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# Two-Dimensional Mixed Convection and Radiative $Al_2O_3$-Cu/$H_2O$ Hybrid Nanofluid Flow over a Vertical Exponentially Shrinking Sheet with Partial Slip Conditions

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**ABSTRACT**

Hybrid nanofluid is considered a modern and improvised form of nanofluid which usually used to enhance the performance of heat transfer in fluid flow systems. Previous studies found hybrid nanofluid offered a wide range of applications and this opened up numerous new opportunities to further explore the unknown behaviour of hybrid nanofluid under different body geometries and physical parameters. This paper numerically studied a two-dimensional mixed convection and radiative $Al_2O_3$-Cu/$H_2O$ hybrid nanofluid flow over a vertical exponentially shrinking sheet with partial slip conditions. The main objective is to investigate the effect of mixed convection and radiation on the velocity and temperature profiles, as well as the effect of suction on reduced skin friction and reduced heat transfer with respect to solid volume fraction of copper, velocity, and thermal slips. Exponential similarity variables transformed the governing system of partial differential equations into a system of ordinary differential equations which is solved via MATLAB's bvp4c solver. Outcomes showed that the value of the reduced heat transfer upsurges in the first solution but declines in the second solution when the velocity slip rises. The reduced heat transfer decreases in both dual solutions when thermal slip is enhanced. As the intensity of thermal slip increases, the reduced skin friction rises in the first solution and decreases in the second. As the mixed convection parameter increases, no obvious variation is noticed in the temperature distribution within the first solution, but increasing trend is observed within the second solution. An increment in the temperature distribution also observed within the dual solutions as the thermal radiation parameter increases. In summary, findings from this study are particularly useful to understand various behaviour of $Al_2O_3$-Cu/$H_2O$ hybrid nanofluid under the influence of mixed convection, radiation, and partial slip conditions when it flows over a vertical exponential shrinking sheet.

## 1. Introduction

The study of fluid dynamics has attracted the interest of many experts, researchers, and scholars from various fields in recent decades due to its numerous applications in engineering, science, and

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technology. The boundary layer flow is one of the most frequently discussed topics. Sakiadis [1] was the first to propose the notion of a two-dimensional boundary layer steady flow on a stretching surface. After that, Crane [2] modernized Sakiadis's ideas and applied the two-dimensional steady flow over exponential and linear stretching sheets. He proposed that the velocity of a sheet that stretched from a slit is proportional to the distance between them.

Further, Choi [3] coined the term nanofluid to describe a mixture formed by dispersing nanoparticles into a base fluid. Nanofluids can be developed by evenly dispersed nanoparticles that are 100nm or smaller in size into a base fluid. Base fluids can include water, polymeric solutions, lubricants and oil, organic liquids, and other common fluids (Wang *et al*., [4]). Nanofluids frequently contain nanoparticles with distinct chemical and physical properties, such as oxides, carbon nanotubes, carbides, or metals (Nada *et al*., [5]). Nanofluids are identified to have higher thermal efficiency than their base fluids, especially in terms of raising the thermal conductivity of the base fluids. As a result, nanofluids have numerous real-world applications involving heat transfer. Aerospace, solar energy, drug delivery, agriculture, building heating and cooling, microchips, refrigerators, and automobiles are a few notable examples of applications (Lund *et al*., [6]). Various authors have examined the thermophysical properties of nanofluids during the last two decades, both numerically and experimentally. Miklavĉiĉ and Wang [7] are credited as the first to use suction to study a viscous three-dimensional steady flow over a shrinking sheet. They revealed that in order to sustain flow past a shrinking surface, mass suction is needed. Additionally, Bachok *et al*., [8] explored a two-dimensional steady stagnation-point flow in nanofluid across a stretching/shrinking layer. They discovered non-unique solutions on the shrinking sheet. After that, Bachok *et al*., [9] theoretically investigated a two-dimensional unsteady boundary layer flow in nanofluid through a shrinking/stretching sheet. They revealed that the situation of stretching and shrinking sheets produced dual solutions. After that, in the existence of suction/injection, Naramgari and Sulochana [10] examined the impact of thermal radiation and chemical reaction on a two-dimensional magnetohydrodynamics (MHD) steady nanofluid flow on the stretching/shrinking surface. They concluded that the dual solutions are only present for limited range values of the suction/injection parameters. Ahmad and Pop [11] studied a two-dimensional mixed convective steady nanofluid flow and stated that non-uniqueness of solutions exists in the limited range of parameters. Moreover, Lund *et al*., [12] considered a two-dimensional mixed convection of unsteady nanofluid flow and found four solutions. The fundamental impacts of various kinds of parameters on the behaviour of nanofluid were reported by several other authors [13-28].

In the past few years, the production of advanced heat transfer fluid has earned substantial attention from scientists and scholars. A hybrid nanofluid is a modern kind of nanofluid that is utilized to improve heat transfer efficiency. A hybrid nanofluid is a combination of more than one type of distinct nanoparticles in a base fluid. Suresh *et al.*, [29] suggested the notion of hybrid nanofluid through laboratory experiments. After that, Devi and Devi [30] found out that their computational model of the hybrid nanofluid was reliable because their theoretical findings agreed with the experimental outcomes of Suresh *et al*., [29]. Devi and Devi [30] suggested that hybrid nanofluid increased the performance of cooling, as verified by the fact that hybrid nanofluid has superior heat transfer rate than nanofluid. According to Huminic and Huminic [31], significant applications of hybrid nanofluids include helical coil heat exchanger, microchannels/warmth pipes/ tube-shaped heat exchangers, air conditioning systems, and so on. Waini *et al*., [32] examined a two-dimensional unsteady hybrid nanofluid flow through shrinking/stretching layer. They concluded that increasing copper (Cu) nanoparticle volume fraction in the first solution resulted in greater skin friction coefficient and lower local Nusselt number on the shrinking surface. Additionally, for the second solution, both local Nusselt number and skin friction coefficient were both reduced as copper (Cu) nanoparticle volume fraction increased. Moreover, Waini *et al.*, [33] considered the influence of transpiration steady flow and heat transfer through a shrinking/stretching surface by uniform shear flow on two-dimensional hybrid nanofluid. They revealed that copper (Cu) nanoparticle volume fraction raised the temperature of hybrid nanofluid while lowered its velocity and shear stress. More extended findings on hybrid nanofluid flows over the stretching/shrinking layers may also be studied in the current literature [34-41].

