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MHD Radiative Casson Fluid Flow through Forchheimer Permeable Medium with Joule Heating Influence

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ABSTRACT

In the current study, Casson fluid flow over an inclined non-linear surface through Darcy-Forchhiemer permeable medium is analysed numerically with the influences of effects viscous dissipation, MHD, radiation, chemical reaction, and Joule heating. The governed PDE of the problem are converted to non-linear ODE using suitable similarity conversions. The ODE are solved using the Keller Box method. The influence of parameters is analysed by constructing velocity, temperature, and concentration profiles. The velocity profile decreases for raising values of the Casson parameter and Magnetic parameter and increases for the permeability parameter. Temperature increases for progressive values of the radiation parameter, Dofour parameter, and Concentration profile decreases for the chemical reaction parameter. Also, to validate the numerical method local parameters are also computed and compared with existing work.

1. Introduction

Fluid flows with the influence of Magnetohydrodynamics are useful in several industrial applications such as MHD generators, nuclear reactor designing etc. MHD theory is also used to understand observations from the solar system to distant astrophysical regions. MHD is the science that explores the stream of electrically conducting fluid in presence of magnetic field. MHD would work by stimulating currents in a moving conductive fluid's magnetic field, which polarises the liquid and reverses the magnetic field. MHD is explained mathematically by combining Navier-Stokes equations of fluid dynamics and Maxwell's electro-magnetism equations. The flow of a fluid is controlled by adjusting the observations of electrical conductivities and viscosities of fluid mentioned by Malashetty *et al.,* [1] in their investigation of MHD two-phase flow over inclined channel. The applied magnetic field implies to decrement of the wall temperature gradient and concentration gradient observed by Chen [2] in his study. Alam *et al.*, [3] used RK integration scheme to study influence of MH and heat source on fluid flow over a slanted surface, with increase in magnetic field

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rise in temperature and concentration distributions is observed. Takhar *et al.,* [4] observed in his study that the thermal stratification and magnetic parameters generate notable changes on the skin friction and the heat transfer.

Non-Newtonian fluid phenomena, due to their application in industry and technology fields, such as drilling operations and bio-engineering field, and food processing field attracted many researchers. It is far more difficult to analyse transport phenomenon problems of technical relevance involving non-Newtonian fluid behaviour than it is for Newtonian fluids. In fact, creating an experimental effect in a lab setting using a non-Newtonian fluid is a straightforward process. [5] One such type of non-Newtonian fluid is Casson fluid. The mixed convective course of Casson fluid with adaptable heat conductivity is analyzed by Vajravelu *et al., [*6] using optimal HAM technique. A time-dependent Casson fluid stream on a flat plate is examined. An augmentation in Casson parameter enhancement in the fluid velocity and decrement in the thickness of the marginal layer is mentioned by Mustafa *et al.,* [7]. Thermal radiation will increase the strong thermal diffusivity which causes the growth in temperature discussed by Mukhopadhyay [8]. The radiation influence has significant uses in various engineering fields. It is used in space technology and procedures involving high temperatures and also radiation effect has a vital role in the polymer processing industry, cooling of nuclear reactors, liquid metal fluids, and power generation systems etc. as mentioned by Mukhopadhyay and Gorla [9]. Pramanik [10] studied Casson fluid flow and observed that the exterior shear stress escalates with the advancement in the Casson argument. Manjunatha and Rajashekar [11] investigated the peristaltic transport of Casson fluid in an elastic tube with slip influence and concluded that the velocity slip parameter is utilized in determining flux in an elastic tube. Rehman *et al.,* [12] investigated Casson fluid flow in a corrugated cavity rooted with permeable medium, Casson fluid temperature improves at the upper edge and vertical line with the progressive change of Rayleigh number. Haq *et al.,* [13] investigated Casson nanofluid flow with gyrotactic microorganisms and observed that fluid temperature enhances with higher observations of Eckert number, Prandtl number, and Hartmann parameter. Jawad *et al.,* [14] studied the influence of Willamson Casson nanofluid flow with the influence of bioconvection and radiation, noted that the speed of fluid flow diminishes with the growing observations of magnetic parameter, bio convection parameter. Omar *et al.,* [15] studied time-dependent Casson fluid with radiation and chemical reaction influences using Laplace transforms method, for raising observations of Schmidt number concentration fluid reduces.

Due to its many applications in fields like cooling of electronic equipment, gas turbines and various aircraft propulsion devices, nuclear power plants, filtration, refrigeration, spreading of chemical pollutants in plants and diffusion of medicine in blood veins, and other fields, heat and mass exchange with a chemical reaction influence plays a significant role in fluid problems. Using the RK fourth-order scheme, Reddy [16] investigated Casson fluid flow with the impact of radiation and chemical reaction, concentration falls with the augmentation of the chemical reaction argument. Haq *et al*., [17] investigated the impact of chemical reaction on Oldroyd-B fluid, observed that Sherwood number value enhances for increasing observations of Prandtl number and decreases for ratio parameter. Chu *et al*., [18] analysed the thermally developed flow of Maxwell fluid between two elongated disks and observed that thermophoretic diffusion enhances the concentration of fluid. Khan *et al*., [19] analysed MHD Micropolar fluid with the influence of the chemical reaction and heat source, with the progressive observation of chemical reaction argument Sherwood number increases Chu *et al*., [20] investigated the squeezing flow of a Jeffrey nanofluid with the influence of chemical reaction, the temperature declines for augmented observations of Prandtl number. Liu *et al*., [21] analysed Maxwell nanofluid over an elongated cylinder through Darcy Forchhiemer absorbent media, with enhanced observations of Brownian diffusion intensification in entropy is observed. Safdar *et al., [*22] studied Mixed convective Maxwell nanofluid with MHD influence and observed that increasing observations of Pecelet number microorganism field decreases. Haq *et al., [*23] analysed the Cattaneo-Christov model on nanofluid with the influence of chemical reaction, magnetic effect using ND solve code observed that nanofluid fluid velocity decays with incremental observations of magnetic parameter. Majeed *et al.,* [24] studied MHD nanofluid flow over an elongated cylinder with radiation, MHD and Brownian motion influences enhancement occurs in the concentration profile is witnessed for incremental observations of thermal stratification argument. When Hakeem *et al.,* [25] studied the effect of an inclined Lorentz force on Casson fluid, they found that as the aligned angle of the magnetic field escalates, so does the velocity of the non-Newtonian fluid. Later, using the RK-Fehlberg method, Gopal *et al.,* [26] looked into how viscosity dissipation and Joule heating affected Casson fluid. According to Shamshuddin *et al.,* [27] observation of Casson fluid course across an inclined absorbent plate with chemical reaction influence, the concentration distribution drops as the strength of the reaction enhances due to species molecular diffusion. Raju [28] observed a considerable reduction in the Casson fluid stream across a slanted surface under the impact of radiation and chemical reaction, With enhanced observations of the Schmidt number and Chemical reaction argument.

