Penerbit Akademia Baru

CFD Letters

Journal homepage: www.akademiabaru.com/cfdl.html
ISSN: 2180-1363

Magnetic Field Effect on Mixed Convection Heat Transfer in a Lid-Driven Rectangular Cavity

Open Access

Norhaliza Abu Bakar[1], Rozaini Roslan[2,*], Arash Karimipour[3]

[1] Department of Science and Mathematics, Centre for Diploma Studies, Universiti Tun Hussein Onn Malaysia, 84600 Pagoh, Johor, Malaysia
[2] Centre for Research in Computational Mathematics, Faculty of Sciences and Technology, Universiti Tun Hussein Onn Malaysia, 84600 Pagoh, Johor, Malaysia
[3] Department of Mechanical Engineering, Faculty of Engineering, Najafabad Branch Islamic Azad University, 8514143131 Isfahan, Iran

ARTICLE INFO

Article history:
Received 21 November 2019
Received in revised form 16 January 2020
Accepted 21 January 2020
Available online 29 January 2020

Keywords:
Cavity; finite volume method; lid-driven; magnetic field; mixed convection

ABSTRACT

This study aims to analyze numerically the fluid flow and heat transfer in a two-dimensional (2D) rectangular cavity in the presence of magnetic field. The bottom and top cavity walls are kept at T_h and T_c respectively, where $T_h > T_c$. Meanwhile, the vertical walls are insulated. The top wall is moving at a constant speed in the positive horizontal direction. The dimensionless governing equations are solved using the finite volume method and the SIMPLE algorithm. The influence of Hartmann number (*Ha*) (ranges from 0 to 60) on the thermal characteristics and fluid flow is analyzed. The simulated streamlines and isotherm plots, as well as the variation of local Nusselt numbers, are then presented. It is found that the *Ha* has a significant effect on the fluid flow structure and temperature field. As *Ha* increases, the flow convection is attenuated and therefore the heat transfer rate decreases.

1. Introduction

Mixed convection is commonly found in engineering systems such as heat exchanger, building, solar collectors, insulation materials or heat pump [1]. Numerous simulations have been performed to study the mixed convection usingdifferent boundary conditions and shapes [2-4]. Currently, the presence of magnetic field inside a thermofluid system has been considered by many researchers as magneto-hydrodynamics (MHD) is important in many applications such as astrophysics, geophysics, aeronautics, metallurgy, chemical and petroleum industries, crystal growth in liquid, cooling of nuclear reactor, microelectronic devices and solar technology [5].

The combined effect of magnetic field and heat convection (with internal heat generation) in a lid-driven square cavity has been investigated using the finite volume method (FVM) [6]. The presence of internal heat generation was found to decrease the average Nusselt number significantly in aiding flow and vice-versa. The effects of Reynolds number and Prandtl number on the MHD mixed convection in a lid-driven rectangular cavity occupied by a heat conducting circular

* *Corresponding author.*
E-mail address: rozaini@uthm.edu.my (Rozaini Roslan)

block have been studied using FVM [7]. It was reported that both streamline and isotherm would vary in accordance with Reynolds number and Prandtl number. The laminar mixed convection in a top sided lid-driven square cavity heated by a corner heater in the presence of magnetic field has been studied as well [8]. By imposing a sinusoidal boundary temperature at the sidewalls of a square cavity, the mixed convection in the presence of magnetic field has been studied numerically using FVM [9]. It was observed that magnetic field would strongly affect the flow behavior and heat transfer rate inside the cavity. The laminar MHD mixed convection in an inclined lid-driven square cavity with opposing temperature gradients has been investigated [10]. It was found that by increasing either Hartmann number or inclination angle, the rate of heat transfer along the heated walls could be enhanced. The effect of magnetic field on the convective heat transfer rate and entropy generation in an inclined square cavity with a heat-conducting fin and thermal radiation was studied by Alnaqi *et al.,* [11]. The studies of MHD flows past a thin needle [12] and over a stretching sheet [13] have been performed as well. Furthermore, the effect of moving lid direction on the MHD mixed convection in a square cavity where the bottom wall was linearly heated has been reported [14,15]. The effect of lid direction on heat transfer and fluid flow is more pronounced for mixed convection.