Convection is a type of heat transmission that happens due to the mass movement of the fluid particles. Free convection occurs when heated fluid molecules become less dense and migrate from one location to another, carrying the heat with them owing to temperature differences. It is a significant source of heat transmission that occurs by diffusion, advection, or both (Lund *et al.*, [42]). On the other hand, the fluid movement in forced convection is generated by an external factor such as a pump or fan. The occurrence of forced convection is very essential and has various applications in industries (Waini *et al.*, [43]). For instance, forced convection occurs when electronic parts in a computer are being cooled. A small fan is mostly fitted to the side or rear of the chassis to cool the electronic components with openings on the side surface for easy air circulation. Mixed convection is the mixture of both forced and free convection. It is a very effective heat transfer mode that rises in many transport processes in engineering devices as well as in nature. In mixed convection, both free and forced convection act together in the heat transfer procedure (Prasad *et al.*, [44]).

Merkin [45] is the first researcher who considered two-dimensional mixed convection effect on boundary layer steady flow. Later in 1986, he extended his work on porous medium and found dual solutions (Merkin [46]). After that, Ghalambaz *et al.*, [47] investigated the mixed convection effect on a two-dimensional $Al_2O_3$-Cu/water steady hybrid nanofluid flow and heat transfer through a vertical plate. They exposed that the structure of hybrid nanoparticles affected the mixed convection parameter critical values at which the dual solutions met. Besides, Waini *et al.*, [48] examined a two-dimensional hybrid nanofluid flow with mixed convection over a vertical sheet in a porous medium. They demonstrated that in the case of opposing flows, there are two solutions. Furthermore, the addition of hybrid nanoparticles caused slower separation of the boundary layer. Further, they observed that the velocity of the first and second solutions become closer when the mixed convection parameter is declined. Yashkun *et al.*, [49] studied the heat transfer of a two-dimensional hybrid nanofluid flow through an exponentially stretching/shrinking sheet, with the influence of mixed convection and Joule heating. A variety of fluid flow models have been used by other scholars to study the mixed convection flow in hybrid nanofluids [50, 51].

A broad review on the available researches indicates that the impact of slip parameter on the hybrid nanofluid flow has not been given much consideration. Many important applications of fluid display boundary slip states as in the creation of heart valves and inner cavities, and artificial heart valves cleaning (Jamil *et al.*, [52]). Andersson [53] is the first to present the impact of slip parameter on boundary layer flow. Hayat *et al.*, [54] studied a three-dimensional rotating Ag-CuO/water hybrid nanofluid flow with partial slip boundary and radiation. They discovered that when the slip parameter is enhanced, the temperature and velocity quantities are reduced. Additionally, Aly and Pop [55] investigated the heat transfer of a two-dimensional MHD stagnation point flow in hybrid nanofluid through a stretching/shrinking layer with viscous dissipation and partial slip. They concluded that hybrid nanofluid can become a good cooler by increasing the stretching and slip parameters. The slip effect along various flow configurations and physical consequences may also be studied in the current literature [56-58].

At high operating temperature, thermal radiation has major effect that cannot be ignored. Numerous engineering processes take place at extremely high temperature, so thermal radiation sensitivity is critical in the design of appropriate device. It also plays a significant part in a variety of industrial applications such as glass processing and furnace construction, and space development applications which include spaceships, internal combustion engines, propulsion systems, ship compressors, spacecraft, solar radiations, combustion procedures and plasma physics (Khan *et al*., [59]). Hayat *et al*., [60] investigated a three-dimensional heat transfer enhancement steady flow involving Ag-CuO/water hybrid nanofluid. They observed an increment in the temperature profile alongside with the value of radiation parameter. Besides, Waini *et al*., [61] observed the influence of radiation on the heat transfer of a two-dimensional steady hybrid nanofluid flow over an exponentially shrinking surface. When they increased the effect of radiation, they noticed that the temperature in both solutions is raised. After that, Lund *et al*., [62] investigated a two-dimensional unsteady MHD hybrid nanofluid flow over a shrinking and stretching sheet with radiation. They concluded that the temperature in both solutions has increased due to the enhancement of radiation. Likewise, Anuar *et al*., [63] described a three-dimensional rotating and radiative hybrid nanofluid flow across a stretching and shrinking sheet. They discovered that as radiation raised, the thickness of the thermal boundary layer increased in both solutions. Additional references of radiative hybrid nanofluid can also be found in the existing literature [64, 65].

Waini *et al*., [43] investigated a two-dimensional hybrid nanofluid flow over a vertical exponentially stretching/shrinking sheet with the impact of mixed convection, but without taking into account the effect of thermal radiation and partial slip conditions. On the other hand, Yan *et al*., [66] addressed a two-dimensional magnetized hybrid nanofluid flow over a vertical exponentially shrinking surface with Joule heating and multiple slip conditions without considering the effect of mixed convection and thermal radiation. Therefore, this current study intents to fill up the gaps found in Waini *et al.*, [43] and Yan *et al.*, [66] and further extends their works by considering the joint effect of thermal radiation, mixed convection, and partial slip conditions, but without the presence of magnetic field and Joule heating using Tiwari-Das model (see Tiwari and Das [67]). Hence, a novel physical model of two-dimensional mixed convection and radiative $Al_2O_3$-Cu/$H_2O$ hybrid nanofluid over a vertical exponentially shrinking sheet with partial slip conditions is obtained for this study. The current study aims to investigate the variation effects of solid volume fraction copper (Cu), velocity slip, and thermal slip for reduced skin friction and reduced heat transfer. Besides, the velocity and temperature profiles with respect to various Prandtl number, mixed convection parameter, and radiative parameter are observed and presented as well. The solid nanoparticles considered in this study are alumina ($Al_2O_3$) and copper (Cu). The $Al_2O_3$-Cu/$H_2O$ hybrid nanofluid can be formed by suspending both nanoparticles in water ($H_2O$). For comparison purposes, the current numerical findings are compared with those results published in previous studies. To the best of our knowledge, the current proposed model is unique and has not yet been issued elsewhere.

## 2. Methodology
### *2.1 Mathematical Modelling*

A two-dimensional Cartesian coordinate system with steady mixed convectional and incompressible flow of $Al_2O_3$-Cu/$H_2O$ hybrid nanofluid on a vertical exponentially shrinking sheet is as shown in Figure 1. The temperature of the surface is $T_w(x)$, being represented with $T_w(x) = T_\infty + T_0\, e^{2x/l}$, where $T_\infty$ is the free stream temperature and $T_0$ is the characteristic temperature. Furthermore, $l$ is the characteristic length. This physical model possessed the following assumptions:

i. The surface velocity is $u = u_w(x) = U_w\, e^{x/l}$ in the direction of the $x$-axis.
ii. The acceleration caused by gravity is indicated by the symbol $g$.

iii. The heated plate ($T_w(x) > T_\infty$) and the cooled plate ($T_w(x) < T_\infty$) are both taken into account.