Al-Hadhrami *et al.,* [29] suggested a novel model and corresponding derivation for the viscous dissipation term in the Brinkman equation. Later Cortell [30] considered the effect of viscous dissipation over a non-linearly extending surface and observed that an enhancement in Eckert number leads to an augmentation of the temperature profile. Nusselt number declines with the rising of Eckert number mentioned by Sheikholeslami *et al.,* [31]. Kumar *et al.,* [32] analysed the impact of Joule heating on the Casson fluid stream over a vertical flat plate and perceived that the solutal marginal layer width of the fluid increases as the Prandtl number upsurges. Swain *et al.,* [33] studied Joule heating influence on flow across an extending sheet, for larger values of joule heating argument heat transfer pace decreases. Qayyum *et al.,* [34] investigated Williamson fluid flow with the influence of radiation and thermal slip using the bvp4c technique and noted that with the enhanced observation of radiation and magnetic constraints change in entropy is witnessed. Hamrelaine *et al.,* [35] analysed Jefrey Hamel flow with the impact of MHD using HAM technique concludes that with enhanced observations of Hartmann parameter augmentation in velocity is noted. Haq *et al.,* [36] investigated hybrid nanofluid flow over a cylindrical surface and concludes that the thermal conductivity of nanofluid is higher when compared with nanofluids. Ganesh and Sridhar [37] considered Casson nanofluid flow through Darcy-Forchheimer permeable medium, the momentum marginal layer lessens for a greater inertial effect and the resisting force offered by the permeable channel to the fluid flow. Jawad *et al.,* [38] examined nanofluid flow with gyrotactic microorganisms noted that the velocity profile for nanofluids decreases by increasing the suction parameter. Abbas *et al.,* [39] examined Mixed convective radiative secondary grade nanofluid flow and witnessed that thermal radiation enhances temperature distribution. Haq *et al.,* [40] studied entropy exploration on Jeffery nanofluid with the homotopy analysis method. Shafiq *et al.,* [41] studied Cattaneo-Christov of secondary grade nanofluid with the influence of viscous dissipation studied that enhancement in skin friction values is observed with Darcy medium implementation. Safdar *et al.,* [42] studied Buongiorno nanofluid flow and enhancement in temperature profile is noted for the growing observations of mixed convection argument. Jawad *et al.,* [43] studied Maxwell bio nanofluid with Neild boundary conditions and observed that the temperature profile increments with the progressive values of radiation Thermophoresis arguments. Gupta and Bhargavi [44] investigated the magnetic field influence on laminar forced convection in a thermally developing region enclosed in the porous medium, enhancement I Nusselt number is observed for augmentation of Hartman number. Jawad *et al.,* [45] investigated Maxwell nanofluid flow through Darcy-Forchhiemer medium, with upliftment of the activation energy and thermal conductivity argument, decrement in local

Nusselt number is observed. Later Ganesh *et al.,* [46] examined Casson nanofluid stream with an electric field, magnetic field, viscous dissipation chemical reaction influences and noted that viscous dissipation will behave as an energy source, so temperature raises. Later Sharma and Gandhi [47] studied the influence of Joule heating, the enhancement in permeability argument and the Forchhiemer number shows a declination trend in the velocity profile. After that Bezawada *et al.,* [48] studied radiation influence on Casson fluid course over a slanted surface through Darcy Forchhiemer porous medium and noted that Nusselt and Sherwood number Rises and Skin friction declines for incremental observations of Prandtl number and Radiation parameter.

In the present study, Casson fluid stream over a slanted extending surface through Forchhiemer permeable media along with the effects of Magnetic field, radiation, chemical reaction, viscous dissipation, slip effects and Joule heating is considered. The present research is applicable in the fields of transmission of magnetic materials and chemical technology systems. The corresponding equations are converted into nonlinear ordinary differential equations are further solved using the Keller box scheme.

2. Mathematical Formulation

A two-dimensional Steady Casson fluid flow on a non-linear extending surface in Forchhiemer porous media is considered. The sheet is inclined with an angle α vertically. The stretching surface is taken along the x-axis. Y axis and uniform magnetic field applied are normal to the surface (see Figure 1). Fluid flow on the surface is influenced by MHD, Chemical reaction, viscous dissipation, radiation, Joule heating, and slip effects. Upon these assumptions, the governed partial differential equations [48] are

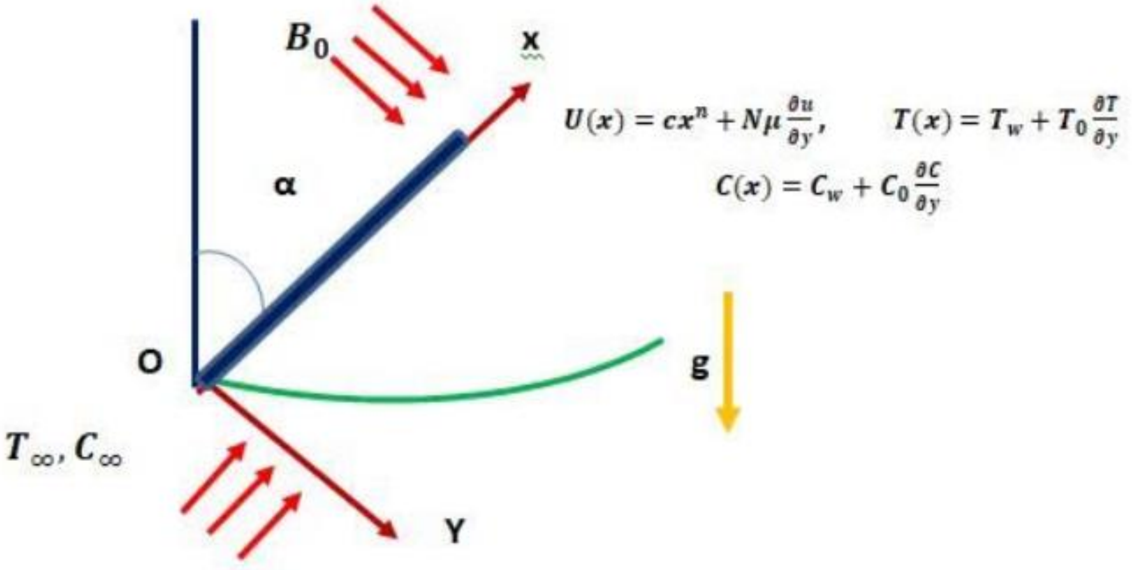

Fig. 1. Flow model of the problem.

