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Study on Flow Structure Behind Multiple Circular Cylinders in a Tandem Arrangement Under the Effect of Magnetic Field

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ABSTRACT

The fast-moving technologies and the increasing rate of growth population indicates that the demand for energy will continue to be spiking and prominent in the discussion of the upcoming future. Therefore, to cater to the need for sustainable and clean energy, the idea of nuclear fusion is proposed and studied. Because the nuclear fusion reaction happens at a high temperature, the concept of magnetic field is adapted to the nuclear or plasma fusion reaction. The energy will be harnessed inside a blanket module of the fusion reaction plant. However, the presence of the magnetic field affects the fluid flow inside the blanket module where it reduces the heat transfer efficiency in the channel. This research examines the flow structure behind multiple bluff bodies arranged in tandem in a channel under the influence of a magnetic field with the aim to increase the heat transfer efficiency inside the channel. The effect of gap ratio, G/h = [1-2.4] and Hartmann friction parameter, H = [0-800], were analysed to determine the critical Reynolds number and Nusselt number. It was found that the presence of the downstream cylinder with gap ratios, G/h = 1.2, 1.4 and 1.6, causes the flow to be unsteady at a lower Reynolds number compared to those of a single cylinder. The multiple cylinders proved to increase the Nusselt number. Increasing the Hartmann friction parameter increases the critical Reynolds number and decreases the Nusselt number.

1. Introduction

The Magnetohydrodynamic (the branch of physics that studies the behaviour of an electrically conducting fluid acted on by a magnetic field.) flow in rectangular ducts has its significance in metallurgical processing applications as explained by Hussam and Sheard [1]. However, the effect of MHD within the cooling blanket enveloping magnetic confinement fusion reactor must be avoided. Previous studies show that the strong uniform magnetic field has an application that provides a convenient means for the contactless damping of turbulent fluctuations and the laminarisation of flows, which may further reduce heat transfer efficiency in the fusion blanket [2,3].

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According to Mück *et al.,* [4], the interaction between the liquid metal in motion with the magnetic field induces currents. The induced currents in the fluid then interact with the magnetic field and produce Lorentz forces that significantly change the flow pattern. This behavioural flow change is seen for the situation of laminar-turbulent transition in the magnetohydrodynamic duct, pipe, and channel flows with the homogeneous magnetic field and electrically insulating walls. This is proven by few authors where it can be concluded that under a sufficiently strong magnetic field, the electrically conducting fluid interacts with the applied magnetic field in such a way that disturbances parallel to the magnetic field are strongly suppressed, giving vortices a propensity to elongate and align parallel with the magnetic field [5,6]. To further dive into the effects of the magnetic field, a study is made by Hadid and Henry [7] where the flow structures in a rectangular cavity filled with an electrically conducting liquid are investigated when it is subjected to a constant magnetic field. The result shows that the magnetic field stabilizes the flow, where the flow at first is unstable and modifies the nature of the instability.

According to previous research, the magnetic field influences the flow structure by stabilizing the flow and lowering the heat efficiency. To introduce vortex dynamics into a flow, Thompson *et al.,* [8] proposed that bluff bodies can be added to the channel. Many studies have been conducted on circular cylinder bluff bodies such as study by Mittal and Kumar [9], and adding two circular cylinders as bluff bodies further increase the channel's unsteadiness. Between two arrangements, it is discovered that the second cylinder behaves almost identically to an isolated cylinder in almost all cases when the arrangement is staggered. At the same time, very large amplitude oscillations are observed for the second cylinder in a tandem arrangement based on the findings by Mittal and Kumar [9]. Previously, a study was conducted on the vortex induction of two cylinders in tandem arrangement. According to Borazjani and Sotiropoulos [10], when compared to an isolated cylinder, the tandem arrangement has larger motion amplitudes and a wider lock-in region, and above a threshold reduced velocity, large-amplitude motion is excited for the rear cylinder with amplitudes significantly greater than those of the front cylinder. In terms of unsteadiness, multiple cylinders are preferred because it allows for the observation of oscillations with significant amplitude.

