian motion parameter has enhanced the flow while rising values of the buoyancy and thermophoresis parameter will decelerate the flow. The temperature profile is increases when Brownian motion, buoyancy and thermophoresis parameter increases and concentration profile increase with an increases in buoyancy and thermophoresis parameter.

Keywords:
Free convection; stagnation point; three-dimensional; nanofluid; numerical solution

1. Introduction

Nowadays, problems that are related to boundary layer near stagnation point can be found in many high technology products. Various aspects of the flow and heat transfer problem for boundary

* *Corresponding author.*
E-mail address: khairy@unimap.edu.my (Khairy Zaimi)

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layer flow near stagnation point have been explored in many investigations. Amin and Riley [1], Seshadri *et al.,* [2] and Nazar *et al.,* [3] had been extensively investigating about the unsteady laminar free and mixed convection boundary layer flow in the neighbourhood of a two-dimensional and axisymmetric stagnation point. In steady two-dimensional stagnation-point flow of an incompressible fluid over a stretching sheet, Pop *et al.,* [4] investigated theoretically by taking into account radiation effects using the Rosseland approximation to model the radiative heat transfer. This approximation leads to a considerable simplification in the radiation flux.

The steady laminar flow of an incompressible non-Newtonian micropolar fluid at a two-dimensional stagnation point with heat transfer has been studied by Attia [5]. This author focused on the effect of uniform suction or blowing directed normal to the wall [5]. Salleh *et al.,* [6] presented the free convection boundary layer flow on a solid sphere with Newtonian heating. Ahmad *et al.,* [7], Panigrahi *et al.,* [8], Yasin *et al.,* [9], Shateyi *et al.,* [10] and Dessie *et al.,* [11] presented a problem of MHD near stagnation-point and Alkasasbeh [12] explored the behaviour of the Casson fluid under various effect and circumstances.

On the other hand, another work had been done by Raj *et al.,* [13]. They focused on the effects of thermal radiation and variable fluid viscosity on stagnation point flow past a porous stretching sheet embedded in a porous medium with partial slip condition. They solved the governing equation numerically using the same method by Dessie *et al.,* [11] but along with the shooting technique and result, they are present in graphical form [11,12]. Since then, many researchers have been working on the stretching or shrinking sheet with various physical conditions such as Dero *et al.,* [14] and Yashkun *et al.,* [15]. Non-axisymmetric stagnation point flow and heat transfer of a viscous fluid with variable viscosity on a cylinder in constant heat flux studied by Alizadeh *et al.,* [16]. Sharma *et al.,* [17] considered the unsteady two-dimensional stagnation point flow of a viscous and incompressible nanofluid over a permeable flat plat. They investigated the heat transfer characteristics caused by the stagnation-point flow of a nanofluid over a permeable flat plate using finite element method.

Nanofluids are of great scientific interest because these new thermal transport phenomena surpass the fundamental limits of conventional macroscopic theories of suspensions. These fluids are engineered colloidal suspensions of nanoparticles in a base fluid, which is the term proposed by Choi [18] to describe the new class of nanotechnology-based heat transfer fluids that exhibit thermal properties superior to those of their base fluids or conventional particle fluid suspensions. Hassani *et al.,* [19] studied an analytical research for boundary layer flow of a nanofluid past a stretching sheet by using Homotopy Analysis Method to solve the resulting equations. Next, laminar convective heat transfer of CuO/water nanofluid through an equilateral triangular duct at constant wall heat flux was investigated by Edalati *et al.,* [20]. Furthermore, Hajipour *et al.,* [21] studied about transient two-dimensional mixed convection of nanofluids in the entrance region of a vertical channel.

A research by Farooq *et al.,* [22] presented an analytical result on steady free convection flow near the stagnation point of a three-dimensional body over a general curved isothermal surfaced placed in a nanofluid. The previous research about free convection flow near stagnation point in three-dimensional body was investigated by Admon *et al.,* [23] by using the Keller-box method [22,23]. Meanwhile, Hayat *et al.,* [24] dealt with the boundary layer flow of nanofluid over power-law stretched surface. Khan *et al.,* [25] investigated the three-dimensional flow of nanofluid over a bi-directional exponentially stretching sheet.

In this paper, we analyze the stagnation-point flow and heat transfer in a nanofluid. This study is an extension of the previous article by Admon *et al.,* [23]. Different from that considered by Admon *et al.,* [23], we study three-dimensional flow near the stagnation point in a nanofluid. The impacts of nanofluid on the skin friction coefficient, velocity profile, temperature profile and concentration profile will be presented and analyzed. In practical, the investigations of the stagnation-point flow and heat transfer over general curved isothermal surface in a nanofluid effects is very important and useful mainly in many industrial manufacturing processes. Even though many investigations on the stagnation-point fluid flow problems have been examined, there are still limited results found.

