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The Effects of Magnetic Blood Flow in an Inclined Cylindrical Tube Using Caputo-Fabrizio Fractional Derivatives

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ABSTRACT

In this paper, the flow of blood through an inclined cylindrical tube subjected to an inclined magnetic field was analysed. The blood flow was considered under the influence of uniformly distributed magnetic particles. The Caputo-Fabrizio fractional derivative was used to study the flow of magnetic blood in an inclined cylindrical tube. The blood flow was driven by an oscillating pressure gradient and an external magnetic field. The analytical solutions were obtained by means of the Laplace and finite Hankel Transforms. The effects of fluid parameters such as Hartmann (Ha) and Reynolds (Re) numbers on the velocities of blood and magnetic particles were graphically presented using MATHCAD. The results show that magnetic field would reduce the velocities of blood and magnetic particles due to Lorentz forces. Meanwhile, the velocities of blood and magnetic particles increase with respect to Re. The velocity of magnetic particles is always lesser than that of blood regardless of the presence of magnetic field.

Keywords:
Caputo Fabrizio derivative; blood flow; magnetohydrodynamics; cylindrical domain

1. Introduction

Biomagnetic Fluid Dynamics (BFD) revolves around the study of fluid flow under the interference of magnetic field. As reported by Pao *et al.,* [1], biomagnetic fluid exists in human body and its motion is influenced by magnetic field. Many studies in BFD have been carried out since the past few centuries. BFD is manly applied in bioengineering and medical sciences such as developing magnetic tracer, targeting transport of medical drugs using magnetic particles as drug carriers, developing magnetic cell separation devices, treating cancer tumor, etc. The underlying fluid dynamics has attracted the interest of many researchers [2].

Blood is one of the biological fluids that can conduct electricity due to the strong presence of erythrocytes (red blood cells). Erythrocytes are indeed negative charge carriers that can provide magnetic effect on the vessel walls. This may change the pulsatile nature of the blood flow Bansi *et al.,* [3]. According to Haik *et al.,* [4], Ferro-Hydrodynamics (FHD) has the same characteristics as BFD.

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The flow of an electrically conductive liquid is marginally affected by the fluid magnetization and there is no influence of induced current at low Reynolds number. Therefore, similar to Magneto Hydrodynamics (MHD), fluid magnetization is considered in the mathematical model of BHD. Choudhari *et al.,* [5] studied the impact of slip velocity on peristaltic blood flow by using the Herschel-Bulkley model in an elastic tube. They found that the volume flux in a flexible tube diminishes with an increase in the permeable parameter and it increases with an increment in the slip parameter. Blood containing nanoparticles has been previously modelled as the third grade non-Newtonian fluid [6]. The porosity and the existence of magnetic field were considered in the study as well. By using the analytical approach, the researchers found that the velocity increased with respect to the pressure gradient. Nevertheless, thermophoresis increased as the Brownian motion parameter decreased.

The study of MHD is very useful in bio-engineering especially in the study of Magnetic Resonance Imaging (MRI), heart attack and cancer treatment [7]. The mathematical model of blood flow in tiny blood vessel subjected to magnetic field has been previously studied [8]. The authors considered the blood as Newtonian fluid with the suspension of erythrocytes and the cell-free layer surrounding the core. Their findings indicated that the velocity and flow rate reduced as the strength of the magnetic field increased. Sharma *et al.,* [9] studied the blood flow in the catheterized stenosed artery subjected to slip velocity with the existence of a transverse magnetic field. They found that the wall shear stress increased as the strength of the transverse magnetic field increased. Hatami *et al.,* [10] have analyzed the MHD blood flow with gold nanoparticles. The blood was treated as the third grade nanofluid in a hollow blood vessel. This problem was solved by applying the Least Square Method (LSM), the Galerkin method (GM) and the fourth order Runge-Kutta method. It was found that the velocity decreased in the presence of magnetic field. A mathematical model based on fractional derivative was derived to study the effect of magnetic field on the blood flow inside the oscillatory arteries [3]. The blood flow was driven by periodic pressure gradient and body acceleration. The exact solution was obtained by using the Hankel and Laplace Transforms. They found that fractional derivative is valuable in controlling the blood temperature and velocity.

The use of fractional order derivative in mathematical modeling has found numerous applications such as those in physics, fluid mechanics, mathematical biology and electrochemistry [11-13]. Besides that, as compared to the ordinary traditional calculus, the model derived from fractional calculus is more general and accurate. Considering the significance of fractional derivatives, the researchers [14] used fractional calculus to study the flow of Oldroyd-B fluid in stenosed arteries. The derived mathematical model of tapered stenosed artery in the presence of pressure gradient might help medical practitioners in treating cardiovascular diseases. The free convection flow of an incompressible fractional second-grade fluid near the vertical plate has been studied by using Caputo and Caputo Fabrizio derivatives [15]. Based on the comparative study, the effects of flow and fractional parameters on the temperature and velocity profiles were obtained. They found that the temperature values obtained from both fractional models decreased with respect to Prandtl number.