The MHD affects the fluid flow where the flow structure decreases its velocity as studied by Lavanya [11], and according to Esfahani [12], this decreases the heat efficiency in a fusion blanket as it suppresses vortex shedding. As a result, the issue that arises from the magnetic field present in the fusion blanket must be addressed. This research is done by including bluff bodies in tandem arrangements inside the channel to create unsteady flow. Thus, the flow structure and heat transfer are then studied with the effect of the magnetic field.

The bluff bodies in the channel act as a vortex promoter in a tandem manner, and the optimal gap length will cause a steady flow to change its behaviour to unsteady is investigated. Past studies have been made regarding a cylindrical bluff body [1,13]. No attempt has been made to analyze the effect of the additional bluff body in tandem arrangement on the heat transfer efficiency for MHD flow. Thus, to achieve a productive result, the outcome of two cylindrical cylinders is compared with the result of a single cylindrical cylinder that will act as a baseline. The optimum gap length will then be investigated.

2. Methodology

The system of interest under consideration is shown in Figure 1. The geometry is a rectangular channel confining one and two circular cylinders at the centre of the channel perpendicular to the flow direction. The walls of the channel and cylinders are assumed to be non-slip and electrically insulated in this problem. A uniform vertical magnetic field with a strength B is imposed perpendicular to the *x-y* plane. The bottom wall (parallel orientation to the magnetic field) is heated with a constant wall temperature θ_{hot} whereas the top wall is set with $\theta = 0$. The heated wall with the interaction of the cylinders inside the channel is expected to contribute to heat transfer.

Sommeria and Moreau [5] described the magnetic Reynolds number (which represents the ratio between induced and applied magnetic fields) at a high Hartmann number as very small. In this case, the induced magnetic field is negligible, and the resulting magnetic field is imposed in the *z*-direction only. The flow is quasi-two-dimensional under these conditions and consists of a core region. The velocity is invariant in the magnetic field direction, and a thin Hartmann layer at the wall is perpendicular to the magnetic field as in the studies of Hussam *et al.,* [14]. Thus, they derived a two-dimensional model that governs this problem by averaging the flow quantities along the magnetic field direction.

For the study, the blockage ratio will be constant with the value of $\beta = 0.3$ for single cylinder and two-cylinder cases. The height is constant at 1, while the gap ratio will vary at $G/h = 1, 1.2$, 1.4 and 1.6 to achieve an optimum result.

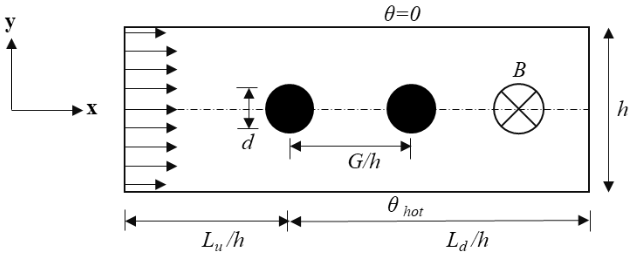


Fig. 1. Schematic illustration of the case under investigation. The magnetic field, *B* is parallel to the cylinder axis and acts in a *z*-direction which is in the out-of-plane direction

In Figure 1, *h* is the height of the channel, *Lu/h* is the upstream length starting from inlet to centre of the upstream cylinder, Ld/h is the downstream length, and *d* is the diameter of the cylinder, respectively. The blockage ratio is $\beta = (d/H)$, and the gap ratio between the cylinders is defined as G/h. Magnetohydrodynamics flow inside the channel in Figure 1 is governed by Eq. (2) for a quasi-2D model. Sommeria and Moreau [5] proposed this model in 1982, where the flow is assumed to be incompressible and laminar. This model approximates the MHD flow in a two-dimensional perspective by averaging the flow quantities in the core flow and the Hartmann layers. The non-dimensional magnetohydrodynamics equations of continuity, momentum and energy equation reduce to