2. Problem Formulation

We consider the unsteady free convection flow near the stagnation point of a heated three-dimensional body located in a viscous and incompressible nanofluid with a uniform temperature T_∞. It is assumed that the constant wall temperature of the body is suddenly changed from T_w to T_∞, where $T_w > T_\infty$. A locally Cartesian orthogonal system(x, y, z) is chosen with the origin O at the nodal stagnation point, where the *x*- and *y*-coordinates were measured along the body surface, while the *z*-coordinates was measured normal to the body surface. Under these assumptions, the governing equations for free convection nanofluid flow in the stagnation-point region of a three-dimensional body are [22]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho_{f\infty}}+v\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\left[(1-C_\infty)\beta(T-T_\infty)-\frac{\rho_p-\rho_{f\infty}}{\rho_{f\infty}}(C-C_\infty)\right]gax, \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{1}{\rho_{f\infty}}+v\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+\left[(1-C_\infty)\beta(T-T_\infty)-\frac{\rho_p-\rho_{f\infty}}{\rho_{f\infty}}(C-C_\infty)\right]gby, \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\tau\left\{\begin{matrix}D_B\left(\frac{\partial C}{\partial x}\frac{\partial T}{\partial x}+\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{\partial C}{\partial z}\frac{\partial T}{\partial z}\right)\\+\left(\frac{D_T}{T_\infty}\right)\left[\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2+\left(\frac{\partial T}{\partial z}\right)^2\right]\end{matrix}\right\}, \tag{4}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=D_B\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}\right)+\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right). \tag{5}$$

The initial and boundary conditions given as follows

$$\begin{aligned}
t<0:\ & u=v=w,\ T=T_\infty,\ C=C_\infty \text{ for any } x, y, z\\
t\geq 0:\ & u=v=0,\ T=T_w,\ C=C_w\ on\ z=0, x\geq 0, y\geq 0,\\
& u=v=0,\ T=T_\infty,\ C=C_\infty\ on\ x=0, y\geq 0, z>0,\\
& u=v=0,\ T=T_\infty,\ C=C_\infty\ on\ y=0, x\geq 0, z>0,\\
& u=v=0,\ T=T_\infty,\ C=C_\infty\ on\ z\to\infty, x\geq 0, y\geq 0.
\end{aligned} \tag{6}$$

Here, *t* represents time, *u*, *v*, *w* are the velocity components along the *x*-, *y*-, *z*-axis, respectively, *T* is the fluid temperature, *g* is the magnitude of the gravity acceleration, αdenoted as the coefficient of thermal diffusivity, while v and β are labelled as kinematic viscosity and volumetric coefficient of thermal expansion. *C* is the nanoparticle volume fraction, the thermophoretic diffusion coefficient and the Brownian diffusion coefficient denoted by D_T and D_B, and *a* and *b* are the parameters of the principal curvature at *O* of the body measured in the plane *x* and *y*, respectively.

A little inspection shows that Eq. (1)-(5) along with the boundary conditions (6) admit a semi-similar solution of the form [22,23]

$$\eta = Gr^{\frac{1}{4}} a \xi^{\frac{-1}{2}} z, u = va^2 x Gr^{\frac{1}{2}} f'(\xi,\eta), v = va^2 cy Gr^{\frac{1}{2}} h'(\xi,\eta),$$
$$w = -vaGr^{\frac{1}{4}} \xi^{\frac{1}{2}}(f + ch), \theta(\xi,\eta) = \frac{(T - T_\infty)}{(T_w - T_\infty)}, \phi(\xi,\eta) = \frac{(C - C_\infty)}{(C_w - C_\infty)}$$
$$\xi = 1 - e^{-\tau}, \tau = va^2 Gr^{\frac{1}{2}} t \tag{7}$$

Substitution of Eq. (7) in Eq. (2) - (5) gives

$$f''' + (1-\xi)\frac{\eta}{2\xi} f'' + \xi[(f+ch)f'' - (f')^2] + \xi\theta - \xi\frac{N_r}{PrGr}\varphi = \xi(1-\xi)\left(\frac{\partial f'}{\partial \xi}\right) \tag{8}$$

$$h''' + (1-\xi)\frac{\eta}{2\xi} h'' + \xi[(f+ch)h'' - (ch')^2] + \xi\theta - \xi\frac{N_r}{PrGr}\varphi = \xi(1-\xi)\left(\frac{\partial h'}{\partial \xi}\right) \tag{9}$$

$$\theta'' + Pr(1-\xi)\frac{\eta}{2}\theta' + \xi Pr(f+ch)\theta' + N_b\theta'\varphi' + N_t(\theta')^2 = \xi Pr(1-\xi)\frac{\partial \theta}{\partial \xi} \tag{10}$$

$$\varphi'' + LePr(1-\xi)\frac{\eta}{2}\varphi' + \xi LePr(f+ch)\varphi' + \frac{N_t}{N_b}\theta'' = \xi LePr(1-\xi)\frac{\partial \varphi}{\partial \xi} \tag{11}$$

where primes denote partial differentiation with respect to η, $Gr = \frac{(1-C_\infty)g\beta(T_w - T_\infty)}{a^3 v^2}$ is the Grashof number, $Pr = \frac{v}{\alpha}$ is the Prandtl number, $c = \frac{b}{a}$ where $v = \frac{\mu}{\rho}$, $N_b = \frac{\tau D_B(C_w - C_\infty)}{\alpha}$ is the Brownian motion parameter, $N_r = \frac{(\rho_p - \rho_{f\infty})(C_w - C_\infty)}{a^3 \alpha\mu}$ is the buoyancy parameter and $N_t = \tau\frac{D_T}{T_\infty}\frac{(T_w - T_\infty)}{\alpha}$ is the thermophoresis parameter.