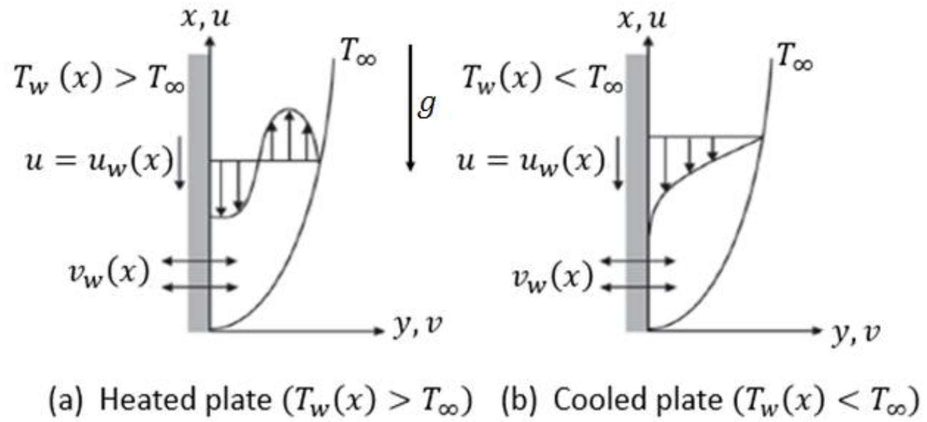


**Fig. 1.** The physical model considered in this study

The governing equations for the above-mentioned physical model are as follows (Waini *et al*., [61]; Yan *et al*., [66]):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{hnf}}{\rho_{hnf}}\frac{\partial^2 u}{\partial y^2} + \beta_{hnf} g(T - T_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{hnf}}{(\rho c_p)_{hnf}}\frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho c_p)_{hnf}}\frac{\partial q_r}{\partial y}. \tag{3}$$

The boundary conditions are (Yan *et al*., [66]):

$$v = v_w(x),\ u = u_w + A\nu_f\frac{\partial u}{\partial y},\ T = T_w + D\frac{\partial T}{\partial y}\ \text{ as } y = 0, \tag{4}$$

$$u \to 0,\ T \to T_\infty,\ \text{as}\ y \to \infty. \tag{5}$$

The velocities along the $x$- and $y$-axes are symbolized by the letters $u$ and $v$, respectively. $T$ denotes the fluid's temperature while $\nu_f$ is the kinematic viscosity. Additionally, $D = D_1\, e^{-x/2l}$ is the thermal slip factor and $D_1$ denotes the initial value of the thermal factor. On the other hand, $A = A_1\, e^{-x/2l}$ is the factor of velocity slip and $A_1$ represents the initial value of the velocity factor. Besides, $v_w = -\sqrt{\frac{\nu_f U_w}{2l}}\, e^{x/2l} S$ is the velocity of mass flux, where $S$ is the parameter of suction/injection such that $S > 0$ corresponds to suction and $S < 0$ for injection. The radiative heat flux $q_r$ is given by (Waini *et al*., [61]; Anuar *et al*., [63]):

$$q_r = -\frac{4\sigma_1}{3k^1}\frac{\partial T^4}{\partial y}. \tag{6}$$

The Stefan-Boltzmann constant and the coefficient of mean absorption are specified by $\sigma_1$ and $k^1$, respectively. By ignoring the higher order terms and extending $T^4$ with a Taylor series about $T_\infty$, and further converting $T^4 \cong 4T_\infty^3\, T - 3T_\infty^4$, Eq. (3) becomes (Waini *et al*., [61]; Anuar *et al*., [63]):

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \left[\frac{k_{hnf}}{(\rho c_p)_{hnf}} + \frac{16\sigma_1 T_\infty^3}{3k^1(\rho c_p)_{hnf}}\right]\frac{\partial^2 T}{\partial y^2}. \quad (7)$$

Additionally, the constants such as $c_p$, $\mu$, $\rho$, $\beta$, $k$, and $\rho c_p$ are specific heat capacity, dynamic viscosity, density, thermal expansion coefficient, thermal conductivity, and effective heat capacity, respectively. The fluid, nanofluid, hybrid nanofluid, solid nanoparticle $Al_2O_3$, and solid nanoparticle Cu are indicated by the subscripts "$f$", "$nf$", "$hnf$", "*Al2O3*", and "*Cu*", respectively. Following the definition of the subscripts, the thermophysical properties of hybrid nanofluid used in Eqs. (2)-(3) are given in Table 1. From Table 1, the volume fractions of $Al_2O_3$ and Cu are stated by $\phi_{Al_2O_3}$ and $\phi_{Cu}$, respectively.

**Table 1**
The thermophysical properties of hybrid nanofluid (Yashkun *et al*., [49]).

| Names | Properties |
|---|---|
| Dynamic viscosity | $\mu_{hnf} = \dfrac{\mu_f}{(1-\phi_{Cu})^{2.5}(1-\phi_{Al_2O_3})^{2.5}}$ |
| Density | $\rho_{hnf} = (1-\phi_{Cu})\left[(1-\phi_{Al_2O_3})\rho_f + \phi_{Al_2O_3}\rho_{Al_2O_3}\right] + \phi_{Cu}\rho_{Cu}$ |
| Thermal conductivity | $k_{hnf} = \frac{k_{Cu}+2k_{nf}-2\phi_{Cu}(k_{nf}-k_{Cu})}{k_{Cu}+2k_{nf}+\phi_{Cu}(k_{nf}-k_{Cu})} \times (k_{nf})$, where $k_{nf} = \frac{k_{Al_2O_3}+2k_f-2\phi_{Al_2O_3}(k_f-k_{Al_2O_3})}{k_{Al_2O_3}+2k_f+\phi_{Al_2O_3}(k_f-k_{Al_2O_3})} \times (k_f)$ |
| Heat capacity | $(\rho c_p)_{hnf} = (1-\phi_{Cu})\left[(1-\phi_{Al_2O_3})(\rho c_p)_f + \phi_{Al_2O_3}(\rho c_p)_{Al_2O_3}\right] + \phi_{Cu}(\rho c_p)_{Cu}$ |
| Thermal expansion coefficient | $\beta_{hnf} = (1-\phi_{Cu})\left[(1-\phi_{Al_2O_3})\beta_f + \phi_{Al_2O_3}\beta_{Al_2O_3}\right] + \phi_{Cu}\beta_{Cu}$ |

The thermophysical properties of the nanoparticles (i.e., $Al_2O_3$ and Cu) and base fluid (water) are shown in Table 2.

