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Stagnation Point Flow over a Stretching/shrinking Sheet in a Carbon Nanotubes with Suction/Injection Effects

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ABSTRACT

This research is to explore the stagnation point flow problem in carbon nanotubes (CNTs) with suction/injection impacts over a stretching/shrinking sheet. Nonlinear ordinary differential equations systems are transformed from the governing partial differential equations by applying similarity transformation, thus resolved numerically using a bvp4c solver in MATLAB. Two types of CNTs are used which are SWCNTs (single-walled) and MWCNTs (multi-walled) and water as the base fluids. Influence of the velocity, temperature, skin friction and Nusselt numbers are studied and displayed in the graphs form. It is concluded that the suction effect could widen the range of solutions exist and the effect of injection acted on the contrary. Also, the solutions are dual for shrinking sheet while unique for stretching sheet.

Keywords:
Stagnation point flow; stretching/shrinking sheet; carbon nanotubes; suction/injection effects; dual solutions

1. Introduction

Stagnation point flow define as a fluid flow near solid surface stagnation area, where fluid entering the surface divides into different streams. First person who studied 2D stagnation point flow through a stationary semi-infinite wall was Heimenz [1] where he reduced the Navier-Stokes equations. After that, problem in the study [1] was extended by Homann [2] in consideration of axisymmetric stagnation point flow. Mahapatra and Gupta [3,4] investigated both stagnation point flows and stretching surface effects. Crane [5] studied the 2D steady boundary layer flow due to the stretching surface moving in the plane. But Khan and Pop [6] are among the earliest who considered stretching sheet in nanofluids. After this seminal work, the research problem on stagnation point flow considering a stretching/shrinking surface has been examined by several researchers [7-13].

As many studies related to nanofluid, CNTs also show excellent results due to its properties of electrical and mechanical. So, Choi *et al.,* [14] researched oil-based CNTs' heat conductivity. CNTs is an allotrope of carbon, tube-shaped material, made of carbon. CNTs suspensions offer a higher thermal property contrasted with those different nanoparticles with a similar volume fraction [15-

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16]. So CNTs can improve both heat transfer of convection and base fluids' thermic conductivity. Imtiaz *et al.,* [17] studied flow convection of CNTs between rotating disks, while Hayat *et al.,* [18] researched on 3D rotating flow of CNTs. Other than that, many other researchers also come across the advantages of CNTs and investigated various boundary layer problem on CNTs [19-22].

There have been a lot of research involve with stagnation point flow over a permeable plate [23-27]. While in this present work, we study the stagnation point flow by a linearly stretching/shrinking surface. We are also studying the boundary impacts of suction and injection. As a consequence, the fluid's suction or injection via the surface bounding, such as in cooling mass transfer, can result to flow field to change and thus impact the transfer rate of heat.

2. Methodology

Incompressible steady flow toward a stretching/shrinking surface in CNTs with the existence of suction/injection was investigated. The velocity of stretching/shrinking $U_w(x)$ and ambient fluid $U_\infty(x)$ are assumed to vary linearly from the stagnation point, i.e., $U_w(x) = ax$ and $U_\infty(x) = bx$, where a and b are constants. The boundary layer equations are as follows [28]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_\infty \frac{dU_\infty}{dx} + \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

subject to the boundary conditions

$$\begin{gathered} u = U_w, v = V_w, T = T_w \; at \; y = 0 \\ u \to U_\infty, T \to T_\infty \text{ as } y \to \infty \end{gathered} \tag{4}$$

The component's velocity in directions of x and y are called u and v respectively and T, T_w and T_∞ are the temperature of the nanofluid, sheet and free stream, respectively. It should be mention that variable suction/injection velocity is $V_w = -(v_f b)^{1/2} S$, where $S > 0$ for suction and $S < 0$ for injection. Additionally, L denotes the slip length, whereas $\mu_{nf}, \alpha_{nf}, \rho_{nf}$ are the viscosity, thermal diffusivity and density of the nanofluid, respectively, which are given by Oztop and Abu-Nada [29]:

$$\begin{gathered} \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}}, \rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_{CNT}, \\ (\rho C_p)_{nf} = (1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_{CNT}, \quad \frac{k_{nf}}{k_f} = \frac{1-\varphi+2\varphi\frac{k_{CNT}}{k_{CNT}-k_f}ln\frac{k_{CNT}+k_f}{2k_f}}{1-\varphi+2\varphi\frac{k_f}{k_{CNT}-k_f}ln\frac{k_{CNT}+k_f}{2k_f}} \end{gathered} \tag{5}$$

