CFD Letters

Journal homepage:
https://semarakilmu.com.my/journals/index.php/CFD_Letters/index
ISSN: 2180-1363

Transient Free Convection Mass Transfer of Second-grade Fluid Flow with Wall Transpiration

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ARTICLE INFO

Article history:
Received 11 June 2021
Received in revised form 7 October 2021
Accepted 8 October 2021
Available online 20 November 2021

Keywords:
non-Newtonian fluid; second-grade fluid; transient free convective flow; wall transpiration; mass transfer

ABSTRACT

The study of mass transfer in the non-Newtonian fluid is essential in understanding the engine lubrication, the cooling system of electronic devices, and the manufacturing process of the chemical industry. Optimal performance of the practical applications requires the appropriate conditions. The unsteady transient free convective flow of second-grade fluid with mass transfer and wall transpiration is concerned in the present communication. The behavior of the second-grade fluid under the influence of injection or suction is discussed. Suitable non-dimensional variables are utilized to transform the governing equations into non-dimensional governing equations. A Maple solver "pdsolve" that is using the centered implicit scheme of a finite difference method is utilized to solve the dimensionless governing equations numerically. The effects of wall injection or suction parameter, second-grade fluid viscoelastic parameter, Schmidt number, and modified Grashof number on the velocity and concentration profiles are graphically displayed and analyzed. The results show that with increasing wall suction, viscoelastic parameter, and Schmidt number, the velocity and concentration profiles decrease. Whereas, the velocity profiles show an opposite tendency in situations of wall injection. The wall suction has increased the skin friction and also the rate of mass diffusion in the second-grade fluid

1. Introduction

Theoretically, fluid is divided into two types which are Newtonian and non-Newtonian fluids. A Newtonian fluid is a fluid that obeys Newton's second law of viscosity [1]. Their shear stress at each point is linearly proportional to the strain rate at that point and a constant of proportionality is known as the viscosity. This concept was first introduced by Isaac Newton. The behaviour of Newtonian fluid can be shown as follow [2]

$$\tau_{yx} \propto \frac{du}{dy} \text{ or } \tau_{yx} = \mu \frac{du}{dy}, \tag{1}$$

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https://doi.org/10.37934/cfdl.13.11.3552

where τ_{yx} is shear stress, μ is dynamic viscosity, and du/dy is the rate of strain or velocity gradient. In common terms, this means the fluid continues to flow, regardless of the forces acting on it. For example, all gases are Newtonian, as are most common liquids such as water, hydrocarbons, oils, and air [3].

However, fluids in which shear stress is not proportional to the velocity gradient are called non-Newtonian fluids [4]. For example, ketchup, custard, toothpaste, starch suspensions, paint, blood, and shampoo [5]. This behavior may be represented by the power-law model [2],

$$\tau = k\left(\frac{du}{dy}\right)^n ; n \neq 1, \tag{2}$$

where n is called flow behavior index and k is the flow consistency index. When the values of $n=1$ and $k=\mu$, Eq. (2) reduces to Eq. (1) called Newton's law of viscosity. Although the concept of viscosity is commonly used in fluid mechanics to characterize the shear properties of the fluid, it can be inadequate to describe non-Newtonian fluids. This fluid is usually divided into three main categories which are differential type, rate type, and integral type [6]. Differential and rate type models are used to describe the response of fluids that have slight memory such as dilute polymeric solutions, while the integral models are used to describe materials such as polymer melts that have considerable memory. For example, Rajput *et al.,* [7] had investigated the Casson-type fluid over a vertical plate under the impact of chemical reaction with an Arrhenius temperature, thermal radiation, and heat source/sink. Micropolar fluid past a vertical porous plate with heat absorption, chemical reaction, and Joule heating was discussed by Shamshuddin *et al.,* [8]. Another type of non-Newtonian fluid known as Oldroyd 8-constant fluid was analyzed by Salawu *et al.,* [9]. There are many kinds of non-Newtonian fluid and second-grade fluid is one of the well-known non-Newtonian fluids due to its wide practical applications in manufacturing and industry.

Second Grade Fluid is one of the most popular subclasses of differential type of non-Newtonian fluid. Coleman and Noll [10] were the first to introduce the second-grade model in this sector in 1960. Due to its relatively simple structure, this model is employed to study various problems lately. The study of free convection flow of second-grade fluid is a challenging problem that being introduce in fluid mechanics. Free convection or natural convection is the flow induced by buoyancy forces that arise from density differences caused by temperature variation in the fluid [11]. Nowadays, many researchers have investigated the problem in fluid flow in either Newtonian or non-Newtonian fluid [12-19]. Among them, few have considered free convection flow on second-grade fluid. The elastic properties of the second-grade flow are studied with certain conditions. The conditions that often discussing are heat transfer, flow on vertical plates, stretching sheet, incompressible, steady and unsteady [20-22]. Furthermore, the second-grade fluid has also the characteristic of viscoelastic [23-25].

Aman *et al.,* [26] had analytically analyzed the heat and mass transfer behavior of a second-grade fluid flow over a flat plate with wall suction and injection. The authors found that the velocity and temperature distribution decreases when the suction/injection parameter increase. Recently, Dwivedi *et al.,* [27] acknowledged the transient effect of a second-grade fluid flows past a vertical cylinder. The research output indicated that the velocity profiles and the mass flow rate were dismissed by the second-grade fluid parameter. Krishna *et al.,* [28] discussed the effect of the Hall and ion slip on an unsteady rotating MHD convective second-grade fluid flows over a vertical moving permeable surface with heat generation. The authors applied the perturbation methodology to solve the governing equations and determined that the second-grade fluid parameter has reduced the primary velocity throughout the fluid region. Many researchers have studied the flow behavior of such kind of fluid due to its plenty of applications in the industry such as in enhancing lubricant viscosity, manufactured products, and technological advancement [29-31].

On the other hand, the phenomenon of mass transfer has gained the attention of plenty of researchers recently due to wide applications in many processes, such as absorption, evaporation, adsorption, drying, precipitation, membrane filtration, and distillation [32]. Mass transfer is the net movement of mass from one location, usually meaning a stream, phase, fraction, or component, to another. In mass transfer problems especially free convection cases, the velocity conditions at the wall play an important role in several industrial applications. The fluid convection resulting from the wall temperature has attracted the interest of researchers to investigate and produce new problems. Many researchers have studied the effect of wall transpiration mathematically where many methods are induced to solve the equation to get the real picture of the fluid flow [33-37]. Jang *et al.,* [33] have researched wall transpiration effects on developing mixed convection heat transfer in inclined rectangular ducts where they assume the entering upward flow of the liquid to be steady and laminar. Ishak *et al.,* [34] had studied the effect of wall transpiration on the boundary layer problem with both theoretical and numerical methods were used.

Furthermore, Weidman *et al.,* [35] have done a numerical study for the effect of transpiration on self-similar boundary layer flow over moving surface with the transpiration considered the flow at the free stream. Chang *et al.,* [36] discussed the effect of the bouncy force on the transient Walters-B' viscoelastic boundary layer flow over a vertical porous plate with species diffusion. The authors found that the viscoelastic parameter has enhanced the velocity distribution close to the plate surface. However, the physical quantities such as the skin friction and Sherwood number were not analyzed by Chang *et al.,* [36].

