



## Numerical Investigation of Flow Around Finite Height Rectangular

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### ABSTRACT

Three-dimensional flow features of the slender rectangular prism with cross-section height ( $H$ ) to streamwise depth ( $D$ ) ratio or side ratio ( $D/H = 0.5$ ) were investigated numerically using the Large-eddy simulations (*LES*) turbulence model with Reynolds number  $Re = 22000$ . Four different aspect ratios ( $L/H = 2.5 - 10$ ) were employed in this research to study the effect of the spanwise variation of the prism model on the flow pattern around the prism. Moreover, the instability-induced motion of the prism was modeled to predict the alteration of flow characteristics of stationary to vibrating states of the test model. The global quantities such as drag force, pressure coefficient, and Strouhal frequency characteristics are presented, which suggests that the structure end tip effect plays an essential role in the dependency of flow features variation. The velocity vector variations at streamwise and spanwise positions are also demonstrated. The prism model with a small aspect ratio ( $L/H = 2.5$ ) exhibited Karman vortex suppression at the prism's vicinity, and the streamwise vortices region shrank. The flow features of the vibrating prism show different behaviors from the stationary prism model.

## 1. Introduction

Flow behavior around the blunt prism is considerably paid attention because of its effect on many engineering structures, such as rigid or dynamic structures, heat exchangers, masts, long piping systems, marine risers, chimneys, cooling towers, and tall buildings. The behavior of the flow structure around the cylindrical or prismatic shapes can be found in literature, both numeric and experimental studies, such as references [1-3]. Structure failure due to the flow-induced vibration is closed related to the structure response on the incoming flow stream. For instance, the failure of the Tacoma narrow bridge [4]. Flow stream around the cylinder can also create aeolian tone noise, which makes environmental noise issues. Dynamic response behaviors over the structure have been mitigated using the turbulent control parts such as using multiple cylinders in tandem or staggered, stepped, attached endplate, splitter plate, etc. Zdravkovich [5] presented the various ways to suppress vortex shedding over the circular cylinder.

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Moreover, using multiple cylinders to control the circular cylinder's dynamic response can be found in the researches by Griiffith *et al.*, [6], Ding *et al.*, [7], and Kim and Alam [8]. The prismatic structure equipped with a turbulent control has demonstrated the alteration of flow pattern and dynamic response of elastically mounted structure [9-11]. Flow behavior around the prismatic body has different characteristics from the circular body, which depends on the body shape, length, presence of end tip, and Reynolds number [12-14]. In the case of a rectangular prismatic body with a sharp edge, flow separation occurs at the leading edge, and flow behaviors at the side and behind the prism are considerably influenced by the prism's depth (side ratio). It differs from a circular cylinder because the separation point varies with the Reynolds number [15].

The effect of the cylinder's depth on the flow field characteristics demonstrated that the alteration of flow features such as drag force, pressure coefficient, and Strouhal number is influenced by the prism's depth and Reynolds number. Prism's critical depth to height ratio is around 0.6, as presented in researches by Nakaguchi [15], Okajima *et al.*, [16], and Nakamura and Hirata [17]. Moreover, recent investigations on a depth ratio effect are found in researches by Bruno *et al.*, [18], Tian *et al.*, [19], Liu *et al.*, [20], and Wang *et al.*, [21]. In nature, separated shear layers at the prism's leading edge roll up and reattach on the surface. The symmetric or asymmetric vortices shed behind as the regular Karman vortex. Consequently, Reynolds number, depth to a height ratio or a side ratio, and vortex interaction of flow wake structure are essential to the flow behaviors around prismatic structures.

The rectangular prismatic body is a common shape for a second structure member widely used in construction systems. The presence of an end tip structure alters flow characteristics in the wake, which is different from a two-dimensional one. The end tip condition is also commonly found in civil and architectural construction buildings, airplane winglets, etc.

Studies on the flow characteristics around the prismatic body with an end tip are widely shown by various investigation approaches, such as grounded at a base, pivoted, or mounted at a ceiling tunnel [22-25]. Flow characteristics analysis has shown that the end tip vortex alters the spanwise vortex in the wake. Meanwhile, the alteration of the spanwise vortex is closed related to the prism's length. The critical length is defined as the ratio of span length to cross-section height  $L/H$  is 5.0. The flow investigations around prismatic structure with free end focus on the square prism. Studies on the free-end effect of the slender rectangular are still entirely unexplored. He *et al.*, [26] presented a numerical approach to the end effect of a circular cylinder with a low aspect ratio. They showed that the presence of an end tip affects fluid force characteristics and the stability response of the cylinder's motion. The span length variation changes the local velocity, wake zone, and vortex periodicity around the prism with a small aspect ratio, mainly at the prism with an aspect ratio smaller than the critical aspect ratio [14].

Meanwhile, there is insufficient analysis of the unsteady flow behavior prediction of flow around slender rectangular prism by the numerical method in the past. Detailed analysis of the flow features around 3D slender rectangular is a challenge to provide partial parts to the comprehensive analysis of the end tip effect. Tian *et al.*, [19] showed that the two-dimensional numerical approach using the unsteady RANS model has been unsuccessful in predicting aerodynamic components around a slender rectangular prism.

Three-dimensional Large Eddy Simulation (*LES*) turbulence model is a robust tool to analyze and visualize the flow-structure interaction of a blunt body from the perspective of academic research [27]. In this study, we investigated the effect of spanwise variation (aspect ratios, *AR*) of a slender rectangular prism on the flow characteristics numerically. Moreover, the instability-induced motion of the rectangular prism was also predicted.

## 2. Computational Method

The finite volume-based *CFD* code (ANSYS Fluent v18) with an academic license was employed to model the flow behaviors past the rectangular prism. Three-dimensional analysis of turbulent flow characteristics around the free-end rectangular prism was solved using the large eddy simulation turbulence model. The governing equations of incompressible Newtonian flow problems with constant physical properties, i.e., mass and momentum conservations, were solved by the equations below (see also the common governing equations of mass and momentum conservation by Tey *et al.*, [29]).

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0, \quad (1)$$

and spatial filtering Navier-Stokes equation as

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} (\overline{u_i u_j}) = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \nu \nabla^2 \bar{u}_i, \text{ where } \bar{u} \text{ is solenoidal.} \quad (2)$$

Eq. (2) is adjusted in the common form by considering residual or subgrid-scale stress tensor ( $\tau_{ij}^R$ ) due to the filtering effect.

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} (\overline{u_i u_j}) = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \frac{1}{\rho} \frac{\partial \tau_{ij}^R}{\partial x_j} + \nu \nabla^2 \bar{u}_i, \quad (3)$$

$$\tau_{ij}^R = \rho [\bar{u}_i \bar{u}_j - \overline{u_i u_j}] \quad (4)$$

where  $\bar{u}$ ,  $\bar{p}$ , and  $\nu$  are filtered average velocity vector, pressure, and fluid kinematic viscosity, respectively.

The effect of turbulent motion is lumped into a turbulent viscosity where kinetic energy dissipation at the sub-grid scale (SGS) is considered a molecular diffusion. Nicoud and Ducros [28] introduced near-wall treatment of eddies motion to adapt local eddy viscosity by introducing the *Wall-Adapting Local Eddy-Viscosity* (WALE) model. This model maintains the asymptotic ( $\gamma^3$ ) near-wall scaling behavior for the turbulent viscosity. Consequently, it yields zero eddy viscosity in wall-bounded flows and is considered to reproduce the transition flow regime. Moreover, it is solely a product of the characteristic length and velocity scale, as shown in Eq. (5) to Eq. (8) [30].

$$v_t = L_s^2 \frac{(S_{ij}^d S_{ij}^d)^{\frac{3}{2}}}{(S_{ij}^d S_{ij}^d)^{\frac{5}{2}} + (S_{ij}^d S_{ij}^d)^{\frac{5}{4}}} \quad (5)$$

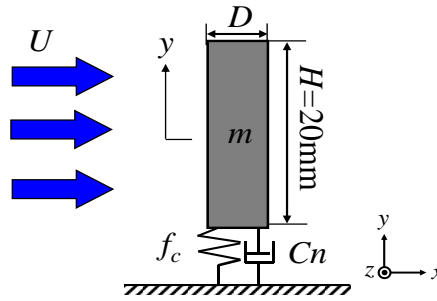
$$\overline{S_{ij}} = \frac{1}{2} (\overline{g_{ij}} + \overline{g_{ji}}), \quad (6)$$

$$S_{ij}^d = \frac{1}{2} (\overline{g_{ij}^2} + \overline{g_{ji}^2}) - \frac{1}{3} \delta_{ij} \overline{g_{kk}^2} \quad (7)$$

$$\bar{g}_{ij} = \frac{\partial \bar{u}_i}{\partial x_j}, \quad (8)$$

where the strain tensor and velocity gradient tensor rate ( $\bar{s}_{ij}, \bar{g}_{ij}$ ) are represented as  $L_s = \min(\kappa d, C_w V^{1/3})$ ,  $C_w = 0.235$ ,  $\kappa = 0.4187$  where  $L_s, \kappa, d, C_w, V, \delta_{ij}$  are the length scale, the Karman's constant energy, minimum wall spacing, WALE constant, cell volume, and Kronecker delta ( $\delta_{ij} = 0$  if  $i \neq j$ , otherwise 1 if  $i = j$ ), respectively [30]. This WALE method is adopted in the present study.