where φ is the CNTs volume fraction, $(\rho C_p)_{nf}$ and k_{nf} are the heat capacity and thermal conductivity of nanofluid, while $(\rho C_p)_{CNT}$, ρ_{CNT} and k_{CNT} are the capacity of heat, density and thermal conductivity of CNTs, respectively, and ρ_f and k_f are the density and thermal conductivity of the fluid. The term for k_{nf}/k_f were taken from Xue [30] where Maxwell's theory model considers the thermal conductivity impact of space distribution of CNTs.

A similarity transformation for Eqs. (1) – (4) can be written as follows:

$$\eta = \left(\frac{b}{\nu_f}\right)^{1/2} y\,,\psi = \left(\nu_f b\right)^{1/2} x f(\eta)\,,\theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty} \tag{6}$$

where η is the variable 0f similarity and ψ is the function of stream described as $u = \partial\psi/\partial y$ and $\nu = -\partial\psi/\partial x$, which comply with Eq. (1). Using Eq. (6), Eqs. (2) and (3) reduce to the following ordinary differential equations as:

$$\frac{1}{(1-\varphi)^{2.5}(1-\varphi+\varphi\rho_{CNT}/\rho_f)} f''' + ff'' - f'^2 + 1 = 0 \tag{7}$$

$$\frac{1}{Pr}\frac{k_{nf}/k_f}{\left[1-\varphi+\varphi(\rho C_p)_{CNT}/(\rho C_p)_f\right]}\theta'' + f\theta' = 0 \tag{8}$$

Thus, subject to the boundary conditions in Eq. (4), we have

$$\begin{gathered} f(0) = S\,, f'(0) = \varepsilon, \theta(0) = 1 \\ f'(\eta) \to 1\,, \theta(\eta) \to 0 \text{ as } \eta \to \infty \end{gathered} \tag{9}$$

where S is the suction/injection parameter, Pr is the number of Prandtl and ε is the parameter of stretching/shrinking which illustrate as

$$Pr = \frac{\nu_f}{\alpha_f}\,,\ \varepsilon = \frac{a}{b} \tag{10}$$

where $\varepsilon > 0$ for stretching and $\varepsilon < 0$ for shrinking.

Physical interest's quantities in this research are the skin friction coefficient C_f and the local Nusselt number Nu_x, are described as

$$C_f = \frac{\tau_w}{\rho_f U_\infty^2}\,, \qquad Nu_x = \frac{x q_w}{k_f(T_w-T_\infty)} \tag{11}$$

in which the surface shear stress τ_w and the surface heat flux q_w are likely shown as

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0}\,, \qquad q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{12}$$

with μ_{nf} is the viscosity of nanofluids and k_{nf} being the thermal conductivity of the nanofluids. Using the similarity variables in Eq. (6) into Eq. (11), we obtain

$$C_f Re_x^{1/2} = \frac{1}{(1-\varphi)^{2.5}} f''(0), \tag{13}$$

$$Nu_x/Re_x^{1/2} = -\frac{k_{nf}}{k_f}\theta'(0), \tag{14}$$

where $Re_x = U_\infty x/\nu_f$ is the local Reynolds number.

3. Results

The system of Eqs. (7) to (8) together with conditions in Eq. (9) are solved numerically using the bvp4c package in Matlab. Following to Oztop and Abu-Nada [28], we have considered the range of CNTs volume fraction φ as $0 \leq \varphi \leq 0.2$, where $\varphi = 0$ is regular fluid with Prandtl number $Pr = 6.2$. The thermophysical properties of the base fluid and the CNTs are listed in Table 1.

Table 1
Thermophysical properties of CNTs [31]

Physical properties	Base fluids	Nanoparticle	
		SWCNT	MWCNT
$\rho\ (kg/m^3)$	997	2600	1600
$c_p\ (J/kgK)$	4179	425	796
$k\ (W/mK$	0.613	6600	3000

Figures 1 and 2 illustrate the $f''(0)$ and $-\theta'(0)$ graphs for ε and φ, where $\varphi = 0, 0.1$ and 0.2 for water-SWCNTs at $S = 0.5$. There exist dual solutions when $\varepsilon_c < \varepsilon \leq -1$, unique solution when $\varepsilon > -1$ and no solutions when $\varepsilon < \varepsilon_c < 0$, where ε_c is the critical value of ε.