The flow of this fluid through a porous medium has gained the interest of researchers in fluid studies [38-41]. The interest in this field is due to the wide range of applications either in engineering and technology such as boundary layer control, transpiration cooling, gaseous diffusion, and reducing the frictional resistance between two solid surfaces moving relative to each other. In transpiration cooling, the porous media will form a surface to make an easy diffusion or heat exchanger. Other than that, the flow in the channel is also quite famous such as vertical and asymmetric. This research has been carried out by previous studies [42-47]. This research is being carried out because many applications have been discovered for this problem for example in transpiration cooling, the walls of a channel carrying a hot fluid are made of a porous material through which fluid is injected to form a protective layer of cooler fluid near the wall.

Motivated by Chang *et al.,* [36], the present study extends the work done by novelty considering the second-grade fluid. The second-grade model has almost the same as the model of Walter-B fluid which involves viscosity and elastic properties but the second-grade model involves two derivatives in the stress-strain tensor relationship. Besides, the second-grade fluid has a wide application in polymer processing industries, power engineering, petroleum production, and chemical engineering [27,28]. In addition, the study of fluid flow with the effect of wall transpiration has got attracted many people due to its application. This effect has been investigated by many researchers previously and applied to certain conditions by using the controlled variable. For example, researchers have to look up the flow of this fluid near the wall. The fluid is an injection from one point and suction with another direction. This will give an understanding of the flow of viscoelastic fluid along the wall. Based on the above discussion, it is interesting to study the behavior of the fluid motion influenced by the transpiration effect. Therefore, the present study aims to investigate the second-grade fluid of free convection flow with mass transfer and transpiration effect. The developed governing equations are firstly converted into a dimensionless system by using some suitable dimensionless variables. The transformed equations are then solved numerically by using the Maple solver “pdsolve”. The effect of the pertinent parameters such as the second-grade parameter, suction/injection parameter, and modified Grashof parameter on the velocity and concentration profiles is displayed in graph form and discussed. Besides, the variation of the wall shear stress and Sherwood number regarding the various values of the parameters are tabulated demonstrated.

2. Mathematical Formulation

The free convection mass transfer flow induced by mass transpiration in a second-grade fluid is considered. The wall suction and injection represent the permeable wall is taken into account. The cartesian coordinate system with an $x-$axis parallel to an infinite rigid plate and $y-$axis normal to the plate is practiced. An unsteady unidirectional flow of an incompressible and nonlinear viscoelastic second-grade fluid at the region $y>0$ is investigated. The viscoelastic parameter k_0 is chosen small. Initially, both the fluid and plate are starting to flow with constant U_0. A constant suction velocity is present at the plate, V_w. At the plate, the concentration is setted to be C_w and the concentration in the ambient fluid at initially is fixed to be C_∞. The physical model of the fluid system is presented in Figure 1.

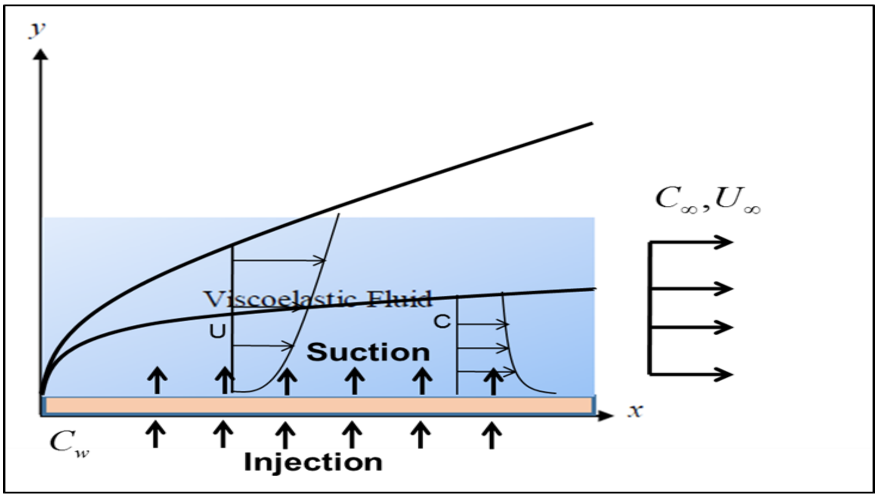


Fig. 1. Physical diagram and coordinate system

Based on boundary layer and Boussinesq approximation subjected to effect considered, the fluid problem is governed mathematically as [26,36],

$$\frac{\partial u}{\partial t}-V_w\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+k_0\left(\frac{\partial^3 u}{\partial y^2\partial t}-V_w\frac{\partial^3 u}{\partial y^3}\right)+g\beta(C-C_\infty), \tag{3}$$

$$\frac{\partial C}{\partial t}-V_w\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}, \tag{4}$$

with the subjected initial and boundary conditions [36]

$$\begin{aligned} &u(y,t)=0, \quad C(y,t)=C_\infty; \quad y>0, t\le 0 \\ &u(0,t)=0, \quad C(0,t)=C_w; \quad t>0 \\ &u(\infty,t)=0, \quad u_t(\infty,t)=0, \quad C(\infty,t)=C_\infty \end{aligned} \tag{5}$$

where $u, \upsilon, \rho, k_0, g, \beta, C$ and D is the velocity in the $x-$direction, kinematic viscosity, fluid density, second-grade viscoelasticity parameter, acceleration due to gravity, volumetric coefficient of mass expansion, species concentration, and chemical molecular diffusivity respectively. The dimensionless variable as given in this Refs. [26,36] is then introduced,

$$u^* = \frac{u}{U_0},\ y^* = \frac{U_0 y}{\upsilon},\ t^* = \frac{U_0^2 t}{4\upsilon},\ \phi = \frac{C-C_\infty}{C_w - C_\infty} \tag{6}$$

where ϕ, C, C_∞ and C_w is the dimensionless species concentration in the fluid near the wall, species concentration in the fluid near the wall, species concentration in the fluid far away from the wall, and constant species concentration in the fluid at the wall respectively. The Eq. (6) is rewritten as,

$$u = u^* U_0,\ t = \frac{4t^*\upsilon}{U_0^2},\ y = \frac{y^*\upsilon}{U_0}, C = \phi\left(C_w - C_\infty\right) + C_\infty. \tag{7}$$

Using Eq. (7), the dimensionless partial differential Eq. (3)-(4) can be written as,

$$\frac{\partial u^*}{\partial t^*} - 4V_0 \frac{\partial u^*}{\partial y^*} = 4\frac{\partial^2 u^*}{\left(\partial y^*\right)^2} + \Gamma\left(\frac{\partial^2 u^*}{\partial t\left(\partial y^*\right)^2} - 4V_0 \frac{\partial^3 u^*}{\left(\partial y^*\right)^3}\right) + G_m \phi, \tag{8}$$

$$\frac{\partial \phi}{\partial t^*} - 4V_0 \frac{\partial \phi}{\partial y^*} = \frac{4}{S_c}\frac{\partial^2 \phi}{\left(\partial y^*\right)^2}, \tag{9}$$

where

$$V_0 = \frac{V_w}{U_0}, \quad \Gamma = \frac{k_0 V_0^2}{\upsilon^2}, \quad G_m = \frac{g\beta\left(C_w - C_\infty\right)\upsilon}{V_0^2}, \quad S_c = \frac{\upsilon}{D}.$$