$$u_x + v_y = 0 \tag{1}$$

$$u(u_x) + v(u_y) = \upsilon\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u - \frac{\upsilon}{k_1}u - \frac{C_b}{\sqrt{k_1}}u^2 \pm g\left[\beta_T\left(T-T_\infty\right)+\beta_c\left(C-C_\infty\right)\right]\mathrm{Cos}\,\alpha \tag{2}$$

$$u(T_x)+v(T_y)=\alpha(T_{yy})+\frac{D_m k_t}{C_s C_p}(C_{yy})-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y}+\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2\left(1+\frac{1}{\beta}\right)+\frac{\sigma B_0^2}{\rho C_p}u^2 \tag{3}$$

$$u(C_x)+v(C_y)=D_B(C_{yy})+\frac{D_T}{T_\infty}\left(T_{yy}\right)-k_2(C-C_\infty) \tag{4}$$

Corresponding boundary conditions are

At $y=0$, $\quad u=U(x)=cx^n+N\mu\frac{\partial u}{\partial y}, v=-V(x), T=T_w+N\mu\frac{\partial u}{\partial y}$ (5)

As $y\to\infty, u\to 0, T\to T_\infty, C\to C_\infty$

Similarity transformations are

$$u=cx^n f'(\eta), v=-\sqrt{\frac{c\upsilon(n+1)}{2}}x^{\frac{n-1}{2}}\left[f(\eta)+\frac{n-1}{n+1}\eta f'(\eta)\right]$$
$$\theta=\frac{T-T_\infty}{T_w-T_\infty}, \phi=\frac{C-C_\infty}{C_w-C_\infty} \tag{6}$$

Employing similarity transformations Eq. (6), Eq. (2), Eq. (3), Eq. (4) are transformed to

$$\left(1+\frac{1}{\beta}\right)f'''+ff''-\frac{2N}{N+1}f'^2-\frac{2}{N+1}\left[\left[M+\frac{1}{k_1}\right]f'-Frf'^2+G_1\theta\cos\alpha+G_2\phi\cos\alpha\right]=0 \tag{7}$$

$$\left(1+\frac{4}{3}Rd\right)\theta''+\Pr f\theta'+\Pr Du\phi''+\Pr Ec\left(1+\frac{1}{\beta}\right)f''^2+\frac{2}{N+1}\Pr Jf'^2=0 \tag{8}$$

$$\phi''+Scf\phi'+\frac{Nt}{Nb}\theta''-\frac{2}{N+1}Sc\gamma\phi=0 \tag{9}$$

Boundary conditions are transformed to

$$f(0)=S, f'(0)=1+\lambda_u f''(0), \theta(0)=1+\lambda_t\theta'(0), \phi(0)=1+\lambda_c\phi'(0)$$
$$f'(\infty)\to 0, \theta(\infty)\to 0, \phi(\infty)\to 0 \tag{10}$$

The local skin friction can be calculated using $Cf_x=\frac{1}{\sqrt{Re_x}}\sqrt{\frac{N+1}{2}}\left(1+\frac{1}{\beta}\right)f''(0)$

Local Nusselt number is given by $Nu_x=-\frac{1}{\sqrt{Re_x}}\sqrt{\frac{N+1}{2}}\left(1+\frac{4}{3}Rd\right)\theta'(0)$

Local Sherwood number is given by $Sh_x=-\frac{1}{\sqrt{Re_x}}\sqrt{\frac{N+1}{2}}\phi'(0)$

3. Numerical Methodology

The implicit finite difference scheme called the Keller Box method is utilised to convert Eqs. (7)-(9) into ordinary differential equations of the first order. By using finite differences, the subsequent equations are linearized and then written in matrix form by introducing Newton's method. Finally, by applying the tri-diagonal elimination method, the linear system of equations is solved.

Introducing, $\frac{df}{d\eta} = p, \frac{dp}{d\eta} = q, g = \theta, \frac{dg}{d\eta} = t, s = \phi, \frac{ds}{d\eta} = n$, (11)

Eq. (7), Eq. (8) and Eq. (9) transformed to

$$\left(1+\frac{1}{\beta}\right)q' + fq - \frac{2N}{N+1}p^2 - \frac{2}{N+1}\left(M+\frac{1}{k_1}\right)p - \frac{2}{N+1}Frp^2 + \frac{2}{N+1}G_1 g\cos\alpha + \frac{2}{N+1}G_2\cos\alpha = 0 \quad (12)$$

$$\left(1+\frac{4}{3}Rd\right)t' + \Pr\, ft + \Pr Dun' + \Pr Ec\left(1+\frac{1}{\beta}\right)q^2 + \frac{2}{N+1}\Pr Jp^2 = 0 \quad (13)$$

$$n' + Scfn + \frac{Nt}{Nb}t' - \frac{2}{N+1}Sc\gamma s = 0 \quad (14)$$

By using finite differences, and Newton's method, Eqs. (11) - (14) reduces to

$$\delta f_j - \delta f_{j-1} - \frac{h_j}{2}(\delta p_j + \delta p_{j-1}) = (r_1)_j \quad (15)$$

$$\delta p_j - \delta p_{j-1} - \frac{h_j}{2}(\delta q_j + \delta q_{j-1}) = (r_2)_j \quad (16)$$

$$\delta g_j - \delta g_{j-1} - \frac{h_j}{2}(\delta t_j + \delta t_{j-1}) = (r_3)_j \quad (17)$$

$$\delta s_j - \delta s_{j-1} - \frac{h_j}{2}(\delta n_j + \delta n_{j-1}) = (r_4)_j \quad (18)$$