**Table 2**
Thermophysical properties of water and nanoparticles (Yashkun *et al*., [49]).

| | Copper (Cu) | Alumina ($Al_2O_3$) | Water ($H_2O$) |
|---|---|---|---|
| $\rho$ ($\mathrm{kgm^{-3}}$) | 8933 | 3970 | 997.1 |
| $c_p$ ($\mathrm{Jkg^{-1}K^{-1}}$) | 385 | 765 | 4179 |
| $k$ ($\mathrm{Wm^{-1}K^{-1}}$) | 400 | 40 | 0.613 |
| $\beta \times 10^{-5}$ ($\mathrm{K^{-1}}$) | 1.67 | 0.85 | 21 |

The following similarity transformation variables convert the governing equations in Eqs. (2)-(3) and the boundary conditions in Eqs. (4)-(5) to a system of nonlinear ordinary differential equations (ODEs) (Waini *et al*., [61]; Yan *et al*., [66]):

$$\psi = \sqrt{2\nu_f l U_w}\, e^{x/2l} f(\eta);\ \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty};\ \ \eta = y\sqrt{\frac{U_w}{2\nu_f l}}\ e^{x/2l}. \quad (8)$$

The velocities along the $x$- and $y$-axes can be transformed via the stream function $\psi$ found in Eq. (8). The following transformations are obtained by satisfying $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$:

$$u = U_w e^{x/l} f'(\eta); \;\; v = -\sqrt{\frac{U_w \nu_f}{2l}} e^{x/2l} (f(\eta) + \eta f'(\eta)). \tag{9}$$

Eq. (1) is completely satisfied. The following system of nonlinear ODEs in Eqs. (10)-(14) is derived by substituting Eqs. (8)-(9) into Eqs. (2)-(5):

$$f''' + \varsigma_1 \varsigma_2 \{f'' f - 2(f')^2 + 2\lambda_1 (\beta_{hnf}/\beta_f)\theta\} = 0, \tag{10}$$

$$\frac{1}{Pr \varsigma_3}\left[\frac{k_{hnf}}{k_f} + \frac{4Rd}{3}\right]\theta'' + \theta' f - 4\theta f' = 0, \tag{11}$$

where

$$\begin{cases} \varsigma_1 = \left\{(1-\phi_{Cu})\left[1-\phi_{Al_2O_3} + \phi_{Al_2O_3}\left({}^{\rho_{Al_2O_3}}/_{\rho_f}\right)\right] + \phi_{Cu}\left({}^{\rho_{Cu}}/_{\rho_f}\right)\right\}, \\ \varsigma_2 = (1-\phi_{Cu})^{2.5}\left(1-\phi_{Al_2O_3}\right)^{2.5}, \\ \varsigma_3 = \left\{(1-\phi_{Cu})\left[1-\phi_{Al_2O_3} + \phi_{Al_2O_3}\frac{(\rho c_p)_{Al_2O_3}}{(\rho c_p)_f}\right] + \phi_{\mathrm{Cu}}\frac{(\rho c_p)_{Cu}}{(\rho c_p)_f}\right\}, \end{cases} \tag{12}$$

together with the boundary conditions:

$$f(0) = S, f'(0) = -1 + \delta f''(0), \theta(0) = 1 + \delta_T \theta'(0), \tag{13}$$

$$f'(\eta) \to 0; \;\; \theta(\eta) \to 0 \text{ as } \;\; \eta \to \infty, \tag{14}$$

where $\lambda_1 = \frac{\beta_f T_0 l}{U_w^2} g$ is the mixed convection parameter, $Pr = \frac{\mu_f (c_p)_f}{k_f}$ is the Prandtl number, $Rd = \frac{4\sigma_1 T_\infty^3}{k^1 k_f}$ is the thermal radiation parameter. In addition, $\delta = A_1 \sqrt{\frac{\nu_f U_w}{2l}}$ and $\delta_T = D_1 \sqrt{\frac{U_w}{2l\nu_f}}$ is the velocity slip parameter and the thermal slip parameter, respectively.

The significant physical elements are skin friction coefficient $C_f$ and local Nusselt number $Nu_x$ that are described as:

$$C_f = \frac{\mu_{hnf}}{\rho_f u_w^2}\left(\frac{\partial u}{\partial y}\right)_{y=0}, \;\; Nu_x = \frac{2l}{k_f (T_w - T_\infty)}\left[-k_{hnf}\left(\frac{\partial T}{\partial y}\right)_{y=0} + (q_r)_{y=0}\right]. \tag{15}$$

By using Eq. (9) into Eq. (15), the following are obtained:

$$\sqrt{Re_x} C_f = \frac{1}{\varsigma_2} f''(0), \;\; \sqrt{\frac{1}{Re_x}} Nu_x = -\left[\frac{k_{hnf}}{k_f} + \frac{4Rd}{3}\right]\theta'(0), \tag{16}$$

where $Re_x = \frac{2lU_w \, e^{x/l}}{\nu_f}$ is the local Reynolds number.

*2.2 Numerical Method*

The numerical solution for the system of higher-order nonlinear ODEs in Eqs. (10) and (11) with the boundary conditions (13) and (14) can be obtained using the bvp4c solver, which executes on the MATLAB computational platform. The bvp4c solver was created by Jacek Kierzenka and Lawrence F. Shampine of Southern Methodist University, Texas (Hale, N. P. [68]). The bvp4c solver is a finite difference algorithm that uses the three-stage Lobatto IIIA implicit Runge–Kutta technique and generates numerical solutions with fourth-order precision. The following steps have been taken in order to implement bvp4c solver to our physical model.

**STEP 1:** In Eqs. (10)-(11), new variables are incorporated for the system of higher-order nonlinear ODEs:

$$y(1) = f, y(2) = f', y(3) = f'', y(4) = \theta, y(5) = \theta'. \tag{17}$$

**STEP 2:** Using the new variables in Eq. (17), convert the system of higher-order nonlinear ODEs in Eqs. (10)-(11) to a system of first order nonlinear ODEs:

$$\begin{aligned} f' &= y(2), \\ f'' &= y(3), \\ f''' &= \varsigma_1\varsigma_2\left(2\big(y(2)\big)^2 - y(1)y(3) - 2\lambda_1\big(\beta_{hnf}/\beta_f\big)y(4)\right), \\ \theta' &= y(5), \\ \theta'' &= \frac{Pr\varsigma_3}{(k_{hnf}/k_f + 4Rd/3)}\big(4y(2)y(4) - y(1)y(5)\big). \end{aligned} \tag{18}$$

**STEP 3:** Express the boundary conditions (13) and (14) in terms of the new variables in Eq. (17):

$$y(1)_a = S, y(2)_a = -1 + \delta y(3)_a, y(4)_a = 1 + \delta_T y(5)_a, y(2)_b = 0, y(4)_b = 0. \tag{19}$$

Note that the subscript '$a$' indicates the position of the surface at $\eta = 0$ while subscript '$b$' indicates the location away from the surface for a certain value of $\eta$. This location is set at $\eta = 20$ in this research.