The boundary conditions (6) become

$$f(\xi,0) = f'(\xi,0) = 0, h(\xi,0) = h'(\xi,0) = 0, \theta(\xi,0) = \phi(\xi,0) = 1,$$
$$f' \to 0, h' \to 0, \theta \to 0, \phi \to 0 \ \ as \ \ \eta \to \infty$$
$$\text{for } 0 \le \xi < 1. \tag{12}$$

The physical quantities of practical interest in this problem are the skin friction coefficient in the *x*- and *y*-directions, C_{fx} and C_{fy} and the Nusselt number, Nu, and Sherwood number, Sh, that are defined as

$$C_{fx} = \frac{\mu\left(\frac{\partial u}{\partial z}\right)_{z=0}}{\rho v^2 a^3 x}, C_{fy} = \frac{\mu\left(\frac{\partial v}{\partial z}\right)_{z=0}}{\rho v^2 a^3 cy},$$
$$Nu = \frac{a^{-1}\left(\frac{\partial T}{\partial z}\right)_{z=0}}{T_w - T_\infty}, Sh = \frac{a^{-1}\left(\frac{\partial C}{\partial z}\right)_{z=0}}{C_w - C_\infty}, \tag{13}$$

where ρ and μ are the density and dynamic viscosity, respectively. In terms of the non-dimensional variables, we have

$$\frac{C_{fx}\xi^{\frac{1}{2}}}{Gr^{\frac{3}{4}}} = f''(\xi,0), \frac{C_{fy}\xi^{\frac{1}{2}}}{Gr^{\frac{3}{4}}} = h''(\xi,0),$$
$$\frac{Nu\xi^{\frac{1}{2}}}{Gr^{\frac{1}{4}}} = -\theta'(\xi,\eta), \frac{Sh\xi^{\frac{1}{2}}}{Gr^{\frac{1}{4}}} = -\phi'(\xi,\eta). \tag{14}$$

For the unsteady-initial flow case, where ξ is small that is $\xi \approx 0$, Eq. (8) - (11) become to the following form

$$f''' + \frac{\eta}{2}f'' = 0,\ h''' + \frac{\eta}{2}h'' = 0,$$
$$\theta'' + Pr\frac{\eta}{2}\theta' + N_b\theta'\phi' + N_t(\theta')^2 = 0,$$
$$\phi'' + LePr\frac{\eta}{2}\phi' + \frac{N_t}{N_b}\theta'' = 0. \tag{15}$$

subject to the boundary conditions

$$f(0) = f'(0) = 0, h(0) = h'(0) = 0, \theta(0) = 1, \varphi(0) = 1$$
$$f'(\infty) = 0, h'(\infty) = 0, \theta(\infty) = 0, \varphi(\infty) = 0 \tag{16}$$

For final steady-state case, where $\xi = 1$, Eq. (8) - (11) become to the following form

$$f''' + (f + ch)f'' - (f')^2 + \theta - \frac{N_r}{PrGr}\varphi = 0, \tag{17}$$

$$h''' + (f + ch)h'' - (ch')^2 + \theta - \frac{N_r}{PrGr}\phi = 0, \tag{18}$$

$$\theta'' + Pr(f + ch)\theta' + N_b\theta'\phi' + N_t(\theta')^2 = 0, \tag{19}$$

$$\varphi'' + LePr(f + ch)\phi' + \frac{N_t}{N_b}\theta'' = 0, \tag{20}$$

subject to the boundary conditions (16). These equations are identical with those first found by Poots [26].

3. Results and Discussions

The two sets of Eq. (8) - (11) and (17) -(20) subject to the boundary conditions (12) and (16) were solved numerically using Keller-box method described in book by Cebeci *et al.,* [27]. Result are obtained of Prandtl number, Pr ($Pr = 0.72, 1$), Grashof number, $Gr(Gr = -1, 1)$, Lewis number, $Le(Le = 1, 5, 10)$, Brownian motion parameter, $N_b(N_b = 0.00001, 0.1, 0.2)$, buoyancy parameter, $N_r(N_r = 0, 0.1, 0.2)$, thermophoresis parameter, $N_t(N_t = 0, 0.1, 0.2)$, *c* = 0 (plane stagnation point), 0.5 and 1 (axisymmetric stagnation point) and the values of $\eta = 10$ is enough to satisfied the boundary conditions. To access the exact of the solutions, the present results for the reduced skin friction coefficients $f''(0)$ and $h''(0)$, and heat transfer from the surface of the body, $-\theta'(0)$ are compared with Admon *et al.,* [23]. Table 1 is presented for $Pr = 0.72$ and *c* = 0 (plane stagnation point) and 1 (axisymmetric stagnation point). It can be conclude that agreement between the present results for the case $\xi = 1$ (final steady-state flow), and Admon *et al.,* [23] obtained using finite-difference method are excellent. It can be concluding that all these results are in very good agreement.

Table 1
Comparison of the skin frictions $f''(0), h''(0)$ and heat flux rate $-\theta'(0)$ for $\xi = 1$ (final steady-state), *Pr* = 0.72 and various values of *c*

	c = 0		*c* = 1	
	Admon *et al.,* [23]	Present result	Admon *et al.,* [23]	Present result
$f''(0)$	0.855909	0.856447	0.764685	0.764712
$h''(0)$	1.080763	1.077572	0.764685	0.764712
$-\theta'(0)$	0.374102	0.373870	0.462223	0.462259

Figure 1 illustrates the graph of the skin friction coefficients, local Nusselt and local sheerwood number for various values of *c* keeping the other parameters such as *Pr* = *Gr* = *Le* = 1.0 and $N_t = N_r = N_b = 0.1$. It can be seen in this figure that the skin friction coefficients reach their maximum values in the stagnation point under the boundary layer effects and the increase in the value of *c* raised the skin friction. Besides that, the analysis of the parameter *c* on the heat and mass transfer depicts the increase in the values of *c* will increases the Nusselt number and the Sherwood number but when values of *c* more than 0.9 and reaches to 1 (axisymmetric stagnation point) result shows the values of Nusselt and Sherwood number slowly decrease.