Furthermore, the governing equations were discretized based on the finite volume method approach, with the convection term as the bounded center difference. The second-order precision implicit method and the pressure-velocity coupling *PISO* logarithm were employed for time integral and momentum balance as outlined in Issa [31]. The turbulent intensity at the inlet boundary condition was 2%, similar to the experimental method employed by Lyn and Rodi [32] and Mizukami [33]. The instability-induced motion is derived from Newton's law of motion, defined by the schematic of Figure 1 below.



**Fig. 1.** The schematic of one DoF fluid-prism interaction

The instability-induced motion was calculated using the equation of motion of one degree of freedom, as shown in Eq. (6) to Eq. (11).

$$m\ddot{y} + c\dot{y} + ky = F_y \quad (9)$$

$$c = 2m\delta \frac{U}{V_r H} \quad (10)$$

$$k = 4\pi^2 m \left( \frac{U}{V_r H} \right)^2 \quad (11)$$

where  $c$  and  $k$  are the fluid damping and spring stiffness constants, respectively. For convenience, the equation of motion subsequently was discretized to cater governing components, i.e., acceleration, velocity, and displacement. The initial velocity of the prism displacement was set as  $dy/dt = A\omega_c (A = 0.1H)$ , where  $A$ ,  $\omega_c$ , and  $H$  denote the amplitude, angular frequency, and prism's height normal to the flow stream, respectively. The temporal solution of the ordinary differential equations was solved by the explicit method of the Adams-Bashforth multi-time step. The Adams-Bashforth predictor-corrector was verified for convergence and stability as in the study by Mizukami [33]. Therefore, second-order accuracy is adopted in this study.

## 2.1 Computational Domain

The schematic model of the computational domain of the cantilevered rectangular prism is shown in Figure 2. The finite rectangular prism with a cross-section height ( $H$ ) normal to the flow stream is 20 mm, depth ( $D$ ) of 10 mm, and span lengths variation ( $L$ ) of 50, 100, 150, and 200 mm. A three-dimensional O-grid domain was developed with a diameter of  $40H$ . The total number of grid points is a maximum of  $4.41 \times 10^6$  with a minimum grid width of  $5 \times 10^{-3}H$ . The Reynolds number of  $2.2 \times 10^4$  is employed in this study. The no-slip condition was applied at the prism and wall surface, and then the symmetry plane or frictionless wall was adopted at the bottom and top surfaces of the domain.

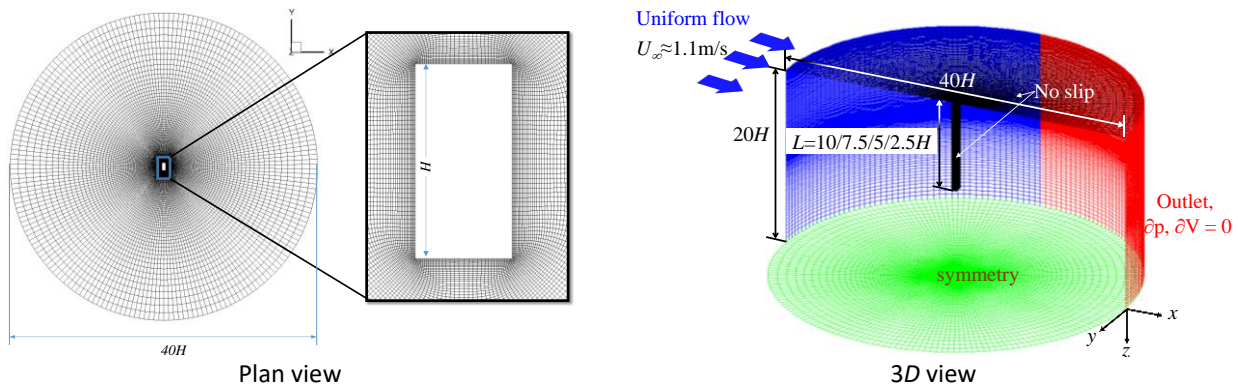


Fig. 2. Computational domain

## 2.2 Model Validation and Verification

The grid configuration was determined by considering the first cell height and growth rate of the mesh construction, which is determined by considering eddy motions at the viscous sublayer ( $y^+ < 5$ ). The result showed that the  $y^+$  calculation was 2.1, with the maximum value around 7.0. The value of  $y^+$  considers the viscous stress linear to the wall distance. It shows a good agreement with the initial  $y^+$  calculation as outlined in the studies by Versteeg and Malalasekera [34] and Kajishima and Taira [35]. The residual and mass balance were monitored for convergence criterion as outlined by Oberkampf and Trucano [36] and Oberkampf and Barone [37].

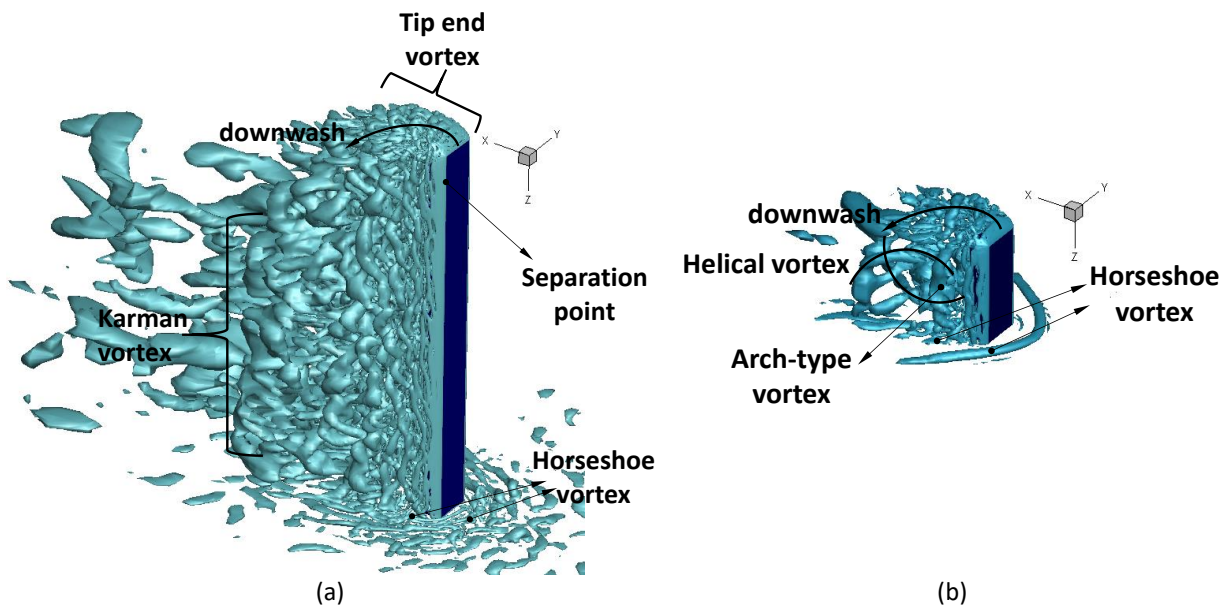
The flow feature around the infinite rectangular prism was analyzed first and then extended to the finite model with a similar method. The flow features or global quantities comparison of flow past the slender prism are shown in Table 1. The drag and base pressure coefficient are sensitive to the side ratios ( $D/H$ ) variation, where the depth  $D = \sim 0.6 H$  is the critical depth [42]. The Strouhal number of the rectangular prism with  $D/H \leq 1.0$  is less sensitive than the hydrodynamics forces. The infinite prism's drag and base pressure coefficients agree with other numerical calculations and slightly differ from the experimental investigations. The Strouhal frequency is also reasonable, in agreement with both numeric and experimental studies. However, the finite prism presents different flow characteristics from the infinite model that the global quantities are sensitive to the presence of the end tip.

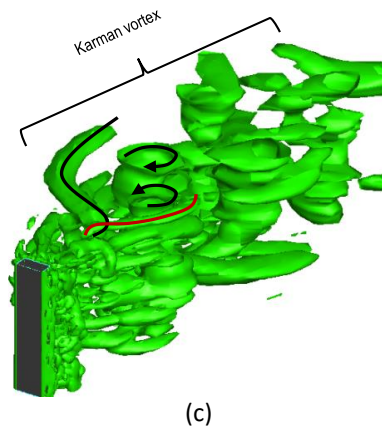
**Table 1**  
 The flow quantity parameters for the rectangular prism with a side ratio of 0.5

		$CD (= \frac{F_D}{0.5\rho \cdot U^2 H \cdot L})$	$St (= \frac{U \cdot H}{f_w})$	$-Cpb (= \frac{P}{0.5\rho \cdot U^2})$	$Re (= \rho \frac{U}{\mu \cdot H})$	Remark
Tamura and Itoh [38]	Simulation	2.81	0.13		10000	2.5D Infinite
Haque <i>et al.</i> , [39]	Simulation	2.39		1.67	22000	2.5D infinite
Knisley [40]	Experiment	2.33	0.14		22000	infinite
Nakaguchi [15]	Experiment	2.5		1.29	$2 \sim 6 \times 10^4$	Infinite
Hiroaki [41]	Experiment	2.46	0.13	1.74	38000	Infinite
Mizukami [33]	Simulation	2.84	0.132	2.33	22000	2.5D LES Infinite
Barata <i>et al.</i> , [11]	Simulation	2.80	0.14	2.40	22000	2.5D LES Infinite
Present	Simulation	1.4 – 1.5	0.11	0.6 – 0.66	$2.2 \times 10^4$	LES Finite

### 3. Results and Discussion

Figure 3 shows the local vortex region of the rectangular prisms with aspect ratios ( $L/H$ ) of 10 and 2.5. The flow structure is recognized for the tip end, von Karman, and base swirling vortex at the prism with an aspect ratio ( $L/H$ ) of 10. The fluid separation point is located at the leading edge, a common feature of the prismatic body with a sharp edge. A Sheared flow from the tip end immediately induces a downwash flow to merge to a spanwise vortex or regular Karman vortex. For the short prism ( $L/H = 2.5$ ), the tip-end vortex dominates the flow structure and diminishes the spanwise vortex. The base and symmetrical horseshoe vortex formations are recognized at the ground. The horseshoe vortex is not sensitive to the aspect ratio for all test models ( $AR = 10 - 2.5$ ) and is still recognized at the slender prism. These data are consistent with experimental findings studied by Sumner *et al.*, [43]. The downstream horseshoe vortex existence is also presented by Rastan *et al.*, [44]. The helical vortex induced by the base and tip-end vortices is clearly seen further downstream of the prism with  $L/H = 2.5$ , which is considered to replace the regular Karman vortex downstream. The arch-type vortex is located at the prism's base. The free-end absence of the finite-length prism provides a strong spinning local vortex at the trailing edge (see Figure 3(c)). It is considered to affect the fluid force quantities shown in Figure 4.