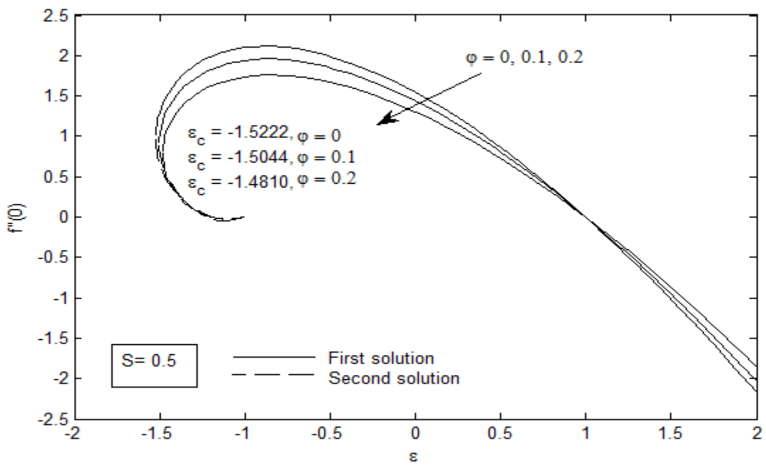

Fig. 1. $f''(0)$ graph for ε and φ with water-SWCNT

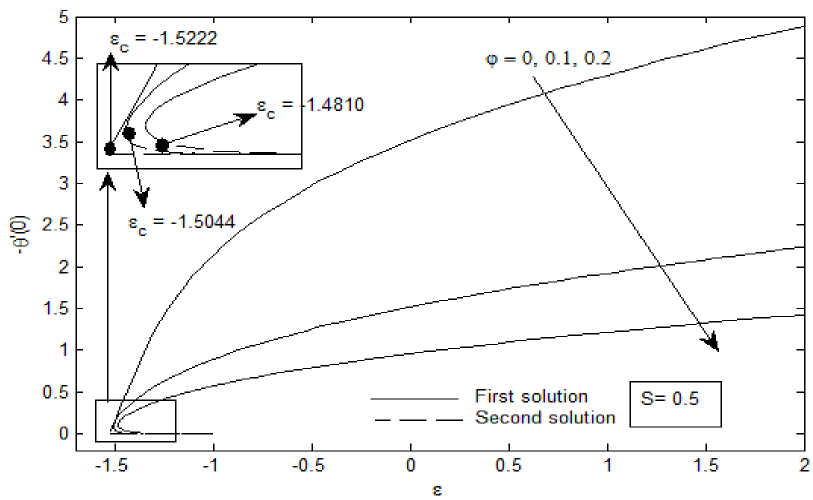

Fig. 2. $-\theta'(0)$ graph for ε and φ with water-SWCNT when $Pr = 6.2$

Figures 3 and 4 show $f''(0)$ and $-\theta'(0)$ graphs for ε and S, where $S = -0.5, 0$ and 0.5 for water base fluid at $\varphi = 0.1$. When S increases, the range of solutions exist becomes larger. When $\varphi = 0$ and $S = 0.5$, the value of ε_c= -1.5222, which results in good agreement with Bhattacharyya and Layek [32]. Furthermore, as S increases, the skin friction and heat loss from the surface increases. Hence, suction slows the separation of boundary layer, while injection speeds it up.