Separately, the performance of nanofluid in a lid-driven cavity with the presence of magnetic field has been numerically investigated [16-19]. It was reported that heat transfer rate was dependent on the strength of magnetic field while the suspended nanoparticles would improve the heat transfer significantly. The mixed convection nanofluids flow in a lid-driven and inclined square cavity has been investigated as well [20-21]. It was found that the orientation and strength of the magnetic field would affect the heat transfer rate. The heat transfer performance of MHD nanofluids flow in a lid-driven cavity with partially heated wavy wall has been determined [22]. The effect of inclined magnetic field on the mixed convection in a trapezoidal cavity has been investigated also [23].

Based on the literature review, most of the studies revolved around laminar mixed convection in a square cavity subjected to heating and magnetic field. The present study is performed to investigate numerically the effect of magnetic field on the mixed convection heat transfer in a rectangular cavity filled with water. As compared to previous studies, the main objective of the current investigation is to examine the effect of mixed convection of Newtonian fluid as the top rectangular cavity wall is heated. Meanwhile, the vertical walls are treated as adiabatic and the flow is subjected to a horizontal magnetic field. Different magnetic field strengths are considered while the Richardson number is fixed to 1. The numerical results obtained using FVM are presented in terms of streamlines, temperature contours and local Nusselt number.

2. Methodology

2.1 Mathematical Modelling

A shallow rectangular cavity with a top lid is shown in Figure 1. The top lid is movable from left to right at constant speed U_0. The width and height are denoted as L and H, respectively. The cavity is filled with Newtonian fluid that it is heated from the bottom wall at temperature T_h. The top lid is kept at cold temperature, T_c such that $T_h > T_c$. Both vertical walls are adiabatic. A uniform magnetic field strength $B$is imposed onto the fluid in the positive x-direction.

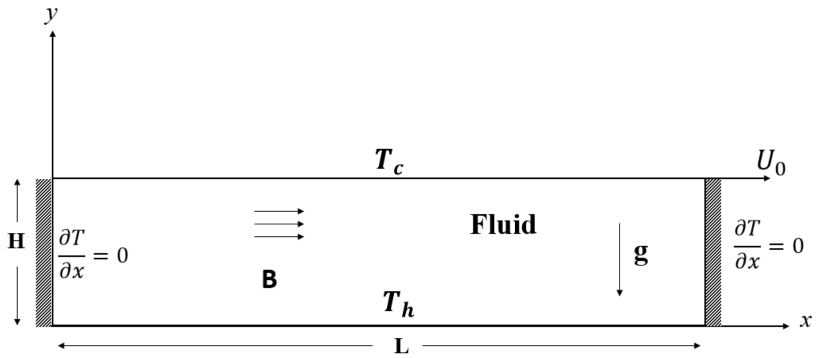

Fig. 1. Geometrical configuration

The induced secondary magnetic field due to the motion of the electrically conducting fluid is neglected. Additionally, the effects of imposed and induced electrical fields, Joule heating of the fluid and viscous dissipation are ignored in the current work. The working fluid is incompressible, steady, two-dimensional and laminar. The gravitational force acts vertically downward. It is assumed that the thermo-physical properties of the fluid are constant. The variation of density with temperature is fluid-dependent. In order to represent the variation of density with temperature, the well-known Boussinesq approximation is used. After rearranging the terms, the governing equations can be expressed as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \qquad (2)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + g\beta(T - T_c) - \frac{\sigma B^2}{\rho}v \qquad (3)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \qquad (4)$$

where x and y are the Cartesian coordinate directions. The variables u, v, p and T are x-velocity, y-velocity, fluid pressure and fluid temperature, respectively. The parameters β, g, σ, ν, and ρ are fluid thermal expansion coefficient, gravity, fluid electrical conductivity, fluid kinematic viscosity, and fluid density, respectively. The parameter B is the magnetic induction coefficient. The thermal diffusivity is defined as $\alpha = k/\rho c$, where k is the thermal conductivity and c is the heat capacity. The early stage boundary conditions of the problem are given as

$$\begin{aligned} &\text{Top wall:} && u = U_0,\ v = 0,\ T = T_c \\ &\text{Bottom wall:} && u = v = 0,\ T = T_h \\ &\text{Left and right walls:} && u = v = 0,\ \frac{\partial T}{\partial x} = 0 \end{aligned} \qquad (5)$$