**STEP 4:** Code the system of first-order nonlinear ODEs in Eq. (18) as well as the boundary conditions in Eq. (19) in the bvp4c solver.

**STEP 5:** Obtain dual solutions by supplying two distinct initial guesses to the bvp4c solver, one at a time. The first solution can be obtained with less restrictive initial guesses, but this is not always true when getting the second solution. This procedure is continued until the numerical solutions asymptotically satisfy the boundary conditions at infinity (i.e., Eq. (14)).

## 3. Results and Discussion

Before starting to discuss the results of the current study, we have to validate the numerical results generated by bvp4c solver. In order to validate these results, the reduced skin friction $f''(0)$ and reduced heat transfer $-\theta'(0)$ for pure water $\left(\phi_{Al_2O_3} = \phi_{Cu} = 0\right)$ for varying values of Prandtl number $(Pr)$, are compared with those equivalent results found in Lund *et al.*, [12] for suction $S = 5$, mixed convection parameter $\lambda_1 = -0.5$, and without the occurrence of velocity slip, thermal slip, and radiation $Rd$ (i.e. $\delta = \delta_T = Rd = 0$). Such comparisons are tabulated in Table 3. Furthermore, the reduced skin friction $f''(0)$ for $\phi_{Al_2O_3} = 0.1$ subjects to varying values of $\phi_{Cu}$, are compared with those corresponding results obtained by Yan *et al.*, [66] for suction $S = 3$, thermal slip $\delta_T = 0.1$, Prandtl number $Pr = 6.2$, and without the occurrence of mixed convection and velocity slip (i.e. $\lambda_1 = \delta = 0$). These evaluations are summarised in Table 4. The physical model proposed in this current study has been validated through Table 3 and Table 4 as the numerical findings from this current study agreed with those numerical results found in the previous studies.

**Table 3**
Values of $f''(0)$ and $-\theta'(0)$ for various values of $Pr$ when $S = 5$, $\phi_{Al_2O_3} = \phi_{Cu} = 0$, $\lambda_1 = -0.5$, $\delta = \delta_T = Rd = 0$

| $Pr$ | Lund *et al.*, [12] | | Present Results | |
|---|---|---|---|---|
| | $f''(0)$ | $-\theta'(0)$ | $f''(0)$ | $-\theta'(0)$ |
| 1 | 4.449203 | 4.447507 | 4.449203 | 4.447507 |
| 1.6 | 4.540536 | 7.334577 | 4.540536 | 7.334577 |
| 2 | 4.570372 | 9.284828 | 4.570372 | 9.284828 |
| 2.4 | 4.590011 | 11.247347 | 4.590011 | 11.247347 |
| 6.2 | - | - | 4.648148 | 30.10742 |

**Table 4**
Values of $f''(0)$ for various values of $\phi_{Cu}$ when $\phi_{Al_2O_3} = 0.1$, $\lambda_1 = \delta = 0$, $S = 3$, $\delta_T = 0.1$, $Pr = 6.2$

| $\phi_{Cu}$ | Yan *et al.*, [66] | | Present Results | |
|---|---|---|---|---|
| | First solution $f''(0)$ | Second solution $f''(0)$ | First solution $f''(0)$ | Second solution $f''(0)$ |
| 0.01 | 2.48626 | -1.10767 | 2.48626 | -1.10767 |
| 0.03 | - | - | 2.65001 | -1.35195 |
| 0.05 | 2.81888 | -1.62610 | 2.81888 | -1.62610 |
| 0.08 | - | - | 2.88110 | -1.73155 |
| 0.1 | 3.07486 | -2.08072 | 3.07486 | -2.08072 |

Figures 2-3 show the effect of $\phi_{Cu}$ on the behaviour of reduced skin friction coefficient $f''(0)$ and rate of reduced heat transfer $-\theta'(0)$, respectively. In Figure 2, an increment in $f''(0)$ was found within the first solution, but an opposite trend was noticed within the second solution as the values of $\phi_{Cu}$ rises. On the other hand, $-\theta'(0)$ declined in both solutions when $\phi_{Cu}$ enhances. In other words, $-\theta'(0)$ rises in both solutions when $S$ improves. It is observed that fluid is flowing towards $S$ till it arrived at a point $S_{ci}$, $i = 1,2,3$ where $S_{ci}$ is the critical point of $S$ such that the connection of the first and second solutions exist. There is no solution when $S < S_{ci}$. It is worth mentioning here that when $\phi_{Cu} = 0$, it is purely an $Al_2O_3$-water nanofluid and $S_{c1} = 2.1885$. After that, 3% of $\phi_{Cu}$ is added, and the critical point of $S$ becomes $S_{c2} = 2.1001$. Additionally, the value of $S_{c3} = 2.0447$ appeared as a result of adding 6% of $\phi_{Cu}$ into the hybrid nanofluid. Furthermore, the addition of $\phi_{Cu}$ extended the boundary layer separation. The chosen values of $\phi_{Cu}$ in Figures 2-3 are within the range of $\phi_{Cu} \in [0,0.2]$ as suggested by Tiwari and Das [67]. Similar findings can also be noticed in Waini *et al.*, [32].