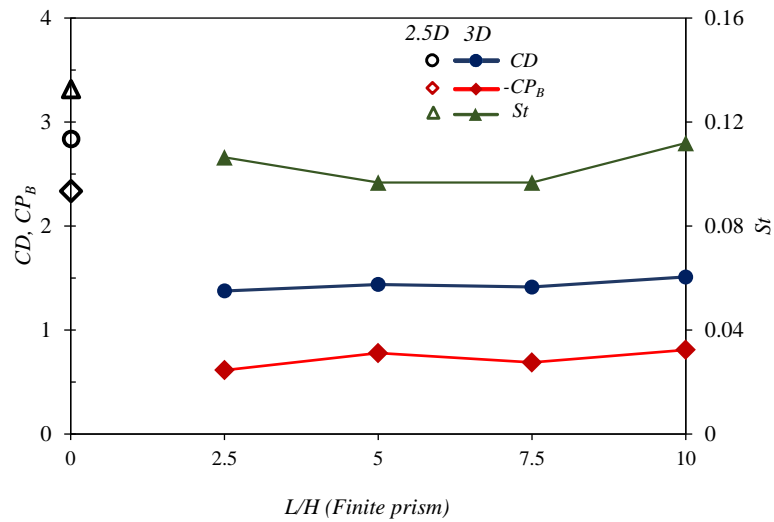


**Fig. 3.** The spanwise local vortex region on the finite length rectangular prisms; (a)  $L/H = 10$ , (b)  $L/H = 2.5$ . (c) The spinning Local vortex region in the infinite length rectangular prism (adopted from Barata *et al.*, [11])

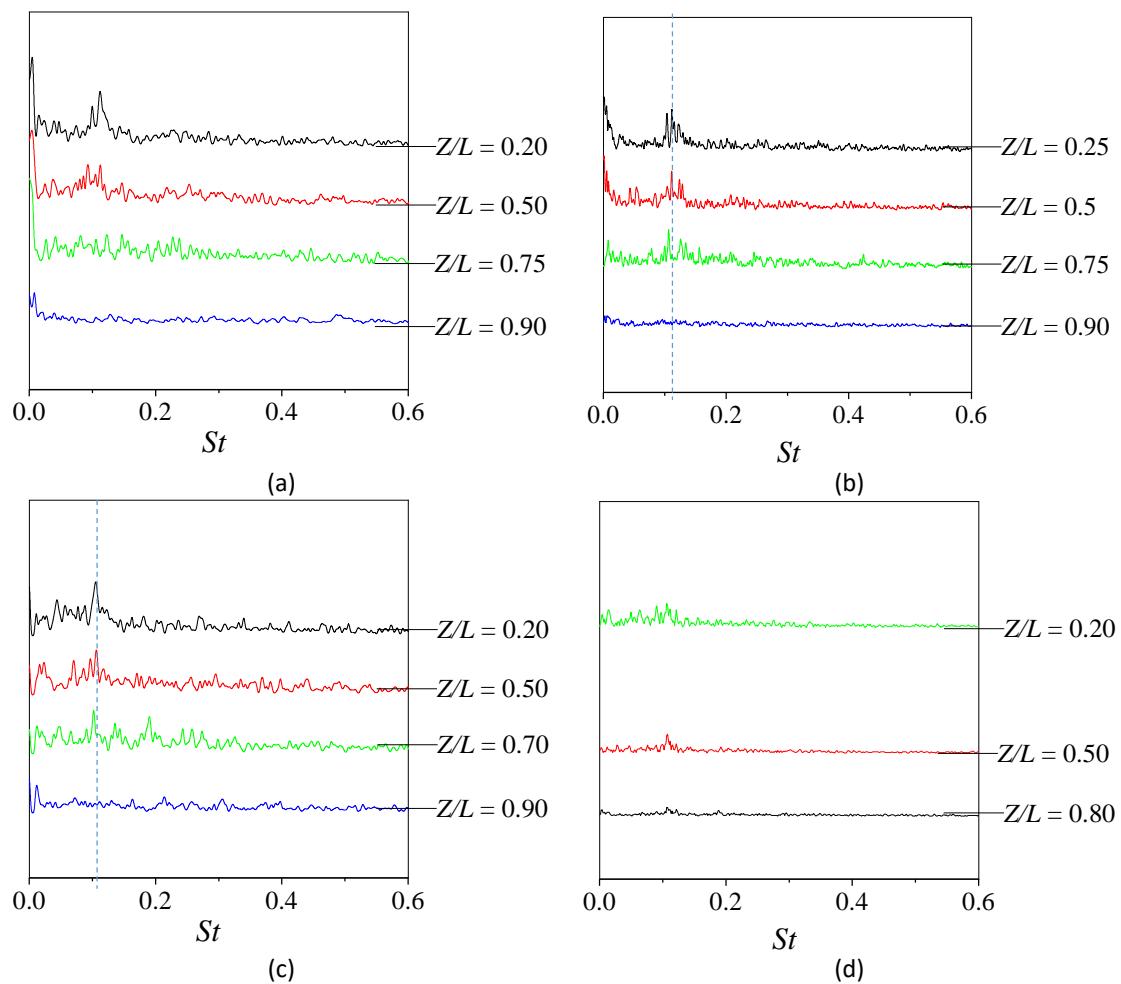
### 3.1 Fluid Forces

Figure 4 presents the variation of the fluid force components for the finite and infinite prism. All the parameters were taken at the prism's mid-length to avoid the effect of the base or tip swirling vortex. The infinite prism (2.5D) exhibited a base pressure and drag coefficient larger than the 3D prism. It implies that the end tip condition influences spanwise vortex behavior, which is a flow structure of the prism's base. Decreasing the base pressure along the span length reduces the drag force component, which correlates to the sheared layer separation at the side surface of the prism. The wake fluctuation frequency of the 3D prism is weaker than the infinite or 2.5D model (see also Figure 3). However, increasing the span length ( $L/H$ ) slightly increases the frequency of vortex shedding at the prism streamwise, as indicated in Figure 4. The velocity component ( $u$ ) fluctuation was also confirmed by calculating the lift fluctuation. This phenomenon is similar to the circular cylinder studied by Kawamura *et al.*, [45]. The fluid force components from studies of the slender rectangular prism are presented in Table 1. The Strouhal frequency along the prism spanwise is presented in Figure 5. The fluctuation velocity vector was measured at  $4H$  and  $1H$  in downstream and transverse points, respectively. It is consistent with the method suggested by Tamura and Dias [46]. The lift fluctuation calculation's result confirmed the velocity fluctuation at the mid-span. Near the prism's tip, the fluctuation component weakens, which suggests the contribution of the presence of the end tip. Unsymmetrical vortices likely exhibit their existence near the end tip. A weakened fluctuation is recognized along spanwise of the  $AR = 2.5$ , and no clear peak frequency at the selected span's planes. It suggests that turbulence intensity is lower than its counterparts. In the case of the slender prismatic body of rectangular prisms, the fluctuation frequency is not sensitive with respect to the aspect ratio, mainly for  $AR \geq 2.5$ .