By dropping the * sign for simplicity, dimensionless Eq. (8) and Eq. (9) is represented as,

$$\frac{\partial u}{\partial t} - 4V_0 \frac{\partial u}{\partial y} = 4\frac{\partial^2 u}{\partial y^2} + \Gamma\left(\frac{\partial^2 u}{\partial t \partial y^2} - 4V_0 \frac{\partial^3 u}{\partial y^3}\right) + G_m \phi, \tag{10}$$

$$\frac{\partial \phi}{\partial t} - 4V_0 \frac{\partial \phi}{\partial y} = \frac{4}{S_c}\frac{\partial^2 \phi}{\partial y^2}. \tag{11}$$

Again, using Eq. (7) into the initial boundary condition in Eq. (5), the dimensionless initial and boundary conditions becomes,

$$
\begin{aligned}
&u(y,0)=0, \quad y>0, \qquad u(0,t)=0, \quad u(\infty,t)=0, \quad u_t(\infty,t)=0 \;\; t>0 \\
&\phi(y,0)=0, \quad y>0, \qquad \phi(0,t)=1, \quad \phi(\infty,t)=0, \quad t>0.
\end{aligned} \tag{12}
$$

In mathematics, finite-difference methods (FDM) are numerical methods for solving differential equations by approximating them with difference equations, in which finite differences approximate the derivatives. Today, FDMs are the dominant approach to numerical solutions of partial differential equations. Many research had solved the fluid problems by using the FDMs. Siddiqui *et al.,* [44] has studied an unsteady second-grade fluid model between circular plates. FDMs have been used to solve the governing equation. The results have shown that the stability and convergence of the method for this kind of problem. Sharma *et al.,* [45] has studied the forced flow of a viscoelastic second-grade fluid between two infinite discs. The method that is used was Range-Kutta (FDMs). The result shows that the finite difference method has high consistency and stability for this type of problem. Fadlun *et al.,* [46] produce the numerical result for combined immersed-boundary in three-dimensional complex flow simulations using finite difference method. The study illustrated that for the second-order derivative, the FDM has high accuracy and is efficient for unsteady three-dimensional incompressible flow in complex geometry.

Therefore, the differential equation in Eq. (10) and (11) under boundary conditions in Eq. (12) are solved using the centered implicit scheme of the finite difference method that is practiced by the Maple solver "pdsolve". The results are then presented graphically and analyzed critically. Besides, the physical quantities such as wall shear stress and Sherwood number have also been computed. The quantities are defined as

$$
\tau_x = \left.\frac{\partial u}{\partial y}\right|_{y=0} \qquad \text{and} \qquad Sh = -\left.\frac{\partial \phi}{\partial y}\right|_{y=0}. \tag{13}
$$

3. Results and Discussion

To enhance the confidence of the present solutions determined by the developed Maple code, the present result of a velocity profile for limiting cases with $\Gamma=0, V_0=1, Sc=0.3$ and $Gm=1$ at $t=1$ is compared with the obtained result in Ref. [36]. As displayed in Figure 2, the current results are in outstanding agreement with the former numerical results. The numerical solution for the problem of incompressible second-grade fluid flow with wall transpiration is then calculated. The effect of the dimensionless transpiration velocity (V_0) is divided into two-component that is wall suction $(V_0>0)$ and wall injection $(V_0<0)$, the second-grade viscoelastic parameter (Γ), Schmidt number (Sc), modified Grashof number (Gm), time (t), and space (y) on the dimensionless velocity and concentration are analyzed.

Graphical results are presented to show the variation behavior of the fluid field due to the various values of the pertinent parameters. The value of wall suction $(V_0>0)$, wall injection $(V_0<0)$, and the viscoelastic second-grade fluid parameter (Γ) is chosen arbitrarily as shown by Hayat *et al.,* [22] in a range between 0.1 to 0.4. Further, the values of Sc are taken according to a different type of material such as $Sc=0.3$ corresponds to Helium, $Sc=0.78$ represents the Ammonia and $Sc=2$ for the hydrocarbon derivatives. Furthermore, the Gm is choose arbitrarily along range 0.2 to 1 as agreed in Ref. [29].

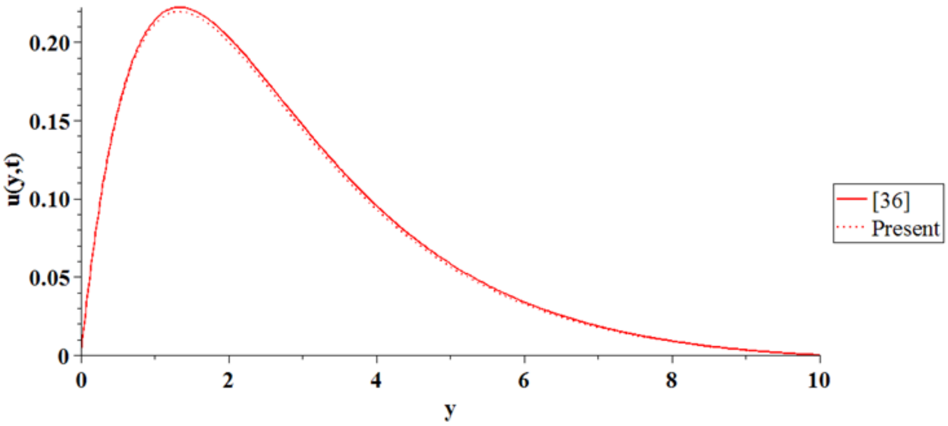
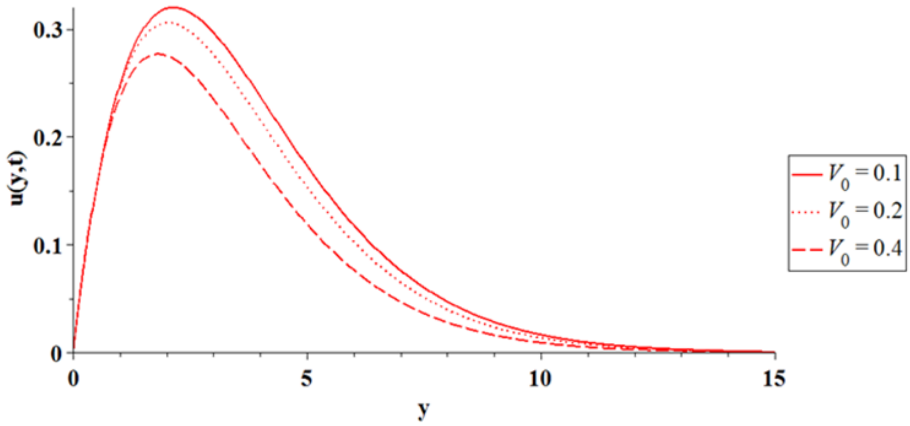
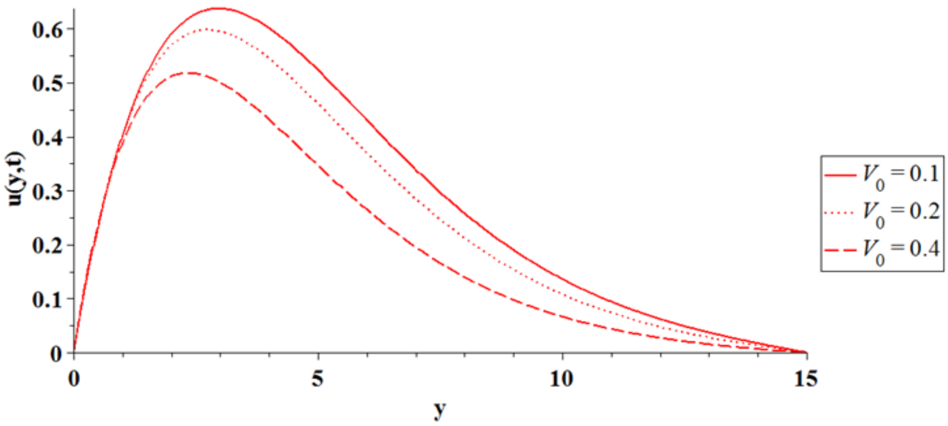


