



## Significance of Cattaneo-Christov Heat Flux on Chemically Reacting Nanofluids Flow Past a Stretching Sheet with Joule Heating Effect

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### ABSTRACT

This paper examined the significance of Cattaneo-Christov's theories on the flow of chemically reacting fluid past a stretching surface with thermophysical parameters. The mathematical modelling of the physical problem was represented by partial differential equations. The set of partial differential equations was simplified by employing suitable similarity variables to obtain the system of coupled nonlinear ordinary differential equations. The transformed equations were later solved using the spectral relaxation method. The spectral relaxation method employs the basic concept of the Gauss-Seidel relaxation techniques. The outcome of this method was presented in graphs and tables. The thermal radiation parameter was found to enhance the velocity and temperature distributions. Also, the effect of the magnetic field parameter was found to decline the velocity profile. It was found that the Brownian motion parameter greatly influences the velocity as well as temperature profiles.

## 1. Introduction

Cattaneo-Christov thermal flux system is utilized to designate the thermal transfer in viscoelastic flow encouraged by an exponentially extending mass. Idowu *et al.*, [1] have demonstrated that flow dissipation can be communicated by altering viscosity and heat conductivity. The MHD Falkner-Skan-Sutterby nanofluid was studied using the nanofluid model and the Cattaneo-Christov heat flux theory by Khan *et al.*, [2]. Williamson hybrid engine oil nanofluids and Cattaneo-Christov heat flux. The MHD Casson-Ferro fluid's heat radiative transport was modelled numerically by Ali and Sandeep [3]. Zhang and colleagues [4] investigated the melting heat reaction in a von Karman circulating motion of hybrid nanofluids by employing a Cattaneo-Christov heat flux. The Cattaneo-Christov model and a chemical process on an exponentially stretchable surface were used by Hayat and Nadeem [5] to address the motion of 3D Eyring-Powell. The Cattaneo-Christov model was used to investigate the flow of Carreau fluid across a thin sheet of material. The Cattaneo-model Christov's was used in the work of Shihao *et al.*, [6] to connect viscoelastic fluid flow with heat transport processes. The importance of the Cattaneo-Christov heat flux model was examined by Tanveer *et al.*, [7]. Upadhya *et al.*, [8]

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investigated the effect of MHD motion past a stretchy cylinder on heat flux changed into a Fourier form. An unstable fluid flow was studied by Falodun *et al.*, [9].

The magnetic and electric field-induced flow of highly conducting fluids is explained by MHD. Astrophysicists and geophysicists use a variety of techniques to study astrophysics, geophysics, MHD power generation and heat exchanger design. A large number of researchers have investigated MHD flows on non-Newtonian fluids over stretched surfaces. Plasma research, flow meters, aerodynamics, and solar energy devices are examples of MHD processes. As a result of the wide range of MHD applications, the studies listed below have documented the flow phenomena associated with MHD. In their paper, Alao *et al.*, [10] studied the flow of a chemically reactive fluid past a half-infinite upright plate with heat radiation and Soret-Dufour significance. Multiple slides on the relevance of MHD and the non-Newtonian flow of nanofluids across a stretchable cylinder were explored by Tlili *et al.*, [11]. According to Salawu and Dada [12], different thermo-physical variables affected magnetic inclination and dissipation in an area outside the Darcy zone. The Peristalsis of a fluid with chemical reaction, wall characteristics, and heat plus mass transportation was studied by Tanveer *et al.*, [7]. Geometries for MHD Cattano-Christov flow with Brownian motion and thermophoresis were shown. According to Karimi *et al.*, [13], the MHD nano layer travels via elastic surfaces in a permeable medium. Using a stretchy sheet, Ramana Reddy *et al.*, [14] investigated the effects of MHD on the simultaneous flow of Casson and Maxwell fluids. MHD mass transport via a slanting plate with thermophoresis, a non-constant heat source/sink, chemical reaction, and Soret-Dufour significance was studied by Mondal *et al.*, [15]. Ramzan *et al.*, [16] used heat transport analysis across a stretchy sheet with thermal and velocity slip limitations to investigate MHD hybrid nanofluids with heat transport. According to Waqas *et al.*, [17], MHD flow over a circulating disk of hybrid nanofluids is important. In 2018, Fagbade *et al.*, [18] elucidated the MHD flow of viscosity-elastic fluid past an accelerating penetrable surface. The unsteadiness of MHD heat transfer layer motion in an electrically conducting and noncompressible fluid was studied by Falodun and Fadugba [19]. A thermally stratified porous medium was studied by Falodun and Omowaye [20] for MHD convective double-diffusive flow. Alias *et al.*, [21] considered the flow past a fixed vibrating drilling riser and proposed the auxiliaries required in a laminar flow. Khan *et al.*, [22], Ouru *et al.*, [23], Koriko *et al.*, [24], Oke [25], Juma *et al.*, [26,27], Ali *et al.*, [28] and Aljaloud *et al.*, [29] considered MHD flows in a double stratified micropolar fluid, a stagnation point flow, over an inclined surface, over a nonuniform surface and in a porous cylindrical tank.