**Fig. 4.** Effect of the span lengths variation on the hydrodynamic force components

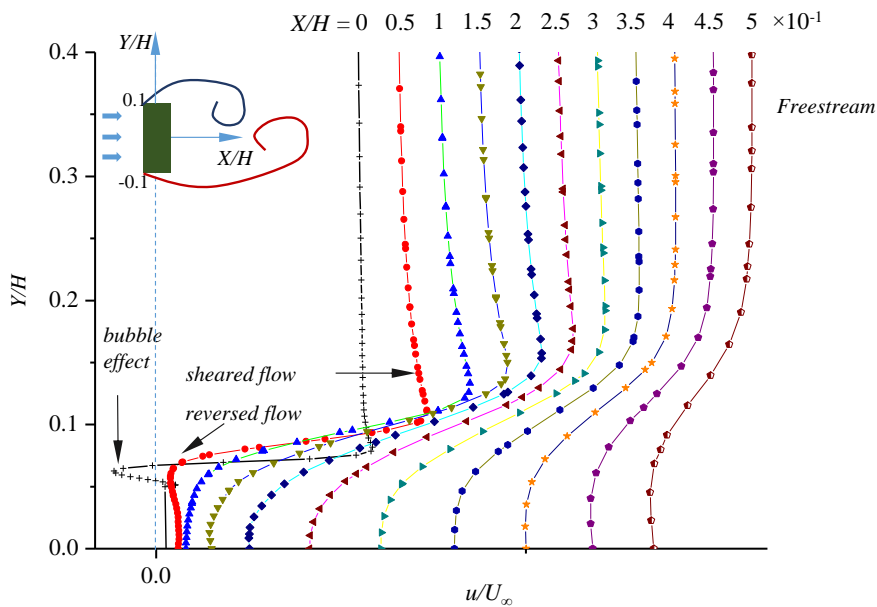


**Fig. 5.** Flow stream fluctuation around the bluff body at various the span planes ( $Z/L$ ); (a) AR = 10, (b) AR = 7.5, (c) AR = 5, (d) AR = 2.5



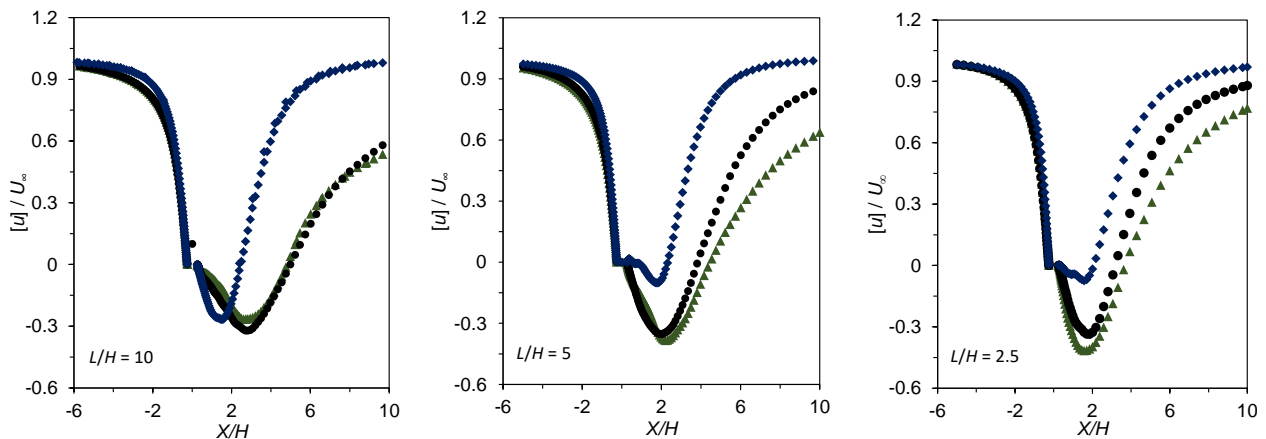
### 3.2 Velocity Vector

Figure 6 presents the velocity vector distribution along the streamwise positions at the prism's mid-plane. The velocity profile at the prism's trailing edge ( $X/H = 0.5$ ) shows different behavior from the downstream  $u$ -component because of the bubble's effect. The separated sheared layer at the side surfaces induces fluid to accelerate to a  $2H$  streamwise position, and the viscous boundary is clearly defined. The negative base pressure in the viscous layer causes potent flow entrainment at the prism's base, which is indicated by reserved flow at the prism's vicinity. The reserved flow length descends by increasing the downstream flow regime. The velocity vector deformation effect varies with the  $x$ -coordinate position.



**Fig. 6.** Streamwise mean velocity profile with respect to the transverse direction for  $L/H = 2.5$

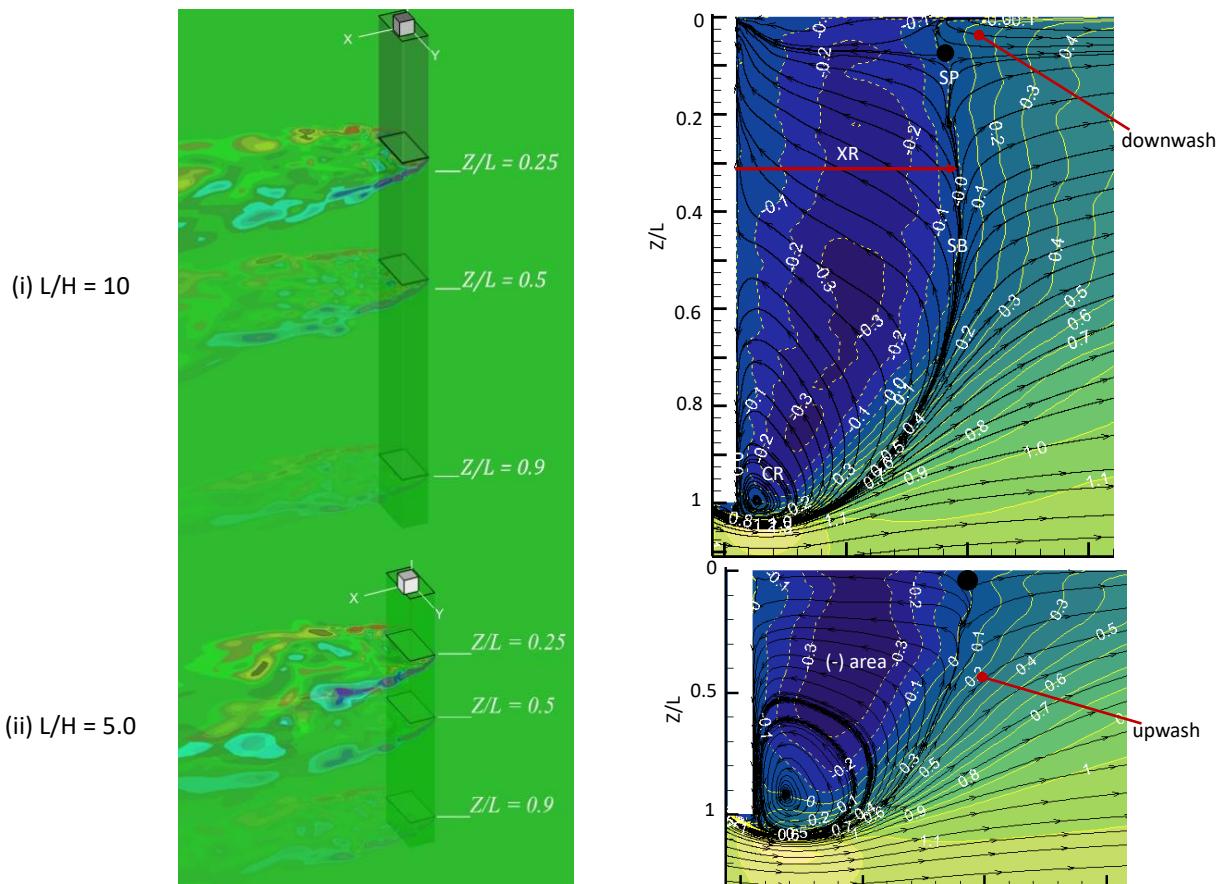
Figure 7 shows the mean velocity vector distribution at various span lengths with respect to the downstream locations. The effect of the separated shear layer from the end tip on the velocity distribution is absent at the mid-plane of  $L/H = 10$ . Moreover, the velocity ( $u$ -component) profile at the midplane is similar to the wall boundary (top boundary condition) with a no-slip condition.

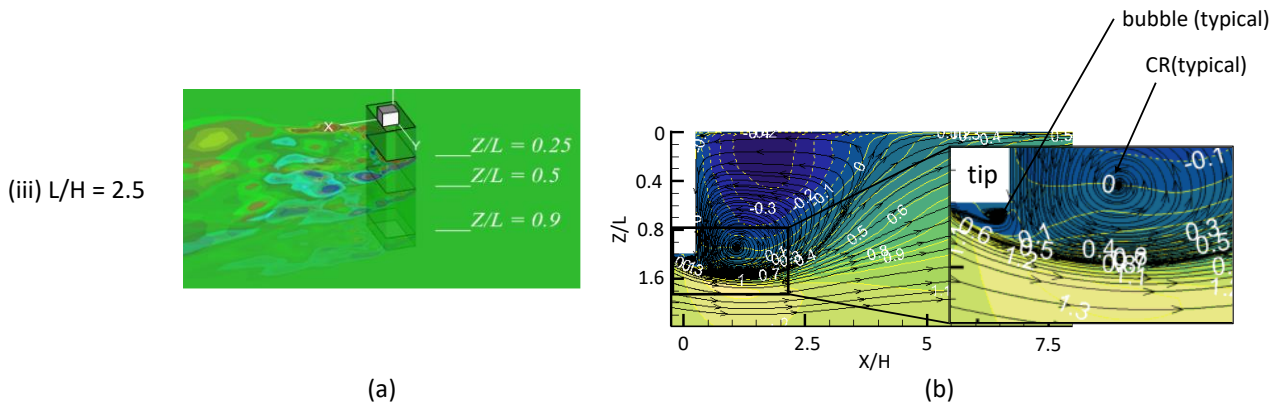


**Fig. 7.** Normalized velocity vector profile at the Z-planes variation

The reversed flow intensity near the tip ( $Z/L = 0.9$ ) descends for the prism with  $L/H \leq 5.0$ , while it is more negative at  $Z/L = 0.5$ . The upwash effect dominates the spanwise vortex (see Figure 8(b)), so the velocity profile near the tip distinctly differs from the prism with  $L/H = 10$ . The velocity profile at the mid-spanwise ( $Z/L = 0.5$ ) tends to approach the flow near the end tip at  $X > 4H$  locations. The steepness of the velocity profile at mid-plane slightly increases by reducing the aspect ratio. Far from the prism (at least  $X = 6H$ ), the upwash effect (induced by the tip's separated layer) weakens at  $L/H = 10$ , which is different from  $L/H \leq 5.0$ .