In a general physiological system, some arteries are not perfectly horizontal or vertical. As a result, the effect of gravity should be considered in the flow calculation involving inclined arteries. The unsteady non-Newtonian blood flow in an inclined, catheterized and overlapping stenosed artery has been analyzed [16]. The finite difference method (FDM) was used to solve the stated problem. They showed that axial velocity, flow rate and resistance impedance were heavily dependent on parameters such as wall slip and inclination angle. The pulsatile flow of Herschel-Bulkley fluid through multiple inclined multiple stenoses with periodic body acceleration has been analyzed [17]. The study revealed that the velocity increased as the body acceleration increased. The influence of heat transfer on the peristaltic transport of magnetohydrodynamic through a porous medium has been studied [18]. The blood was modeled as an incompressible MHD second grade fluid that travelled in an inclined asymmetric channel. The authors used the analytical perturbation method to solve the problem. Based on the study, the pressure gradient and the size of trapped bolus increases as the inclination angle increased.

Based on our review, most of researchers did not considered inclined cylindrical tube in their studies. It can be seen easily that in physiological systems all arteries are not horizontal, in which few of them are inclined. In the current work, motivated from the above studies, the blood vessel was modeled as an inclined cylindrical tube subjected to an inclined magnetic field. Blood flow was driven by the oscillating pressure gradient in the z-direction. A fractional mathematical model of Caputo-Fabrizio was developed in the cylindrical coordinate system to study the magnetic blood flow. The fractional models of blood flow and particle motion under the influence of magnetic field were solved analytically by using the joint method of Laplace and finite Hankel Transforms. The analytical solutions were then plotted using the mathematical software, i.e. Mathcad.

2. Methodology

The circular section of the artery was shown in Figure 1. In this study, blood flow occurred in an inclined cylindrical tube of radius R_0 under the influence of uniformly distributed magnetic particles. The magnetic blood particles that travelled in the axial direction (i.e. z-direction) were driven by an oscillating pressure gradient. Meanwhile, an external magnetic field was applied perpendicularly throughout the inclined cylindrical vessel.

In this study, the magnetic Reynolds number was taken small enough as the entire blood flow stream was subjected to an external magnetic field. In other words, the induced magnetic field was assumed negligible as compared to the applied magnetic field. At *t=0*, the cylindrical tube, the blood and the magnetic particles were treated as stationary.

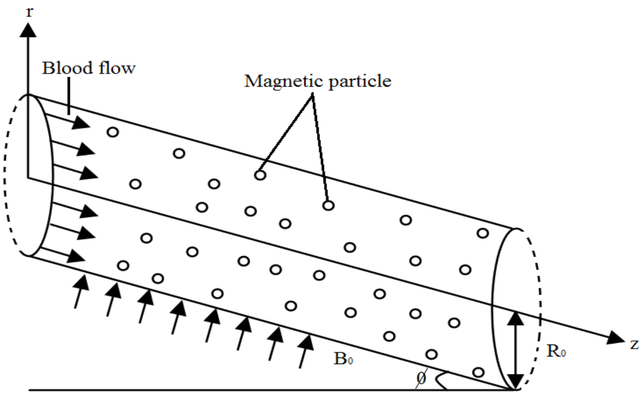


Fig. 1. Geometry figure of the magnetic blood flow

The governing equations are the Navier-Stokes equations describing the blood flow, the Maxwell's relations describing the magnetic field and the Newton's second law describing the particle motion. The Maxwell equations are

$$\nabla \cdot \vec{B} = 0, \nabla \times \vec{B} = \mu_0 \vec{J}, \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \tag{1}$$

where $\vec{B}$ is the magnetic flux intensity, μ_0 is the magnetic permeability, $\vec{E}$ is the electric field intensity and $\vec{J}$ is the current density given by [10], [16]

$$\vec{J} = \sigma(\vec{E} + \vec{V} \times \vec{B}), \tag{2}$$

Here σ is the electrical conductivity and $\vec{V}$ is the velocity field. The electromagnetic force $\vec{F}_{em}$ is defined as [3]

$$\vec{F}_{em} = \vec{J} \times \vec{B} = \sigma\left(\vec{E} + \vec{V} + \vec{B}\right) \times \vec{B} = -\sigma B_0^2 u(r,t)\vec{k} \tag{3}$$

where $\vec{k}$is the unit vector in the z-direction and $\vec{V} = u(r,t)\vec{k}$ is the axial velocity of the blood. The force $\vec{F}_{em}$ is included in the momentum equations.

The unsteady blood flow in an axisymmetric cylindrical tube of radius R_0 under the influence of uniform transverse magnetic field and pressure gradient of the form [17]

$$-\frac{\partial p}{\partial z} = A_0 + A_1 \cos(\omega t), \quad A_0 > 0 \tag{4}$$

was considered. Here, the constants A0 and A1 are the amplitudes of the pulsatile magnetic field and pressure gradient that give rise to systolic or diastolic pressure.

The momentum equation for fluid flow in the cylindrical coordinate system(*r,θ, z*) is [8,19]

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \upsilon\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) + \frac{KN}{\rho}(v - u) - \frac{\sigma B_0^2 \sin\theta}{\rho} + g \sin\phi, \tag{5}$$

where ρ is the fluid density, υ is the kinematic viscosity, p is the pressure, N is the number of magnetic particles per unit volume, K is the Stokes constant, u is the fluid velocity and ν is the velocity of the particle. The term $(KN/\rho)(\nu - u)$ was introduced to model the force due to the relative motion between fluid and magnetic particles. It was assumed that the Reynolds number of the relative velocity was small. Hence, the force between the magnetic particles and the blood is proportional to the relative velocity.