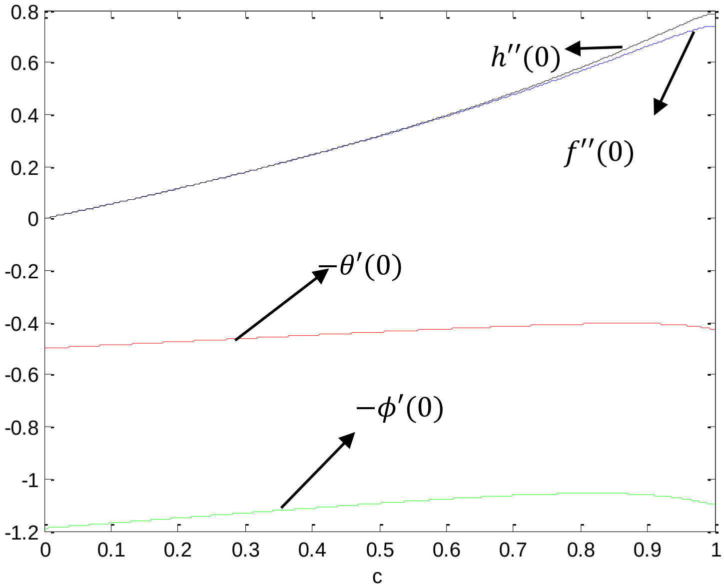

Fig. 1. Graph of skin friction coefficient, Nusselt and Sherwood number. $N_b = N_t = N_r = 0.1$ and *Le* = 1.0

Graph of skin friction coefficient for varying N_r in Figure 2 reveals the effects of N_r and N_t on the skin friction for value *Gr* = -1.0 and 1.0. For *Gr* = 1.0, the temperature at the stagnation points T_w is greater than the ambient fluid temperature T_∞ that is $T_w > T_\infty$. When the values of N_r increase, the values of density and nanoparticles mass difference also increase. For *Gr* = -1.0, the ambient fluid temperature greater than temperature at stagnation point, $T_\infty > T_w$ the result also shows the values of density and nanoparticle mass difference increase.

Figure 3 describes the effect of N_t on skin friction coefficient. For *Gr* = 1.0, the increase in the values of N_r will increase the skin friction for both $f''(0)$and $h''(0)$. For *Gr* = -1.0, the result also increase for both $f''(0)$and $h''(0)$ same like *Gr* = 1.0. It can be summarizing that both values of *Gr* increase when N_r = 0 and 0.2 but the value of *Gr* = 1.0 higher than *Gr* = -1.0 in the graph. For Figure 4, the result demonstrates that *Gr* =1.0 increase for both skin friction $f''(0)$and $h''(0)$for values of N_r = 0 and 0.2. For *Gr* =-1.0, the result also shows raised the skin friction but it lower than *Gr* =1.0.

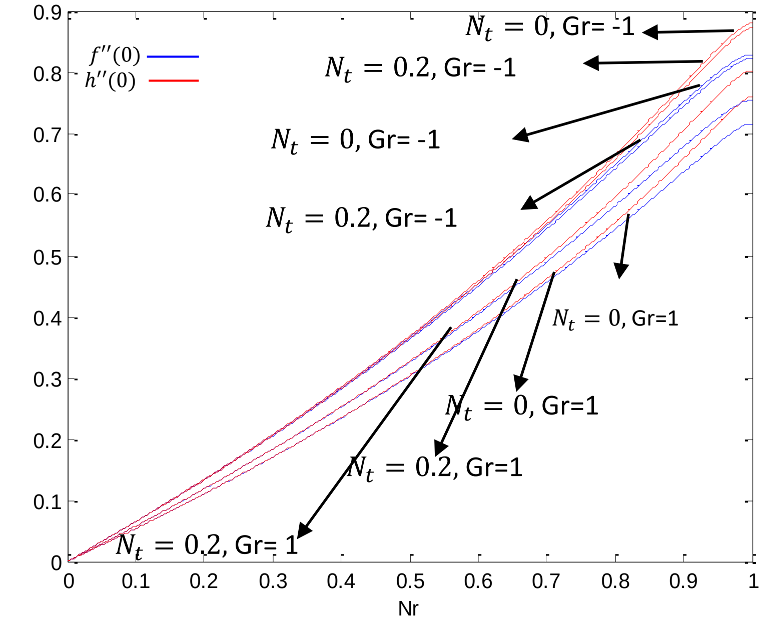

Fig. 2. Graph of skin friction coefficient for N_b = 0.2, *Le* = *Pr* = 1.0 and *c* = 0.5 with varying N_t and *Gr*