Reddy and Chamkha [30] studied the Soret-Dufour effects on an MHD convective flow of Al<sub>2</sub>O<sub>3</sub>-water and TiO<sub>2</sub>-water nanofluids past a stretching sheet. Rashidi *et al.*, [31] did a comprehensive review on energy analysis of shell and tube heat exchangers. Farooq *et al.*, [32] recently examined the computation of nonlinear thermal radiation in magnetized nanofluid flow with entropy generation. Bhatti *et al.*, [33] studied natural convection non-Newtonian EMHD dissipative flow through a microchannel containing a non-Darcy porous medium using homotopy perturbation method. Waqas *et al.*, [34] explored Cattaneo-Christov heat flux and entropy generation on hybrid nanofluid flow in a nozzle of a rocket engine with melting heat transfer. The impact of MHD radiative flow of hybrid nanofluid over a rotating disk was investigated by Waqas *et al.*, [17]. Farooq *et al.*, [35] studied the thermally radiative bioconvection flow of Carreau nanofluid with modified Cattaneo-Christov expressions and an exponential space-based heat source. Li *et al.*, [36] explored the numerical exploration of modified second-grade nanofluid with motile microorganisms, thermal radiation and Wu's slip. Dawar *et al.*, [37] studied a convective flow of Williamson nanofluid through cone and wedge with non-isothermal and non-isosolutal conditions. Alqahtani *et al.*, [38] used molecular dynamics to estimate the atomic arrangements in a fluid by considering the effects of atomic obstacle size on the hydrogen flow inside a nanochannel. Hejazi *et al.*, [39], Li *et al.*, [40] and

Oke [41, 42] investigated the flows of nanofluids by considering several factors; including inclined surface slip with fractional derivatives, vacancy defect, nonuniform surface and rotating surface. Oke [43] proposed a modified Eyring Powell fluid and identified that the fluid can exhibit both shear thickening and shear thinning properties. Vaidya *et al.*, [44] investigated the flow of reactive peristaltic nanofluid through an inclined channel under the influence of non-constant thermal conductivity and Vyakaranam [45] examined the Casson-based nanofluid in a permeable surface. A further study was carried out by Tan Jian *et al.*, [46] to unravel the self-diffusion of nanoparticles by using dissipative particle dynamics. Recently, Oke *et al.*, [47] explored the influence of thermal radiation on water-based nanofluids over an exponentially stretching and rotating plate.