The motion of magnetic particles is governed by the Newton's second law [20]

$$m\frac{\partial v}{\partial t} = K(u - v) \tag{6}$$

where m is the average mass of the magnetic particles.

In order to consider the time-fractional model, Eqs. (5) and (32) can be multiplied by$\lambda = \sqrt{(R_0\rho/A_0)}$ to yield a term with the dimension of time t. Therefore, the governing equations of the time-fractional model are

$$\lambda^\alpha D_t^\alpha u = -\frac{\lambda}{\rho}(A_0 + A_1 \cos(\omega t)) + \lambda\upsilon\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) + \frac{KN\lambda}{\rho}(v - u) - \frac{\sigma B_0^2 \lambda}{\rho} + g\lambda \sin\phi \tag{7}$$

and

$$\lambda^\alpha D_t^\alpha v = \frac{K\lambda}{m}(u - v) \tag{8}$$

where the Caputo-Fabrizio derivative operator is

$$ {}^{CF}D_t^{\alpha}u(r,t) = \frac{1}{1-\alpha}\int_0^t exp\left(-\frac{\alpha(t-\tau)}{1-\alpha}\right)\frac{\partial u(r,\tau)}{\partial \tau}d\tau \tag{9} $$

The Laplace transform of the Caputo-Fabrizio time derivative can be written as

$$ L\{{}^{CF}D_t^{\alpha}u(r,t)\} = \frac{sL\{u(r,t)\}-u(r,0)}{(1-\alpha)s+\alpha} \tag{10} $$

The initial boundary conditions of the fluid inside the cylindrical domain of radius R_0 are

$$ u(r,0) = 0,\ \ v(r,0) = 0,\ \ \mathrm{r} \in [0, R_0], u(R_0,t) = 0,\ \ v(R_0,t) = 0,\ \ t > 0 \tag{11} $$

For dimensionless study, the following non-dimensional parameters can be introduced

$$ r^* = \frac{r}{R_0},\ t^* = \frac{t}{\lambda},\ u^* = \frac{u}{u_0},\ v^* = \frac{v}{u_0}, A_0^* = \frac{\lambda A_0}{\rho u_0},\ A_1^* = \frac{\lambda A_1}{\rho u_0},\ \omega^* = \lambda\omega,\ g^* = \frac{g}{u_0^2/R_0} \tag{12} $$

where u_0 is the characteristics velocity.

By introducing the above parameters and dropping the * notation the non-dimensional forms of Eqs. (7), (8), and (11) are

$$ D_t^{\alpha}u = A_0 + A_1\cos(\omega t) + \frac{1}{Re\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r^*}\frac{\partial u^*}{\partial r^*}\right]^2\frac{\sin\phi}{F}} \tag{13} $$

$$ G \cdot D_t^{\alpha}v = u - v \tag{14} $$

where $Re = R_0/\lambda v$ is the Reynolds number, $R = KN\lambda/\rho$ is the particles concentration parameter, $Ha^2 = \sigma B_0^2\lambda sin\theta/\rho$ is the Hartmann number and $F = R_0/\lambda u_0 g$is the inclination angle parameter. The non-dimensional boundary conditions are

$$ u(r,0) = 0,\ v(r,0) = 0,\ \mathrm{r} \in [0,1],\ u(1,t) = 0,\ v(1,t) = 0,\ t > 0 \tag{15} $$

The use of Laplace Transform is suitable when the temporal variable t is adopted in the blood flow model (see Eqs. (7), (8)) and the boundary conditions (15). After the transformation process, we have

$$ \frac{s\bar{u}(r,s)}{s+\alpha(1-s)} = \frac{A_0}{s} + \frac{A_1 s}{s^2+\omega^2} + \frac{1}{Re}\left[\frac{\partial^2\bar{u}(r,s)}{\partial r^2} + \frac{1}{r}\frac{\partial\bar{u}(r,s)}{\partial r}\right] + R\bar{v}\ -\ (R + Ha^2)\bar{u}(r,s) + \frac{\sin\phi}{sF} \tag{16} $$

$$ G\frac{s\bar{v}(r,s)}{s+\alpha(1-s)} = \bar{u}(r,s) - \bar{v}(r,s) \tag{17} $$

$$ \bar{u}(1,s) = 0,\ \ \bar{v}(1,s) = 0 \tag{18} $$

From Eq. (17), the following equation can be obtained

$$ \bar{v}(r,s) = \frac{s+\alpha(1-s)}{Gs+s+\alpha(1-s)}\bar{u}(r,s) \tag{19} $$

Upon substituting $\bar{v}(r,s)$ from Eq. (19) into Eq. (16)

$$\left[\frac{s}{s+\alpha(1-s)} - R\left(\frac{s+\alpha(1-s)}{s+sG+\alpha(1-s)}\right) + R + Ha^2\right]\bar{u}(r,s) = \frac{A_0}{s} + \frac{A_1 s}{s^2+\omega^2} + \frac{1}{Re\left[\frac{\partial^2 \bar{u}}{\partial r^2}+\frac{1}{r}\frac{\partial \bar{u}}{\partial r}\right]}\frac{\sin\phi}{sF} \tag{20}$$