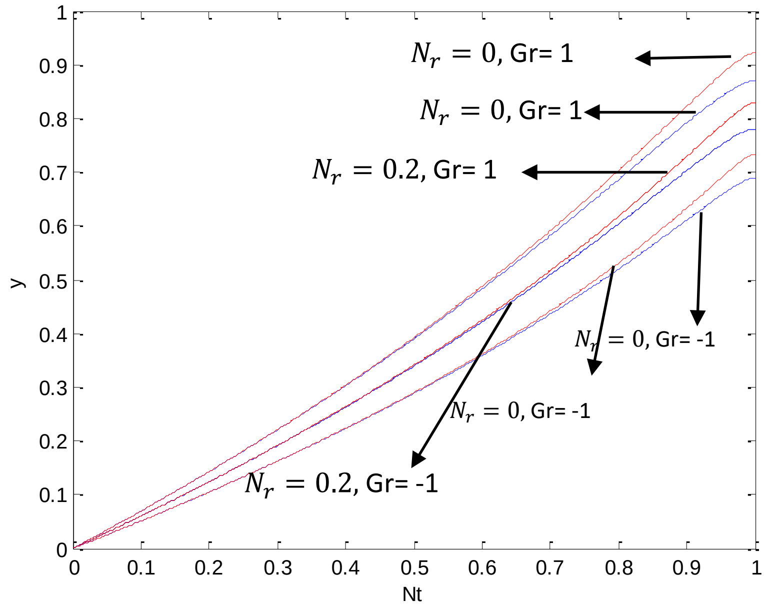

Fig. 3. Graph of skin friction coefficient for N_b = 0.2, *Le* = *Pr* = 1.0 and *c* = 0.5 with varying N_r and *Gr*

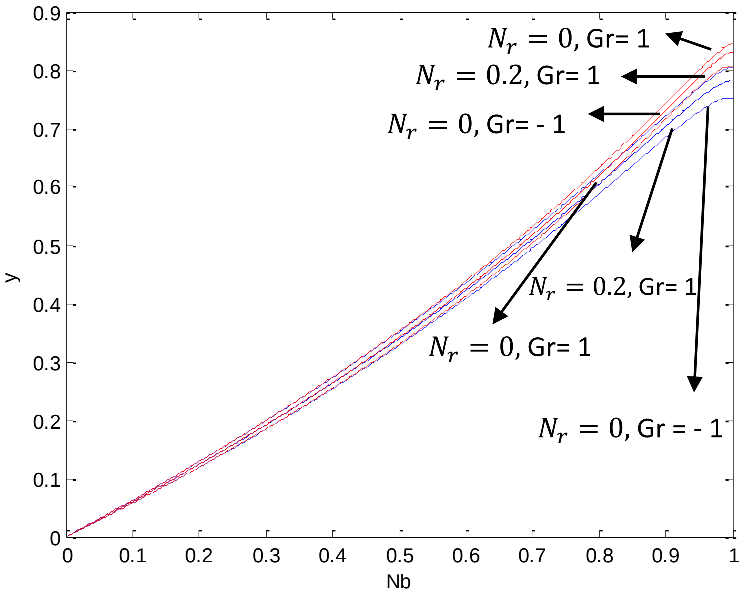

Fig. 4. Graph of skin friction coefficient for N_t=0.2, Le = Pr = 1.0 and c = 0.5 varying N_r and Gr

Figure 5 presented that general trend of velocity profile and the behaviour of increase in the values of Le on the velocity field. From this figure, it can be seen that at stagnation point the effect of velocity become zero and velocity tends to increase when particles is moving from cold to hot region. When values of Le increase, then the velocity and momentum boundary layer thickness will increase too. Besides that, thermal diffusivity in free convection increase when Le increase. Therefore, flow field become higher when higher in Lewis number. On the other hand, Figure 6 describes the effects of dimensionless temperature and concentration profile for various values of Le. Temperature is maximum at wall but notice that it reduces and approaches to the free stream temperature. It is detected that the temperature and concentration profiles become decrease throughout boundary layer when the value of Le is increase. It can be concluding that reduce the mass diffusivity because of higher value in Lewis number. Therefore, heat transfer and mass transfer slow down through fluid.

Further, Figure 7 shows the strong effect of buoyancy coefficient N_r on velocity, temperature and concentration profiles. In the absence of buoyancy parameter, the density difference is zero hence the flow is maximum as we increase the buoyancy parameter. The density difference increase will make fluid flow decrease. The increase in the buoyancy parameter will increase the density difference and mass concentration difference as shown in Figure 8. Hence, it improves the temperature and concentration profiles. In free convection, the thermophoresis parameter acts as important parts in the flow, heat transfer, and mass transfer properties.

From Figure 9, the values of N_t is increase with the reduced in the fluid flow. It is clearly state that the increase in values of N_t will increase the temperature difference. Therefore, fluid flow will decrease. Figure 10 depicts the effect of the thermophoresis parameter on the fluid temperature and concentration profile. The increase in the values of N_t will affect the temperature and concentration profile also increase. Hence, the nanoparticle enhances heat and mass transfer.

Figure 11 shows Brownian motion parameter tends to increase the mass diffusivity and at the same time enhances the momentum transfer when the values of N_b increase. Finally, Figure 12 illustrates increase the values of Brownian motion will increase mass diffusivity and heat transfer process. So we can conclude that temperature profiles increase when the values of N_b increase. In addition, increase values of N_b will reduce thermal diffusivity therefore decrease the mass transfer and concentration profiles.