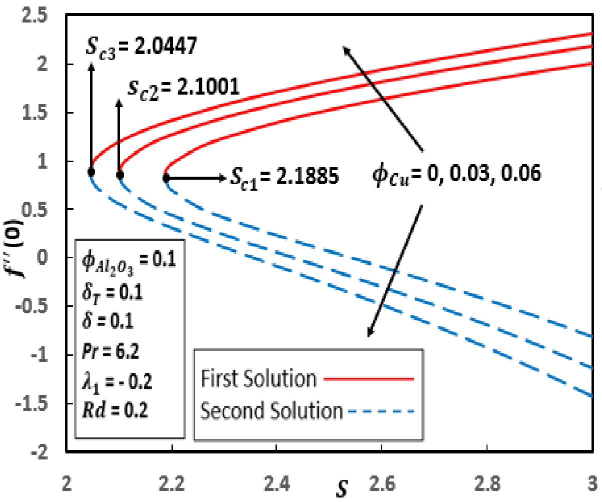

**Fig. 2.** Effect of $\phi_{Cu}$ on $f''(0)$ with respect to $S$

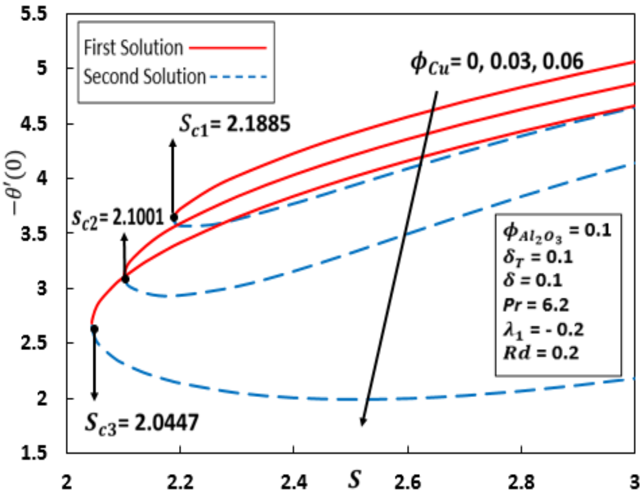

**Fig. 3.** Effect of $\phi_{Cu}$ on $-\theta'(0)$ with respect to $S$

The various sort of velocity slip conditions on $f''(0)$ and $-\theta'(0)$ is highlighted in Figures 4-5. Partial slip conditions are examined in this study where $\delta$ show the effect of the velocity slip condition of the first order. Physically, it is evidently pragmatic that no-slip condition (when $\delta = 0$) has a lower effect on the boundary layer separation as compared to the existence of velocity slip $\delta$ in a hybrid nanofluid. When $\delta = 0,\ 0.1$, and $0.3$, the critical values are $S_{c1} = 2.1316$, $S_{c2} = 2.0447$, and $S_{c3} = 1.9043$, respectively. The values of $\delta$ used in Figures 4-5 are the same as those picked by Yan *et al.*, [66]. The fluid is flowed till a critical point $S_{ci}$, $i = 1,2,3$ while no flow of fluid is possible when $S < S_{ci}$. In Figure 4, when the value of $\delta$ increases, $f''(0)$ decreases in both solutions, while an opposite trend is noticed for $S > 2.4$ in the second solution of $f''(0)$. As seen in Figure 5, when the value of $\delta$ moves from 0 to 0.3, values of $-\theta'(0)$ upsurges in the first solution but drops in the second solution.

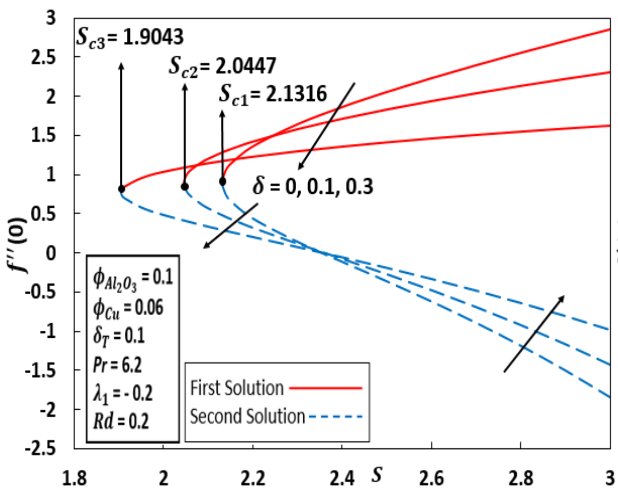

**Fig. 4.** Effect of $\delta$ on $f''(0)$ with respect to $S$

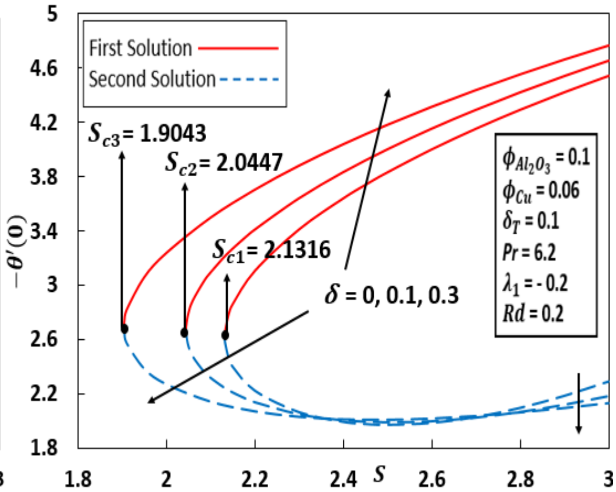

**Fig. 5.** Effect of $\delta$ on $-\theta'(0)$ with respect to $S$

Figures 6-7 demonstrate the effect of first order thermal slip $\delta_T$ with suction $S$ on $f''(0)$ and $-\theta'(0)$, respectively. Suction is employed to increase the performance of diffusers with strong compression ratios of the flow. When $\delta_T = 0, 0.1$, and $0.3$, the critical points are $S_{c1} = 2.0557$, $S_{c2} = 2.0447$, and $S_{c3} = 2.0339$, respectively. The values of $\delta_T$ in Figures 6-7 follow the values of the same parameter chosen by Yan *et al.*, [66]. Besides, an upsurge of $\delta_T$ causes an increment (decrement) within the first (second) solution of $f''(0)$. The values of $\delta_T$ are improved by means of developing early separation of the boundary layer. It is noted that the duality of the solutions happens when $S <$

$S_{ci}$ and no solution exist beyond $S_{ci}$ for $i = 1{,}2{,}3$. Heat transfer $-\theta'(0)$ reduces in both solutions when $\delta_T$ is improved. Substantially, this decreasing trend is due to the fact that heat is transferring fast from the surface to cold areas of the hybrid nanofluid.