Figure 8(a) shows vorticity variation along the prism's spanwise (three different planes,  $Z/L$ ), i.e., near the wall (top boundary condition), mid-plane, and near the prism's tip. It can be observed that the vorticity's width shrinks at the tip's vicinity. The  $x$ -mean velocity distribution at the span's mid-plane ( $Y=0$ ) is presented in Figure 8(b). The center of recirculation ( $CR$ ) is located immediately at the prism's end tip, induced by the separated shear layer from the end tip (bubble effect). Further, the secondary  $CR$  is located at the base, followed by the negative velocity area. The secondary  $CR$  position shifts away from the end tip by reducing  $AR$  for  $AR \leq 5.0$ . The existence of secondary  $CR$  at the base is also enclosed between the separated shear layer from the tip and the negative span area. The downstream reversed flow ( $XR$ ) length is typically around  $4H$ . This area is bounded by the stagnation borderline ( $SB$ ) where stagnation velocity exists. It is noted that  $SB$  varies with the prism's span coordinate with respect to the downstream position.  $SB$  line from downwash slack at the critical point (saddle point,  $SP$ ) where upwash meets at the point. The saddle point tends to approach the base by reducing  $AR$ . It is consistent with the experimental data studied by Sumner *et al.*, [43].

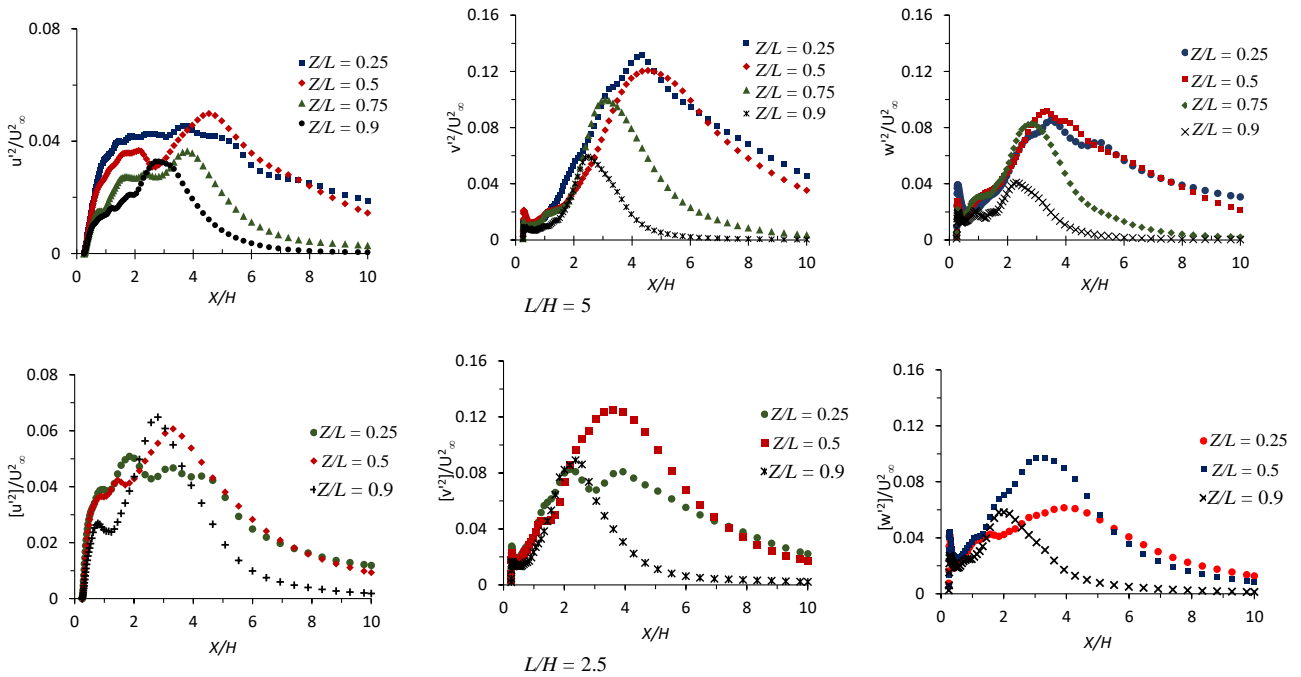




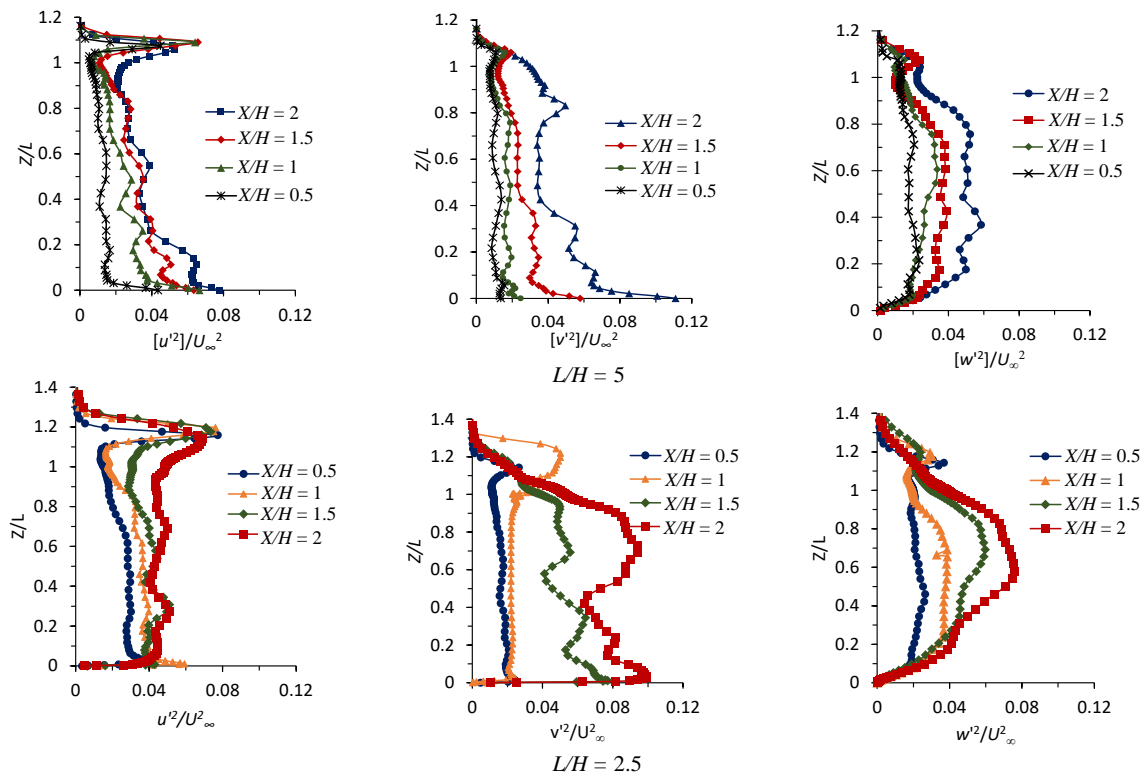
**Fig. 8.** (a) The spanwise vorticity at various  $Z$ -planes for the rectangular prisms with different aspect ratios  $L/H$  ( $=10, 5.0,$  and  $2.5$ ). (b) The mean velocity vector at the mid vertical plane ( $Y = 0$ ) which is featured by velocity contour scale behind for various aspect ratios  $L/H$  ( $=10, 5.0,$  and  $2.5$ )

### 3.3 Reynolds Stress Components

The Reynolds normal stress components ( $u'^2$ ,  $v'^2$ , and  $w'^2$ ) for incompressible flow are presented in Figure 9 and Figure 10. For brevity's sake, Figure 9 and Figure 10 only show the Reynolds normal stress components along the prism's span of  $AR = 5$  and  $2.5$ . Figure 9 presents the Reynolds normal stress components for several  $Z$ -planes with respect to the downstream positions. Generally, the peak of turbulence shrinks near the end tip and moves close to the prism. The peak of streamwise normal stress  $[u'u']$  near the tip of  $AR = 2.5$  is slightly higher than that of the mid-plane ( $Z/L=0.5$ ). It confirms that the interference of the end tip vortex to the spanwise vortex diminishes the regular Karman vortex. It is also confirmed in Figure 5. The peak shrinkage of the streamwise normal stress  $[u'u']$  indicates the reduction of the bubble region. Similarly, it is also found in the transverse normal stress  $[v'v']$  and spanwise normal stress  $[w'w']$  components. Reynolds normal stress variations along the spanwise with respect to the downstream positions are presented in Figure 10. A dominant peak of streamwise normal stress is observed near the tip region, both  $AR = 5$  and  $2.5$ , due to the separated shear layer from the prism's tip. However, it is not found in transverse ( $v'v'$ ) and spanwise ( $w'w'$ ) normal stress. The intensity of streamwise, transverse, and spanwise normal stress is decreased with respect to the downstream locations for both aspect ratios ( $AR$ ).