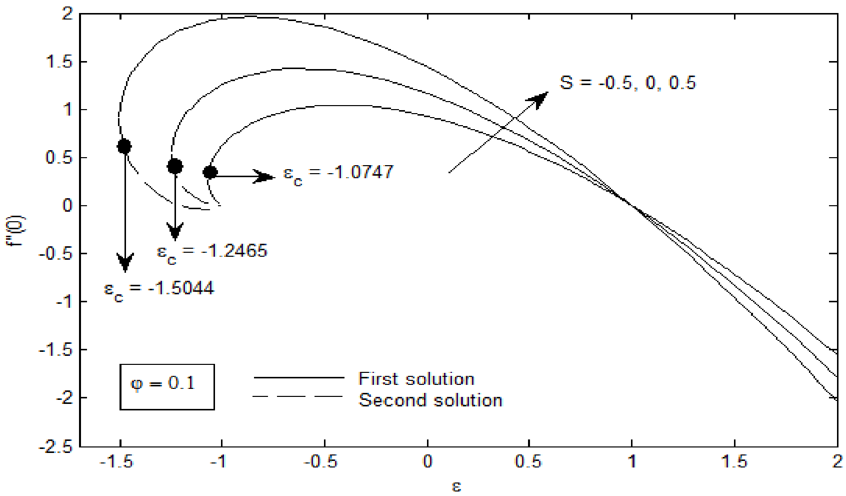

Fig. 3. $f''(0)$ graph for ε and S with water-SWCNT

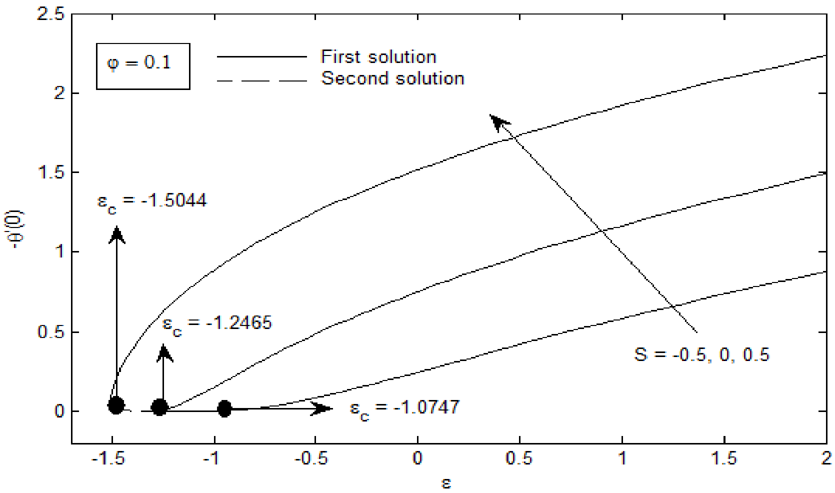

Fig. 4. $-\theta'(0)$ graph for ε and S with water-SWCNT when $Pr = 6.2$

Figures 5 and 6 explain the coefficient of skin friction and the local Nusselt number graphs, as per Eqs. (13) - (14) for φ and S where $S = -0.5, 0$ and 0.5 with $\varepsilon = 0.5$. It is concluded that, when S increases, the coefficient of skin friction increases as well as the local Nusselt number. The coefficient of skin friction and local Nusselt number of SWCNTs is found to be higher than MWCNTs because density and thermal conductivity of SWCNTs are found to be higher than MWCNTs, refer to Table 1. Besides that, when φ increases, the $C_f Re_x^{1/2}$ and $Nu_x/Re_x^{1/2}$ are also increases.

