



Flow And Heat Transfer of Unsteady Two-Phase Boundary Layer Flow Past an Inclined Permeable Stretching Sheet with Electrification of Particles

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ABSTRACT

In the present study, an analysis has been carried out for a particle laden boundary layer flow with existence of electrification of particles has been studied over an inclined permeable stretching sheet. In most of the MHD fluid flow problems, either the plate is externally supplied by the magnetic/electric field or the fluid is electrically conducting. In the present problem, neither the plate is electrified nor the fluid is electrically conducted, but due to the random motion of the particles, collision of particle-particle and particle-wall, the particles are electrified. This electric field affects the fluid flow and heat transfer of the flow problem. Again, in the previous literatures, Buoyancy force is considered in momentum equations of fluid phase only. But in reality, both the phases are affected by the buoyancy force. For this reason, a reasonable mathematical model for two-phase buoyancy driven flow has been formulated with the consideration of electrification of particles in both fluid and particle phase. The governing system of PDEs are transferred to system of ODEs by applying similarity transformations and then computed by implementing Runge-Kutta method. The impact of electrification and other fluid parameters on flow and heat transfer has been studied. The results are represented through graphs and tables.

1. Introduction

Heat transfer phenomena of two-phase flow [1] over a stretchable surface have huge applications in engineering process and manufacturing plants. The concept of unsteady two-phase flow that occurs due to suddenly stretching of sheets is very much used at fiber technology and in extrusion process. Many industrial applications based on stretching sheets are such as the extrusion of plastic sheet in aerodynamic industry, extrusion of polymer sheet in dyeing industry and sedimentation applications. The quantity of heat flux over a plane is very much essential for determining quality of the product. The rate of stretching is also having a great role for getting good quality materials. As the area of applications is very large; many people are interested to work on the topic “boundary layer flow and heat transfer over a stretching sheet”.

Gurbkaand Bobba [2] has investigated for the heat transfer phenomena in the flow over a stretchable sheet. Similarity solution of unsteady flow and heat transfer over stretchable surface has

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been studied by Sharidan, Mohmood and Pop [3]. With existence of magnetic field, a mixed convection of power law of fluid past over a stretching sheet was analyzed by Chen [4]. Rashidi et.al [5] have investigated on the “free convective heat and mass transfer for MHD flow over a vertical stretchable sheet”. Hady, Mohamed and Hilal [6] analyzed on the impacts of deformation on MHD viscoelastic fluid flow over a nonlinear stretchable sheet. Mukhopadhaya and Anderson [7] have studied a viscous incompressible fluid flow problem in cylindrical coordinate system with presence of uniform magnetic field. Reddy, M. Reddy and Naramgiri [8] have given a numerical computation result of heat and mass transfer in a clear fluid over a permeable stretchable sheet. Electrification of the solid particle always happens when particles are contact with each other and apart from each other in different surface conditions. In the fluid flow, electrification occurs by the collision of particle with each other and hitting of particles with the wall of the boundary layer. Since the electrification of particle have a great effect on boundary layer flow like skin friction, heat transfer etc. so it is very much important to merge these phenomena in the modeling of fluid flow problem. Tripathy *et al.*, [9] has studied heat transfer in permeable stretching sheet. Samantara [10-12] have studied impact of electrification of particles in flow geometry of horizontal plate and jet flow. Also many researchers like Ishak *et al.*, [13], Abel *et al.*, [14] and Gireesha *et al.*, [15] have given a great contribution towards the research process of fluid flow over a stretching sheet with presence of non-uniform heat source/sink and radiation.

2. Modeling of the Problem

A boundary layer dusty flow near a “permeable” inclined unsteady stretching sheet is assumed in Figure 1. The wall is stretched with a linear, due to the application of two-interacting opposite forces on the wall. x-axis is considered along the flow and y-axis is normal to it. Due to tribo-electrification, both the phases are influenced by electric force.

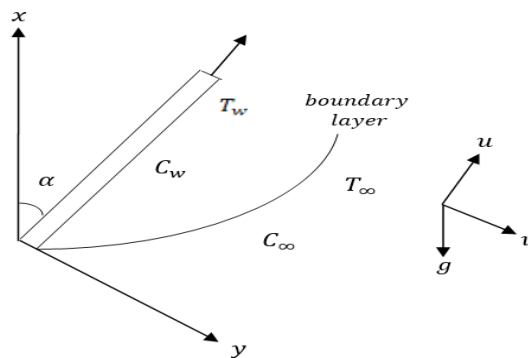


Fig. 1. Geometrical configuration of flow problem

The prevailing equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial}{\partial t} \rho_p + \frac{\partial}{\partial x} (\rho_p u_p) + \frac{\partial}{\partial y} (\rho_p v_p) = 0 \tag{2}$$