$$\begin{aligned}&(a_1)_j\,\delta q_j + (a_2)_j\,\delta q_{j-1} + (a_3)_j\,\delta f_j + (a_4)_j\,\delta f_{j-1} + (a_5)_j\,\delta p_j + (a_6)_j\,\delta p_{j-1}\\ &+(a_7)_j\,\delta g_j + (a_8)_j\,\delta g_{j-1} + (a_9)_j\,\delta s_j + (a_{10})_j\,\delta s_{j-1} = (r_5)_j\end{aligned} \quad (19)$$

$$\begin{aligned}&(b_1)_j\,\delta t_j + (b_2)_j\,\delta t_{j-1} + (b_3)_j\,\delta f_j + (b_4)_j\,\delta f_{j-1} + (b_5)_j\,\delta n_j + (b_6)_j\,\delta n_{j-1}\\ &+(b_7)_j\,\delta q_j + (b_8)_j\,\delta q_{j-1} + (b_9)_j\,\delta p_j + (b_{10})_j\,\delta p_{j-1} = (r_6)_j\end{aligned} \quad (20)$$

$$\begin{aligned}&(c_1)_j\,\delta n_j+(c_2)_j\,\delta n_{j-1}+(c_3)_j\,\delta f_j+(c_4)_j\,\delta f_{j-1}+(c_5)_j\,\delta t_j+(c_6)_j\,\delta t_{j-1}\\&+(c_7)_j\,\delta s_j+(c_8)_j\,\delta s_{j-1}=(r_7)_j\end{aligned} \tag{21}$$

Where,

$$(a_1)_j=1+\frac{\beta h_j}{4(\beta+1)}(f_j+f_{j-1}) \qquad (a_2)_j=-1+\frac{\beta h_j}{4(\beta+1)}(f_j+f_{j-1})$$

$$(a_3)_j=\frac{\beta h_j}{4(\beta+1)}\left(q_j+q_{j-1}\right) \qquad (a_4)_j=\frac{\beta h_j}{4(\beta+1)}\left(q_j+q_{j-1}\right)$$

$$(a_5)_j=-\frac{2N\beta h_j}{2(N+1)(\beta+1)}\left(p_j+p_{j-1}\right)-\frac{\beta h_j}{4(\beta+1)(N+1)}\left(M+\frac{1}{k_1}\right)-\frac{2Fr\beta h_j}{2(M+1)(\beta+1)}\left(p_j+p_{j-1}\right)$$

$$(a_6)_j=-\frac{2N\beta h_j}{2(N+1)(\beta+1)}\left(p_j+p_{j-1}\right)-\frac{\beta h_j}{4(\beta+1)(N+1)}\left(M+\frac{1}{k_1}\right)-\frac{2Fr\beta h_j}{2(M+1)(\beta+1)}\left(p_j+p_{j-1}\right)$$

$$(a_7)_j=\frac{2G_1\cos\alpha}{N+1}\frac{\beta h_j}{\beta+1} \qquad (a_8)_j=\frac{2G_1\cos\alpha}{N+1}\frac{\beta h_j}{\beta+1}$$

$$(a_9)_j=\frac{2G_2\cos\alpha}{N+1}\frac{\beta h_j}{\beta+1} \qquad (a_{10})_j=\frac{2G_2\cos\alpha}{N+1}\frac{\beta h_j}{\beta+1}$$

$$(b_1)_j=1+\frac{3\Pr h_j}{4(3+4Rd)}(f_j+f_{j-1}) \qquad (b_2)_j=-1+\frac{3\Pr h_j}{4(3+4Rd)}(f_j+f_{j-1})$$

$$(b_3)_j=\frac{3\Pr h_j}{4(3+4Rd)}(t_j+t_{j-1}) \qquad (b_4)_j=\frac{3\Pr h_j}{4(3+4Rd)}(t_j+t_{j-1})$$

$$(b_5)_j=\frac{3\Pr Du}{3+4Rd} \qquad (b_6)_j=\frac{-3\Pr Du}{3+4Rd}$$

$$(b_7)_j=\frac{3\Pr Ec(\beta+1)h_j}{2\beta\left(3+4Rd\right)}\left(q_j+q_{j-1}\right) \qquad (b_8)_j=\frac{3\Pr Ec(\beta+1)h_j}{2\beta\left(3+4Rd\right)}\left(q_j+q_{j-1}\right)$$

$$(b_9)_j=\frac{6\Pr Jh_j}{2(N+1)(3+4Rd)}\left(p_j+p_{j-1}\right) \qquad (b_{10})_j=\frac{6\Pr Jh_j}{2(N+1)(3+4Rd)}\left(p_j+p_{j-1}\right)$$

$$(c_1)_j=1+\frac{Sch_j}{4}\left(f_j+f_{j-1}\right) \qquad (c_2)_j=-1+\frac{Sch_j}{4}\left(f_j+f_{j-1}\right)$$

$$(c_3)_j=\frac{Sch_j}{4}\left(n_j+n_{j-1}\right) \qquad (c_4)_j=\frac{Sch_j}{4}\left(n_j+n_{j-1}\right)$$

$$(c_5)_j=\frac{Nt}{Nb} \qquad (c_6)_j=-\frac{Nt}{Nb}$$

$$(c_7)_j=\frac{-Sc\gamma h_j}{N+1} \qquad (c_8)_j=\frac{-Sc\gamma h_j}{N+1}$$

$$(r_1)_j=f_{j-1}-f_j-\frac{h_j}{2}(p_j+p_{j-1})$$

$$(r_2)_j=p_{j-1}-p_j-\frac{h_j}{2}(q_j+q_{j-1})$$

$$\left(r_3\right)_j = g_{j-1} - g_j - \frac{h_j}{2}(t_j + t_{j-1})$$

$$\left(r_4\right)_j = s_{j-1} - s_j - \frac{h_j}{2}(n_j + n_{j-1})$$

$$\left(r_5\right)_j = q_{j-1} - q_j - \frac{\beta h_j}{4(\beta+1)}\left(f_j + f_{j-1}\right)\left(q_j + q_{j-1}\right) + \frac{2N\beta h_j}{4(N+1)(\beta+1)}\left(p_j + p_{j-1}\right)^2$$
$$+\frac{2}{N+1}\left(M + \frac{1}{k_1}\right)\frac{\beta h_j}{2(\beta+1)}\left(p_j + p_{j-1}\right) + \frac{2}{N+1}\frac{Fr\beta h_j}{4(\beta+1)}\left(p_j + p_{j-1}\right)^2$$
$$-\frac{2}{N+1}G_1\cos\alpha\,\frac{\beta h_j}{2(\beta+1)}\left(g_j + g_{j-1}\right) - \frac{2}{N+1}G_2\cos\alpha\,\frac{\beta h_j}{2\left(\beta+1\right)}\left(s_j + s_{j-1}\right)$$