Eqs. (1) – (5) have been non-dimensionalized using the following variables

$$\begin{aligned} &X = x/H,\ Y = y/H,\ U = u/U_0,\ V = v/U_0,\ \theta = T - T_c/T_h - T_c, \\ &Gr = g\beta(T_h - T_c)H^3/\nu^2,\ Pr = \nu/\alpha,\ P = p/\rho U_0^2,\ Re = U_0 H/\nu, \\ &Ha^2 = B^2 H^2 \sigma/\rho v \end{aligned} \qquad (6)$$

where parameters θ, P, Gr, Re, Pr, Ri and Ha^2 are non-dimensional temperature, non-dimensional pressure, Grashof number, Reynolds number, Prandtl number, Richardson number and Hartmann number, respectively. Upon applying Eq. (6), Eqs. (1) – (5) become

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{7}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \tag{8}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + \frac{Gr}{Re^2}\theta - \frac{Ha^2}{Re}V \tag{9}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{RePr}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \tag{10}$$

where Gr/Re^2 is the Richardson number (Ri). The governing equations are subjected to the following dimensionless boundary conditions:

$$\begin{aligned} &\text{Top wall:} && U = 1,\ V = 0,\ \theta = 0 \\ &\text{Bottom wall:} && U = V = 0,\ \theta = 1 \\ &\text{Left and right walls:} && U = V = 0,\ \frac{\partial \theta}{\partial X} = 0 \end{aligned} \tag{11}$$

2.2 Numerical Method

Eqs. (7) – (10) together with the boundary conditions (11) have been solved numerically using the finite volume method (FVM). The set of algebraic equations produced were solved using the iterative SIMPLE algorithm and the tridiagonal matrix algorithm (TDMA). The numerical solutions are obtained on a staggered grid system, such that the velocity components are stored halfway between the scalar storage locations. The calculations of U, V and θ are performed iteratively until the following convergence criterion is attained

$$\varepsilon = \frac{\sum_{j=1}^{m}\sum_{i-1}^{n}\left|\varsigma_{i,j}^{k+1} - \varsigma_{i,j}^{k}\right|}{\sum_{j=1}^{m}\sum_{i-1}^{n}\left|\varsigma_{i,j}^{k+1}\right|} \leq 10^{-7} \tag{12}$$

where ε is the tolerance, m and n are the number of grid points in x- and y-direction, respectively. k is the iteration number and ς is any computed variables. Nusselt number is obtained to investigate the heat transfer rate in the following manner

$$Nu = \frac{1}{L/H}\int_0^{L/H} -\left(\frac{\partial \theta}{\partial Y}\right)_{Y=0,1} dX \tag{13}$$

The computational procedure was implemented using FOTRAN90. Grid independence test was conducted in order to obtain the grid-independent solutions. The test was performed for $Ri = 1.0$, $Pr = 7.0$, and $Ha = 10.0$. As shown in Figure 2, a total of 100×20 grid points with clustering towards the walls were generated and the calculation was done for $Ri = 1.0$ and different values of Ha. The average Nusselt number along the horizontal walls was compared with those reported previously [24-26] and the agreement is promising (see Figure 3 and Table 1).

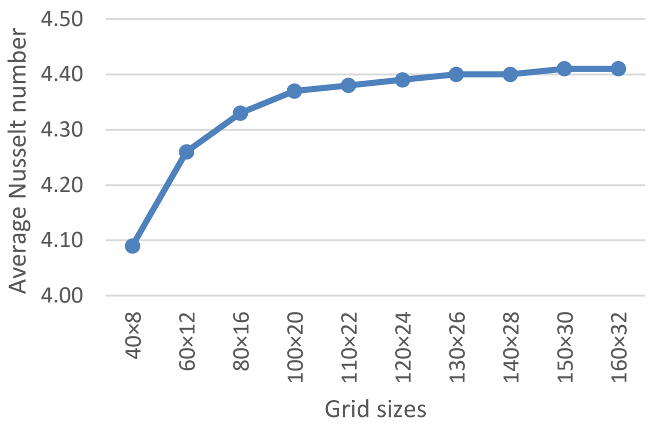

Fig. 2. Variation of mean Nusselt number with grid mesh size for $Ri = 1.0$, $Pr = 7.0$, and $Ha = 10.0$