$$\rho(1 - \varphi) \left[\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = (1 - \varphi)\mu \frac{\partial^2 u}{\partial y^2} - \frac{1}{\tau_p} \varphi \rho_s (u - u_p) + (1 - \varphi)\rho g \beta^* (T - T_\infty) \cos \alpha + \varphi \rho_s \left(\frac{e}{m} \right) E \quad (3)$$

$$\varphi \rho_s \left(\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} \right) = \frac{\partial}{\partial y} \left(\varphi \mu_s \frac{\partial u_p}{\partial y} \right) + \frac{1}{\tau_p} \varphi \rho_s (u - u_p) + \varphi (\rho_s - \rho) g + \varphi \rho_s \left(\frac{e}{m} \right) E \quad (4)$$

$$\varphi \rho_s \left(\frac{\partial v_p}{\partial t} + u_p \frac{\partial v_p}{\partial x} + v_p \frac{\partial v_p}{\partial y} \right) = \frac{\partial}{\partial y} \left(\varphi \mu_s \frac{\partial v_p}{\partial y} \right) + \frac{1}{\tau_p} \varphi \rho_s (v - v_p) \quad (5)$$

$$(1 - \varphi)\rho c_p \left[\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = (1 - \varphi)k \frac{\partial^2 T}{\partial y^2} + \frac{1}{\tau_T} \varphi \rho_s c_s (T_p - T) + \frac{1}{\tau_p} \varphi \rho_s (u - u_p)^2 + (1 - \varphi)\mu \left(\frac{\partial u}{\partial y} \right)^2 + \varphi \rho_s \left(\frac{e}{m} \right) E u_p \quad (6)$$

$$\varphi \rho_s c_s \left[\frac{\partial T_p}{\partial t} + u_p \frac{\partial T_p}{\partial x} + v_p \frac{\partial T_p}{\partial y} \right] = \frac{\partial}{\partial y} \left(\varphi k_s \frac{\partial T_p}{\partial y} \right) - \frac{1}{\tau_p} \varphi \rho_s c_s (T_p - T) - \frac{1}{\tau_p} \varphi \rho_s (u - u_p)^2 + \varphi \mu_s \left[u_p \frac{\partial^2 u_p}{\partial y^2} + \left(\frac{\partial u_p}{\partial y} \right)^2 \right] + \varphi \rho_s \left(\frac{e}{m} \right) E u_p \quad (7)$$

With boundary conditions

$$\left. \begin{aligned} u = U_w(x, t) = \frac{cx}{1 - at}, v = V_w(x, t) = -\frac{v_0}{\sqrt{1 - at}} \text{ at } y = 0 \\ \rho_p = \omega \rho, u = 0, u_p = 0, v_p \rightarrow v \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad (8)$$

where ω is the density ratio in the main stream.

For solving Eq. (6) and Eq. (7), the non-dimensional temperature boundary conditions can be considered as below

$$T = T_w = T_\infty + T_0 \frac{cx^2}{v(1-at)^2} \text{ at } y = 0 \text{ and } T \rightarrow T_\infty, T_p \rightarrow T_\infty \text{ as } y \rightarrow \infty \quad (9)$$

For most of the gases $\tau_p \approx \tau_T$, $k_s = k \frac{c_s \mu_s}{c_p \mu}$, $\frac{c_s}{c_p} = \frac{2}{3Pr}$

The non-dimensional variables used to solve Eq. (1) to Eq. (7)

$$u = \frac{cx}{1-at} f'(\eta), v = -\sqrt{\frac{cv}{1-at}} f(\eta), \frac{\varphi \rho_s}{\rho} = \frac{\rho_p}{\rho} = \rho_r = H(\eta)$$

$$u_p = \frac{cx}{1-at} F(\eta), v_p = \sqrt{\frac{cv}{1-at}} G(\eta), \eta = \sqrt{\frac{c}{v(1-at)}} y$$

Where,

$$Pr = \frac{\mu c_p}{k}, \quad \beta = \frac{1-at}{c\tau_p}, \quad \epsilon = \frac{v_s}{v}, \quad \varphi = \frac{\rho_p}{\rho_s}, \quad A = \frac{a}{c}, \quad Ec = \frac{cv}{c_p T_0}, \quad v = \frac{\mu}{\rho}, \quad f_0 = -\frac{v_0}{\sqrt{cv}}$$

$$\theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \quad \theta_p(\eta) = \frac{T_p-T_\infty}{T_w-T_\infty},$$

$$\text{Where } T = T_\infty + T_0 \frac{cx^2}{v(1-at)^2} \theta(\eta), \quad T_p = T_\infty + T_0 \frac{cx^2}{v(1-at)^2} \theta_p(\eta)$$