In literature, studies have been conducted on Cattaneo-Christov heat flux in the presence of heat generation, viscous dissipation and Brownian motion. Most of the aforementioned studies ignored the electromagnetic force as well as the permeability surface. This serves as the gap in this investigation. In the physical configuration, the porous medium is considered within the stretching surface where the electromagnetic force is imposed. The formulated partial differential equations were solved numerically. The numerical outcomes were presented in the form of graphs and tabular forms. The objectives of this study are:

- i. To formulate a mathematical model for a chemically reacting nanofluid flow past a stretching sheet with Joule heating.
- ii. To obtain the solution of the model by utilizing the spectral relaxation method.
- iii. To present outcomes of the simulations as graphs and extensively discuss the outcomes from the physical point of view.

## 2. Methodology

### 2.1 Mathematical Analysis

Consider a steady, laminar, viscous and incompressible nanofluid flow past a vertical porous plate in this paper (see Figure 1). The heat transfer process was examined in the presence of heat generation and thermal radiation. The temperature  $T_w$  was found to be very effective at the wall while far away temperature  $T_\infty$  from the boundary layer is not effective within the boundary layer. Due to the theory Cattaneo-Christov, the Fourier heat flux is put into consideration. However, the nanofluid is examined to be viscous such that the viscous dissipative energy is considered in this study. In the energy transport, the Joule heating was examined on the fluid temperature gradient. The boundary layer approximation is valid and the governing equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_{hf} \frac{\partial^2 u}{\partial y^2} + \frac{\sigma_{hnf}}{\rho_{hnf}} (E_0 B_0 - B_0^2 u) - \frac{\eta_0}{\rho_{hnf}} \frac{\partial^3 u}{\partial y^3} - \frac{\nu}{K} u \quad (2)$$

$$\begin{aligned}
 u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = & \alpha_{hnf} \frac{\partial^2 T}{\partial y^2} + \frac{\mu_{nf}}{(\rho c_p)_{hnf}} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma_{hnf}}{(\rho c_p)_{hnf}} u^2 + \frac{Q_0}{(\rho c_p)_{hnf}} (T - T_\infty) \\
 & - \beta_1 \left[ u \frac{\partial u}{\partial x} \frac{\partial T}{\partial x} + v \frac{\partial v}{\partial y} \frac{\partial T}{\partial y} + u \frac{\partial v}{\partial x} \frac{\partial T}{\partial y} + v \frac{\partial u}{\partial y} \frac{\partial T}{\partial x} + 2uv \frac{\partial^2 u}{\partial x \partial y} + u^2 \frac{\partial T}{\partial x^2} \right. \\
 & \left. + v^2 \frac{\partial^2 T}{\partial y^2} \right]
 \end{aligned} \tag{3}$$

The associated boundary conditions are:

$$u = bx, \quad v = 0, \quad T = T_w, \quad \text{at } y = 0 \tag{4}$$

$$u = v = 0, \quad T \rightarrow T_\infty, \quad \text{as } y \rightarrow \infty \tag{5}$$

The following suitable similarity transformations are defined to simplify the current model:

$$\eta = y \sqrt{\frac{b}{\nu}}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad u = bx f'(\eta), \quad v = -\sqrt{b\nu} f(\eta) \tag{6}$$

Using Eq. (6) on Eq. (1)-Eq. (4) subject to (5) to obtain:

$$f''' + \frac{\mu_f \rho_{hnf}}{\mu_{hnf} \rho_f} \left[ f f'' - f'^2 - K f^{iv} + M_q (E - f') + \frac{1}{P_0} f \right] = 0 \tag{7}$$

$$\begin{aligned}
 \frac{K_{hnf}}{k_f} \theta'' + Pr \frac{(\rho c_p)_{hnf}}{(\rho c_p)_f} f \theta' + Ec Pr \left[ M_q (E - f')^2 + \frac{\mu_{hnf}}{\mu_f} (f'')^2 \right] + Q Pr \theta - \alpha_1 (f f' \theta' + f^2 \theta'') \\
 = 0
 \end{aligned} \tag{8}$$