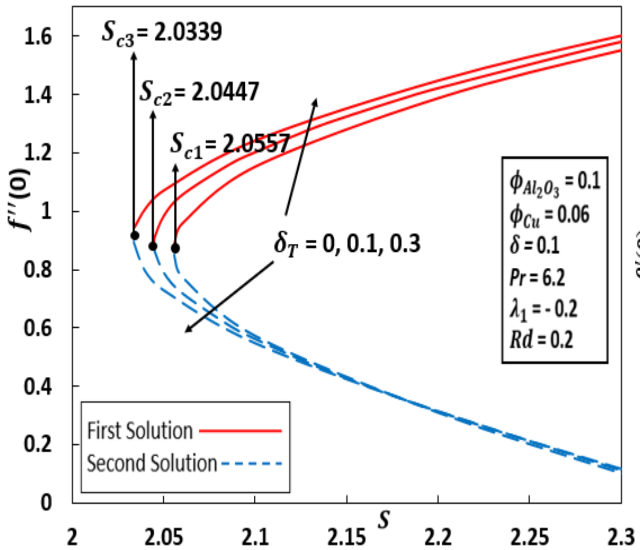


**Fig. 6.** Effect of $\delta_T$ on $f''(0)$ with respect to $S$

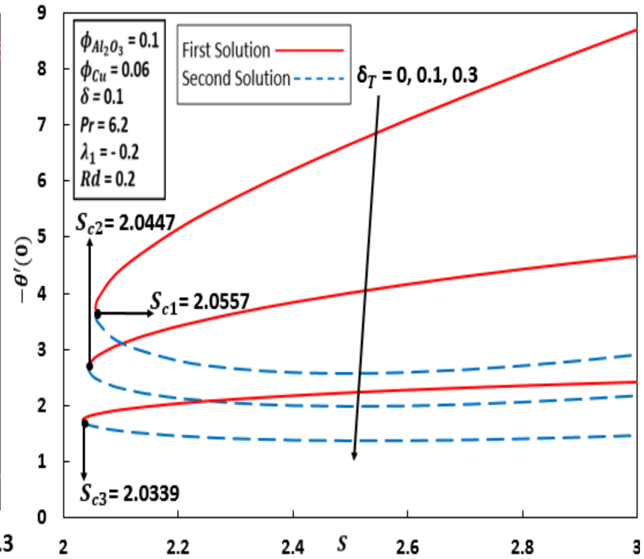


**Fig. 7.** Effect of $\delta_T$ on $-\theta'(0)$ with respect to $S$

Figures 8-9 depict the influence of radiation parameter $Rd$ on the velocity distribution $f'(\eta)$ and temperature distribution $\theta\ (\eta)$. This study chooses the radiation parameter to go between 0 and 0.5 which was found to be acceptable within a wider range values of the radiation parameter previously studied by Waini *et al.*, [61] (i.e. $0 \leq Rd \leq 1$). The velocity distribution for the first solution increases in the range of $0.2 \leq Rd \leq 0.5$, and decreases in the range of $0 \leq Rd \leq 0.2$. Radiation parameter only occurs in the temperature equation in Eq. (11) and it is uncoupled from the momentum equation in Eq. (10). As a consequence, the values of $Rd$ cause fluctuation to the first solution of the velocity distribution. However, an increasing behaviour of the second solution of the velocity distribution is observed when raising the values of $Rd$. Additionally, the temperature distribution constantly rises in the first and second solutions with a rising value of $Rd$. It is indicating that larger radiation values result in a lower temperature gradient at the surface. Due to the occurrence of extreme radiation, a massive quantity of heat energy is generated in the system, which indicates an upsurge in the temperature of the fluid. Similar pattern of temperature distribution shown in Figure 9 can also observed in Waini *et al.*, [61] and Lund *et al.*, [62].

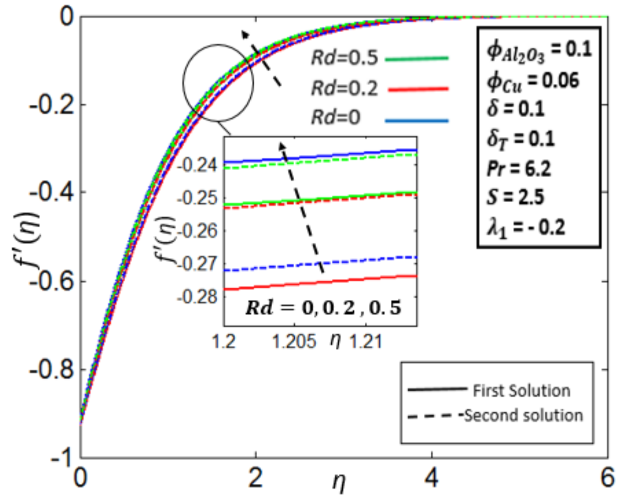


**Fig. 8.** Velocity profile for various values of $Rd$

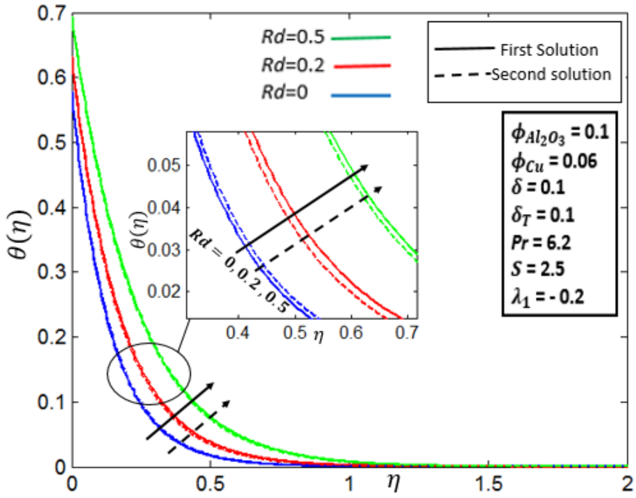


**Fig. 9.** Temperature profile for various values of $Rd$

The plots of velocity distribution $f'(\eta)$ and temperature distribution $\theta(\eta)$ against mixed convection parameter $\lambda_1$ are shown in Figures 10-11, respectively. This study used mixed convection parameter values ranging from $-0.2$ to $0.2$, which were found to be reasonable following the work done by Waini *et al*., [43] who had implemented a wider range of mixed convection parameters which varies between $-0.5$ and $0.5$. It is noticed that the first solution of velocity distribution and temperature distribution neither increase nor decrease with the growing values of mixed convection parameter. However, velocity distribution declines, and temperature distribution rises along with the growing values of the mixed convection parameter for the second solution. Physically, it indicates that dual solutions are conceivable for buoyancy assisting flow.