We get the following

$$f''' = A \left(f' + \frac{\eta}{2} f'' \right) + f'^2 - ff'' + \frac{1}{(1-\varphi)} \beta H (f' - F) - \lambda \theta \cos \alpha - \frac{1}{(1-\varphi)} HE \quad (10)$$

$$F'' = \frac{1}{\epsilon} \left[A \left(\frac{\eta}{2} F' + F \right) + F^2 + GF' - \beta (f' - F) - \frac{1}{Fr} \left(1 - \frac{1}{\gamma} \right) - M \right] \quad (11)$$

$$G'' = \frac{1}{\epsilon} \left[\frac{A}{2} (\eta G' + G) + GG' + \beta (f + G) \right] \quad (12)$$

$$\theta'' = \left[\begin{aligned} &Pr(2f'\theta - f\theta') + \frac{2}{3} \frac{1}{1-\varphi} \beta H (\theta_p - \theta) - \frac{1}{1-\varphi} \beta Pr Ec H (F - f')^2 - Pr Ec f''^2 \\ &+ \frac{A}{2} Pr (\eta \theta' + 4\theta) - \frac{1}{1-\varphi} Pr Ec HMF \end{aligned} \right] \quad (13)$$

$$\theta_p'' = \frac{1}{\frac{\epsilon}{Pr}} \left[\begin{aligned} &\frac{A}{2} (\eta \theta_p' + 4\theta_p) + 2F\theta_p + G + \beta (\theta_p - \theta) + \frac{3}{2} \beta Pr Ec (f' - F)^2 \\ &- \frac{3}{2} \epsilon Pr Ec (FF'' + F'^2) - \frac{3}{2} Pr Ec MF \end{aligned} \right] \quad (14)$$

$$H' = -(HF + HG') / \left(\frac{\eta}{2} A + G \right) \quad (15)$$

3. Numerical Computation

The system of Eq. (10) to Eq. (15) with the conditions Eq. (8) and Eq. (9) are computed by applying Runge-Kutte 4th order method with BVP4C tool of MATLAB. By considering finite value of $\eta \rightarrow \infty$ say $\eta = 15$ with a particular tolerance level of less than $O(10^{-06})$. The investigations of numerical computations has been made for the impact of different physical parameters like unsteady parameter(A), Electrification parameter(M), Prandtl number(Pr), Eckret number(Ec), Inclination parameter(α) and Buoyancy parameter(λ) are represented graphically.

In Table 1 the results are also matched with the results available in previous literature. Here the values of rate of heat transfer are matched with the previous authors. So it proves the validation of our program.

Table 1
 Result validating table

Prandtl number, Pr	Ishak <i>et al.</i> , [13]	Able <i>et al.</i> , [14]	Giressha <i>et al.</i> , [15]	Chen [4]	Gurbka <i>et al.</i> , [2]	Mukhopadhyaya <i>et al.</i> , [7]	Current Study
0.72	-----	1.0885	1.0885	1.0885	1.0885	1.0885	1.0884
1.0	1.3333	1.3333	1.3333	1.3333	1.3333	1.3333	1.3333
3.0	2.5097	-----	2.5097	2.5097	-----	2.5097	2.5097
10.0	4.7969	4.7969	4.7969	4.7969	4.7969	-----	-----

4. Result

4.1 Effect of Prandtl Number (Pr)

Prandtl Number signifies the ratio between momentum boundary layer and thermal boundary layer of a flow field. In this problem, the momentum and energy equations are coupled with bouncy and electrification terms. So, the velocity as well as thermal distribution of the flow problem is influenced by variation of Prandtl number which is shown in Figure 2 & 3. The values of Pr can be chosen as 0.71, 1.0 and 7.0, as are Prandtl number of air, electrolyte solution and water respectively. Figure 2&3 represent the variation of temperature profile of both phase with respect to Pr. From the figure it is observed that the “temperature profile” decreases with increasing of Pr value. It signifies that the flow of fluid along with particle is very high in case of air rather than electrolyte and water. In air, the thermal diffusion dominant momentum diffusion. For electrolyte solution, the momentum and thermal diffusion are equal, and the momentum diffusion dominant the thermal diffusion in case of water.