With the constraints:

$$f(0) = f_w, \quad f'(0) = 1, \quad \theta(0) = 1, \tag{9}$$

$$f(\infty) = 0, \quad \theta(\infty) = 0 \tag{10}$$

The thermophysical properties of the hybrid nanofluids are defined as follows:

$$\begin{aligned}
 \frac{k_{hnf}}{k_f} = & \frac{2\phi_1 \frac{k_{MWCNT}}{(k_{MWCNT} - k_{hnf})} - \phi_1 + 1 - \ln \frac{k_{MWCNT} + k_{hnf}}{2k_{hnf}}}{2\phi_1 \frac{k_{hnf}}{(k_{MWCNT} - k_{hnf})} - \phi_1 + 1 - \ln \frac{k_{MWCNT} + k_{hnf}}{2k_{hnf}}}, \\
 \frac{k_{hnf}}{k_{hnf}} = & \frac{2\phi_2 \frac{k_{SWCNT}}{(k_{MWCNT} - k_{hnf})} - \phi_2 + 1 - \ln \frac{k_{SWCNT} + k_{hnf}}{2k_{hnf}}}{2\phi_2 \frac{k_{hnf}}{(k_{SWCNT} - k_{hnf})} - \phi_2 + 1 - \ln \frac{k_{SWCNT} + k_{hnf}}{2k_{hnf}}}
 \end{aligned}$$

$$\frac{(\rho c_p)_{hnf}}{(\rho c_p)_f} = \left[ (1 - \phi_2) \left( 1 - \left( 1 - \frac{(\rho c_p)_{SWCNT}}{(\rho c_p)_f} \right) \phi_1 + \phi_2 \frac{(\rho c_p)_{MWCNT}}{(\rho c_p)_f} \right) \right]$$

$$\frac{(\rho)_{hnf}}{(\rho)_f} = \left[ (1 - \phi_2) \left( 1 - \left( 1 - \frac{(\rho)_{SWCNT}}{(\rho)_f} \right) \phi_1 + \phi_2 \frac{(\rho)_{MWCNT}}{(\rho)_f} \right) \right]$$

$$\mu_{hnf} = \frac{\mu_f}{(1 - \phi_1)^{2.5} (1 - \phi_2)^{2.5}}, \quad M_q = \frac{\sigma_f B_0^2}{b \rho_f}, \quad Ec = \frac{u_w^2}{c_p (T_w - T_\infty)},$$

$$\alpha_1 = \beta_i b, \quad E = \frac{E_0}{B_0 u_w}, \quad P_0 = \frac{\mu_f}{K \rho_f}, \quad Pr = \frac{\nu_f}{\alpha_f}, \quad Q = \frac{Q_0}{b (\rho c_p)_f}$$

where  $M_q$  is magnetic,  $Ec$  is Eckert,  $\alpha_1$  is thermal relaxation,  $E$  is the electric field factor,  $P_0$  is the porosity parameter,  $Pr$  is Prandtl,  $K$  is the couples stress term, and  $Q$  is the heat source or sink parameter.