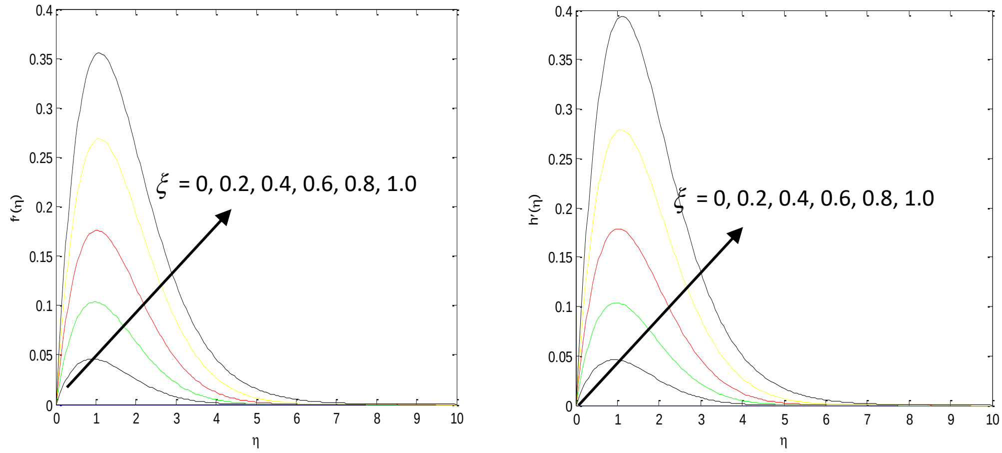

Fig. 5. Velocity profile in *x*- and *y*-direction for various ξ with $N_r = N_t = 0$, N_b= 0.001, $Gr = Pr = 1.0$, $c = 0.5$ and various values of Le

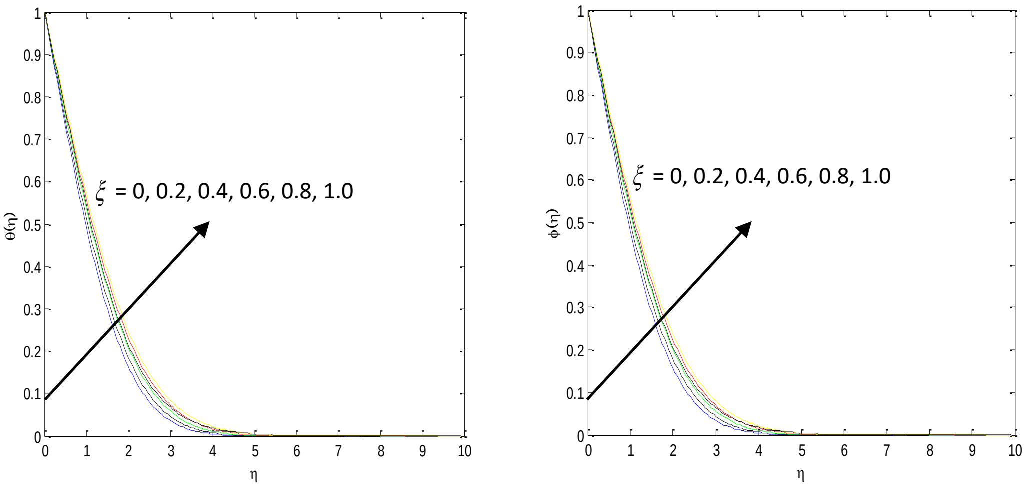

Fig. 6. Temperature and concentration profile $\theta(\eta)$ and $\phi(\eta)$for various ξ with N_r = N_t = 0, N_b = 0.001, $Gr = Pr = 1.0$, $c = 0.5$ and various values Le

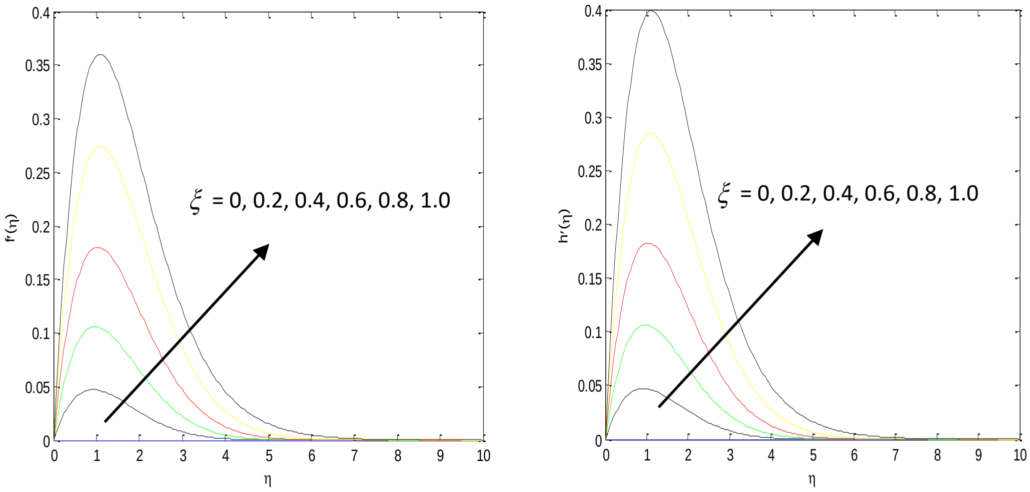

Fig. 7. Velocity profile in *x*- and *y*-direction for various ξ with $N_t = N_b = 0.1$, $Le = Gr = Pr = 1.0$, $c = 0.5$ and various values of N_r