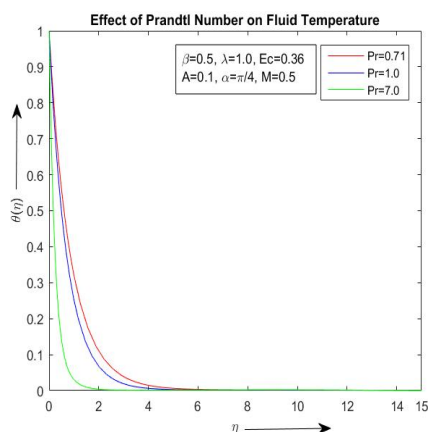


Fig. 2. Effect of Prandtl number (Pr) on fluid temperature

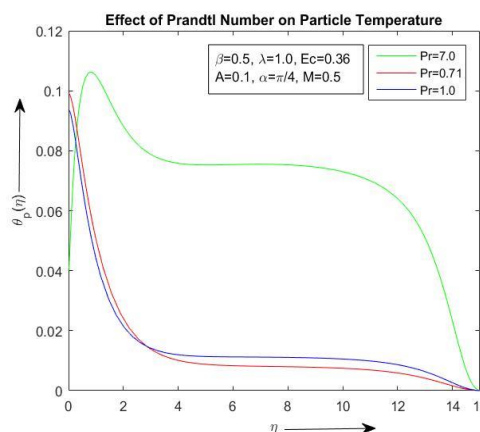


Fig. 3. Effect of Prandtl number (Pr) on particle temperature

4.2 Effect of Eckert Number (Ec)

Figure 4 & 5 represent the variant of temperature contour of fluid and particle phases respectively with respect to Ec. From the figure it is concluded that the temperature profile of both the phase increase with rising of Ec value. In case of particle phase, the impact of Ec clearly visible where as in case of fluid phase it is less distinguished.

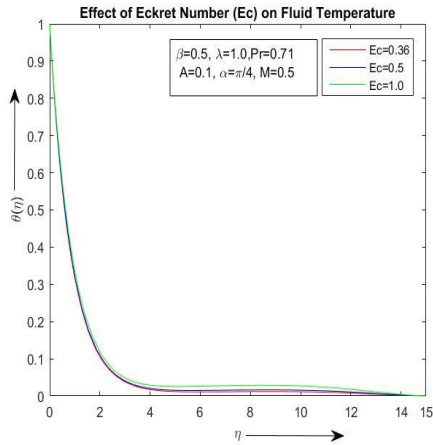


Fig. 4. Effect of Eckert number (Ec) on fluid temperature

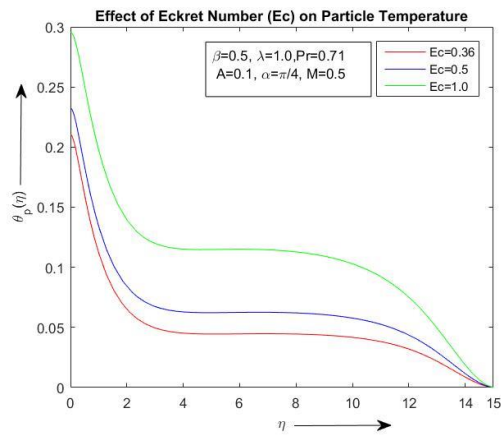


Fig. 5. Effect of Eckert number (Ec) on particle temperature

4.3 Effect of Electrification Parameter (M)

Figure 6 & 7 represent the variant of velocity contour of both the phases with respect to electrification parameter M. From the figure it is marked that the velocity profile in both the-phase increases with enhance of M value. It is prominent in case of particle phase as compare to fluid phase. Figure 8 &9 represent the disparity of temperature profile of both phases with respect to M respectively. From the figure it is concluded that the temperature profile of both the phase increase with increasing of M value but the deviation is negligible in case of fluid phase but very much prominent in case of Particle phase.