Applying the finite Hankel Transform of order zero (i.e. applying the boundary condition (15) in Eq. (20)), the following equation can be obtained

$$\left[\frac{s}{s+\alpha(1-s)} - R\left(\frac{s+\alpha(1-s)}{s+sG+\alpha(1-s)}\right) + R + Ha^2\right]\bar{u}_H(r_n,s) = \left[\frac{A_0}{s} + \frac{A_1 s}{s^2+\omega^2} + \frac{\sin\phi}{sF}\right]\frac{J_1(r_n)}{r_n} - \frac{1}{Re\bar{r}_n{}_{H_n}} \tag{21}$$

where $\bar{u}_H(r_n,s) = \int_0^1 r\bar{u}(r,s)J_0(r_n r)dr$ represents the finite Hankel transform of the velocity function. Here, $\bar{u}(r,s) = LT[u(r,t)]$ are the positive roots of the equation $J_0(x) = 0$, where J_0 is the Bessel function of order zero and it belongs to the first kind. By simplifying the coefficient of $\bar{u}_H(r_n,s)$ in Eq. (21), the following equations can be formulated

$$\bar{u}_H(r_n,s) = \frac{s^2 x_{5n} + s x_{6n} + \alpha^2}{s^2 x_{2n} + s x_{3n} + x_{4n}}\left[\frac{1}{s}\left(A_0 + \frac{\sin\phi}{F}\right) + \frac{A_1 s}{s^2+\omega^2}\right]\frac{J_1(r_n)}{r_n} \tag{22}$$

$$\bar{u}_H(r_n,s) = \left[\frac{x_{9n}}{s-x_{7n}} + \frac{x_{10n}}{s-x_{8n}}\right]\left[\frac{1}{s}\left(A_0 + \frac{\sin\phi}{F}\right) + \frac{A_1 s}{s^2+\omega^2}\right]\frac{J_1(r_n)}{r_n} \tag{23}$$

$$\bar{u}_H(r_n,s) = A_0 + \frac{\sin\phi}{F}\left[\frac{s^{-1}}{s-x_{7n}}x_{9n} + \frac{s^{-1}}{s-x_{8n}}x_{10n}\right]\frac{J_1(r_n)}{r_n} + A_1\frac{s}{s^2+\omega^2}\left[x_{9n}\frac{1}{s-x_{7n}} + x_{10n}\frac{1}{s-x_{8n}}\right]\frac{J_1(r_n)}{r_n} \tag{24}$$

Note, the parameters in Eq. (22) and (23) introduced for simplifying the coefficient of $\bar{u}_H(r_n,s)$ are

$$x_{1n} = Ha^2 + R + \frac{r_n}{Re}$$

$$x_{2n} = 1 + G - \alpha - R - R\alpha^2 + 2R\alpha + x_{1n} + \alpha^2 x_{1n} - 2\alpha x_{1n} + G x_{1n} - G\alpha x_{1n},$$

$$x_{3n} = \alpha + 2R\alpha^2 - 2R\alpha - 2x_{1n}\alpha^2 + 2\alpha x_{1n} + G\alpha x_{1n}, \quad x_{4n} = \alpha^2 x_{1n} - R\alpha^2,$$

$$x_{5n} = 1 + \alpha^2 - 2\alpha + G - G\alpha, \quad x_{6n} = -2\alpha^2 + 2\alpha + G\alpha,$$

$$x_{7n} = \frac{-x_{3n} + \sqrt{x_{3n}^2 - 4x_{2n}x_{4n}}}{2x_{2n}}, \quad x_{8n} = \frac{-x_{3n} - \sqrt{x_{3n}^2 - 4x_{2n}x_{4n}}}{2x_{2n}},$$

$$x_{9n} = \frac{x_{7n}^2 x_{5n} + x_{7n}x_{6n} + \alpha^2}{x_{7n} - x_{8n}}, \quad x_{10n} = \frac{x_{8n}^2 x_{5n} + x_{8n}x_{6n} + \alpha^2}{x_{8n} - x_{7n}}, \tag{25}$$

The Laplace Transform of the image function $\bar{u}_H(r_n,s)$ in Eq. (24) can be obtained by using the Robotnov and Hartley's functions

$$LT^{-1}\left[\frac{1}{s^w+y}\right] = F_w(-y,t) = \sum_{n=0}^{\infty}\frac{(-y)^n t^{(n+1)w-1}}{\Gamma((n+1)w)}, w > 0 \tag{26}$$

$$LT^{-1}\left[\frac{s^z}{s^w+y}\right] = R_{w,z}(-y,t) = \sum_{n=0}^{\infty}\frac{(-y)^n t^{(n+1)w-1-z}}{\Gamma((n+1)w-z)}, Re(w-z) > 0 \tag{27}$$

$$\bar{u}_H(r_n,t) = \frac{J_1(r_n)}{r_n}\left[\left(e^{x_7 n^t}-1\right)\left(\frac{A_0 x_{9n}}{x_{7n}}+\frac{x_{9n}\sin\phi}{x_{7n}F}\right)+\left(e^{x_8 n^t}-1\right)\left(\frac{A_0 x_{10n}}{x_{8n}}+\frac{x_{10n}\sin\phi}{x_{8n}F}\right)+A_1 x_{9n} e^{x_7 n^t} * \cos(\omega t)+A_1 x_{10n} e^{x_8 n^t} * \cos(\omega t)\right] \tag{28}$$