Fig. 2. Comparison of the present velocity result for case with $\Gamma = 0$, $V_0 = 1$, $Sc = 0.3$ and $Gm = 1$ at $t = 1$ with the result in Ref. [36]

Figure 3 present the velocity profiles for different values of wall suction $(V_0 > 0)$ when $\Gamma = 0.1$, $Sc = 0.3$, $Gm = 1$ with two different values of time in Figures 3(a) and 3(b). It is observed that the process of wall transpiration affects the velocity of the fluid especially for the fluid near the wall. This occurs due to the influence of wall friction. Due to the wall friction, the kinetic energy loses while the fluid flows through the boundary layer.



(a) $t = 1.0$



(b) $t = 2.0$

Fig. 3. Velocity profiles for different values of wall suction versus y

From the physical flow of the fluid, the friction between wall and fluid is higher due to the effect of the wall suction. This process is consistent with the physical situation. Thus, the behavior of the fluid is a decreasing function in the wall suction situation as seen in Figure 2. The velocity is increased when V_0 is increased. Furthermore, the velocity keeps increasing until achieving the maximum value at $y \sim 2$ in both panels. Then as $y \to \infty$ the velocity of the fluid is reducing and approach zero as fulfilling the boundary condition. By comparing Figures 3(a) and 3(b), the velocity of the fluid is increased over time. In conclusion, the velocity of the fluid is proportional to the time.

Figure 4 presents the velocity profiles for different values of wall suction versus t. The result shows that the behavior of the fluid flow is affected by time. The velocity profiles of the fluid are increase corresponds to the time at $y = 1$. The result indicates that the velocity profile is directly proportional to time but inversely proportional to V_0. This attribute to the fact that in an unsteady state the velocity will increase much faster compared to the steady-state case.

Figure 5 presents the dimensionless concentration profiles in the fluid region with the different values of wall suction. Physically, the high value of wall suction has reduced the gradient of the concentration more in the fluid results in the decrement of the concentration distribution. Consequently, the thickness of the concentration boundary layer is reduced as the values of wall suction increase (see Figure 5). In many manufacturing processes such as film cooling, polymer extraction, etc, mass transfer cooling is widely be used. The wall suction tends to increase the wall shear stress and the rate of heat transfer from the bounding surfaces due to the low distribution of the concentration [47]. This has significantly altered the cooling or heating of a system and thus affect the productivity and durability of the manufacturing process. Figure 5 indicates the fact that the wall suction is inversely proportional to the concentration.

Figure 6 presents the dimensionless concentration profiles for different values of wall suction versus t. This Figure shows the increase of the velocity consistency with time. We can observe that, at $t < 0.1$, the concentration of the fluid is almost equal at all values of V_0. However, at $t > 0.1$ the concentration of the fluid starts changing for different values of V_0. It is clear from this figure that, the concentration of the fluid is proportional in time but inversely proportional to the wall suction.