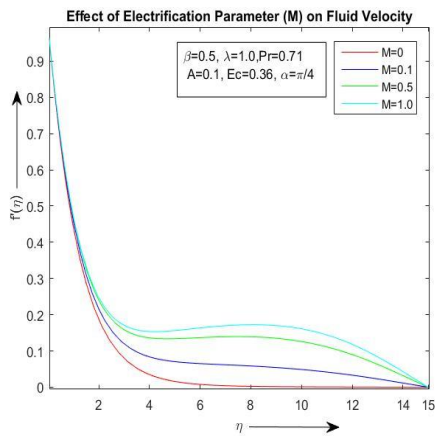


Fig. 6. Effect of Electrification parameter (M) on fluid velocity

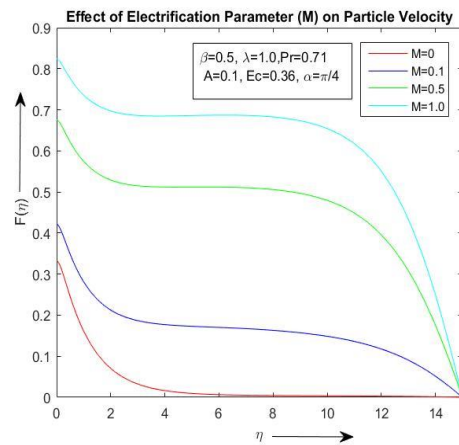


Fig. 7. Effect of Electrification parameter (M) on particle velocity

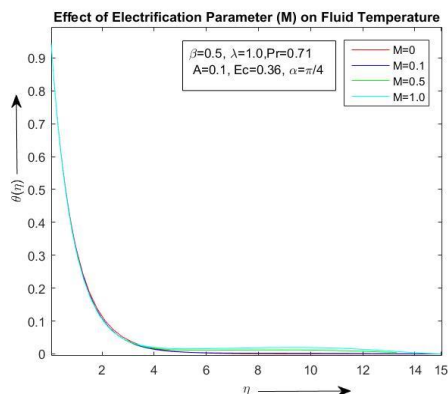


Fig. 8. Effect of Electrification parameter (M) on fluid temperature

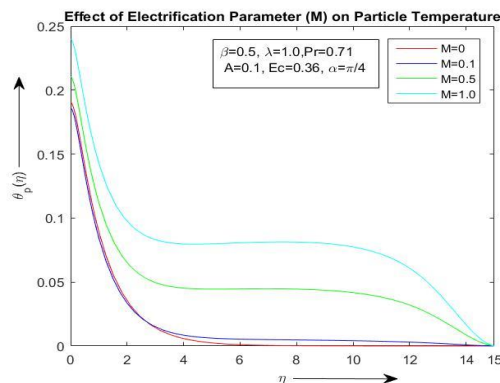


Fig. 9. Effect of Electrification parameter (M) on particle temperature

4.4 Effect of Unsteady Parameter (A)

Figure 10 & 11 represent the variation of velocity profile of both the phases with respect to unsteady parameter “A”. From the graph it is concluded that the velocity profile in both the phase decreases with increase of value of A up to certain distance then its effect reversed. Figure 12 & 13 represent the deviation of temperature contour of both the phases with respect to unsteady parameter “A”. From the figure it is concluded that unsteady parameter “A” has negligible impact on the temperature profile of fluid phase whereas has effect on particle phase.