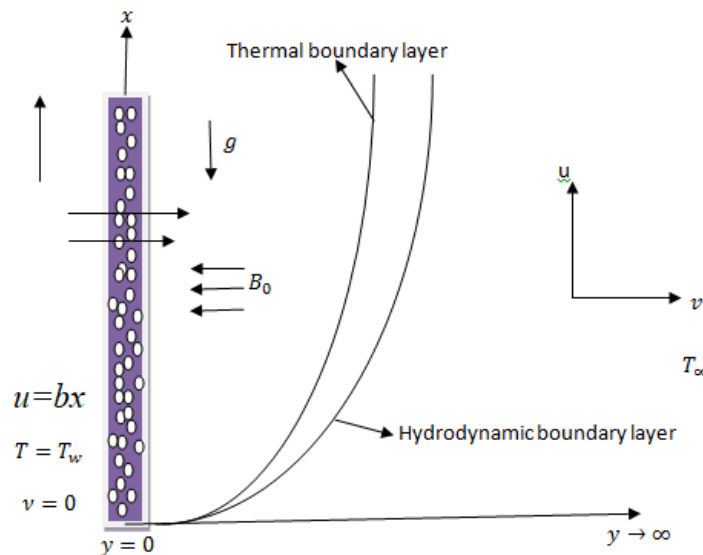


Fig. 1. Physical configuration

## 2.2 Spectral Relaxation Method

By using the SRM, the technique of Gauss-seidel relaxation is utilized in decoupling and linearizing the system of equations. The present iteration represented by  $r + 1$  is utilized on the linear terms while the previous iteration represented by  $r$  is implemented on the nonlinear terms. The Chebyshev collocation approach is further employed on the iterated sequence of equations. See Oke [48] and Oke *et al.*, [49] for a list of other possible methods.

Going by the procedure of SRM on the transformed Eqs. (7) and (8) subject to Eq. (9) to obtain:

$$f'''_{r+1} + a_{0,r} f''_{r+1} + a_{1,r} + a_{2,r} f'_{r+1} + a_{3,r} + a_{4,r} f'_{r+1} + a_{5,r} f_{r+1} = 0$$

$$b_{0,r} \theta''_{r+1} + b_{1,r} \theta'_{r+1} + b_{2,r} + b_{3,r} + b_{4,r} + b_{5,r} \theta'_{r+1} + b_{6,r} \theta''_{r+1} = 0$$

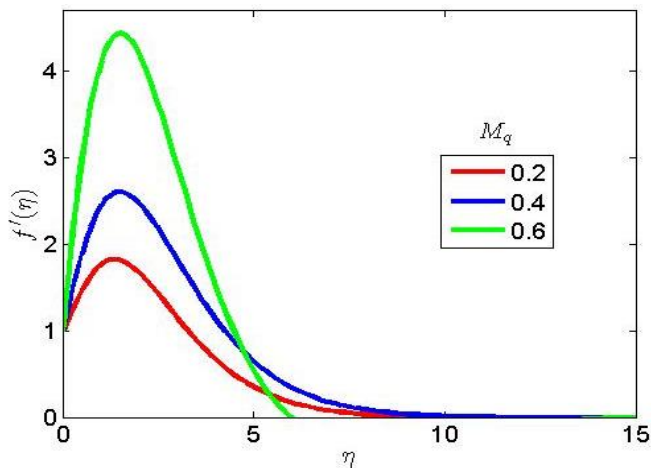
where coefficient parameters are defined as follows:

$$\begin{aligned}
 a_{0,r} &= \frac{\mu_f \rho_{hnf}}{\mu_{hnf} \rho_f} f_r, & a_{1,r} &= -\frac{\mu_f \rho_{hnf}}{\mu_{hnf} \rho_f} f_r'^2, & a_{2,r} &= -K \frac{\mu_f \rho_{hnf}}{\mu_{hnf} \rho_f}, & a_{3,r} &= \frac{M_q \mu_f \rho_{hnf}}{\mu_{hnf} \rho_f} E \\
 a_{4,r} &= -M_q \frac{\mu_f \rho_{hnf}}{\mu_{hnf} \rho_f}, & a_{5,r} &= \frac{\mu_f \rho_{hnf}}{Po \mu_{hnf} \rho_f}, & b_{0,r} &= \frac{K_{hnf}}{K_f}, & b_{1,r} &= Pr \frac{(\rho c_p)_{hnf}}{(\rho c_p)_f} f_r, \\
 b_{2,r} &= Ec Pr M_q E^2, & b_{3,r} &= -2EEcPr f_r' f_{r+1}', & b_{4,r} &= EcPr f_r'^2 + EcPr \frac{\mu_{hnf}}{\mu_f} (f_r'')^2, \\
 b_{5,r} &= -\alpha_1 f_r f_r' f_{r+1}', & b_{6,r} &= -\alpha_1 f_{r+1}'^2
 \end{aligned}$$