$$\nabla \cdot \boldsymbol{u} = 0 \tag{1}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} = -(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} - \nabla p + \frac{1}{Re}\nabla^2 \boldsymbol{u} - \frac{H}{Re}\boldsymbol{u} \tag{2}$$

$$\frac{d\theta}{dt} + (\boldsymbol{u} \cdot \nabla)\theta = \frac{1}{Pr\,Re}\nabla^2 \theta \tag{3}$$

where, *u* represents the velocity, *p* represents the kinematic pressure, and θ is the temperature field $\left(\theta = \frac{\hat{\theta} - \hat{\theta}_{cold}}{\hat{\theta}_{hot} - \hat{\theta}_{cold}}\right)$, and each is projected onto the *x-y* plane. *H* represents the friction term representing the Lorentz force effect on the flow, *Re* represents Reynolds number, and *Pr* is Prandtl number. The viscous dissipation and Joule heating terms are not considered in this case following previous researches [14-16].

Reynolds number is a dimensionless parameter that depends on the value of maximum inlet velocity, U, characteristic length (the channel inlet height), h, and kinematic viscosity, ν where it characterises the ratio of inertial forces to viscous forces. The dimensionless parameters of Reynolds number, Hartmann number, Peclet number and Hartmann friction term are presented below

$$Re = \frac{\rho U h}{\mu} = \frac{Uh}{\nu} \tag{4}$$

$$Ha = aB\sqrt{\frac{\sigma}{\rho\nu}} \tag{5}$$

$$Pe = RePr \tag{6}$$

$$H = n\frac{h^2}{a}\, Ha \tag{7}$$

where $a, B,$ and σ are the half of out-of-plane channel height, the applied magnetic field and the magnetic permeability of the liquid metal, respectively. The Prandtl number in Peclet number, Pe equation is $Pr = \nu/\kappa_T$, where κ_T is the thermal diffusivity of the fluid, characterises the ratio of viscous to thermal diffusion in the fluid.

The local Nusselt number along the lower heated wall of the channel is given by

$$\mathrm{Nu_w}(x,t) = \frac{1}{\theta_f - \theta_w}\frac{\partial\theta}{\partial y}\bigg|_{wall} \tag{8}$$

θ_f is the bulk fluid temperature, which is calculated using the temperature distribution and velocity as

$$\theta_f(x,t) = \frac{\int_0^h u\theta dy}{\int_0^h u dy} \tag{9}$$

A time-averaged Nusselt number for heat transfer through the heated wall of the channel is calculated by finding the average local Nusselt number for few time steps and then integrating over the length of the heated bottom wall, *L* following Hussam *et al.,* [14].

The Strouhal Number can be calculated using the frequency of Vortex shedding (Eq. (10)), where *f* is the frequency of shedding.

$$St = \frac{fh}{u} \tag{10}$$

To proceed with the result analysis, lift coefficient, C_L is also used for this study as shown below in Eq. (11), Where F_L represents lift force.

$$C_L = \frac{F_L}{\frac{1}{2}\rho u^2 h} \tag{11}$$

The flow governed in Figure 1 is studied for its flow structure and heat transfer. The blockage ratio, β is 0.3, Hartmann parameter, H = [0-800] and Re = [1-2500]. Since the quasi-two-dimensional condition established by Sommeria and Moreau [5] and Pothérat *et al.,* [17] is satisfied, the MHD flow in this research is assumed to be quasi-two-dimensional. For the heat transfer study, a Prandtl number, $Pr = 0.022$ is used, which is a representative of Galinstan (GaInSn) liquid metal.

3. Results

3.1 Grid Dependency Study and Validations

This section discusses the grid dependency study and validations made prior to the simulation analysis discussed in this subsection.