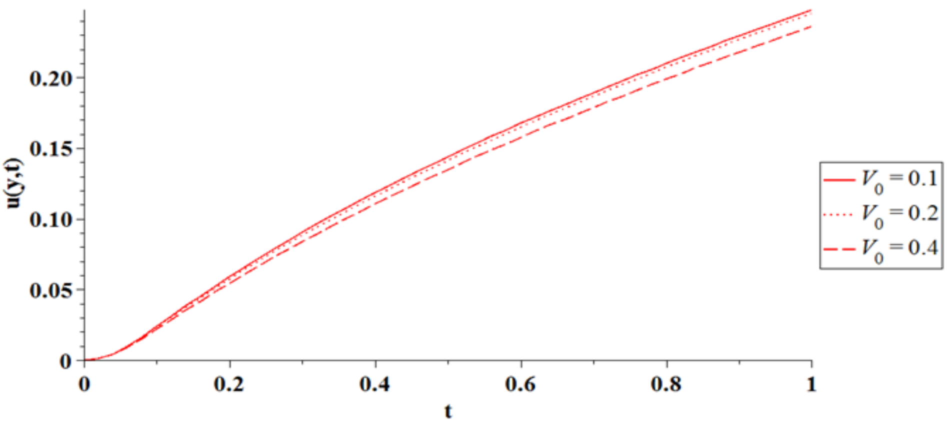


Fig. 4. Velocity profiles for different values of wall suction versus t

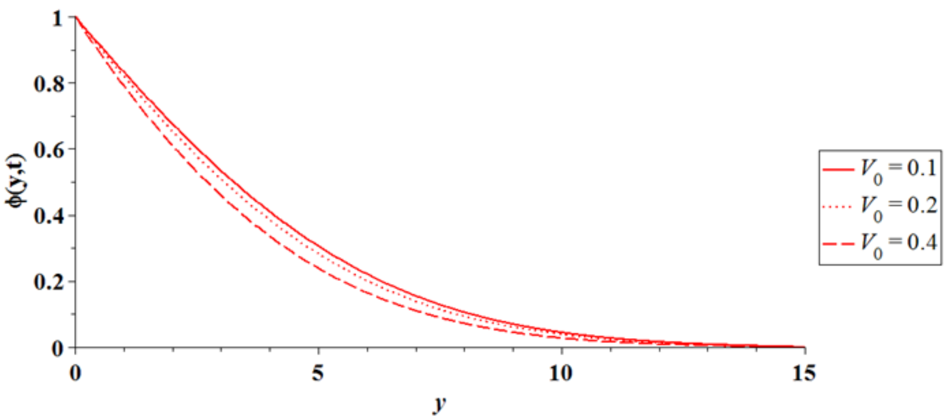


Fig. 5. Concentration profiles for different values of wall suction versus y

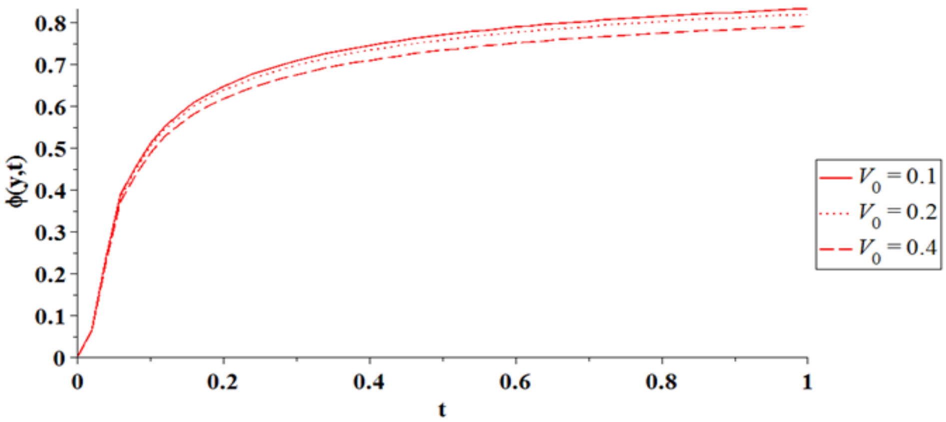
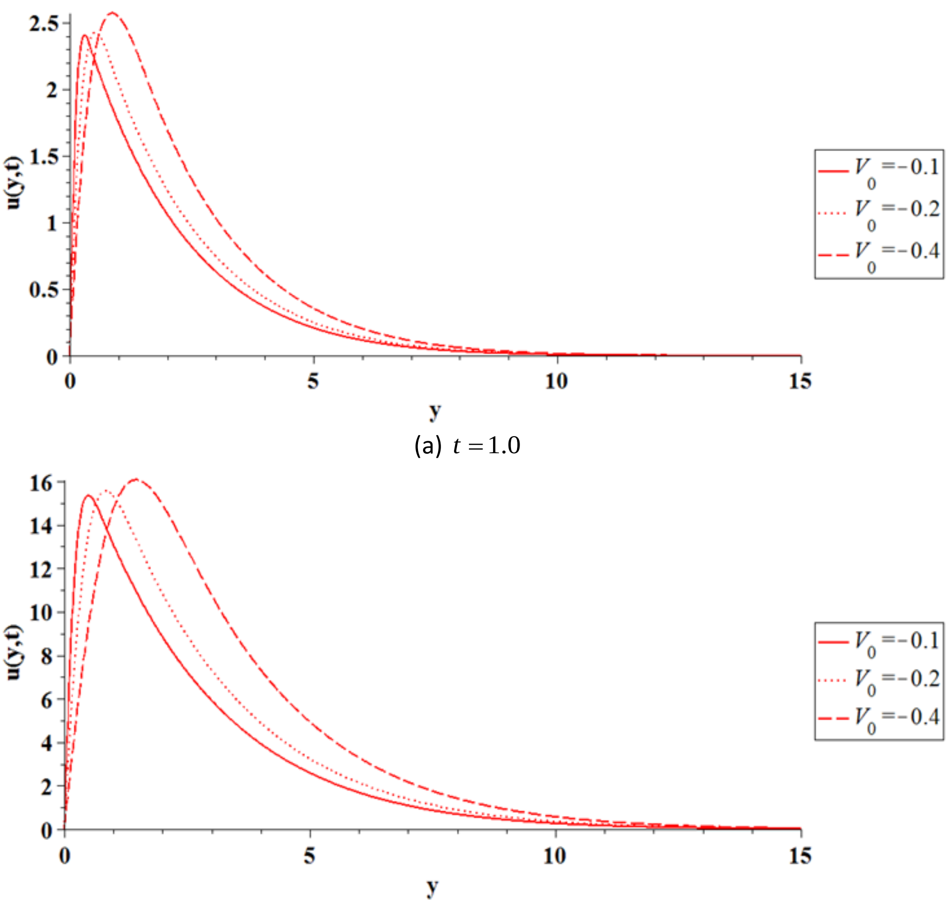


Fig. 6. Concentration profiles with different values of wall suction versus t

Figures 7 and 8 illustrate the velocity profiles for different values of wall injection for y and t. By comparing Figure 7(a) and 7(b), we observe that the dimensionless velocity of the fluid increases when time increase. Furthermore, Figure 7 shows that the small increase of wall injection value (the negative sign indicates the injection) has caused the velocity of the fluid to decrease initially and then increase after reaching the maximum magnitude of the velocity. Wall injection has the opposite behavior of the wall suction, the wall injection has reduced the wall shear stress and therefore enhances the distribution of the velocity. The beginning reduction in velocity is due to the rheology behavior of the second-grade fluid since this nature was not observed in Ref. [36]. In Figure 8, the velocity profile against time t for various values of wall injection in of the fluid is demonstrated. It shows that the velocity of the fluid increases rapidly after $t > 0.4$, the fluid velocity has accelerated exponentially correspond to time. This behavior has been shown by Djenidi *et al.,* [37] who conduct this problem experimentally. From Figure 8, this may attribute to the fact that the velocity of the fluid is directly proportional to time and also the magnitude of the wall injection (the negative sign indicates the injection).

Figures 9 and 10 present the influent of the wall injection for the transient concentration profiles versus y and t respectively. Figure 9 shows that the concentration of the fluid is increased with the enhancement of the magnitude of the wall injection V_0. Figure 10 displays the concentration value increases continuously with time $t < 0.1$, then the concentration reaches the steady-state after $t > 0.1$. Besides, the distribution of the velocity has also elevated with an increment in $|V_0|$. An increase of the magnitude of the wall injection (the negative sign indicates the injection) suppresses the concentration in the boundary layer regime since a higher value of $|V_0|$ will physically manifest the concentration in the viscoelastic fluid. The boundary layer thickness of the concentration is much greater for $V_0 = -0.4$ than $V_0 = -0.1$.

Figures 11 and 12 show the velocity profiles of the fluid flow regarding the various values of the second-grade parameters in the wall suction situation. The parameter Γ is varied from 0.1 to 0.4 and this value is referred to as the elastic effect as state din [22]. Figure 11 depicts that an increase of Γ has slightly decreased the velocity profiles in $V_0 > 0$ but after $y > 1$, an opposite tendency is observed. The viscoelastic parameter $\Gamma = k_0 V_0^2 / \upsilon^2$ is inversely proportional to the square of dynamic viscosity and directly proportional to the square of transpiration velocity and viscoelastic parameter, k_0. In the present case, we considered a weak species buoyancy $(Gm = 1)$, so that the dynamic viscosity ν and wall suction are of constant magnitude. Hence, larger Γ indicates larger viscoelastic parameter, k_0 then more force is needed to overcome the yield stress for generating the fluid movement. Furthermore, the velocity is always positive indicating that no flow reversal or no backflow occurs at any location of the plate or at any time change. In corresponding to time, the velocity profiles are slightly increased when times increase as seen in Figure 12.