By inverting the finite Hankel Transforms (i.e. Eq. (28)), we obtain

$$u(r,t) = 2\sum_{n=1}^{\infty}\frac{J_0(r r_n)}{r_n J_1^2(r_n)} \times u_H(r_n,t) \tag{29}$$

$$u(r,t) = 2\sum_{n=1}^{\infty}\frac{J_0(r r_n)}{r_n J_1(r_n)}\left[\left(e^{x_7 n^t}-1\right)\left(\frac{A_0 x_{9n}}{x_{7n}}+\frac{x_{9n}\sin\phi}{x_{7n}F}\right)+\left(e^{x_8 n^t}-1\right)\left(\frac{A_0 x_{10n}}{x_{8n}}+\frac{x_{10n}\sin\phi}{x_{8n}F}\right)+A_1 x_{9n} e^{x_7 n^t} * \cos(\omega t)+A_1 x_{10n} e^{x_8 n^t} * \cos(\omega t)\right] \tag{30}$$

The magnetic particle velocity can then be obtained from Eq. (17)

$$\bar{v}(r,s) = \frac{s+\alpha-\alpha s}{s+Gs+\alpha-\alpha s}\bar{u}(r,s) \tag{31}$$

$$v(r,t) = x_{12n}(1-x_{11n})[u(r,t) * e^{x_{12n}t}],\ 0<\alpha<1 \tag{32}$$

In Eqs. (30) and (32), $f * g$ represents the convolution product of f and g. The parameters introduced in Eq. (32) are

$$x_{11n} = \frac{1-\alpha}{G-\alpha+1},\quad x_{12n} = \frac{\alpha}{G-\alpha+1} \tag{33}$$

Finally, the convolution product $f * g$can be calculated as

$$(f * g)(t) = \int_0^t f(\tau)g(t-\tau)d\tau \tag{34}$$

3. Results and Discussions

The velocities of fluid and the magnetic particles (Eqs. (30) and (32)) can be solved numerically by using the positive roots of the Bessel function J_0. In the numerical computation, a series consisting of 500 terms was generated for solving the inverse Hankel Transform equation. The following parameters were adopted while performing the simulation using Mathcad: A_0 = 0.5, A_1= 0.6, G = 0.8, R = 0.5, Re = 4, ω = π/4 and Ha = 2 (see [20]). The velocities of blood and magnetic particles in the *r*-direction were shown in Figures (2) – (11). The influence of fractional orderα at various Hartmann numbers *Ha*, Reynolds numbers *Re* and inclination angles $\phi$was studied in order to establish better understanding of the stated problem.

In order to validate the current model, the results presented in Figure (2) were compared against those from previous study [20] (by setting α=1 in Eq. (32)). The agreement is encouraging.

Figures (3) – (5) show the effect of inclined magnetic field on the velocities of blood and magnetic particles. Seemingly, the velocities of the magnetic particles reduce appreciably as Hartmann number increases. Generally, the presence of magnetic field in the blood stream would retard the blood motion due to the rise in resistive drag force. In order to identify the effect of inclined magnetic field on both fluid and particle distributions, Figures (3) – (5) are plotted and the results show that both velocities decrease appreciably as Ha increases. Generally, the presence of magnetic field in the blood flow tends to slow down the particle motion due to the rise in resistive or Lorentz force. This finding agrees well with those found in Ref. [4]. This observation might be useful in some medical treatment processes.

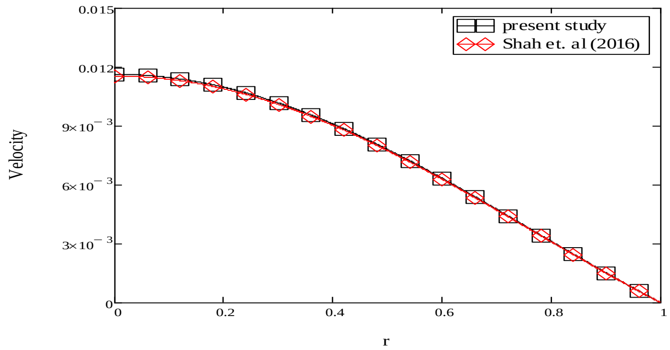
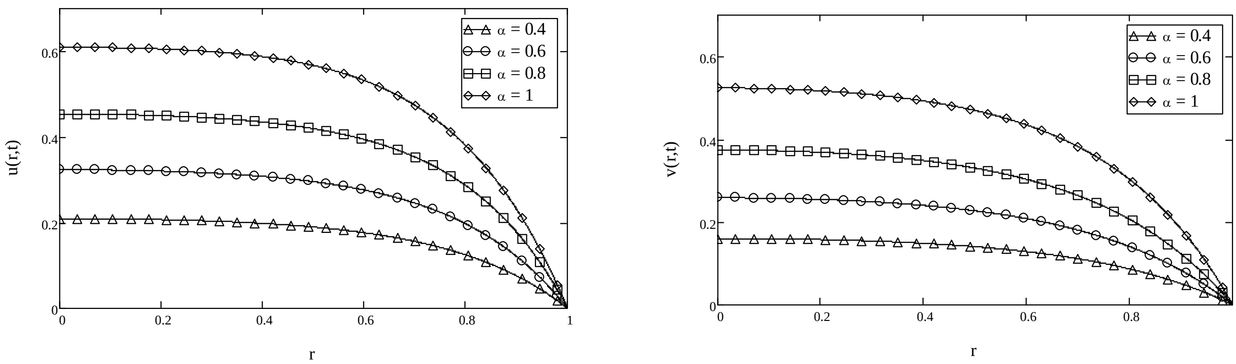
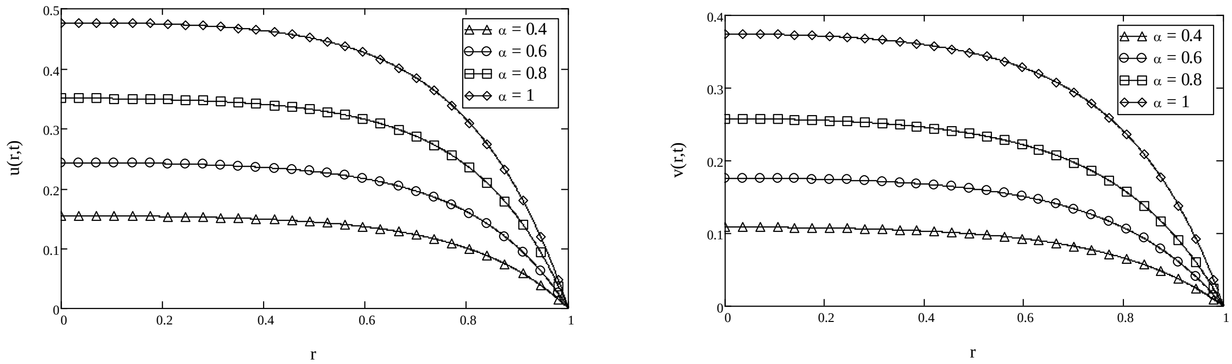