A fully developed velocity profile is imposed at the inlet, and a constant reference pressure is imposed at the outlet. The velocity profile produced from the simulation at various H is verified with the analytical Hartmann fully developed velocity profile. Based on Figure 2, at $H = 0$, the curve can be observed to be a quadratic profile, and as H increases, the profile becomes fuller and flattened towards a "top-hat" profile. The inlet and outlet velocity of the channel seems to be in good agreement. Non-slip boundary conditions are imposed on the walls and the cylinders. The top wall non-dimensional temperature is set at $\theta = 0$, representing cold wall, and the bottom wall is $\theta_{hot} = 1$, while the cylinders are thermally insulated with zero temperature gradient surface.

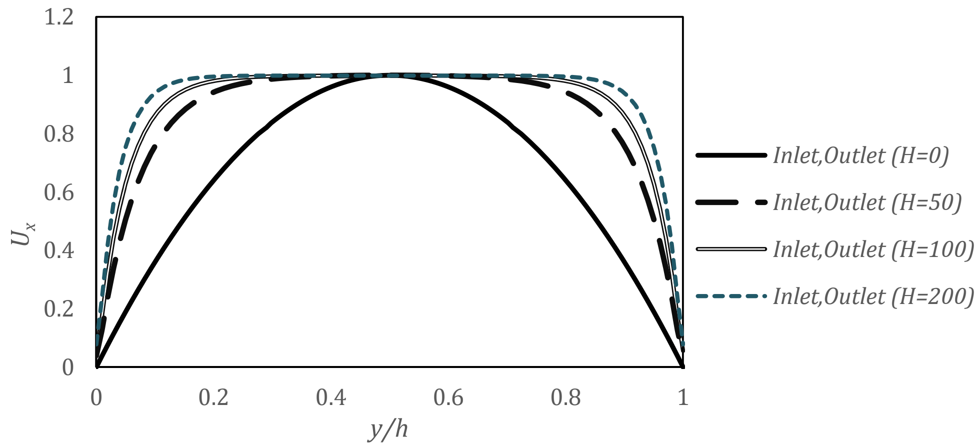


Fig. 2. Normalised velocity at the inlet and out of the channel at *H* = 0, 50, 100 and 200

The grid dependency study is based on the error percentage for lift coefficient and Strouhal number, St. These two terms are essential when analysing unsteady and oscillating flow problems. The number of elements is varied between 20,000-100,000. The desired error threshold was met using mesh M4 in Table 1, which is used hereafter. The number of elements afterwards M4 no longer significantly change the flow based on the percentage of error for *St*. The formulation is also tested to ensure that the solution is independent of the domain upstream length. Table 2 shows that the domain length does not pose any notable enhancement on the result. Thus, the upstream length, $Lu/h = 3$, is chosen ($8d$, eight times the diameter of the cylinder) in reference to Hussam and Sheard [1].

Table 1
Percentage of error for *St* calculated with the different number of elements

Mesh	No. of Elements	*St*	Error, *St* (%)
M_1	20876	0.91743	2.11
M_2	26798	0.89847	0.27
M_3	35096	0.89606	0.36
M_4	52916	0.89286	0.09
M_5	74336	0.89366	0.09
M_6	99356	0.89286	0.18

Table 2
Percentage of error for C_L at upstream, *Lu* = 1, 2, 3, 5, and 7

Length	*Cl*	Error, *Cl* (%)
1	0.0021642	0.70%
2	0.0021794	0.01%
3	0.0021797	0.01%
5	0.0021800	0.01%
6	0.0021798	0.03%

Validation is made against published results to ensure the accuracy of the present formulation and model. The first test concerns critical Reynolds number (Re_c), representing the Reynolds number threshold where the flow transitioned from steady to unsteady flow in a zero Hartmann flow without the effect of the magnetic field. The test is made with regards to blockage ratio, β of 0.1 and 0.2, whereby the critical Reynolds number is investigated. The results obtained are compared with the published numerical results of Hussam *et al.,* [14], Chen *et al.,* [18] and Sahin and Owens [19]. From the two points validated (i.e., β = 0.1 and β = 0.2), a close agreement of the present data and published data is observed (Figure 3).