(a) $t = 1.0$

(b) $t = 2.0$

Fig. 7. Velocity profiles for different values of wall injection versus y

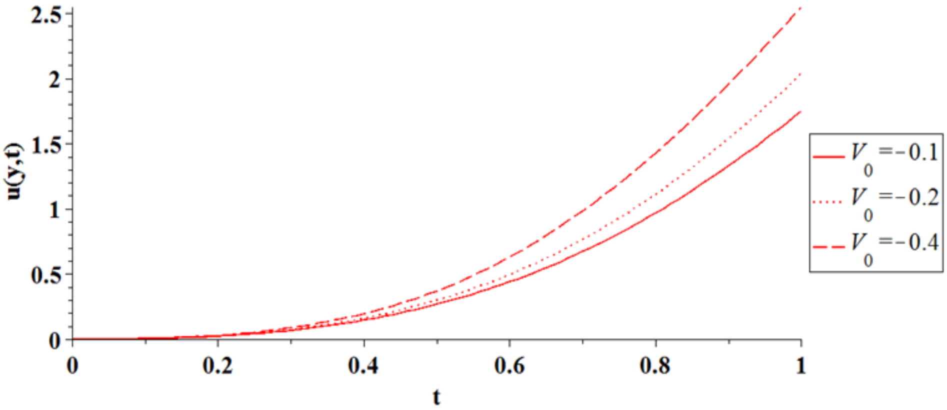


Fig. 8. Velocity profiles for different values of wall injection versus t

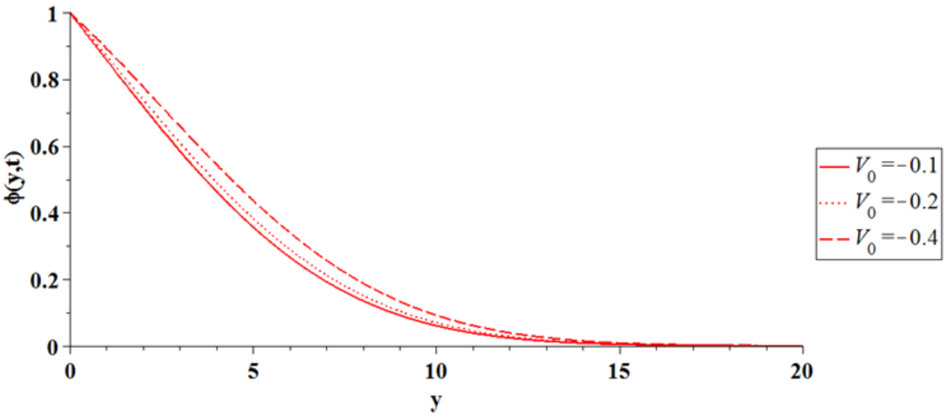


Fig. 9. Concentration profiles for different values of wall injection versus y

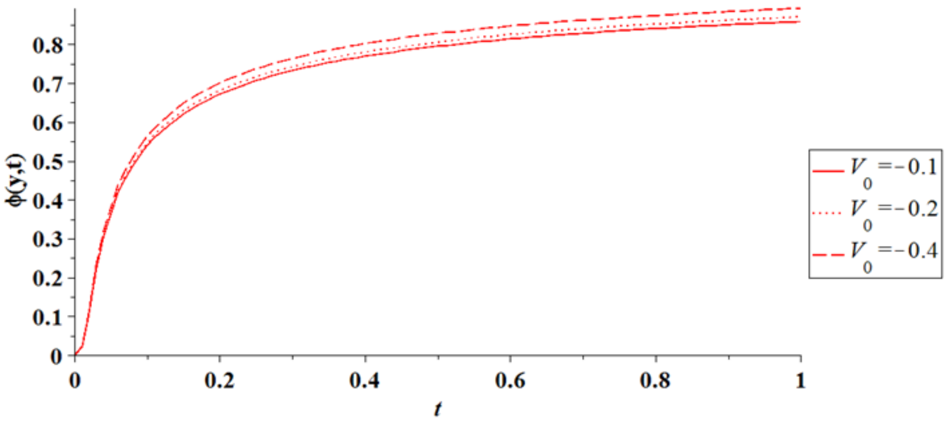


Fig. 10. Concentration profiles for different values of wall injection versus t

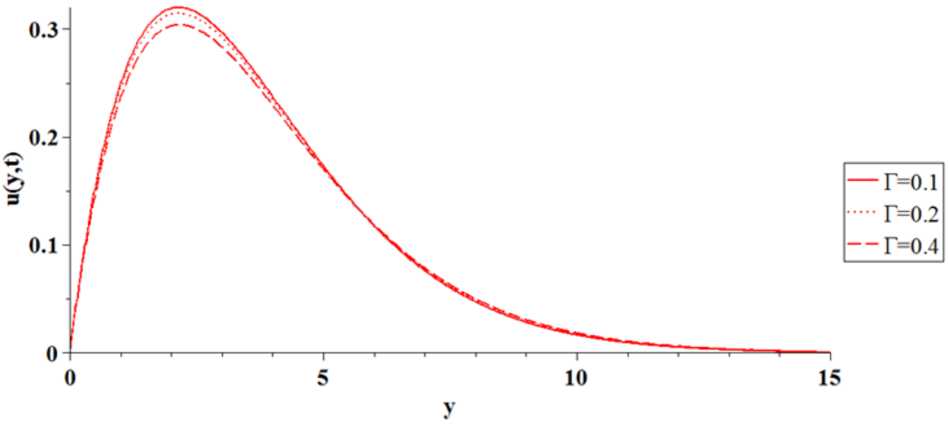


Fig. 11. Velocity profiles for different values of the viscoelastic parameter with the effect of wall transpiration versus y

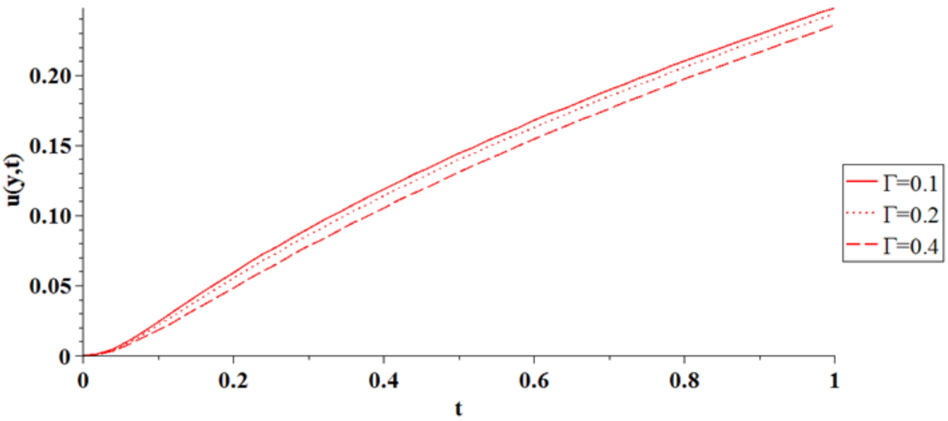


Fig. 12. Velocity profiles for different values of the viscoelastic parameter with the effect of wall transpiration versus t

In Figures 13 and 14, the velocity profiles that are influenced by different values of Schmidt number versus y and t are plotted respectively. The value of the Schmidt number choose is corresponding to certain materials to get the rate of diffusion in viscoelastic fluid. The materials that have been chosen are hydrocarbon derivatives $(Sc=1,2)$, ammonia $(Sc=0.78)$, oxygen $(Sc=0.66)$, and Helium. The Schmidt number is defined as the ratio of the viscous diffusion rate to the mass diffusion coefficient. The increase of the Sc number has increased the viscosity of the fluid, hence the velocity of the fluid is reduced for a high number of Sc as depicted in Figure 13. The velocity decrease with the decrease of molecular diffusivity (rise in Schmidt number). Figure 14 shows that all velocity profiles ascend monotonically from zero $t=0$ to $t=2$. The flow normal to the plate is therefore accelerated as time progresses. The minimum velocity is hydrocarbon derivative $(Sc=2)$ and the maximum velocity profile is Helium $(Sc=0.3)$. Hence the change of Schmidt number has a very dramatic influence on the velocity in the viscoelastic fluid flow regime near the plate.