Fig. 2. Geometry domain of the magnetic blood flow



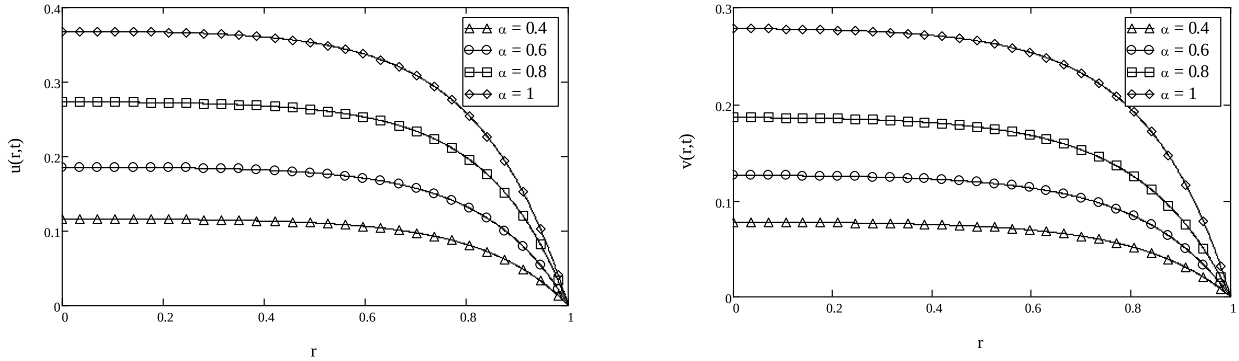
Blood flow velocity at *Ha* = 2 Magnetic particle velocity at *Ha* = 2

Fig. 3. Profiles of axial velocities *u(r,t)* and *v(r,t)* at different fractional parameters against r



Blood flow velocity at *Ha* = 2.5 Magnetic particle velocity at *Ha* = 2.5

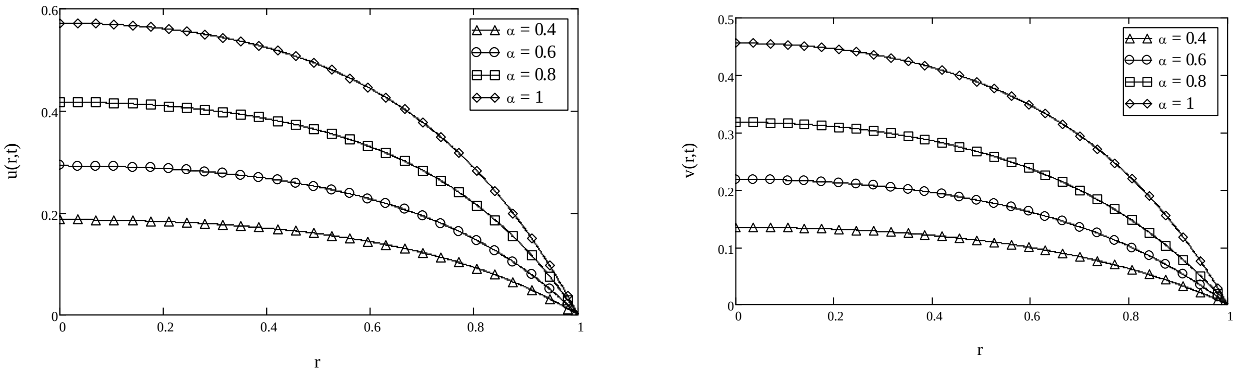
Fig. 4. Profiles of axial velocities *u(r,t)* and *v(r,t)* at different fractional parameters against r