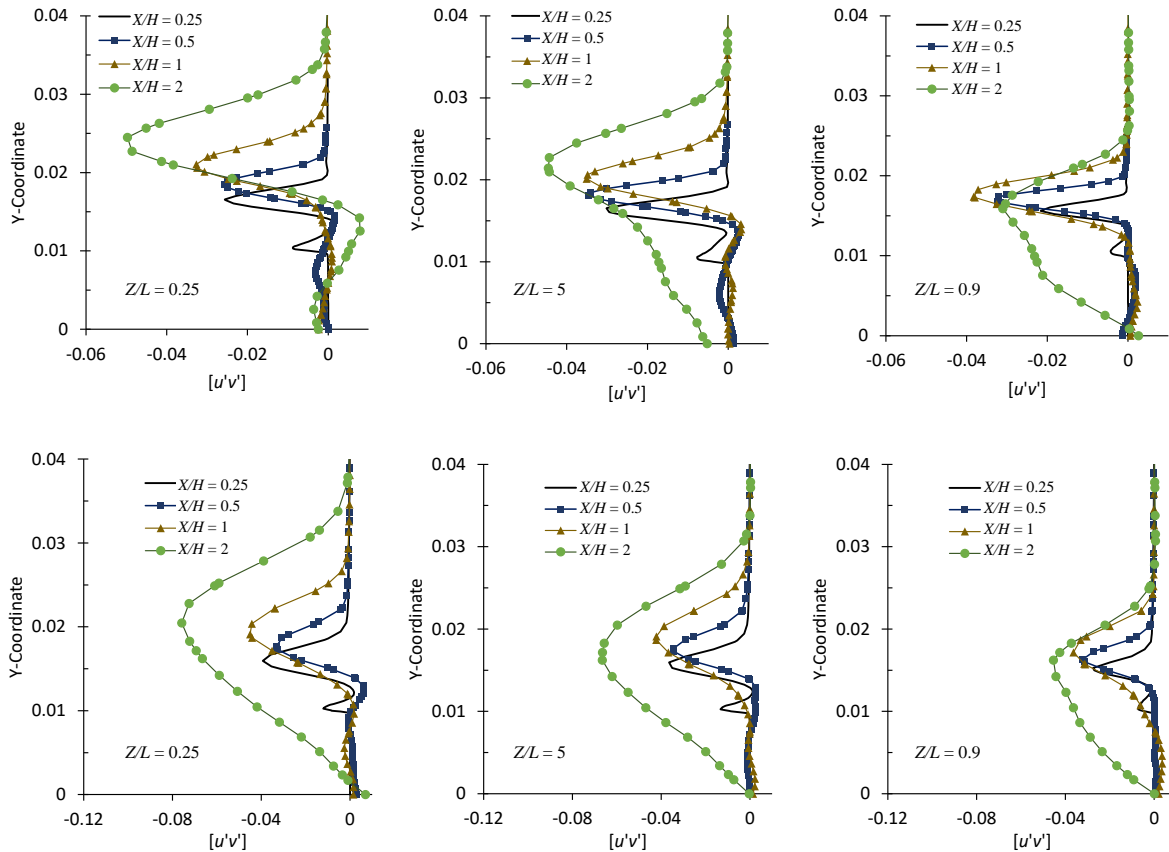


**Fig. 9.** Variation of the streamwise Reynolds normal stress components along the prism's span



**Fig. 10.** Spanwise Reynolds normal stress components at the downstream positions

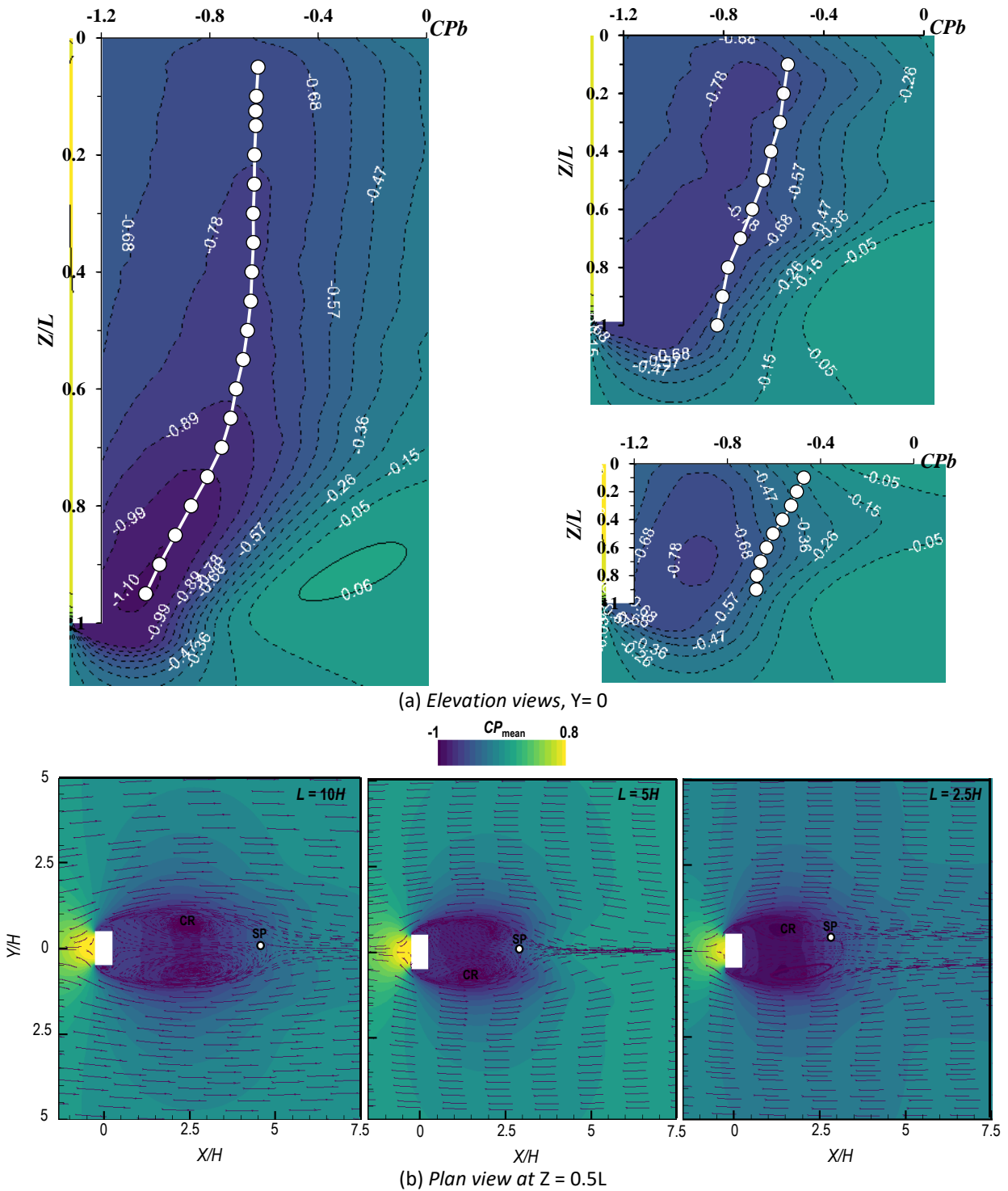
The shrinkage of Reynolds shear stress near the tip-end is also observed, as indicated in Figure 11. The peak of Reynolds shear stress components with respect to downstream locations is far from the prism. The momentum flux at the prism with  $AR = 2.5$  is larger than  $AR = 5$ , which indicates energy transported is large.



**Fig. 11.** Spanwise Reynolds shear stress components at the downstream locations

### 3.4 Pressure Field Components

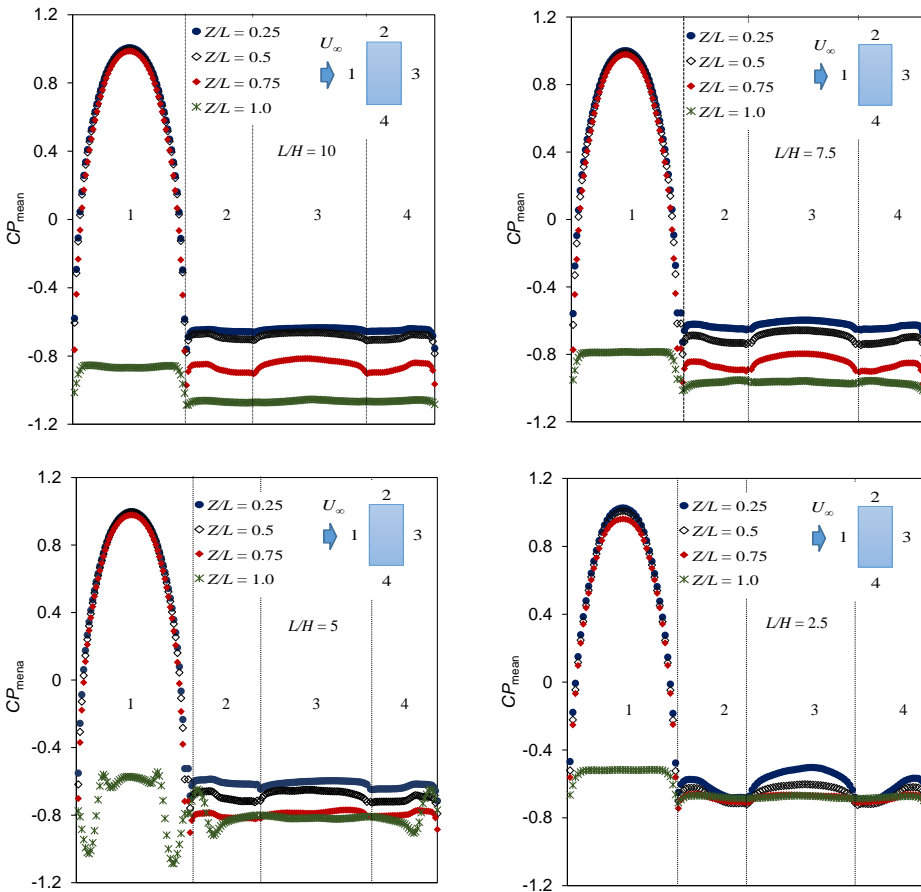
The base pressure distribution along the prism's span length is shown in Figure 12(a). Span length variation affects the base pressure distribution, where the tip end area is the lowest pressure among the flow field. It implies that potent flow entrainment is typically located near the tip end, mainly for  $AR=10$ . Reducing the prism's span length recovers the base pressure mainly near the end tip. It can be said that flow entrainment at the tip's vicinity descends. The section view of pressure contours taken at the prism's midplane of  $Z = 0.5 L$  is shown in Figure 12(b). The most negative base pressure area shrank by reducing the aspect ratio ( $AR$ ). The center of flow recirculation ( $CR$ ) is typically not changed for  $L/H \geq 5$ , as mentioned in the previous section. However, the critical point or saddle point ( $SP$ ) descends by reducing the aspect ratio ( $AR$ ).