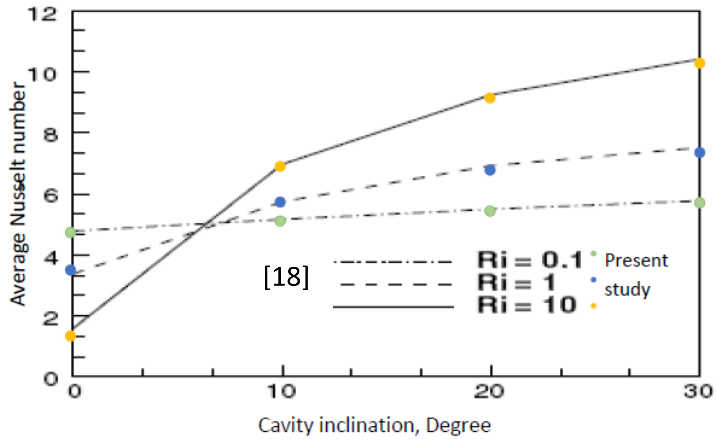

Fig. 3. Comparison of current study and Sharif's study [24]

Table 1
Comparisons of the maximum and minimum values of the horizontal and vertical velocities at the mid-section of the cavity between the present solution and those reported previously [25, 26]

Re = 400.0			
	Iwatsu *et al.,* [25]	Khanafir *et al.,* [26]	Present
U_{min}	-0.3197	-0.3099	-0.3023
U_{max}	1.0000	1.0000	1.0000
V_{min}	-0.4459	-0.4363	-0.4219
V_{max}	0.2955	0.2866	0.2802
Re = 100.0			
U_{min}	-0.2122	-0.2037	-0.2049
U_{max}	1.0000	1.0000	1.0000
V_{min}	-0.2506	-0.2448	-0.2328
V_{max}	0.1765	0.1699	0.1673

3. Results and Discussion

The followings are the controlling parameters used in the simulation: *Re* = 100.0, *Pr* = 6.2, *Gr* = 10^4, *Ha* = 0 – 60 and *Ri* = 1.0. Figure 4 presents the streamlines for different values of *Ha*. A primary clockwise recirculating vortex is visible inside the cavity. The core of the vortex is situated near the

Penerbit Akademia Baru

right wall and it seems that the streamlines are separated halfway. This shows that the fluid flow is shear-dominated due to the moving top lid. By setting *Ha* to10, the strength of the rotating vortex is reduced. It can be seen that the primary vortex is elliptic in shape at increasing magnetic field. As *Ha* increases to 30, the core vortex is pushed to the top part of the cavity due to the conductive heat transfer. As the value of magnetic field increases, the core vortex expands horizontally due to the reduction in flow convection

The effect of *Ha* on the isotherms can be visualized in Figure 5. In the absence of magnetic field, the temperature gradient at the vicinity of the right bottom corner is high due to the top wall movement. When the strength of the magnetic field increases, parameters such as temperature gradient, flow intensity and velocity decrease. The isotherms become almost parallel to the adiabatic wall and evenly distributed near the bottom wall. This phenomenon indicates that conductive heat transfer is dominant.

Figure 6 shows the effect of Hartmann number on the local Nusselt number at the bottom wall. The maximum local Nusselt number increases near the right wall, indicating that the movement of top lid would strongly affect the heat transfer rate inside the cavity. It can be shown that as the Hartmann number increases, the value of local Nusselt number decreases due to the suppression of flow convection.