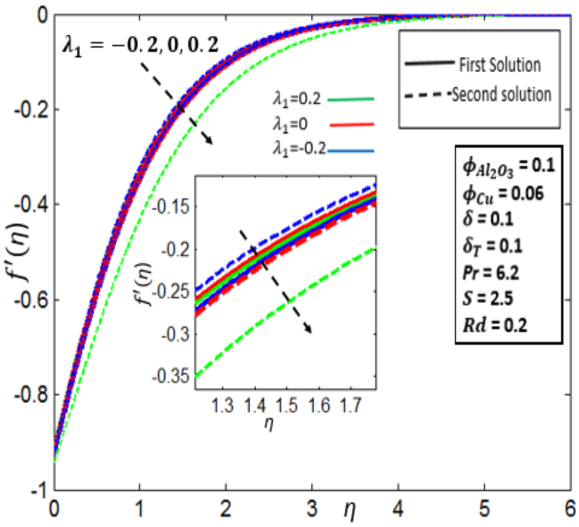


**Fig. 10.** Velocity profile for various values of $\lambda_1$

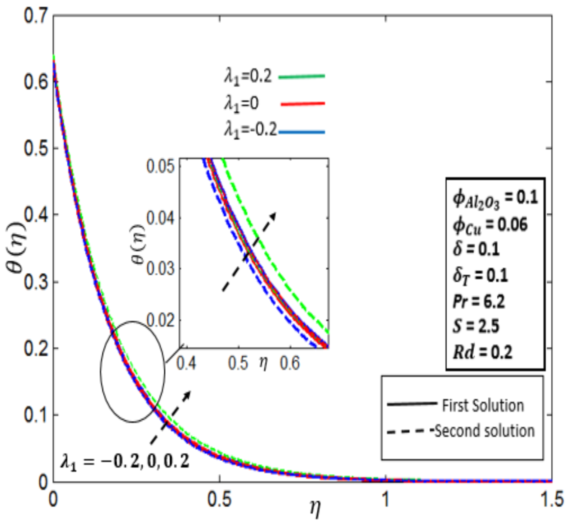


**Fig. 11.** Temperature profile for various values of $\lambda_1$

Figure 12-13 illustrate the effect of Prandtl number $Pr$ on the velocity and temperature distributions of the hybrid nanofluid. Although the values of $Pr$ used in Figures 12-13 fall within the range of $2 \leq Pr \leq 6.2$, but it is also allowable to use a more extended range values of $Pr$ such as $0.5 \leq Pr \leq 10$ as reported in Waini *et al*., [61]. From Figures 12-13, the decreasing behaviour of the first solution in velocity distribution and temperature distribution is observed when improving the values of $Pr$. For the second solution, velocity distribution increases in the range of $2 \leq Pr \leq 4$ and decreases in the range of $4 \leq Pr \leq 6.2$, while temperature distribution is also reduced when the values of $Pr$ is enhanced. Usually, larger Prandtl number possesses lesser thermal diffusivity. Similar outcomes in temperature distribution can also be found in Asghar and Teh [36].

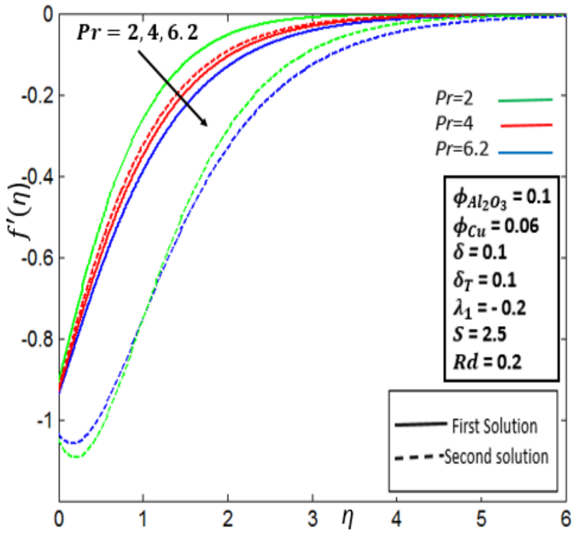


**Fig. 12.** Velocity profile for various values of $Pr$

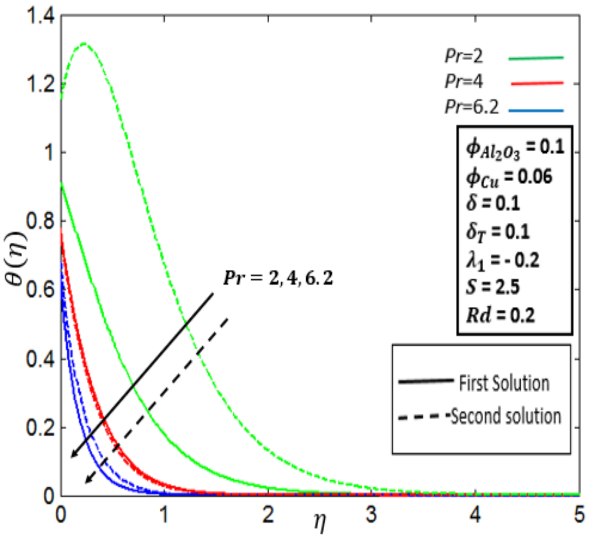


**Fig. 13.** Temperature profile for various values of $Pr$

## 4. Conclusions

In this study, a two-dimensional mixed convective and radiative $Al_2O_3$-Cu/ $H_2O$ hybrid nanofluid flow over an exponential vertical shrinking sheet with partial slips has been investigated using bvp4c solver on the MATLAB computing platform. The physical model suggested in this research has been validated and the bvp4c solver coding has also been verified.

To be more specific, all numerical and graphical outcomes are obtained by solving a system of higher-order ODEs and their boundary conditions that are transformed from the governing PDEs by similarity transformation. This research concentrates on the velocity profile $f'(\eta)$, temperature profile $\theta(\eta)$, behaviour of the reduced skin friction $f''(0)$ and reduced heat transfer $-\theta'(0)$ under the effect of suction, mixed convection, radiation, and partial slips on the above-stated hybrid nanofluid flow. The following are the key findings of the current study: assumptions:

i. Dual solutions have been shown to be feasible with a confident range of applied parameters.
ii. The flow of hybrid fluid is flowed till a critical point $S_{ci}$, while no flow of fluid is possible when $S < S_{ci}$ and when suction parameter $S$ improves, the heat transfer rate increases.
iii. Numerical results have shown that the addition of hybrid nanoparticles also increased the heat transfer rate and the addition of $\phi_{Cu}$ extended the boundary layer separation.
iv. It is detected that when the value of velocity slip $\delta$ varies from 0 to 0.3, values of the $-\theta'(0)$ upsurges in the first solution but declines in the second solution.
v. The heat transfer rate has decelerated with rising values of thermal slip $\delta_T$. This decreasing tendency is due to the fact that heat is moving quickly from the surface to the cold region of the hybrid nanofluid.
vi. This study has also revealed that as the radiation parameter $Rd$ upsurges, the temperature $\theta(\eta)$ increases constantly. The temperature $\theta(\eta)$ also increases in the second solution when increasing the values of mixed convection parameter $\lambda_1$. However, the temperature $\theta$ $(\eta)$ declines as $Pr$ rises.

This study mainly uncovers new insight on the behaviour of $Al_2O_3$-$Cu/H_2O$ hybrid nanofluid under the joint influence of mixed convection, radiation, and partial slip when it flows over a vertical exponential shrinking sheet. Such physical setting has not been investigated in the existing literature. The proposed model might find its application in the air conditioning system. Hybrid nanofluid as the refrigerant is moved through the entire air conditioning system using the compressor. At certain point, the hot refrigerant will enter the micro heat exchanger where slip condition takes place to reduce the fluid flow's resistance. A fan is fixed near the micro heat exchanger. The fan will blow towards the exchanger so that heat from the hot refrigerant can be transferred to the surrounding air through mixed convection. The surrounding warm air will then be released outside the building through radiation. Finally, numerous recommendations for future researches include modifying the current proposed model using other types of physical parameters such as thermal convective conditions and heat source/absorption, or making the proposed model to flow through complex body geometries.

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