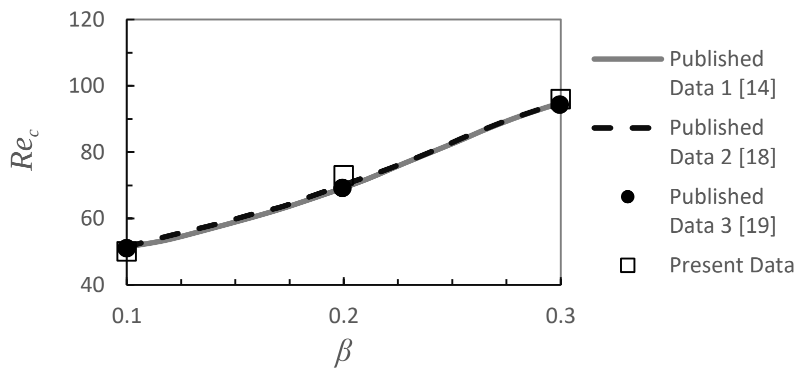


Fig. 3. A graph of critical Reynolds number, Re_c against blockage ratio, β. Hollow symbols show present data, while bullets, solid lines and dashed lines show data published in previous studies [14,18,19]

3.2 The Effect of Hartmann Parameters on the Critical Reynolds Number

For MHD flow to have such vorticity induced, flow with a high Reynolds number must be introduced to the channel as seen in Figure 4.

The figure shows the vorticity contours displayed for $G/h = 1.2$, with $Re = 1500$ at lower H of 200 and higher H of 600. At $H = 200$, the flow managed to induce Karman vortices as in the purely hydrodynamic case. As the Hartmann parameter is increased further ($H = 600$), the vortex shedding is seen to be suppressed entirely, where the flow then becomes a steady flow. The shear layer observed at the upstream cylinder starts to converge and no longer creates attachment with the downstream cylinder.

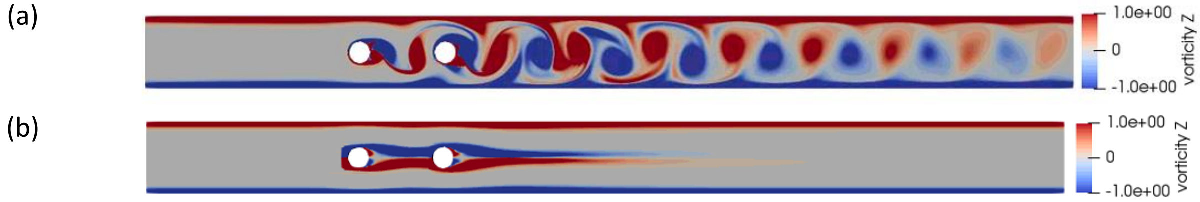


Fig. 4. The z-vorticity contour captured for gap ratio, $G/h = 1.2$, $Re = 1500$ at Hartmann parameter (a) $H = 200$ and (b) $H = 600$

Figure 5 demonstrates the effect of Hartmann parameters on the critical Reynolds number. As the Hartmann friction parameter increases from $H = 0$ to $H = 800$, the critical Reynolds number increases for all the gap ratios, $G/h = 1, 1.2, 1.4$ and 1.6.

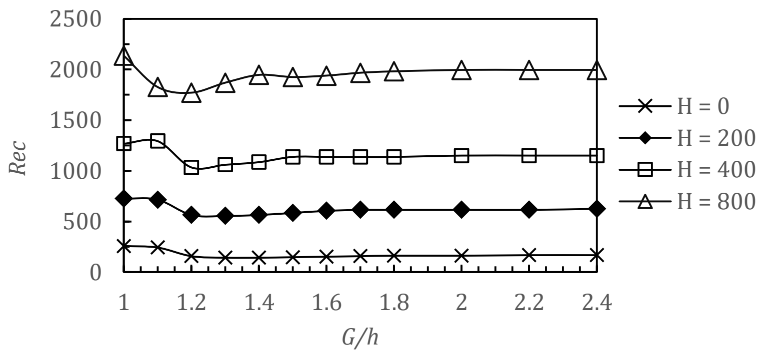


Fig. 5. Graph of critical Reynolds number, Rec plotted against gap ratio, *G/h* at different Hartmann parameters (*H* = 0, *H* = 200, *H* = 400, *H* = 800)