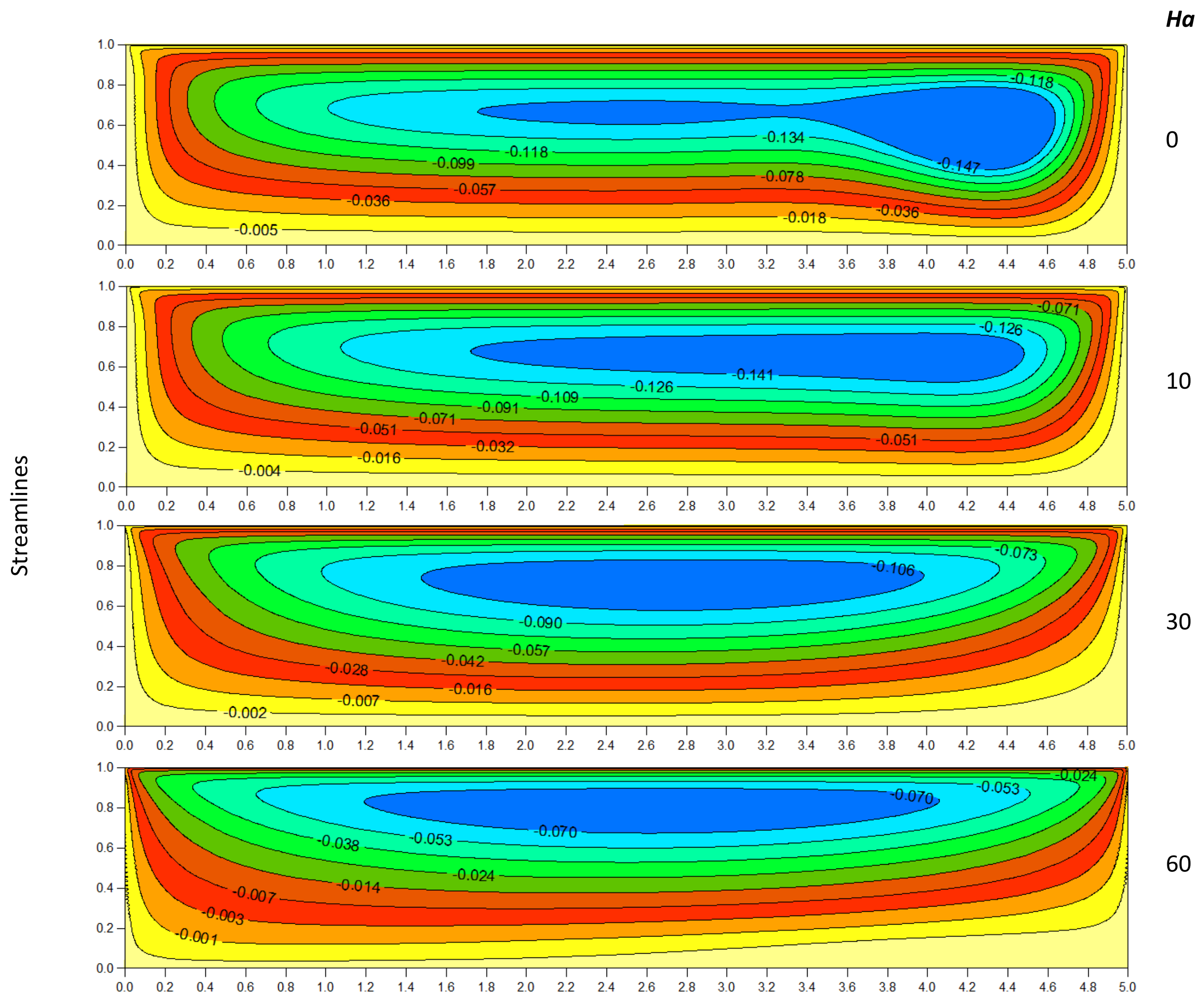

Fig. 4. Variation of streamlines for different *Ha* values for *Ri* = 1.0

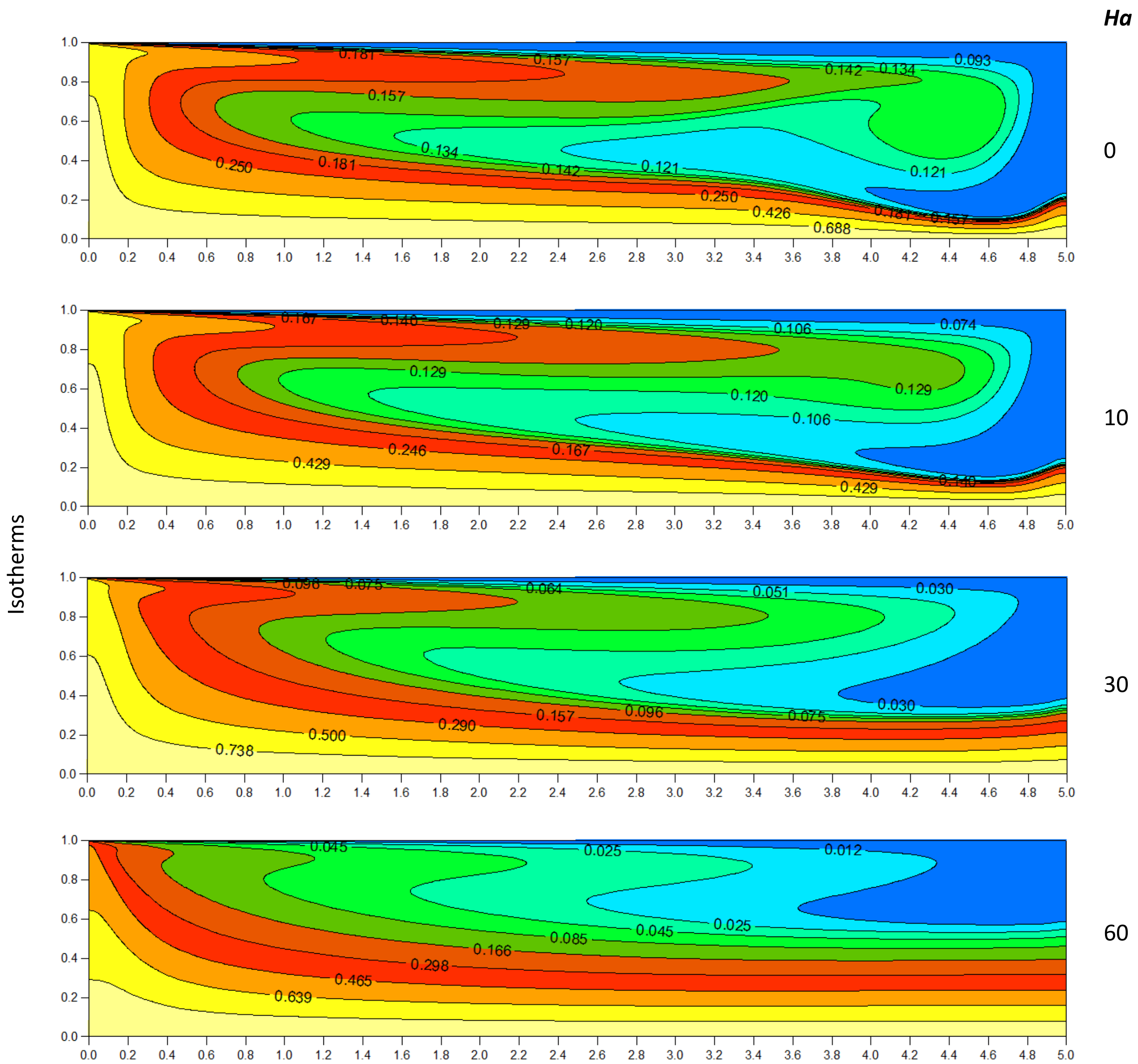

Fig. 5. Variation of isotherms for different *Ha* values for *Ri* = 1.0

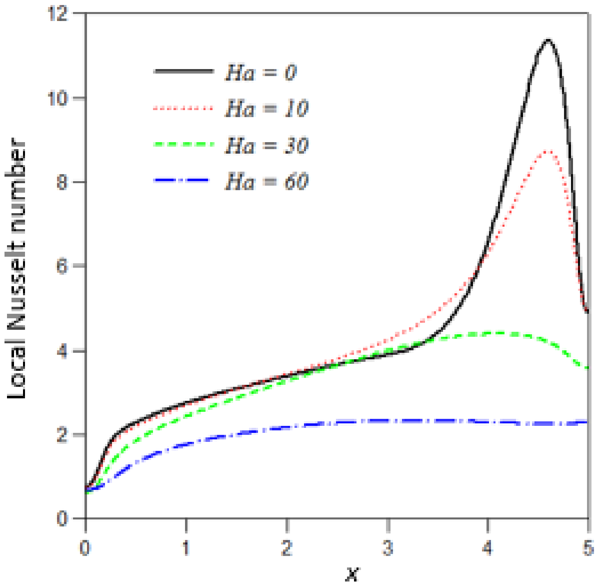

Fig. 6. Variation of local Nusselt number for *Ri* = 1.0

4. Conclusions

In the present study, the mixed convection in a rectangular cavity filled with water in the presence of horizontal magnetic field has been studied numerically using the finite volume method. The bottom and top cavity walls are kept at constant temperatures while the vertical walls are insulated. The top wall is moving at a constant speed in the positive horizontal direction. The effects of mixed convection (*Ri* = 1.0) and *Ha* on the fluid flow and heat transfer mode are investigated. From the numerical results, the following are concluded