$$\left(r_6\right)_j = t_{j-1} - t_j - \frac{3\Pr h_j}{4(3+4Rd)}\left(f_j + f_{j-1}\right)\left(t_j + t_{j-1}\right) - \frac{3\Pr Du}{3+4Rd}(n_j - n_{j-1}) -$$
$$\frac{3\Pr Ec\left(\beta+1\right)h_j}{4\beta\left(3+4Rd\right)}\left(q_j + q_{j-1}\right)^2 - \frac{6\Pr Jh_j}{4(N+1)(3+4Rd)}\left(p_j + p_{j-1}\right)^2$$

$$\left(r_7\right)_j = n_{j-1} - n_j - \frac{Sch_j}{4}\left(f_j + f_{j-1}\right)\left(n_j + n_{j-1}\right) - \frac{Nt}{Nb}\left(t_j - t_{j-1}\right) + \frac{2}{N+1}\frac{Sc\gamma h_j}{2}\left(s_j + s_{j-1}\right)$$

The matrix form of a given system of equations is reduced into a tri-diagonal form

$$\left[A_1\right]\left[\delta_1\right] + \left[C_1\right]\left[\delta_2\right] = \left[r_1\right]$$
$$\left[B_2\right]\left[\delta_1\right] + \left[A_2\right]\left[\delta_2\right] + \left[C_2\right]\left[\delta_3\right] = \left[r_2\right]$$

$$\left[B_{j-1}\right]\left[\delta_1\right] + \left[A_{j-1}\right]\left[\delta_2\right] + \left[C_{j-1}\right]\left[\delta_3\right] = r_{j-1}$$
$$\left[B_j\right]\left[\delta_{j-1}\right] + \left[A_j\right]\left[\delta_j\right] = \left[r_j\right]$$ for j=1,2, …

Where, $$A_1 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -0.5h_j & 0 & 0 & 0 & -0.5h_j & 0 & 0 \\ 0 & -0.5h_j & 0 & 0 & 0 & -0.5h_j & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -0.5h_j \\ (a_2)_1 & 0 & (a_{10})_1 & (a_3)_1 & (a_1)_1 & 0 & 0 \\ (b_8)_1 & (b_2)_1 & 0 & (b_3)_1 & (b_7)_1 & (b_1)_1 & (b_5)_1 \\ 0 & (c_6)_1 & (c_8)_1 & (c_3)_1 & 0 & (c_5)_1 & (c_1)_1 \end{bmatrix}$$

$$A_j = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & -0.5h_j & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & -0.5h_j & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -0.5h_j \\ (a_6)_j & (a_8)_j & (a_{10})_j & (a_3)_j & (a_1)_j & 0 & 0 \\ (b_{10})_j & 0 & 0 & (b_3)_j & (b_7)_j & (b_1)_j & (b_8)_j \\ 0 & 0 & (c_8)_j & (c_3)_j & 0 & (c_5)_j & (c_1)_j \end{bmatrix}$$ where $2 \le j \le J$

$$B_j = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.5h_j & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.5h_j & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.5h_j \\ 0 & 0 & 0 & (a_4)_j & (a_2)_j & 0 & 0 \\ 0 & 0 & 0 & (b_4)_j & (b_8)_j & (b_2)_j & (b_6)_j \\ 0 & 0 & 0 & (c_4)_j & 0 & (c_6)_j & (c_2)_j \end{bmatrix}$$

Where $1 \le j \le J-1$

$$C_j = \begin{bmatrix} -0.5h_j & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ (a_5)_j & (a_7)_j & (a_9)_2 & 0 & 0 & 0 & 0 \\ (b_9)_j & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & (c_7)_j & 0 & 0 & 0 & 0 \end{bmatrix}$$

Where $2 \le j \le J$

This tridiagonal system is further solved using the LU decomposition method. the calculations are performed up to desired convergence criterion is fulfilled and calculations are concluded for $\left|\delta g_0^{(i)}\right| < \varepsilon$ where ε is a very small recommended value.

4. Results and Discussion

The effects of various profiles are analysed by developing velocity, temperature, and concentration profiles using the software MATLAB. The graphs are plotted for $\beta = 5, N = 0.5, \alpha = 0.2, \gamma = 0.2, M = 0.5, k_1 = 1, Sc = 0.6, Rd = 1, Pr = 0.7, Fr = 1, Du = 0.1, k_2 = 1, S = 0.5, \lambda_u = 0.5, \lambda_t = 1, \lambda_c = 1, G1 = 0.2, G2 = 0.5, Q = 1, J = 0.1, Nt = 0.1, Nb = 0.1.$

Figure 2 represents the velocity profiles of the Casson parameter. Declination in graphs of velocity is noted with the growing observations of Casson parameter β. Because with the enhanced values of the Casson parameter, momentum boundary layer thickness reduces.