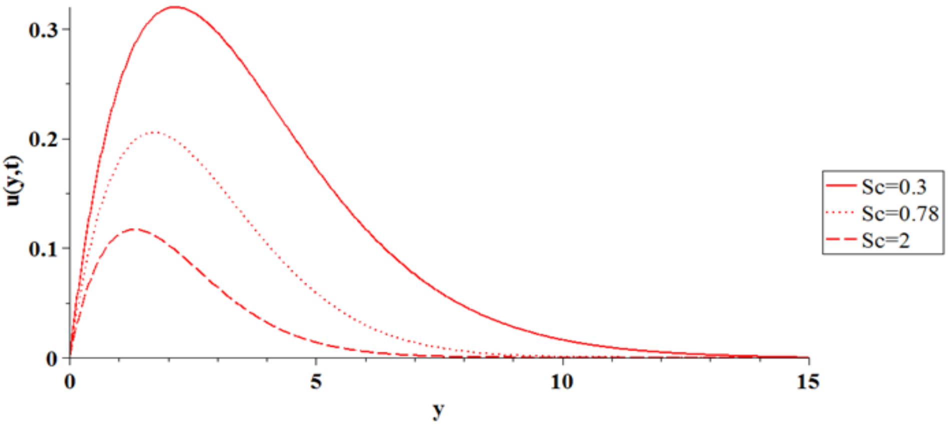


Fig. 13. Velocity profiles with different values of Schmidt number versus y

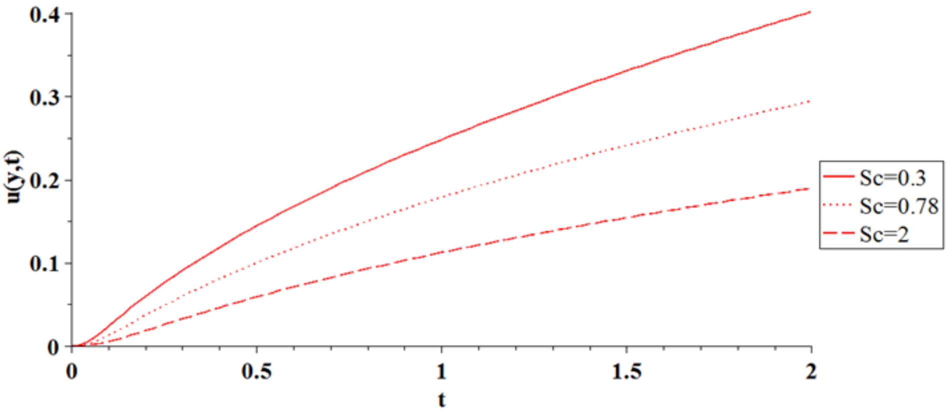


Fig. 14. Velocity profiles with different values of Schmidt number versus t

Figures 15 and 16 present the influent of the Schmidt number for the transient concentration profiles versus y and t respectively. An increase of Schmidt number has reduced the thickness of the concentration boundary layer since a higher Schmidt number has physically decreased the molecular diffusivity of the viscoelastic fluid or the rate of mass diffusion. Figure 15 shows that the concentration of the fluid decreases with the increasing of Schmidt number or in other words decreasing in molecular diffusivity. With the increasing of Schmidt number, we observed that the profile reaches zero progressively faster for a higher value of Schmidt number (see Figure 15). The boundary layer thickness is much greater for $Sc = 0.3$ than $Sc = 2$. Figure 16 shows that the concentration value increases continuously with time. Again, the concentration is computed for a steady-state case at $t = 2$, and the values are also elevated with a reduction in Schmidt number.

Figures 17 and 18 depict the effect of the modified Grashof number (Gm) on velocity distribution for elastic flow cases $(\Gamma = 0.1)$. The modified Grashof number is defined as the ratio of the buoyancy force induced by the concentration gradient to a viscous force acting on a fluid. An increase in Gm implies an increase in the magnitude of species buoyancy forces. This aids flow development in the boundary layer and cause an increase in velocity as displayed in Figure 17. From Figure 17, it is inferred clearly that velocity increased with Gm and the peak value of velocity arises at $y \sim 2$ near the plate surface, after which a smooth decay to the free stream follows. Furthermore, the velocity boundary layer thickness is increased with a rise in Gm values and grows with time as clearly illustrated in Figure 18.

The behavior of wall shear stress and local Sherwood number for different values of relevant variables Sc, Γ, Gm and V_0 are directed in Tables 1 and 2 respectively. Table 1 shows that the wall shear stress is enhanced when a larger value of parameters Gm and V_0 is applied. An opposite trend is observed for parameters Sc and Γ. Additionally, Table 2, the Sherwood number exhibits an increment for a larger value of Sc and V_0.