3.3 The Effect of Gap Ratio on the Critical Reynolds Number

Unsteady flow can be achieved in $G/h = 1.2$ to $G/h = 1.6$, having Reynolds number due at the onset of unsteadiness being lower than that of the single cylinder. This can be further exemplified with the graph presented in Figure 6. The Reynolds number at the onset of steadiness for $G/h = 1$ is higher than the case of flow past single-cylinder, also highest for the result of Rec of flow past two cylinders. This does not suit the purpose of placing two circular cylinder bluff bodies in the channel in introducing a more two-dimensionally unstable flow.

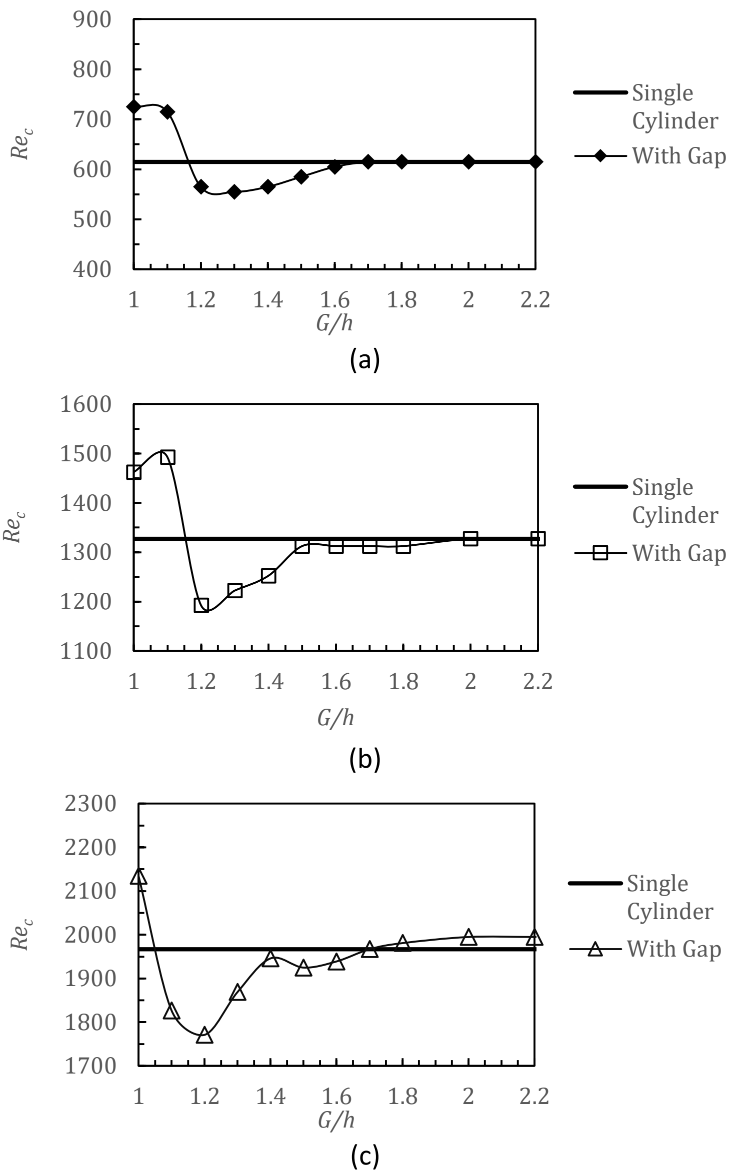


Fig. 6. The comparison of critical Reynolds number for flow past a single cylinder and two cylinders with (a) $H = 200$, (b) 400, and (c) 800, respectively

3.4 The Effect of Gap Ratio on Nusselt Number

In Figure 7, the temperature gradient after the downstream cylinder gives the direction of steeper temperature descent in the temperature field compared to the flow at the upstream channel, which shows that heat is transferred faster at unsteady flow.