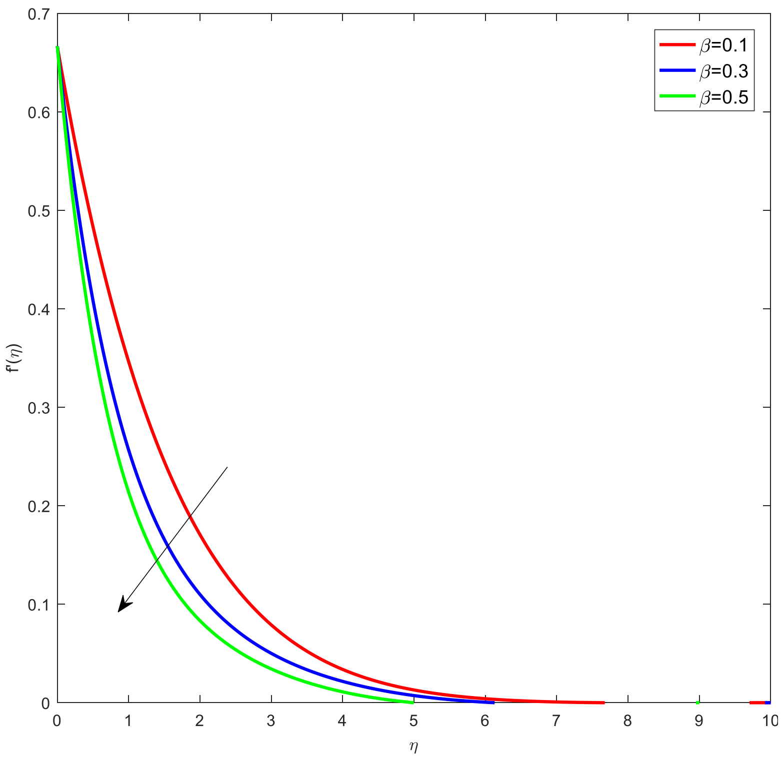

Fig. 2. Velocity graphs of Casson Parameter

Figure 3 depicts velocity profiles of Magnetic parameter. For enhanced observations of Magnetic parameter decrement in velocity profiles is noted. With augmented observations of magnetic parameter, an opposing force named Lorentz force will be generated.

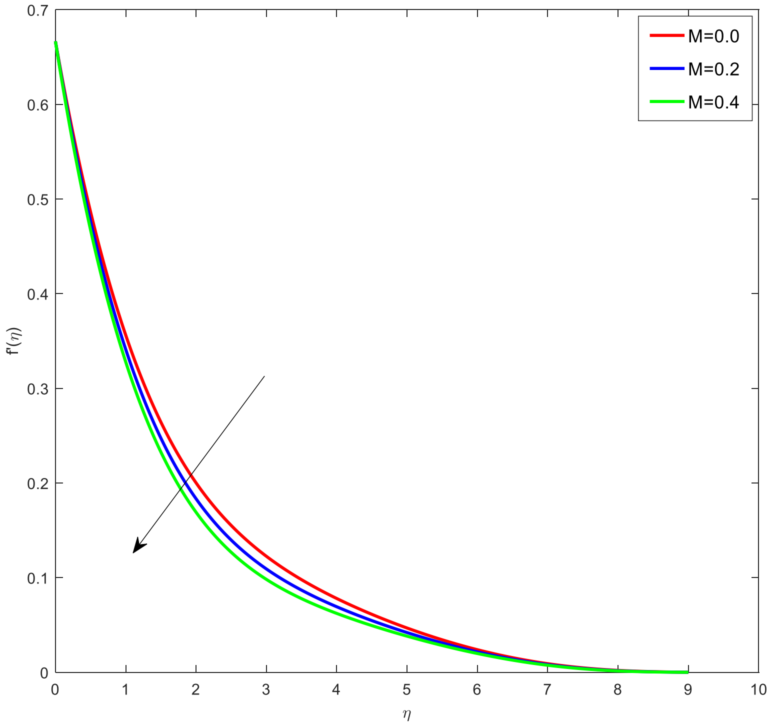

Fig. 3. Velocity profiles of Magnetic parameter

Figure 4 Velocity profiles of buoyancy parameter. Increasing observations of the buoyancy parameter, and enhancement in velocity profiles are observed.

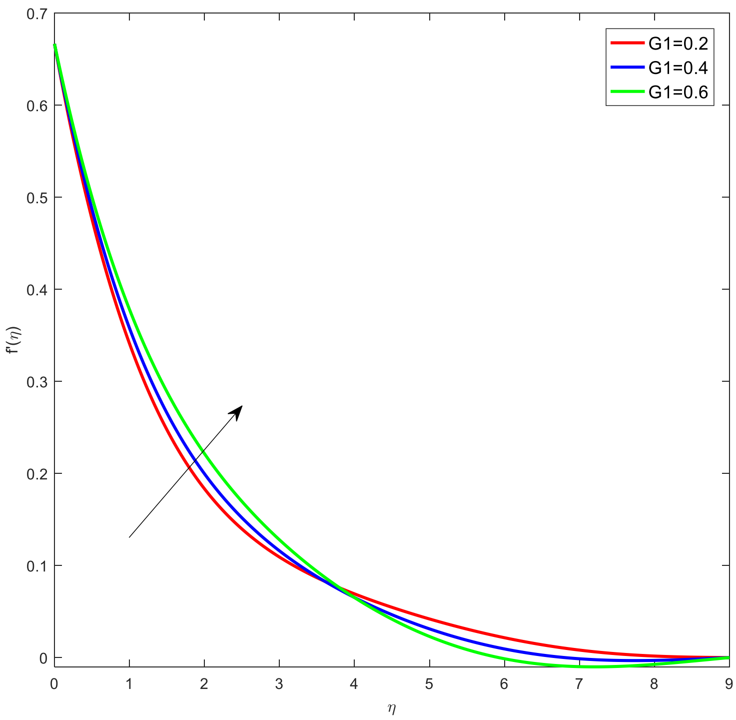

Fig. 4. Velocity graphs of buoyancy parameter

For larger observations of buoyancy parameter momentum marginal layer width enhances. velocity profiles increase for greater observations of solutal buoyancy parameter is portrayed in Figure 5.

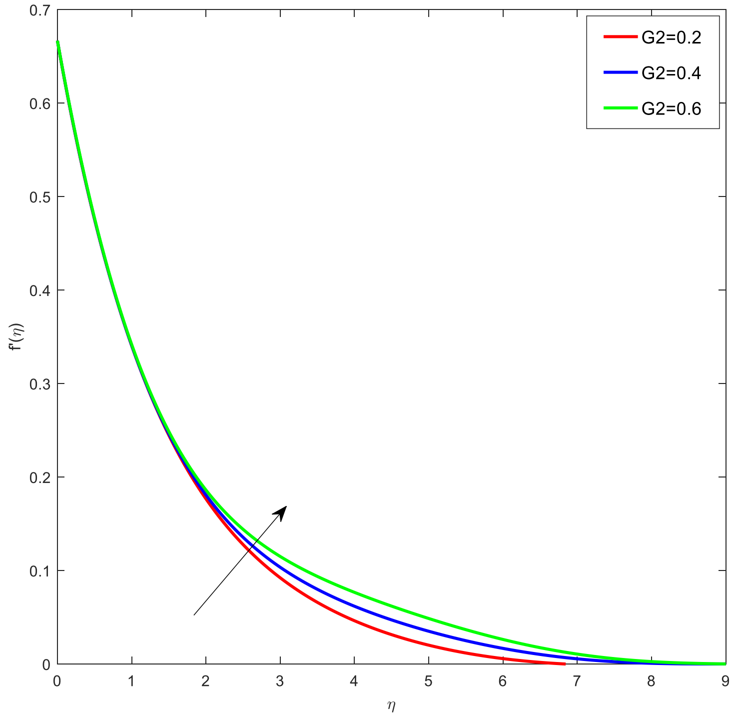

Fig. 5. Velocity graphs of solutal buoyancy parameter

Figure 6 shows velocity graphs of permeability parameter. Enhancing permeability parameter velocity increases because for increasing observations of permeability parameter resistance offered by porous materials deteriorates. Increasing Forchheimer parameter velocity profile decreases.