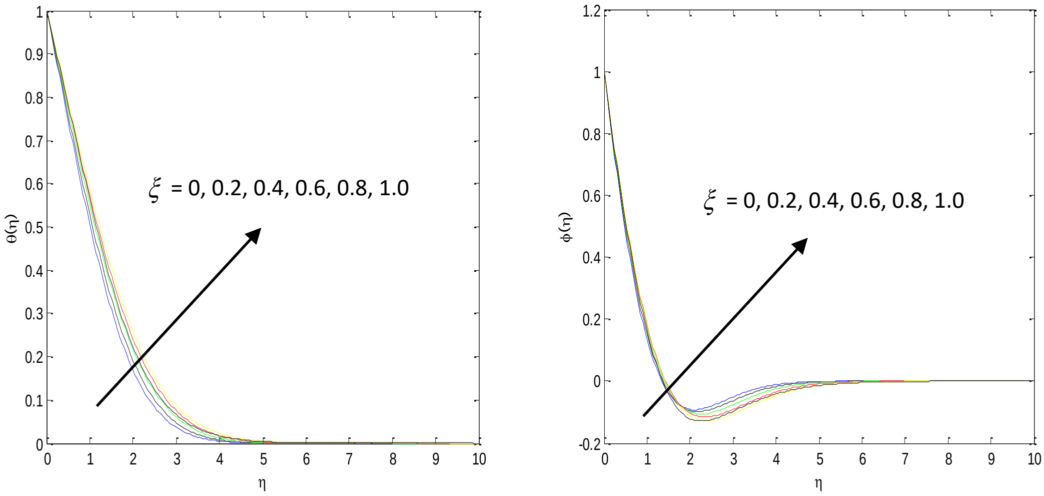

Fig. 8. Temperature and concentration profile $\theta(\eta)$ and $\phi(\eta)$for various ξ with $N_t = N_b$ = 0.1, $Le = Gr = Pr$ = 1.0, c = 0.5 and various value of N_r

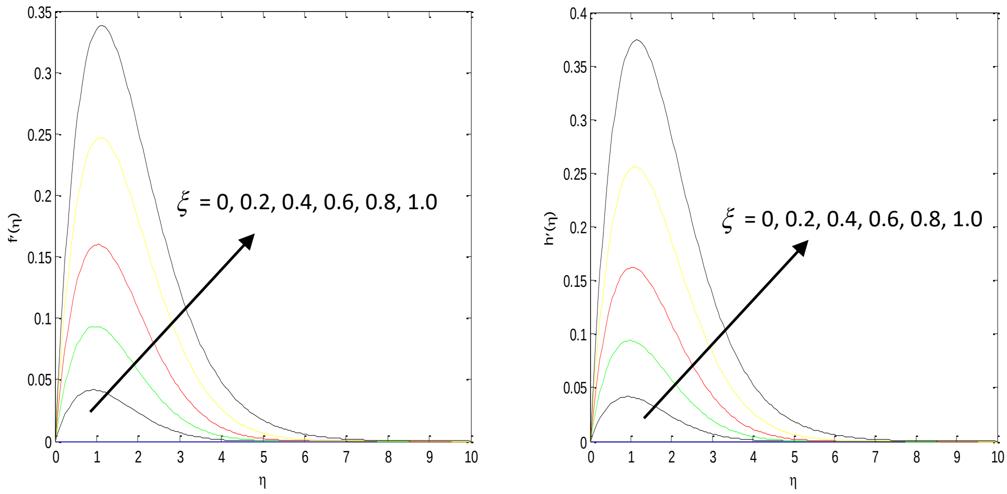

Fig. 9. Velocity profile in x- and y-direction for various ξ with $N_r = N_b$ = 0.1, $Le = Gr = Pr$ = 1.0, c = 0.5 and various values of N_t

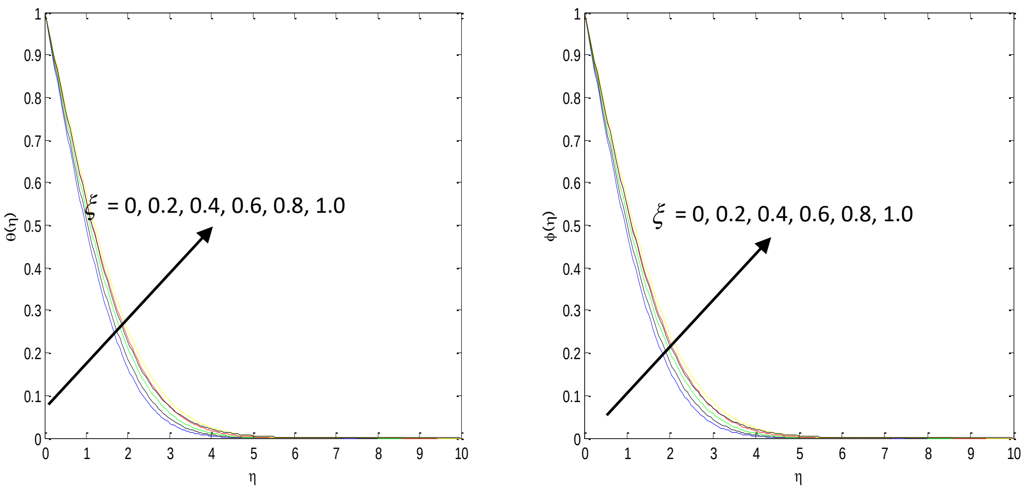

Fig. 10. Temperature and concentration profile $\theta(\eta)$ and $\phi(\eta)$ for various ξ with $N_r = N_b$ = 0.1, $Le = Gr = Pr$ = 1.0, c = 0.5 and various values of N_t