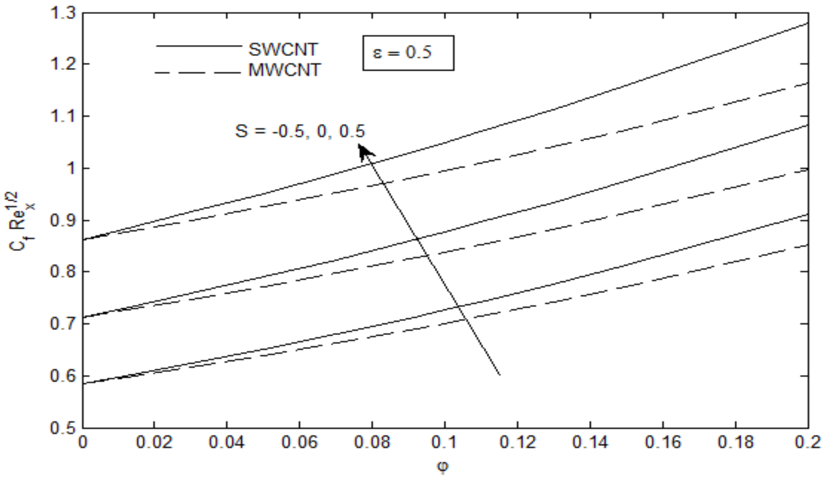

Fig. 5. Coefficient of skin friction graph for φ, S and CNTs

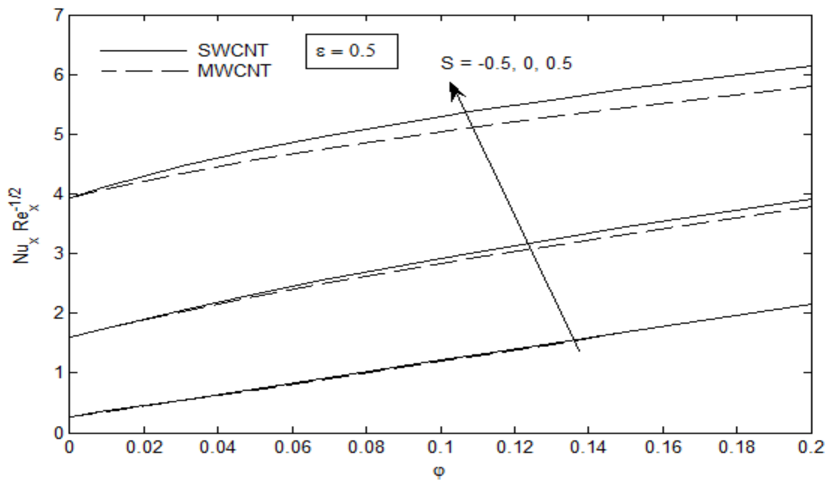

Fig. 6. Local Nusselt number graph for φ, S and CNTs

The outcomes of S and CNTs parameters on the velocity and temperature profiles are presented in Figures 7 to 10. The term for both first and second solutions applies to the curves shown in Figures 1 to 4 and asymptotically these profiles follow the boundary conditions from Eq. (9), which then supporting the existence of dual solutions shown in Figures 1 to 4.