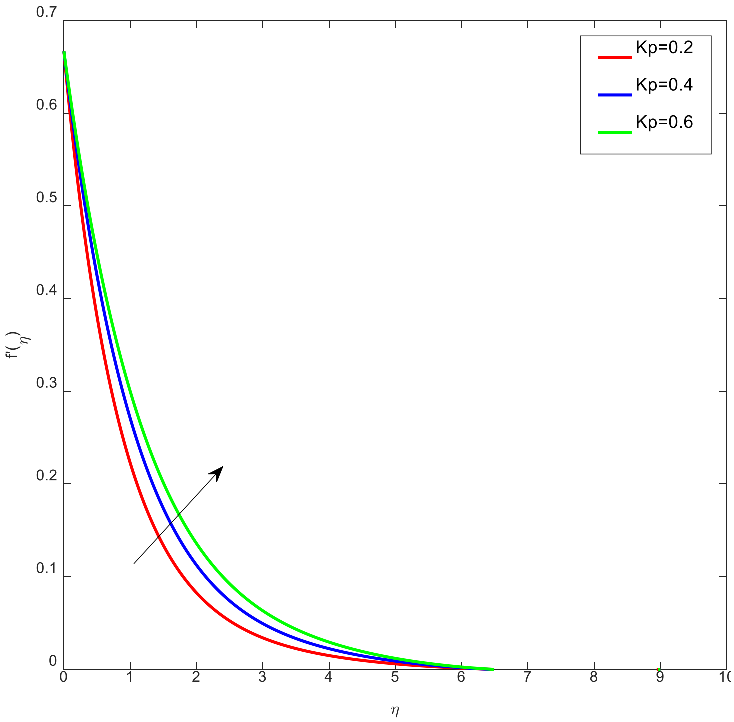

Fig. 6. Velocity graphs of permeability parameter

Because for enhanced observations of Forchheimer parameter frictional force will be generated causing decrement in velocity profiles depicted in Figure 7.

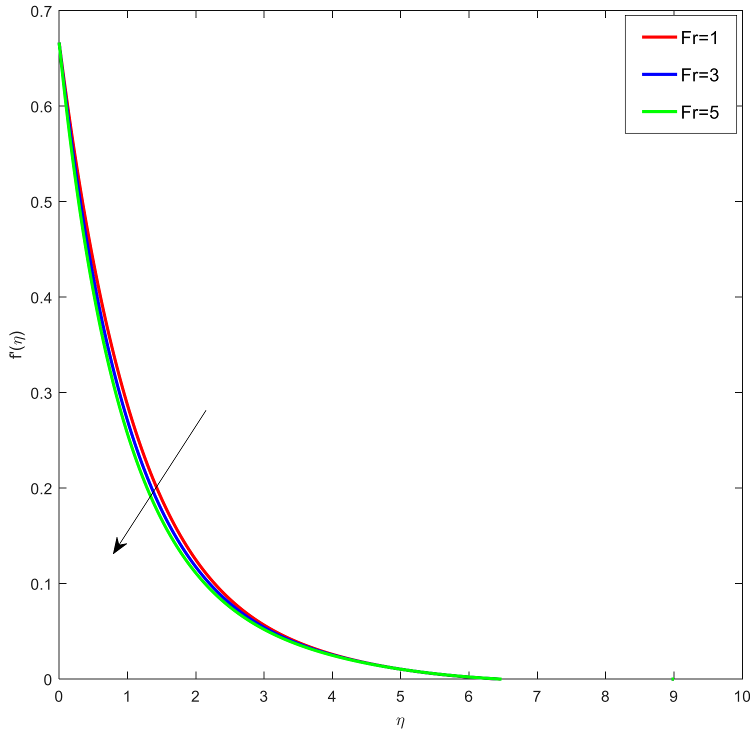

Fig. 7. Velocity graphs of Forchheimer parameter

Figure 8 displays temperature profiles of Prandtl number. For incremental observations of the Prandtl number temperature of the fluid decreases. For enhanced values of Prandtl number heat diffusivity of the fluid reduces so temperature of fluid falls.

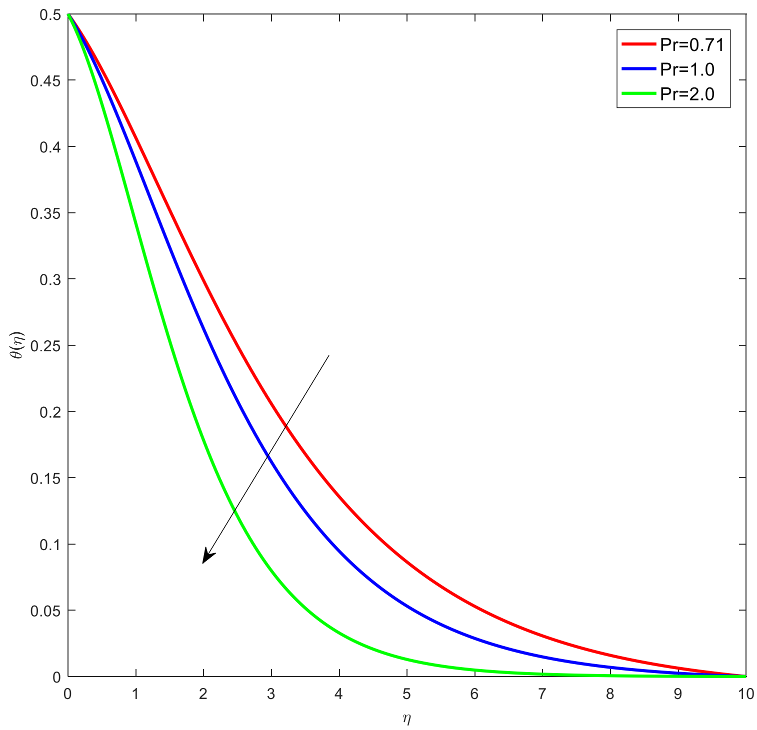

Fig. 8. Temperature graphs of Prandtl number

Increasing radiation parameter, extra heat will be generated inside the fluid. So the temperature of fluid increases which is portrayed in Figure 9.