Blood flow velocity at *Ha* = 3 Magnetic particle velocity at *Ha* = 3

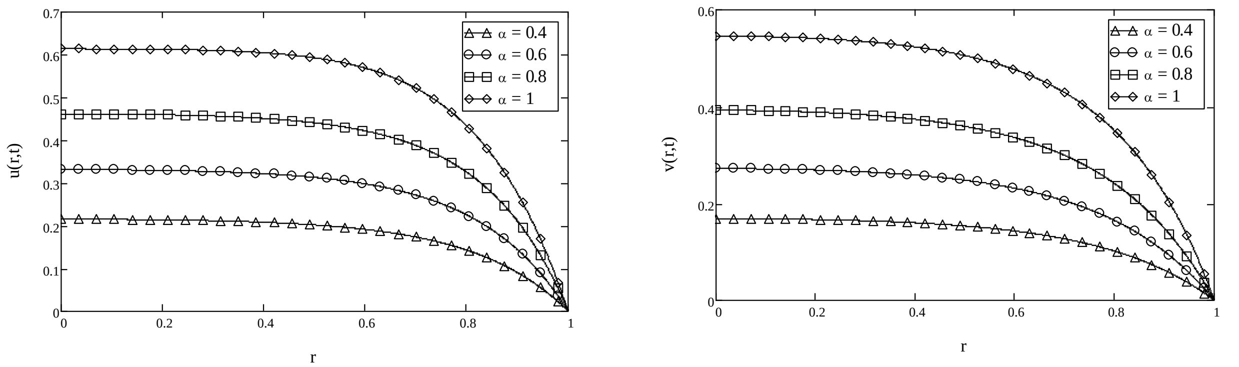
Fig. 5. Profiles of axial velocities *u(r,t)* and *v(r,t)* at different fractional parameters against r

Figures (6) – (8) show the effects of Reynolds number, *Re* on the velocities of blood and magnetic particles. In general, *Re* increased with respect to the velocities, implying that there is a drop in the blood viscosity which would ease the blood flow. This finding is in good agreement with those reported in Bansi *et al.* [3]' study.



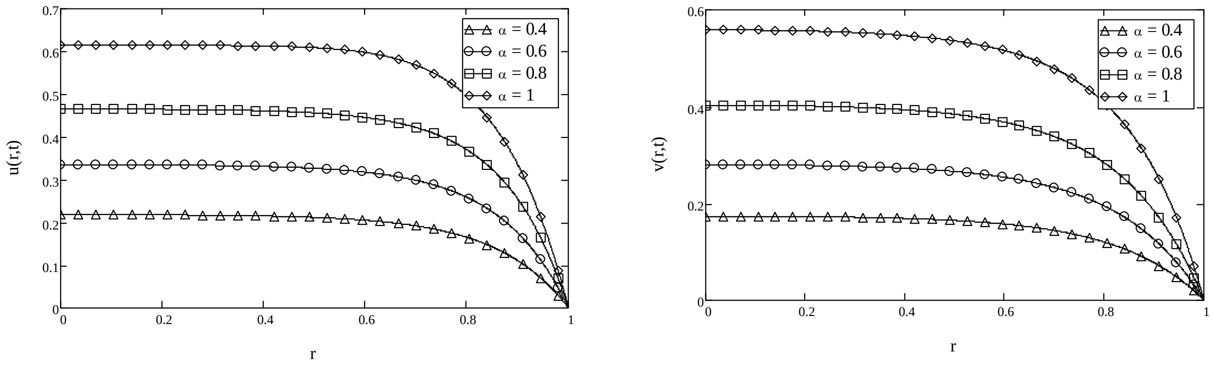
Blood flow velocity at *Re* = 2 Magnetic particle velocity at *Re* = 2

Fig. 6. Profiles of axial velocities *u(r,t)* and *v(r,t)* at different fractional parameters against r



Blood flow velocity at *Re* = 6 Magnetic particle velocity at *Re* = 6

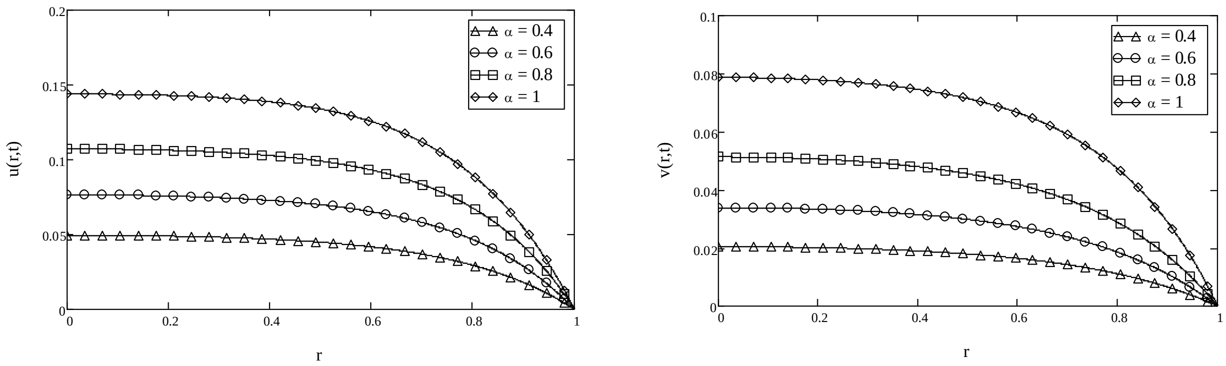
Fig. 7. Profiles of axial velocities *u(r,t)* and *v(r,t)* at different fractional parameters against r