**Fig. 12.** (a) The mean base  $CP$  distribution which is featured mean  $CP$  contour at vertical mid-plane ( $Y=0$ ), and (b) Mid plane of the mean pressure contours which is featured by velocity vector component ( $Z=0.5L$ )

Figure 13 presents the pressure distributions along the prism span length at various section planes. The pressure distribution at the side and base surfaces shows similar characteristics that are varied regularly along the span length. The alteration of base pressure along the prism's span indicates the level of the downwash/upwash flow effect. Meanwhile, the pressure profile at side surfaces is attributed to the bubble effect induced by the separated shear layer from the prism's leading edge. The prism with  $AR \geq 5.0$  shows a similar pressure profile near the tip end ( $Z/L = 0.75$ )

with more negative pressure than other span positions (note that the pressure profile at the tip,  $Z/L = 1$ , is omitted). However, pressure distribution at the prism near the end tip for  $AR \leq 5.0$  show a distinctive pressure profile that indicates flow structure change of  $AR \leq 5.0$  with respect to the end tip end effect. The investigations on the end tip effect suggest the critical length with respect to the cross-section height or diameter is  $AR = \sim 3.0$ . [45,47].

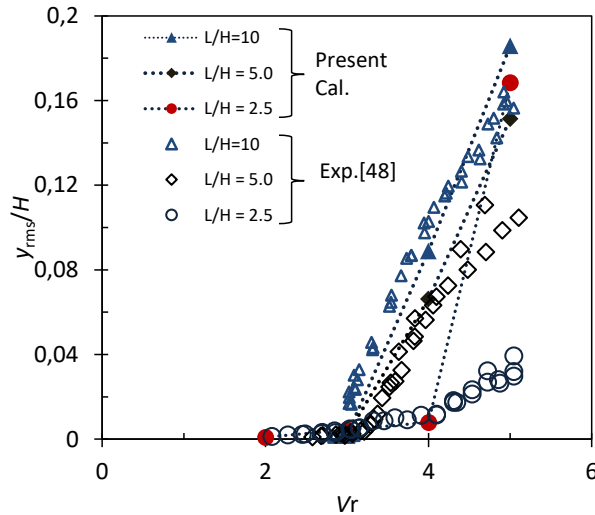


**Fig. 13.** The mean pressure coefficient profiles at the prism’s surfaces along the span

### 3.5 Instability-Induced Motion

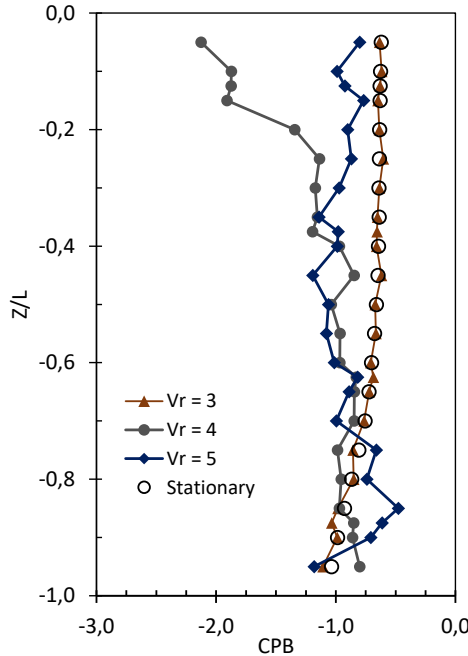
The predicted transverse motion of the prism with respect to non-dimensional velocity is presented in Figure 14. The non-dimensional parameter of the prism's vibration is defined by a reduced velocity  $V_r (= U_\infty/f_c \times H)$  where  $U_\infty$  is the flow stream velocity,  $f_c$  and  $H$  denote frequency characteristic and cross-section height of the prism, respectively. A comparison from experimental data was taken from Barata *et al.*, [48], which measured the tip displacement of a cantilevered type slender rectangular prism ( $D= 0.5H$ ). Mizukami *et al.*, [49] compared the 2.5D predicted motion of a rectangular cylinder to experimental data of a cantilevered rectangular prism. Predicted transverse motion of the prism typically showed a good agreement at low reduced velocity ( $V_r \leq 4$ ). In this case,  $V_r$  calculation was only taken up to  $V_r = 5$ , which shows a deviation from the experimental at  $V_r > 4$ .





**Fig. 14.** Prism’s dynamic response with respect to non-dimensional velocity

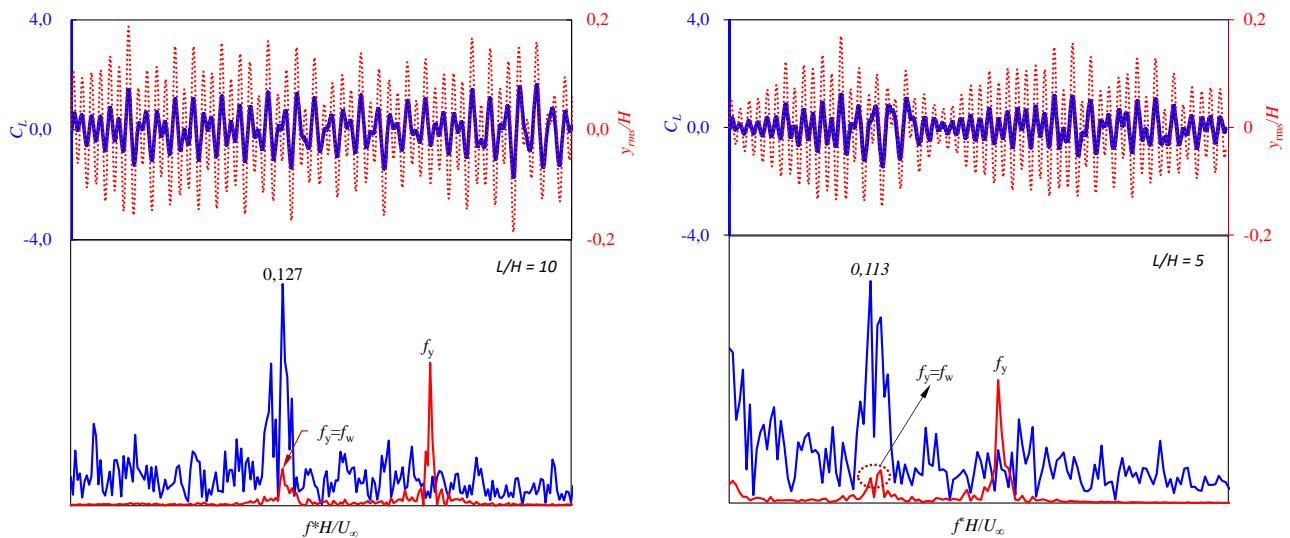
Figure 15 presents the effect of the prism's dynamic response on the mean base pressure profile. Overall, base pressure is sensitive to the prism's dynamic response intensity, which vibrating prism induces the base pressure to be more negative along the prism's span than the stationary prism. However, the base pressure increase near the prism's tip, which suggests the prism's motion reduces the bubble effect at the prism's downstream. In contrast, far from the tip or ( $Z/L \approx 0.5$ ), the prism's dynamic response induces the base pressure to be more negative (see vorticity contours in Figure 18).