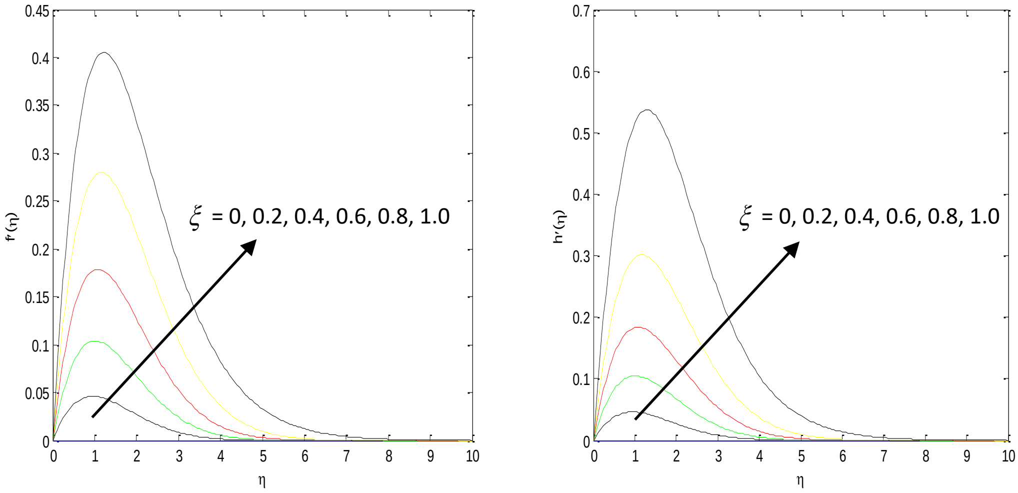

Fig. 11. Velocity profile in *x*- and *y*-direction for various ξ with $N_r = N_t = 0.1$, $Le = Gr = Pr = 1.0$, $c = 0.5$ and various values of N_b

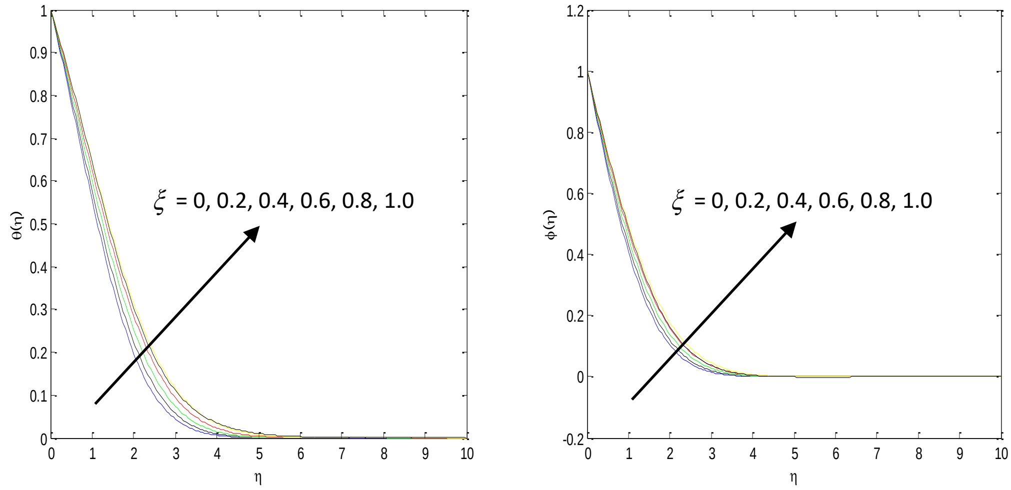

Fig. 12. Temperature and concentration profile $\theta(\eta)$ and $\phi(\eta)$for various ξ with $N_r = N_t = 0.1$, $Le = Gr = Pr = 1.0$, $c = 0.5$ and various values of N_b

4. Conclusions

The problem of unsteady flow and heat transfer of free convection boundary layer flow near the stagnation-point region in the presence of nanoparticles is studied numerically. The governing equations are non-dimensionalized using proper non-dimensional quantities. The resulting boundary value problem is solved by Keller-box method. The effects of the various values of the pertinent parameters c, Nb, Nt, Nr, and Le on the fluid velocity, temperature profile, and concentration profile are illustrated through graphs. We may extract some important findings from our results

i. The increase in the values of curvature parameter, *c,* will increase the skin friction.
ii. The increase in the values of curvature parameter, *c* will increase the local Nusselt and Sherwood number therefore it enhances heat and mass transfer.
iii. The increase in the values of *Le* and N_b will enhance flow in both *x*- and *y*-directions.
iv. The effect of the increase in the values of N_r and N_t is to decelerate the flow.

v. The increase in the values of N_r, N_t, and N_b increase the temperature while quite the opposite is true for *Le*.
vi. The increase in the values of N_r and N_t will increase the concentration profiles.
vii. The increase in the values of *Le* and N_b will decrease the concentration profiles.

We may conclude the flow and heat transfer properties of free convection boundary layer flow in the stagnation-point region in the presence of nanoparticles can be controlled by changing the quantity of the physical parameters. Hence Keller-box is very effective method to solve strongly nonlinear problems.

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