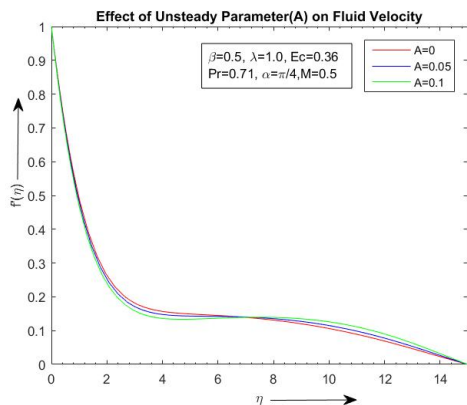


Fig. 10. Effect of Unsteady parameter (A) on fluid velocity

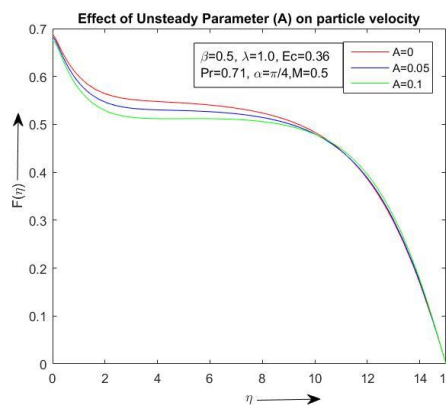


Fig. 11. Effect of Unsteady parameter (A) on particle velocity

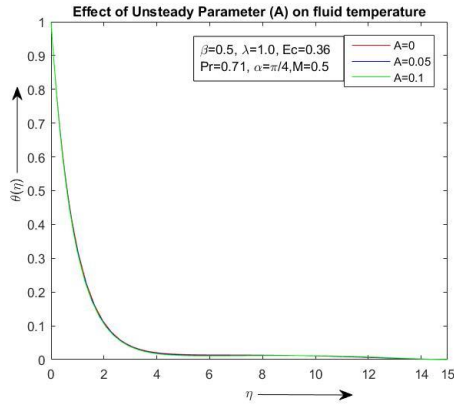


Fig. 12. Effect of Unsteady parameter (A) on fluid temperature

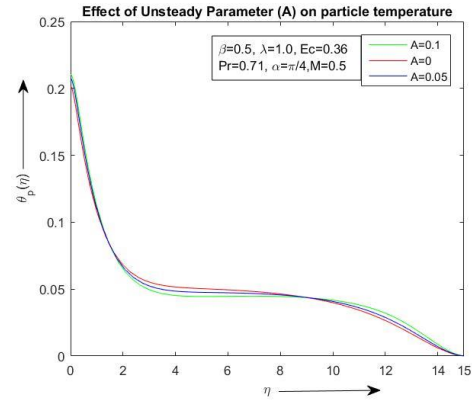


Fig. 13. Effect of Unsteady parameter (A) on particle temperature

4.5 Effect of Angle of Inclined (α)

Figure 14 & 15 represent the variation of velocity profile of both the phases with respect to inclination parameter " α ". From the figure it is concluded that the velocity profile has direct effect on inclination parameter α for both the phases. Figure 16&17 represent the variant of temperature profile of both the phases with respect to inclination parameter α respectively. From the graph it is observed that the temperature profile of both the phase decrease with increase of inclination parameter " α ".

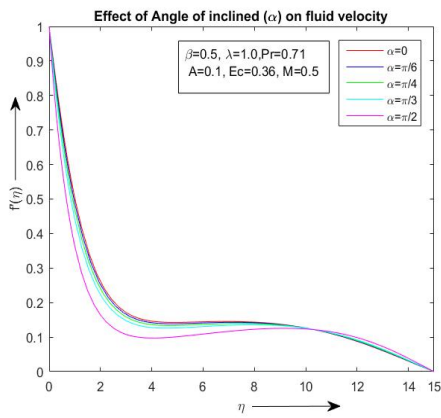


Fig. 14. Effect of Angle of Inclined (α) on fluid velocity

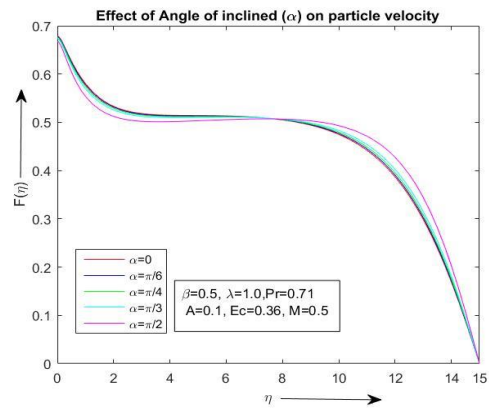


Fig. 15. Effect of Angle of Inclined (α) on particle velocity

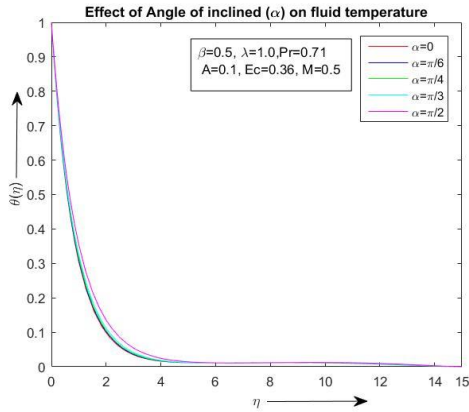


Fig. 16. Effect of Angle of Inclined (α) on fluid temperature

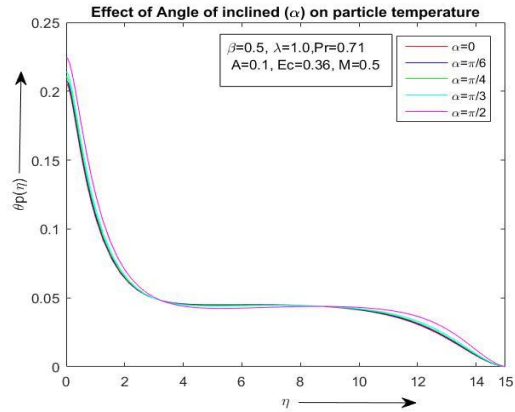


Fig. 17. Effect of Angle of Inclined (α) on particle temperature

4.6 Effect of Bouncy Parameter (λ)

Figure 18 &19 represent the variant of velocity profile of both phases with respect to bouncy parameter λ . From the figure it is concluded that the bouncy parameter λ has direct effect on velocity profile of fluid phase. But has reverse effect on particle phase. Figure 20 & 21 represent the variation of temperature profile of both the phases with respect to bouncy parameter λ . From the figure it is concluded that the temperature profile of both the phases falls with increasing the value of bouncy parameter λ .