**Fig. 15.** The span mid plane ( $Y = 0$ ) of the mean base pressure coefficient variation along the span length of stationary and vibrating prisms with  $AR = 10$

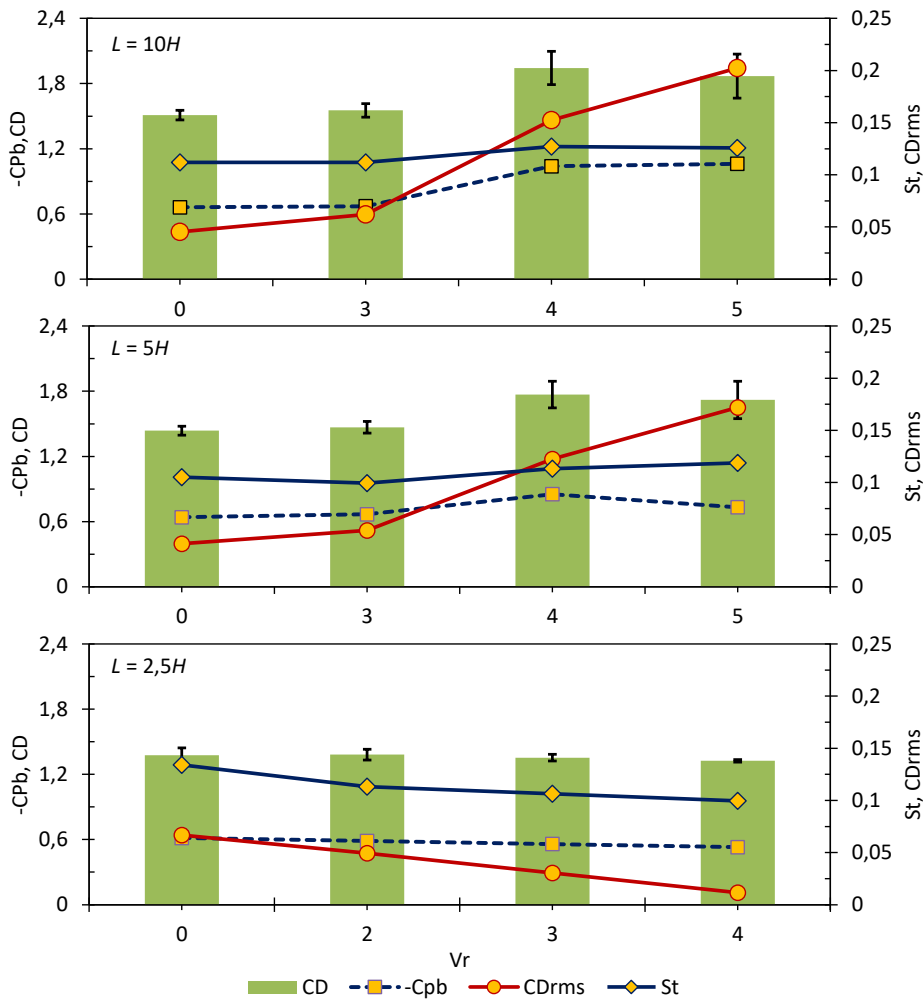
Figure 16 compares time histories of  $CL$  component and prism dynamic response, while  $FFT$  spectra of frequencies are shown downward in Figure 16. Both the  $CL$  component and the prism's

dynamic response are synchronous. *FFT* spectra of vibration response frequency ( $f_y$ ) were calculated based on velocity vector fluctuation at the prism downstream, as outlined in the previous section. The primary peak of wake fluctuation ( $f_w$ ) is different from vibration frequency, both  $L/H = 10$  and 5. It is a common feature of instability-induced motion of rectangular prism with an aspect ratio less than critical depth ( $=D \leq 0.67H$ ). The studies related to this characteristic can be found in researches by Barata *et al.*, [11], Nakamura and Hirata [42], Ohya [50], etc. A secondary peak vibration fluctuation also locked in wake fluctuation for both prism models, as shown in Figure 16. Barata *et al.*, [11,51] presented that fully synchronous vibration frequency occurs at high amplitude response where the secondary peak of vibration *FFT* spectra vanishes. It implies that the vibration motion alters the flow structure in the wake (see Figure 18(b)).



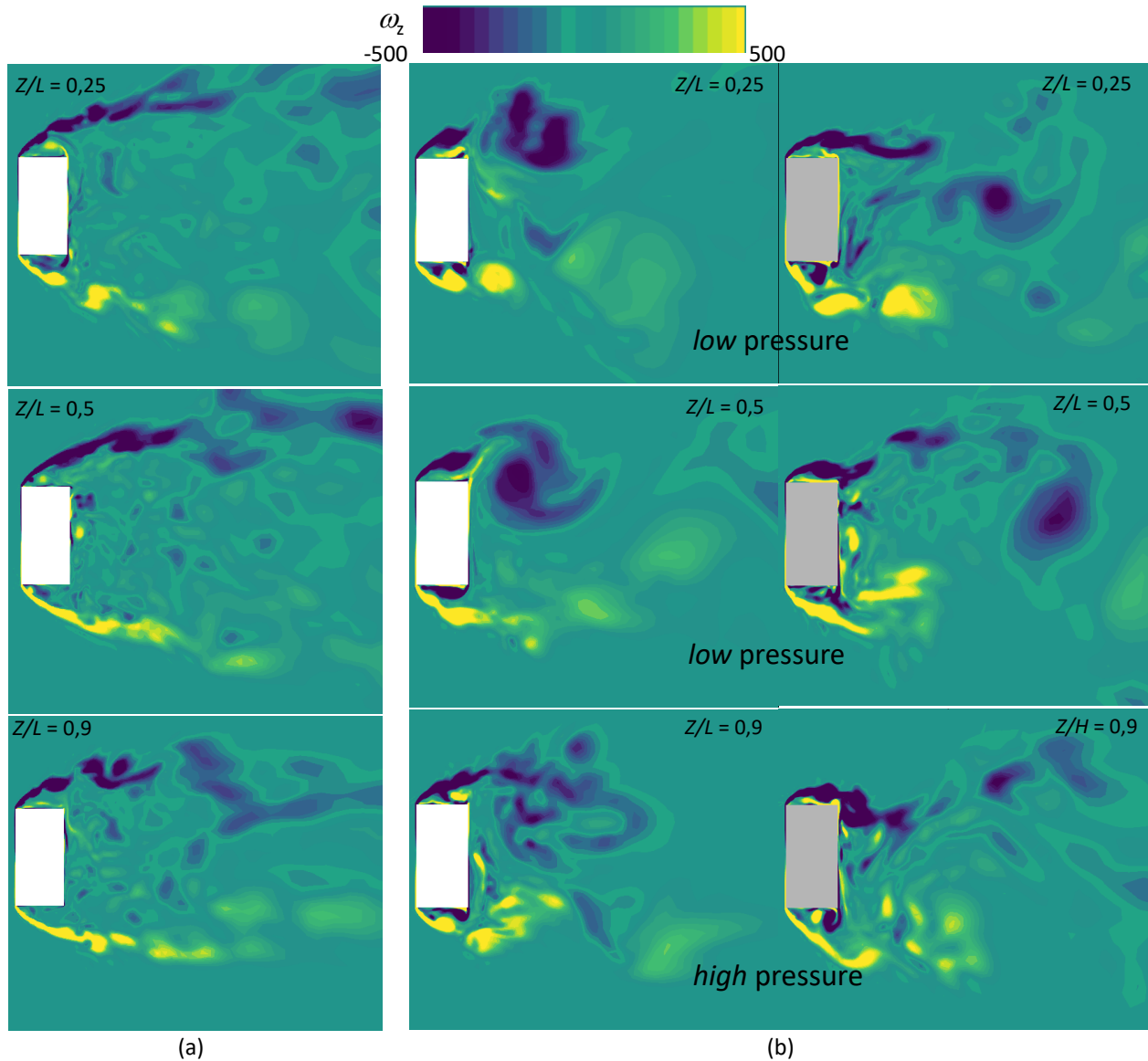
**Fig. 16.** Time histories of lift force and prism dynamic response (upper), and *FFT* spectra of frequencies (bottom) at  $V_r = 4$

The effect of vibration response on the flow quantities is presented in Figure 17. Drag ( $CD$ ) and base pressure coefficient ( $CPB$ ) components changed with  $V_r$  variation, while Strouhal frequency exhibited a small change for  $AR \geq 5.0$ . Moreover,  $CD$  fluctuation is more sensitive to  $V_r$  variation, which suggests a significant deviation in high reduced velocity ( $V_r$ ). The contribution of the end tip effect on the flow quantities alteration is clearly shown at  $AR = 2.5$ , where all flow quantities descended with respect to  $V_r$  variation.



**Fig. 17.** Flow quantities of the prisms model with various vibration intensities

The effect of  $V_r$  variation on the flow field characteristics is shown in Figure 18. The symmetrical vortices can be observed at the stationary prism along the span length planes (see Figure 18(a)). Distinctive flow patterns appear in the vibrating prism along the span-length planes that exhibit unsymmetrical flow patterns. The level of the swirling vortex at the base is dependent on  $V_r$ . The contribution of the end tip effect can be observed in that negative vorticity descend near the tip-end, and the swirling negative vortex is seen at the mid-plane ( $Z/L = 0.5$ ). Shrinkage of the recirculation bubble region is clearly shown at vibrating prism vorticity contours. The global fluid force quantity, such as the drag coefficient, shows significant fluctuation at a high vibrating prism ( $V_r = 4$ ), as confirmed in Figure 17.



**Fig. 18.** Vorticity contours profile at various prism's span planes ( $Z/L$ ). (a) Stationary state for the prism with  $L = 10H$ , (b) Vibrating prism at  $V_r = 4,0$  for the prisms with  $L = 10H$ , and  $L = 5H$

#### 4. Conclusions

3D Large Eddy Simulation turbulence model was used to investigate the flow features around the slender rectangular prism with cross-section height ( $H$ ) to streamwise depth ( $D$ ) ratio or side ratio ( $D/H = 0.5$ ) with Reynolds number  $Re = 22000$ . The presence of the end tip alters the flow features, which is indicated by the alteration of global quantities such as drag, non-dimensional frequency, and base pressure coefficient components. The global quantities of the 3D slender prism are different from the 2D prism, suggesting the flow structure difference between them. The effect span length variation on the flow features unveiled the flow structure change on 3D slender rectangular. It implies that the 3D flow structure changes at  $L < 5H$ , where the separated shear layer for the tip end diminishes the regular Karman vortex and reduces the vorticity area. The flow features of the vibrating prism showed distinctions compared to the stationary structure. The effect of the vibrating prism on the flow structure exhibited potent entrainment at the base where the base pressure is low. However, it did not occur near the tip-end ( $Z/L \sim 0.9$ ), where the base pressure increased at  $V_r = 5$ .

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