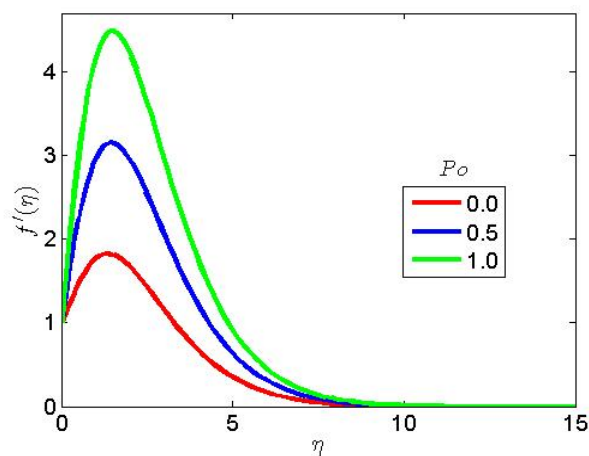
### 3. Results

This paper explored the numerical simulation of chemically reacting nanofluid flow past a stretching sheet with Joule heating effect. The model equations are a set of partial differential equations which were transformed into ordinary differential equations. The set of transformed equations was solved numerically by utilizing the spectral relaxation method. In Figure 2, the impact of the magnetic parameter ( $M$ ) on the velocity profile are illustrated. An increase in the velocity profile was observed because of an increase in the magnetic parameter. Due to the impact of electromagnetic force in the momentum equation, it weakens the Lorentz force produced by the imposed magnetic parameter. The Lorentz force is a drag-like force which always reduces the motion of an electrically conducting fluid. In Figure 2, an unstable effect is noticed as the magnetic parameter increases.

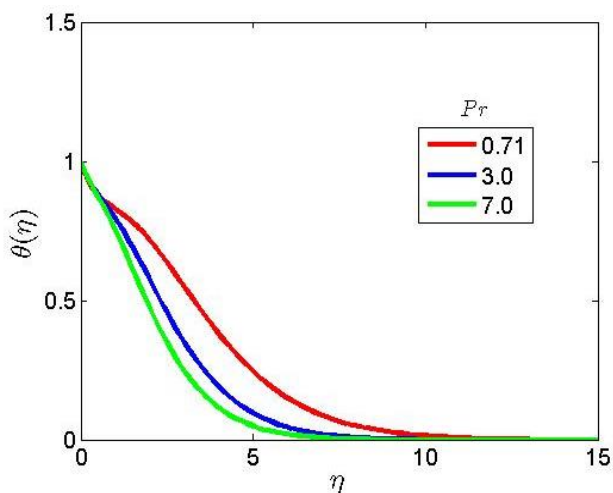
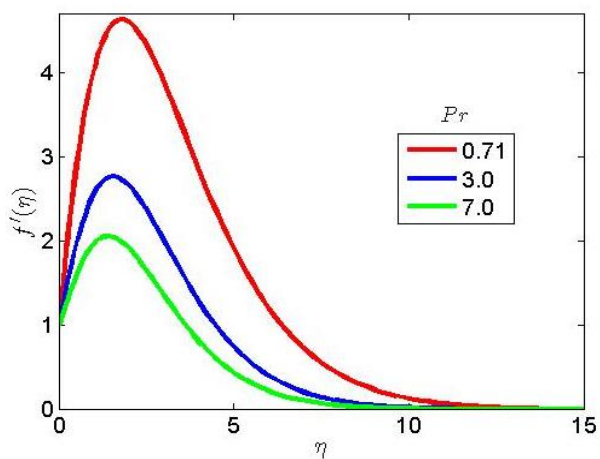
Figure 3 shows the significance of the permeability parameter ( $Po$ ) on the velocity profile. The permeability parameter gives an avenue for the flow of fluid particles easily in the boundary layer. Increasing the permeability parameter expands the holes and speeds up the fluid velocity. As a result of this, a higher permeability parameter leads to higher velocity and the hydrodynamic boundary layer thickness. From Figure 3, a drastic increase is observed very close to the wall while at the free stream, the effect was negligible. Figure 4 shows the impact of Prandtl number ( $Pr$ ) on the velocity and temperature profiles respectively. On the velocity profile, a drastic increase in the profile is observed. As the dimensionless distance ( $\eta$ ) increases, a negligible effect of the Prandtl number was observed. The greater the Prandtl number in a flow phenomenon, the higher the thermal conductivity. Practically, a higher  $Pr$  leads to higher momentum diffusivity and thermal diffusivity. In Figure 4, a decrease in both velocity and temperature is observed. This implies that a higher  $Pr$  leads to greater thickness of both hydrodynamic and thermal boundary layer thickness. Figure 5 shows the significance of Eckert number ( $Ec$ ) on velocity and temperature profiles. A higher value of Eckert number ( $Ec$ ) is observed to increase the velocity and temperature profiles. Physically, an increase in  $Ec$  means an increase in heat energy within the boundary layer. The production of heat energy is maintained because of frictional heating.