i. As *Ha* increases, the flow convection becomes weaker and therefore the heat transfer rate decreases.
ii. The recirculating vortex is pushed towards the top lid as the magnetic field strength increases due to the suppression of flow convection. In this case, the conductive heat transfer is more dominant.
iii. As *Ha* increases, the local Nusselt number decreases.
iv. The value of local Nusselt number increases with respect to the *x*-direction (towards the right wall).

Acknowledgement
The author would like to acknowledge the financial support received from Universiti Tun Hussein Onn Malaysia Grant No. TIER 1/H184 and Ministry of Higher Education of Malaysia (MyBrain15).

References

[1] Hussain, S., S. Ahmad, K. Mehmood, and M. Sagheer. "Effects of inclination angle on mixed convective nanofluid flow in a double lid-driven cavity with discrete heat sources." *International Journal of Heat and Mass Transfer* 106 (2017): 847-860.
[2] Hadavand, Masoud, Shahrouz Yousefzadeh, Omid Ali Akbari, Farzad Pourfattah, Hoang Minh Nguyen, and Amin Asadi. "A numerical investigation on the effects of mixed convection of Ag-water nanofluid inside a sim-circular lid-driven cavity on the temperature of an electronic silicon chip." *Applied Thermal Engineering* 162 (2019): 114298.
[3] Tavakoli, Mohammad Reza, Omid Ali Akbari, Anoushiravan Mohammadian, Erfan Khodabandeh, and Farzad Pourfattah. "Numerical study of mixed convection heat transfer inside a vertical microchannel with two-phase approach." *Journal of Thermal Analysis and Calorimetry* 135, no. 2 (2019): 1119-1134.
[4] Salim, Eutamene, Boudebous Saadoun, Berrahil Farid, Kholai Omar, Dahdi Bachir, and Filali Abdelkader. "Unsteady Mixed Convection in a Cubic Lid-Driven Cavity Partially Heated from the Bottom." *Journal of Advanced Research in FluidMechanics and Thermal Sciences* 57, no. 2 (2019): 275-287.
[5] Oğlakkaya, F. S., and Canan Bozkaya. "Unsteady MHD mixed convection flow in a lid-driven cavity with a heated wavy wall." *International Journal of Mechanical Sciences* 148 (2018): 231-245.
[6] Chamkha, Ali J. "Hydromagnetic combined convection flow in a vertical lid-driven cavity with internal heat generation or absorption." *Numerical Heat Transfer: Part A: Applications* 41, no. 5 (2002): 529-546.
[7] Rahman, M. M., M. M. Billah, M. A. H. Mamun, Rahman Saidur, and M. Hasanuzzaman. "Reynolds and Prandtl numbers effects on MHD mixed convection in a lid-driven cavity along with joule heating and a centered heat conducting circular block." *International journal of mechanical and materials engineering* 5, no. 2 (2010): 163-170.
[8] Oztop, Hakan F., Khaled Al-Salem, and Ioan Pop. "MHD mixed convection in a lid-driven cavity with corner heater." *International Journal of Heat and Mass Transfer* 54, no. 15-16 (2011): 3494-3504.
[9] Sivasankaran, S., A. Malleswaran, Jinho Lee, and Pon Sundar. "Hydro-magnetic combined convection in a lid-driven cavity with sinusoidal boundary conditions on both sidewalls." *International Journal of Heat and Mass Transfer* 54, no. 1-3 (2011): 512-525.
[10] Ahmed, Sameh E., M. A. Mansour, and A. Mahdy. "MHD mixed convection in an inclined lid-driven cavity with opposing thermal buoyancy force: effect of non-uniform heating on both side walls." *Nuclear Engineering and Design* 265 (2013): 938-948.