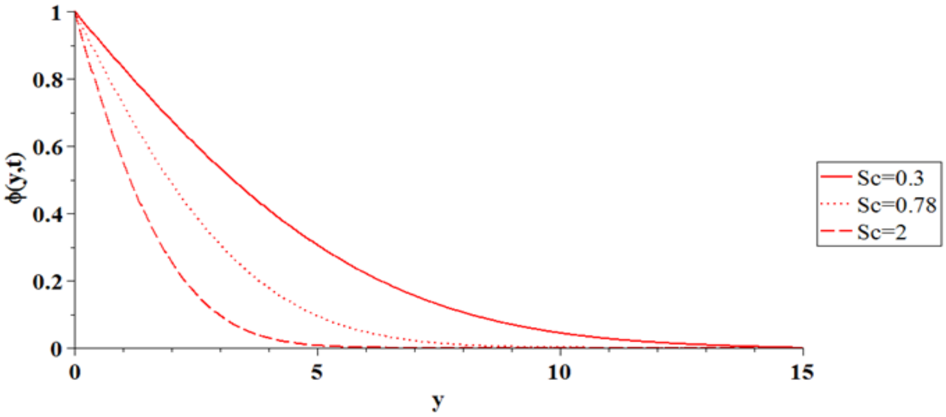


Fig. 15. Concentration profiles with different values of Schmidt number versus y

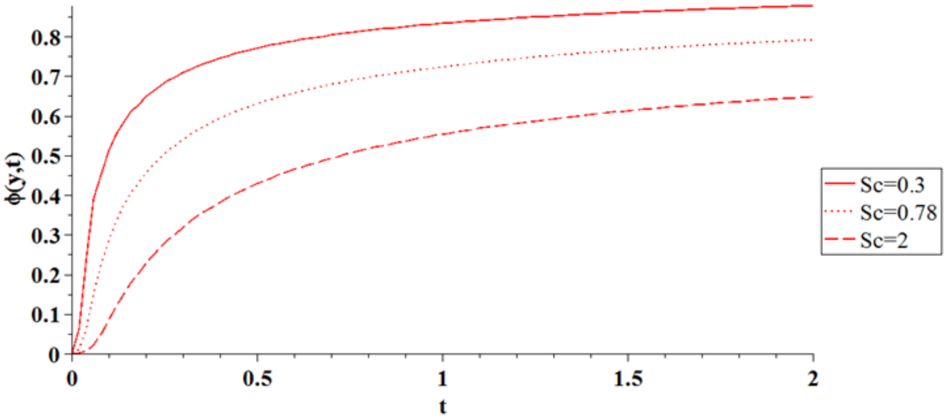


Fig. 16. Concentration profiles with different values of Schmidt number versus t

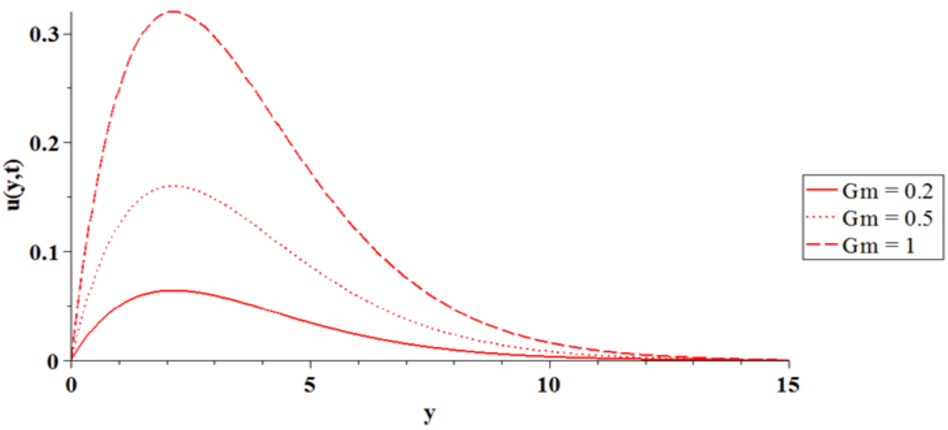


Fig. 17. Velocity profiles for different values of Gm versus y

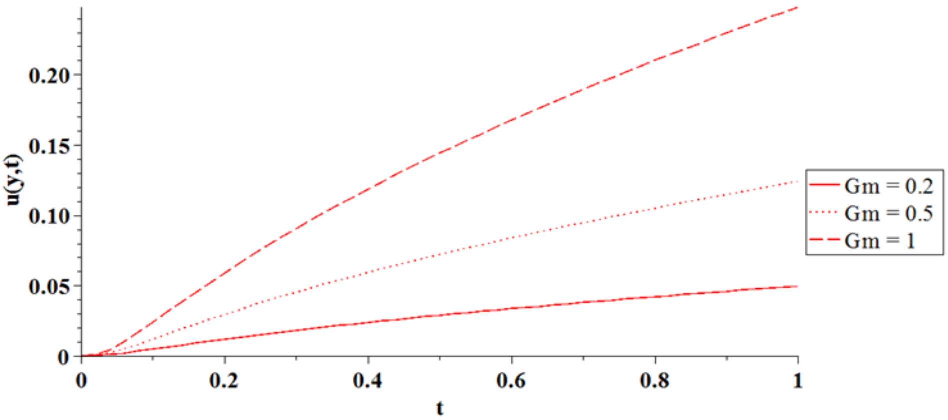


Fig. 18. Velocity profiles for different values of Gm versus t

Table 1
Wall shear stress of the flow field for various values of V_0, Gm, Γ and Sc at $t = 1.0$

V_0	Γ	Gm	Sc	$\left.\frac{\partial u}{\partial y}\right\vert_{y=0}$
0.1	0.1	1	0.3	0.370487
0.2	-	-	-	0.380272
0.4	-	-	-	0.393672
0.1	0.2	-	-	0.364951
-	0.4	-	-	0.353797
-	0.1	0.2	-	0.074097
-	-	0.5	-	0.185243
-	-	1.0	0.78	0.294629
-	-	-	2	0.218126

Table 2
Sherwood number of the flow field for various values of V_0 and Sc at $t = 1.0$

Sc	$\left.\frac{\partial u}{\partial y}\right\vert_{y=0}$		
	$V_0 = 0.1$	$V_0 = 0.2$	$V_0 = 0.3$
0.3	0.040434	0.056817	0.092294
0.78	0.116852	0.161627	0.262212
2.0	0.326944	0.450317	0.739138

4. Conclusions

The boundary layer of unsteady unidirectional flow of incompressible and nonlinear viscoelastic second-grade over a porous flat plate is studied numerically. Mass transfer in Non-Newtonian fluids is of great interest in many operations in the chemical and process engineering industries including coaxial mixers, blood oxygenators, milk processing, steady-state tubular reactors, and capillary column inverse gas chromatography devices, mixing mechanisms, bubble-drop formation processes, dissolution processes, and cloud transport phenomena. The influence of the pertinent parameters such as the transpiration velocity, second-grade viscoelastic parameter, Schmidt number, and modified Grashof number on the velocity and concentration have been considered and analyzed. The present research can be summarised as

i. with the increasing of wall suction $V_0 > 0$, the velocity profile is increased
ii. with the increment of the wall suction $V_0 > 0$, the concentration of the fluid is decreased. The increasing of velocity is not too much with the effect of wall transpiration
iii. the increasing of wall injection $V_0 < 0$ decreases the velocity of the fluid
iv. with the increasing of the second-grade viscoelastic parameter Γ, the velocity of the fluid decreases;
v. for Schmidt number Sc, the increment of the parameter decreases the velocity of the fluid
vi. with the increase of the Schmidt number Sc, the concentration of the fluid is decreased. The effect of the Schmidt number Sc, on the mass transfer, is of great significance with the presence of wall transpiration
vii. for modified Grashof number Gm, the increasing of the number increases the velocity of the fluid

The theory of viscoelastic fluid has attracted the attention of researchers because the characteristic of the fluid differs from the real fluid and is hard to explain by using Newtonian Fluid. In the future, the present communication may be extended with a different type of non-Newtonian fluid such as micropolar fluid, Walter-B fluid, Oldroyd-B fluid, and third-grade viscoelastic fluid. Besides, the investigation can be further by considering additional effects such as the Hall effect, chemical reaction, heat generation, and MHD.

Acknowledgement
The authors would like to acknowledge the Ministry of Higher Education Malaysia and Research Management Centre-UTM, Universiti Teknologi Malaysia (UTM) for financial support through vote numbers FRGS/1/2019/STG06/UTM/02/15, 08G33 and FRGS/1/2019/STG06/UTM/02/22.

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