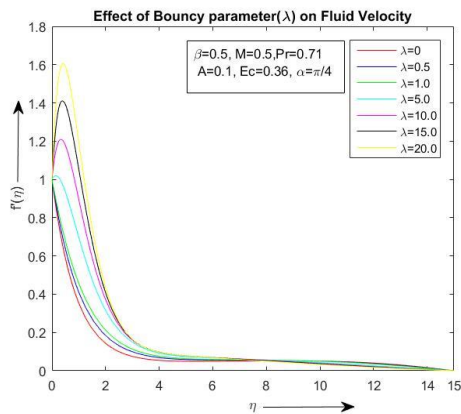


Fig. 18. Effect of Bouncy parameter (λ) on fluid velocity

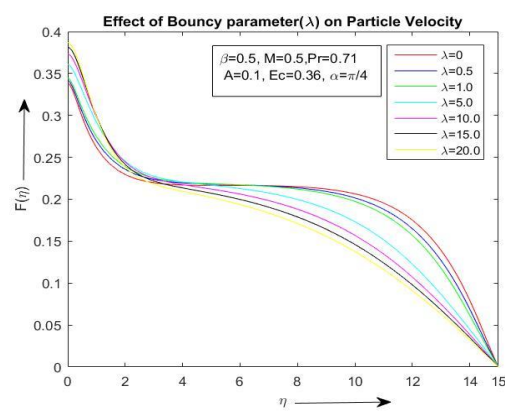


Fig. 19. Effect of Bouncy parameter (λ) on particle velocity

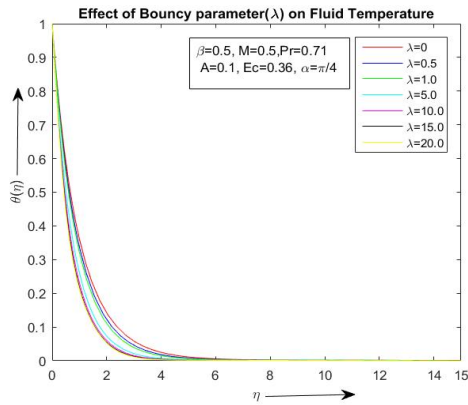


Fig. 20. Effect of Bouncy parameter (λ) on fluid temperature

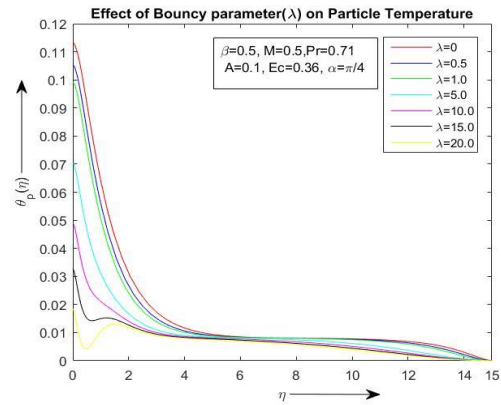


Fig. 21. Effect of Bouncy parameter (λ) on particle velocity

4.7 Skin Friction and Nusselt Number

Due to accelerating flow field, the temperature of both the phases rapidly increases along the surface. i.e. the fluid-particle interaction is more for accelerated fluid and causes the stronger electric field, which significantly enhances the temperature and it occurs more heat transfer on the surface of the stretching surface, shown in Table2.

Table 2

Effect of “Prandtl number”, “Eckret number”, “Angle of inclination” of the sheet and “Electrification parameter” on skin friction and Nusselt number.

Pr	Ec	α	M	$f''(0)$	$-\theta'(0)$
0.71	0.36	$\pi/4$	0.5	-0.70052	0.95802
1.0	--	--	--	-0.73621	1.15040
7.0	--	--	--	-0.90060	3.26596
0.71	0.36	$\pi/4$	0.5	-0.70052	0.95802
	0.5			-0.69711	0.94442
	1.0			-0.68557	0.89775
0.71	0.36	0	0.5	-0.56578	0.98516
		$\pi/6$		-0.62664	0.97332
		$\pi/4$		-0.70052	0.95802
		$\pi/3$		-0.80015	0.93548
		$\pi/2$		-1.06634	0.85917
0.71	0.36	$\pi/4$	0.1	-0.71345	0.95527
			0.5	-0.70052	0.95802
			1.0	-0.69862	0.95891

5. Conclusion

From above graphs and result discussions, it is concluded that the presence of particles in fluid has greater impacts on flow and heat transfer profiles.

- i. The presence of particles in fluid collides with each other and with the wall, thus generates electric force that helps in accelerating body force which acts in the direction of electric field. It results in reducing the skin friction on the surface of stretching sheet.

- ii. It significantly enhances the temperature and it does more heat transfer on the surface of the stretching surface.