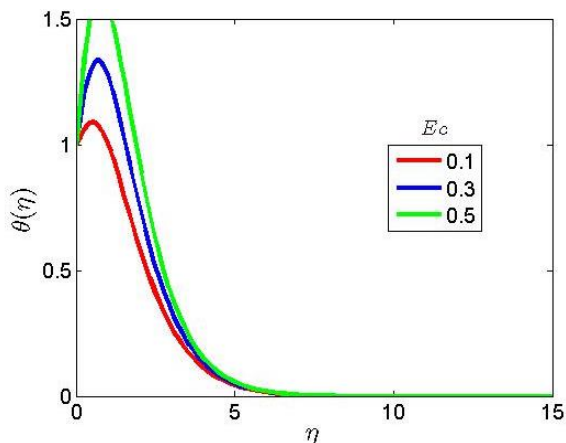
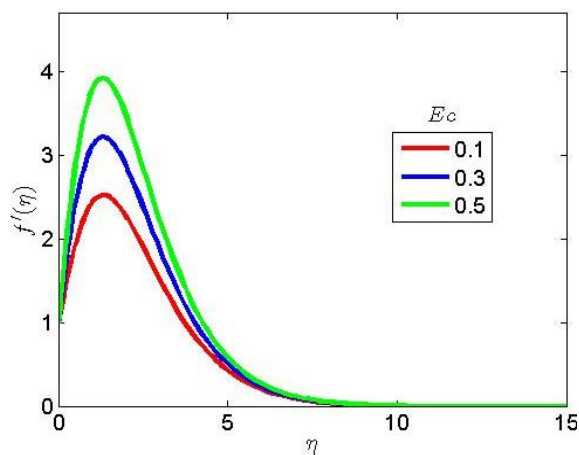
**Fig. 2.** Effect of magnetic parameter on the velocity profile



**Fig. 3.** Effect of permeability parameter on the velocity profile



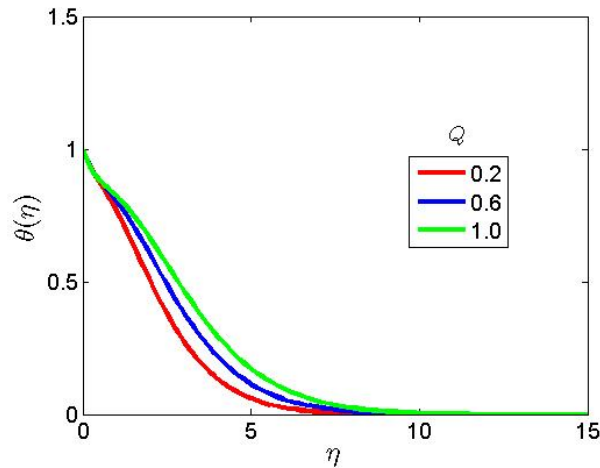
**Fig. 4.** Effect of Prandtl parameter on the velocity and temperature profiles