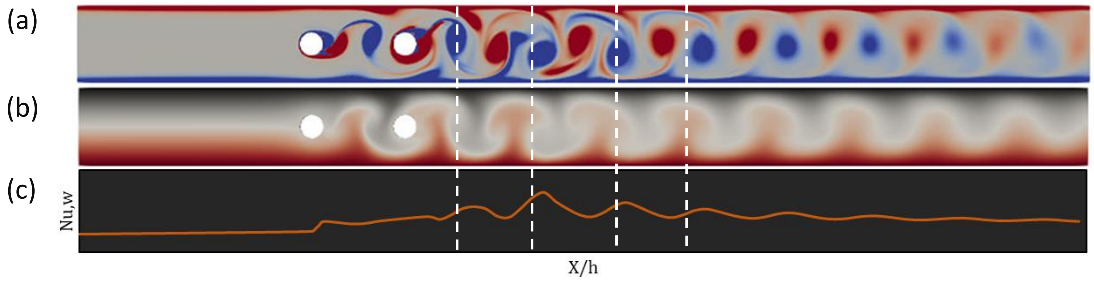


Fig. 7. (a) Vorticity contour, (b) Temperature contour and (c) Local Nusselt number at *x*-locations along the length of the channel for flow at *H* = 200, *G/h* = 1.2, *Re* = 2000

A graph of time-averaged Nusselt number vs Reynolds number for gap ratio, $G/h = 1, 1.2, 1.4$ and 1.6, for $H = 400$, is plotted and presented in Figure 8. The Nusselt number seems to be increasing with Reynolds number and this is adequately reflected in a past study [20]. As seen in the figure, at gap ratio, $G/h = 1$, the Nusselt number is lower than the Nusselt number of the single cylinder. This gap ratio does not optimise the flow in terms of heat transfer. From the same figure, it is apparent that at the gap ratio, $G/h = 1.2, 1.4$ and 1.6, there is a remarkable change in Nu compared to the flow past a single cylinder and flows past two circular cylinders at gap ratio, $G/h = 1$. There is a slight decrease in Nu for $G/h = 1.2, 1.4$ as compared to $G/h = 1.6$. Apart from all that, at $G/h = 1.2, 1.4$ and 1.6, Nu is improved by imposing the two circular cylinders at the gap ratio of $G/h = 1.2$ to $G/h = 1.6$.

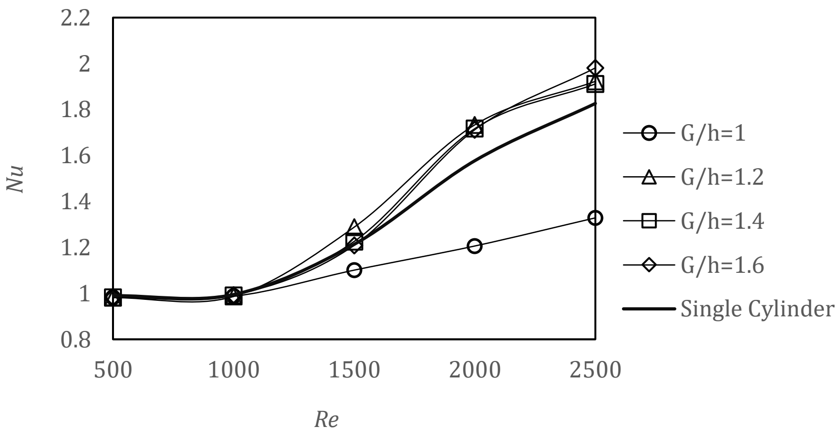
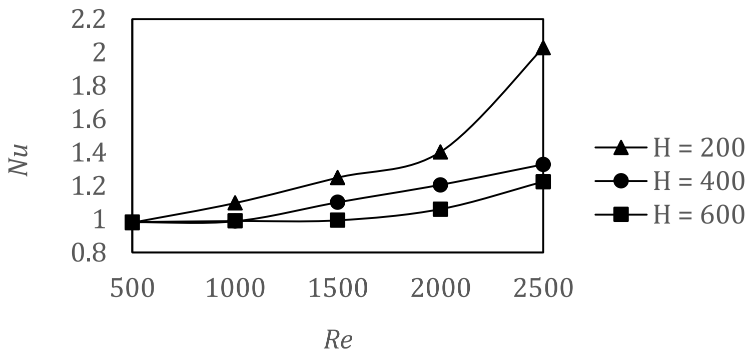