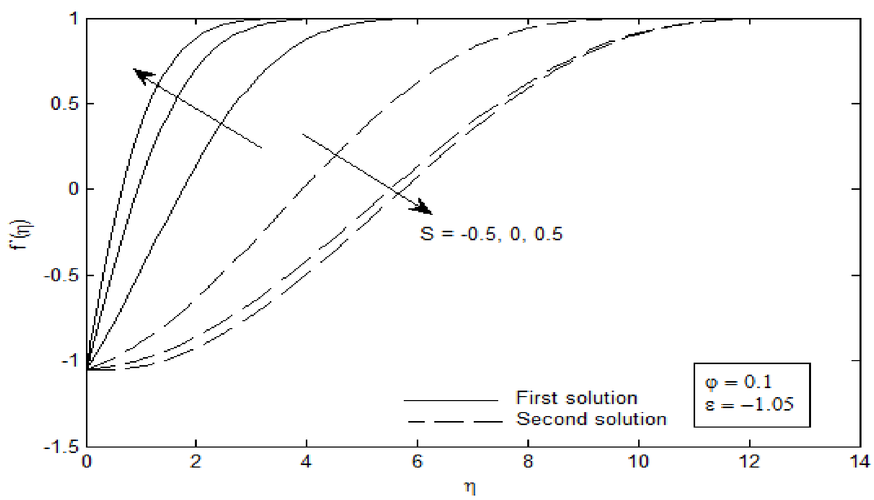

Fig. 7. Profiles of velocity for S with water-SWCNT

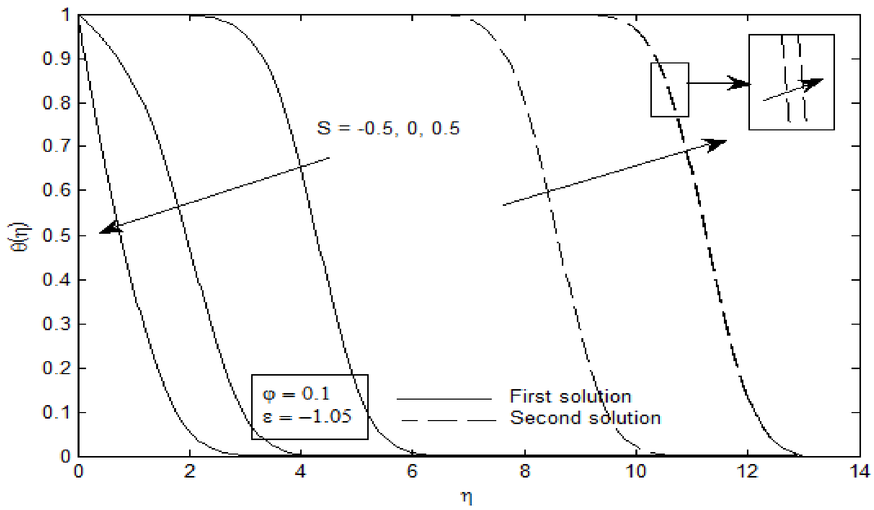

Fig. 8. Profiles of temperature for S with water-SWCNT

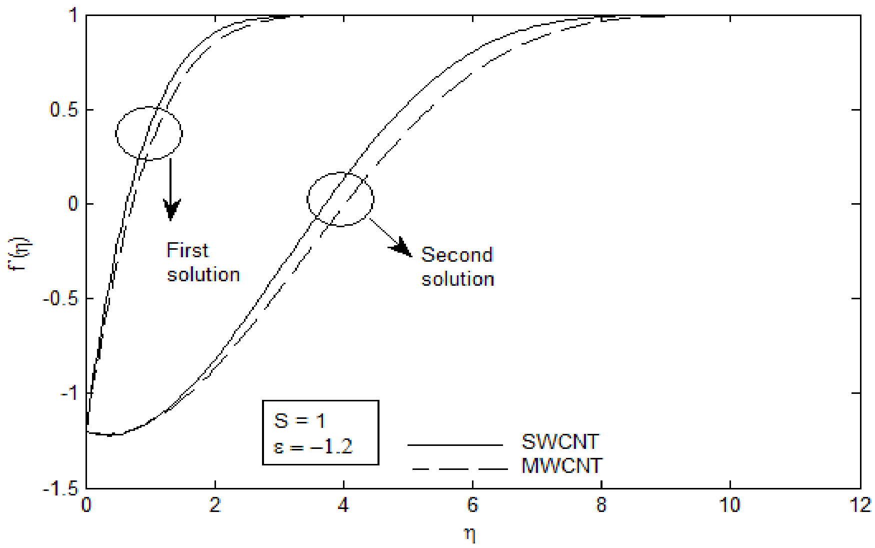

Fig. 9. Profiles of velocity for different types of CNTs with water base fluid

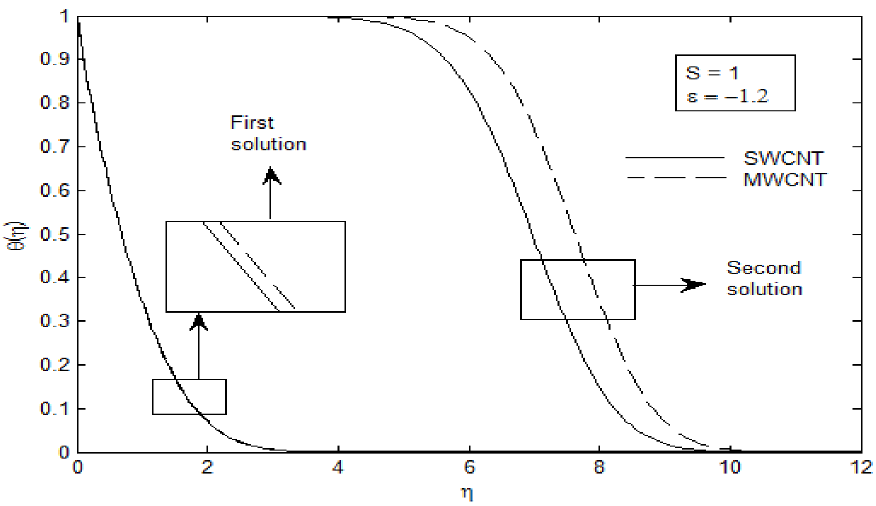

Fig. 10. Profiles of temperature for different types of CNTs with water base fluid

4. Conclusions

We have studied theoretically and analysed the effects of φ and S on the stagnation point flow over a stretching/shrinking sheet. The results indicate that

i. Solutions for a stretching sheet are unique and solutions for a shrinking sheet are non-unique (dual).
ii. The range of solutions are widen with an increases of suction ($S > 0$), while injection parameter acted on contrary.
iii. As φ increases, they accelerate the boundary layer separation.
iv. The skin friction and heat transfer increases with an increase of S and φ.
v. SWCNTs are more efficient than MWCNTs in both skin friction and local Nusselt number.
vi. We conclude that we also get dual solutions when using CNTs as compared to viscous fluid and nanofluid, but CNTs give more advantages as they make skin friction and heat transfer increases more rather than viscous fluid and nanofluid.

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