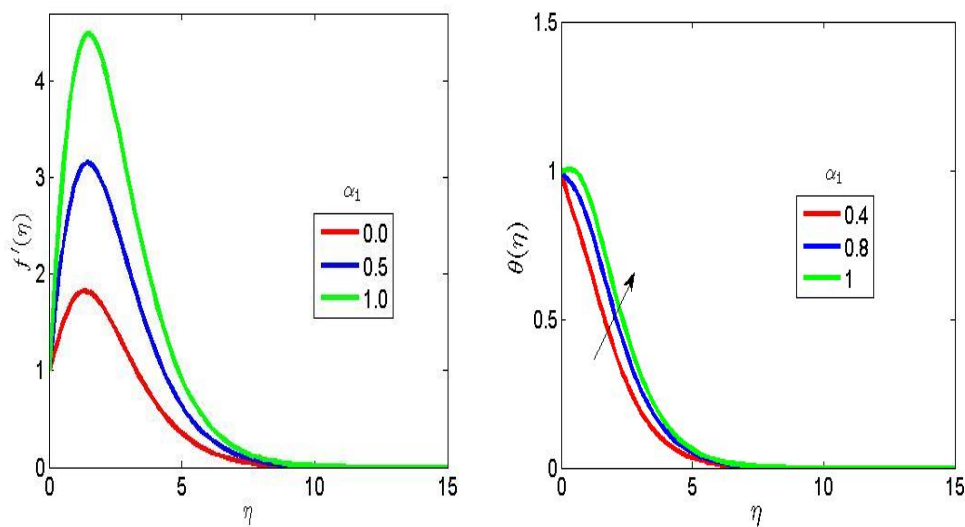
**Fig. 5.** Effect of Eckert number on the velocity and temperature profiles

Figure 6 shows the impact of heat generation parameter on the temperature profile. An increase in the heat generation parameter is observed to increase the temperature profile. Heat generation term finds significance whenever there is a higher temperature. Heat generation finds applications in the production of polymers. Physically, a higher value of the heat generation parameter gives an

increase to the thickness of the thermal boundary layer. Figure 7 shows the impact of heat flux parameter on the temperature profile. The heat flux parameter is the coefficient of the Cattaneo-Christov heat flux theory from the Fourier law. An increase in the heat flux parameter is noticed to enhance the temperature and the entire boundary layer thickness.



**Fig. 6.** Effect of heat generation parameter on the temperature profile



**Fig. 7.** Effect of heat flux parameter on the velocity and temperature profiles

#### 4. Conclusion

In this paper, the numerical simulation of a chemically reacting nanofluid flow past a stretching sheet with Joule heating has been considered. The chemically reacting nanofluid flow phenomenon was considered mathematically by using partial differential equations. The physics of the problem was examined by utilizing the spectral relaxation method. The graphical representation of each parameter is extensively discussed from the physical point of view. The key findings in the paper are:

- i. An increase in the magnetic parameter elevates the velocity profile. This is because the electromagnetic force greatly affects the Lorentz force by reducing the strength to cause an increase;



- ii. The velocity of the fluids within the boundary layer increases because of expansion in the holes due to an increase in the permeability parameter;
- iii. An increase in the Prandtl number was observed to enhance the hydrodynamic and thermal boundary layer thickness;
- iv. An increase in the Eckert number ( $Ec$ ) was noticed to increase the heat energy which enhances both hydrodynamic and thermal boundary thickness; and
- v. The heat flux of time relaxation was observed to increase the temperature and thermal boundary layer thickness.

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