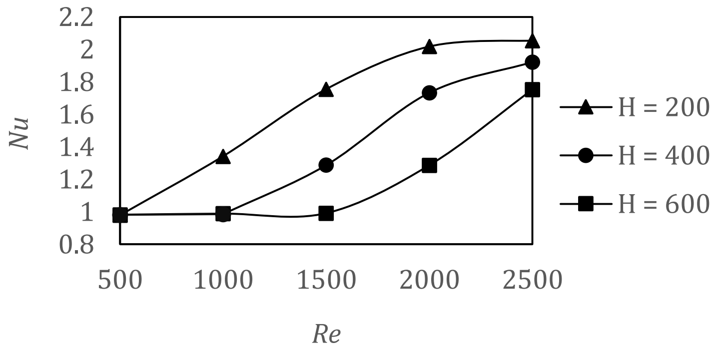
Fig. 8. Time-averaged Nusselt number plotted against Reynolds number for a single cylinder and two circular cylinders with $G/h = 1, 1.2, 1.4$ and 1.6 at $H = 400$

3.5 The Effect of Hartmann Parameters on Nusselt Number

Figure 9 shows the effect of Hartmann parameters on the time-averaged Nusselt number for $G/h = 1$ and 1.2. After the flow reaches the critical *Re*, the flow shows a notable increase in *Nu* due to the unsteady state of the flow regime. From the two graphs, there is a spike in *Nu* for $G/h = 1.2$ as compared to $G/h = 1$. Overall, a conclusion that can be made from Figure 9 is that a higher Hartmann parameter will result in a lower Nusselt number. These findings corresponded well with reviews discovery by Khan *et al.,* [21]. This relationship depends on the gap ratio, *G/h*.



(a)



(b)

Fig. 9. Time-averaged Nusselt number against Reynolds number at $H = 200, 400, 600$ for (a) $G/h = 1$ and (b) $G/h = 1.2$

4. Conclusions

In this study, the effect of gap ratio, $G/h = 1, 1.2, 1.4$ and 1.6 and Hartmann friction parameter, $H = 0, 200, 400$ and 800, are analysed for the critical Reynolds number and Nusselt number. Gap ratio, $G/h = 1.2, 1.4$ and 1.6 improve the critical Reynolds number of the flow to a lower value compared to flow past a single cylinder. While the critical Reynolds number at gap ratio, $G/h = 1$, is higher than the critical Reynolds number of flows past a single cylinder. The gap ratio, $G/h = 1$ and 1.2 are then analysed to determine the Nusselt number of the flow, and only $G/h = 1.2$ has an enhancement of the Nusselt number compared to the single cylinder. In contrast, the flow with $G/h = 1$ has a Nusselt number below the single cylinder. The increase in Hartmann friction parameter from $H = 200$ to 800 increases the critical Reynolds number and decreases the Nusselt number. To conclude, flow past two cylinders at gap ratio, $G/h = 1.2, 1.4$ and 1.6 managed to create an unsteady flow with a critical Reynolds number of lesser than the single-cylinder, contributing to a substantial increment on the Nusselt number. When a higher Hartmann friction parameter is introduced, it suppressed the enhancing effect of the flow gradually.

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