[11] Alnaqi, Abdulwahab A., Saeed Aghakhani, Ahmad Hajatzadeh Pordanjani, Reza Bakhtiari, Amin Asadi, and Minh-Duc Tran. "Effects of magnetic field on the convective heat transfer rate and entropy generation of a nanofluid in an inclined square cavity equipped with a conductor fin: Considering the radiation effect." *International Journal of Heat and Mass Transfer* 133 (2019): 256-267

[12] Souayeh, Basma, M. Gnaneswara Reddy, P. Sreenivasulu, T. Poornima, Mohammad Rahimi-Gorji, and Ibrahim M. Alarifi. "Comparative analysis on non-linear radiative heat transfer on MHD Casson nanofluid past a thin needle." *Journal of Molecular Liquids* 284 (2019): 163-174.

[13] Alarifi, Ibrahim M., Ahmed G. Abokhalil, M. Osman, Liaquat Ali Lund, Mossaad Ben Ayed, Hafedh Belmabrouk, and Iskander Tlili. "MHD flow and heat transfer over vertical stretching sheet with heat sink or source effect." *Symmetry* 11, no. 3 (2019): 297.

[14] Al-Salem, Khaled, Hakan F. Öztop, Ioan Pop, and Yasin Varol. "Effects of moving lid direction on MHD mixed convection in a linearly heated cavity." *International Journal of Heat and Mass Transfer* 55, no. 4 (2012): 1103-1112.

[15] Kefayati, GH R., M. Gorji-Bandpy, H. Sajjadi, and D. D. Ganji. "Lattice Boltzmann simulation of MHD mixed convection in a lid-driven square cavity with linearly heated wall." *Scientia Iranica* 19, no. 4 (2012): 1053-1065.

[16] Muthtamilselvan, M., and Deog-Hee Doh. "Magnetic field effect on mixed convection in a lid-driven square cavity filled with nanofluids." *Journal of Mechanical Science and Technology* 28, no. 1 (2014): 137-143.

[17] Kefayati, GH R. "FDLBM simulation of magnetic field effect on mixed convection in a two sided lid-driven cavity filled with non-Newtonian nanofluid." *Powder technology* 280 (2015): 135-153.

[18] Kefayati, GH R. "FDLBM simulation of mixed convection in a lid-driven cavity filled with non-Newtonian nanofluid in the presence of magnetic field." *International Journal of Thermal Sciences* 95 (2015): 29-46.

[19] Mahalakshmi, Thangavelu, Nagarajan Nithyadevi, Hakan F. Oztop, and Nidal Abu-Hamdeh. "MHD mixed convective heat transfer in a lid-driven enclosure filled with Ag-water nanofluid with center heater." *International Journal of Mechanical Sciences* 142 (2018): 407-419.

[20] Ismael, Muneer A., M. A. Mansour, Ali J. Chamkha, and A. M. Rashad. "Mixed convection in a nanofluid filled-cavity with partial slip subjected to constant heat flux and inclined magnetic field." *Journal of Magnetism and Magnetic Materials* 416 (2016): 25-36.

[21] Selimefendigil, Fatih, and Hakan F. Öztop. "Modeling and optimization of MHD mixed convection in a lid-driven trapezoidal cavity filled with alumina–water nanofluid: effects of electrical conductivity models." *International Journal of Mechanical Sciences* 136 (2018): 264-278.

[22] Öztop, Hakan F., Ahmad Sakhrieh, Eiyad Abu-Nada, and Khaled Al-Salem. "Mixed convection of MHD flow in nanofluid filled and partially heated wavy walled lid-driven enclosure." *International Communications in Heat and Mass Transfer* 86 (2017): 42-51.

[23] Selimefendigil, Fatih, and Hakan F. Öztop. "Mixed convection of nanofluid filled cavity with oscillating lid under the influence of an inclined magnetic field." *Journal of the Taiwan Institute of Chemical Engineers* 63 (2016): 202-215.

[24] Sharif, M. A. R. "Laminar mixed convection in shallow inclined driven cavities with hot moving lid on top and cooled from bottom." *Applied thermal engineering* 27, no. 5-6 (2007): 1036-1042.

[25] Iwatsu, Reima, Jae Min Hyun, and Kunio Kuwahara. "Mixed convection in a driven cavity with a stable vertical temperature gradient." *International Journal of Heat and Mass Transfer* 36, no. 6 (1993): 1601-1608.

[26] Khanafer, Khalil M., and Ali J. Chamkha. "Mixed convection flow in a lid-driven enclosure filled with a fluid-saturated porous medium." *International Journal of Heat and Mass Transfer* 42, no. 13 (1999): 2465-2481.