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References

- [1] Kanungo, Subhrajit, and Tumbanath Samantara. "Numerical Solution of Two-Phase Radiated Unsteady Flow Over a Horizontal Stretching Sheet with Simultaneous Effect of Electrification, Radiation and Non-Uniform Internal Heat Source/Sink." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 100, no. 3 (2022): 11-22. <https://doi.org/10.37934/arfmts.100.3.1122>
- [2] Grubka, L. J., and K. M. Bobba. "Heat transfer characteristics of a continuous stretching surface with variable temperature." *Journal of Heat Transfer* 107, no. 1 (1985): 248-250. <https://doi.org/10.1115/1.3247387>
- [3] Sharidan, S., M. Mahmood, and I. Pop. "Similarity solutions for the unsteady boundary layer flow and heat transfer due to a stretching sheet." *Applied Mechanics and Engineering* 11, no. 3 (2006): 647.
- [4] Chen, C-H. "Laminar mixed convection adjacent to vertical, continuously stretching sheets." *Heat and Mass transfer* 33, no. 5 (1998): 471-476.. <https://doi.org/10.1007/s002310050217>
- [5] Rashidi, Mohammad Mehdi, Behnam Rostami, Navid Freidoonimehr, and Saeid Abbasbandy. "Free convective heat and mass transfer for MHD fluid flow over a permeable vertical stretching sheet in the presence of the radiation and buoyancy effects." *Ain Shams Engineering Journal* 5, no. 3 (2014): 901-912. <https://doi.org/10.1016/j.asej.2014.02.007>
- [6] Hady, F. M., R. A. Mohamed, and Hillal M. ElShehaby. "Thermal Radiation, Heat Source/Sink and Work Done by Deformation Impacts on MHD Viscoelastic Fluid over a Nonlinear Stretching Sheet." *World Journal of Mechanics* 3, no. 04 (2013): 203. <https://doi.org/10.4236/wjm.2013.34020>
- [7] Mukhopadhyay, Swati, and Helge I. Andersson. "Effects of slip and heat transfer analysis of flow over an unsteady stretching surface." *Heat and Mass Transfer* 45, no. 11 (2009): 1447-1452.. <https://doi.org/10.1007/s00231-009-0516-7>
- [8] Machireddy, Gnaneswara Reddy, and Sandeep Naramgari. "Heat and mass transfer in radiative MHD Carreau fluid with cross diffusion." *Ain Shams Engineering Journal* 9, no. 4 (2018): 1189-1204. <https://doi.org/10.1016/j.asej.2016.06.012>
- [9] Tripathy, Pradeep Kumar, Tumbanath Samantara, Jayaprakash Mishra, and Sujata Panda. "Mixed convective radiative heat transfer in a particle-laden boundary layer fluid over an exponentially stretching permeable surface." In *AIP Conference Proceedings*, vol. 2435, no. 1, p. 020030. AIP Publishing LLC, 2022. <https://doi.org/10.1063/5.0083840>
- [10] Samantara, Tumbanath. "Velocity profile of fluid particle suspension over a horizontal plate with electrification of particles." *International Journal of Innovative Technology and Exploring Engineering* 8, no. 11 (2019): 1119-1122. <https://doi.org/10.35940/ijitee.J1220.0981119>
- [11] Samantara, Tumbanath, S. K. Mishra, and T. C. Panda. "Numerical Modeling of Two Phase Jet Flow and Heat Transfer with Charged Suspended Particulate Matter (SPM)." *Modelling, Measurement and Control B* 86: 885-906. https://doi.org/10.18280/mmc_b.860405
- [12] Samantara, Tumbanath, S. K. Mishra, and T. C. Panda. "Numerical Modeling of Two Phase Jet Flow and Heat Transfer with Charged Suspended Particulate Matter (SPM)." *Modelling, Measurement and Control B* 86: 885-906. https://doi.org/10.18280/mmc_b.860405
- [13] Ishak, Anuar, Roslinda Nazar, and I. Pop. "Hydromagnetic flow and heat transfer adjacent to a stretching vertical sheet." *Heat and Mass Transfer* 44, no. 8 (2008): 921-927. <https://doi.org/10.1007/s00231-007-0322-z>
- [14] Abel, M. Subhas, and N. Mahesha. "Heat transfer in MHD viscoelastic fluid flow over a stretching sheet with variable thermal conductivity, non-uniform heat source and radiation." *Applied Mathematical Modelling* 32, no. 10 (2008): 1965-1983. <https://doi.org/10.1016/j.apm.2007.06.038>
- [15] Gireesha, B. J., A. J. Chamkha, S. Manjunatha, and C. S. Bagewadi. "Mixed convective flow of a dusty fluid over a vertical stretching sheet with non-uniform heat source/sink and radiation." *International Journal of Numerical Methods for Heat & Fluid Flow* 23, no. 4 (2013): 598-612. <https://doi.org/10.1108/09615531311323764>