Blood flow velocity at *Re* = 10 Magnetic particle velocity at *Re* = 10

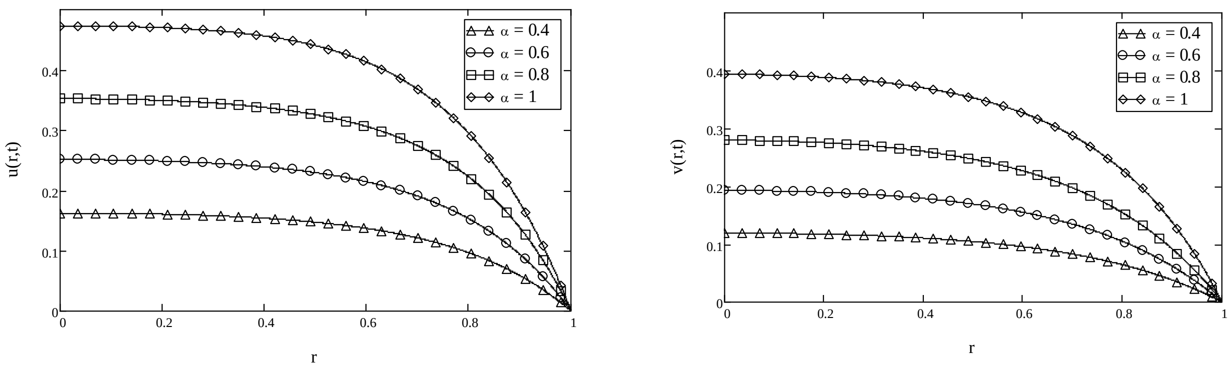
Fig. 8. Profiles of axial velocities *u(r,t)* and *v(r,t)* at different fractional parameters against r

The *u(r,t)* and *v(r,t)* profiles at different inclination angles ϕ are plotted in Figures (9) – (11). As observed, the speeds of fluid and magnetic particles in the inclined artery are higher than those in the non-inclined artery, as it is easier to move the blood through an inclined artery. It should be noted that the magnetic particle velocity is lower than the blood velocity due to the resistive force.



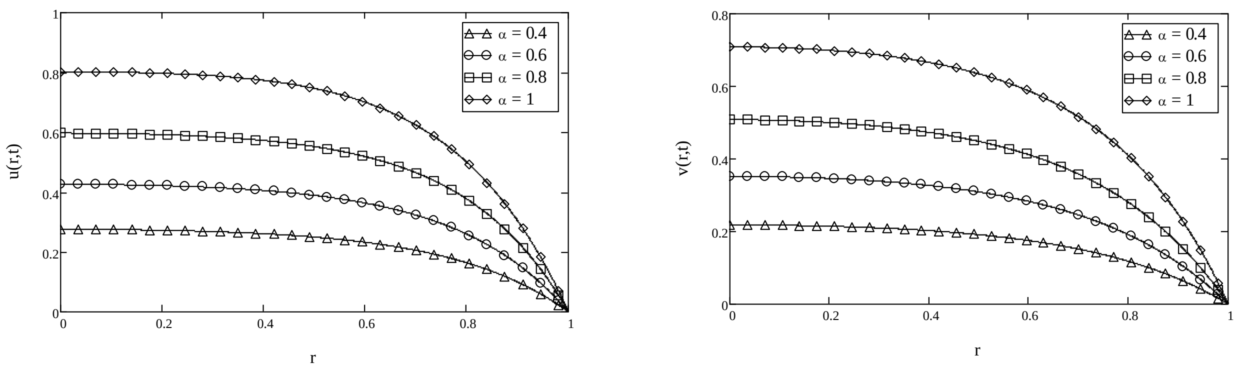
Blood flow velocity at $\phi = 0$ Magnetic particle velocity at $\phi = 0$

Fig. 9. Profiles of axial velocities *u(r,t)* and *v(r,t)* at different fractional parameters against r



Blood flow velocity at $\phi = \frac{\pi}{6}$ Magnetic particle velocity at $\phi = \frac{\pi}{6}$

Fig. 10. Profiles of axial velocities *u(r,t)* and *v(r,t)* at different fractional parameters against r



Blood flow velocity at $\phi = \frac{\pi}{2}$ Magnetic particle velocity at $\phi = \frac{\pi}{2}$

Fig. 11. Profiles of axial velocities *u(r,t)* and *v(r,t)* at different fractional parameters against r

4. Conclusions

A mathematical model has been developed to analyse the blood flow in an inclined cylindrical artery subjected to the transverse magnetic field. The new definition of Caputo-Fabrizio fractional derivative was used to compute the velocities of fluid and magnetic particles. An additional mathematical solution is usually required to derive the ordinary model; however, by using the current approach, the ordinary model ($\alpha = 1$) for the velocity equation can be directly obtained as the equation is fully compatible. The governing non-dimensional fractional partial differential equation has been solved analytically by using both Laplace (with respect to t) and finite Hankel Transforms. Based on the numerical results, the velocities of blood and magnetic particles decrease with respect to Hartmann number. Nevertheless, the velocities increase at increasing Reynolds number. The numerical results show that the inclination angle has a significant impact on the velocities of blood and magnetic particle. This finding might be useful in the diagnosis and therapeutic treatment of some medical problems. Also, the current finding could lead to the better designs of pads and machines.

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