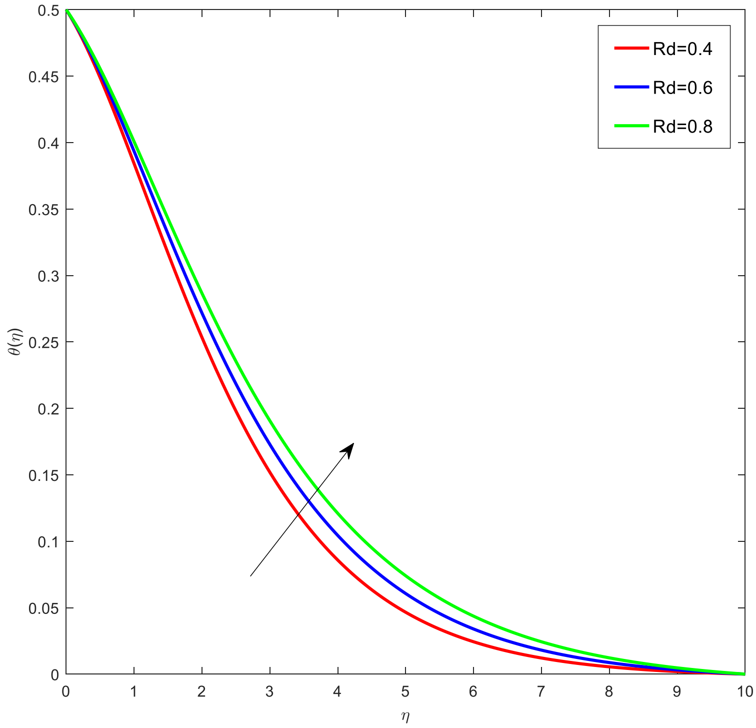

Fig. 9. Temperature graphs of radiation parameter

Figure 10 shows the temperature profiles of the Dofour parameter. Raising the Dofour parameter, temperature increases because thermal boundary layer thickness is raised with progressive observations of the Dofour parameter.

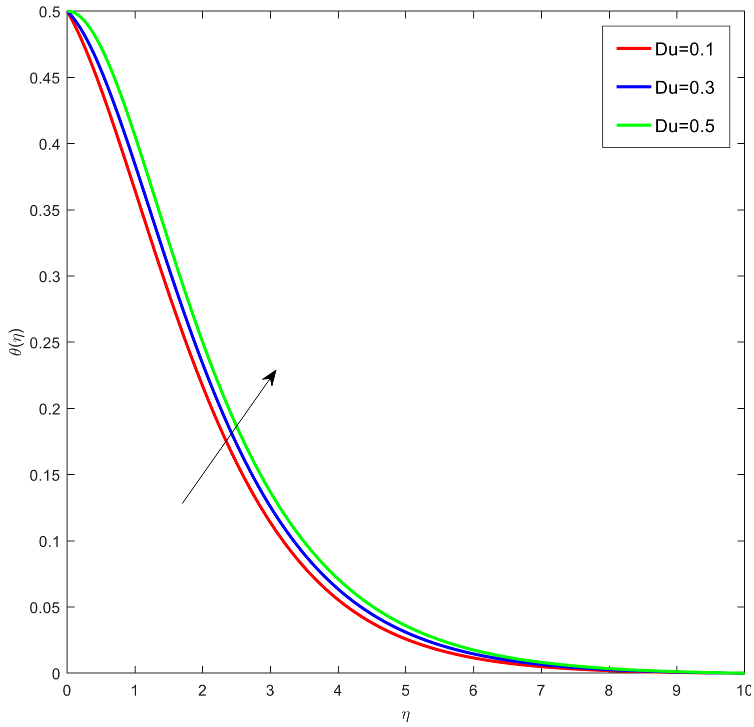

Fig. 10. Temperature graphs of Dufour parameter

Figure 11 displays temperature profiles of Eckert number. Increasing Eckert number kinetic energy of the fluid converts to the internal energy of the fluid which enhances the temperature of the fluid.

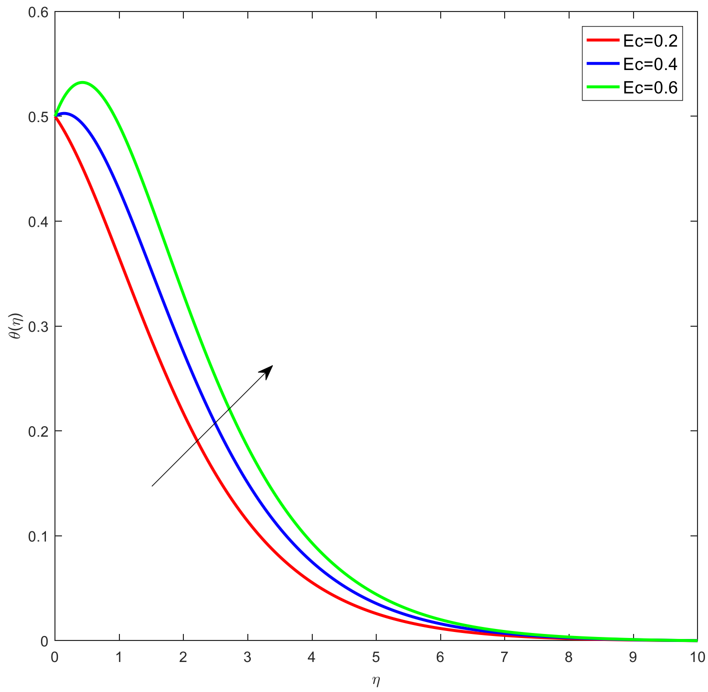

Fig. 11. Temperature graphs of Eckert number

Figure 12 represents temperature profiles of the Joule heating parameter. For progressive observations of Joule heating parameter enhancement in temperature is witnessed. As the Joule heating parameter is a function of Eckert number, increasing joule heating parameter temperature increases.

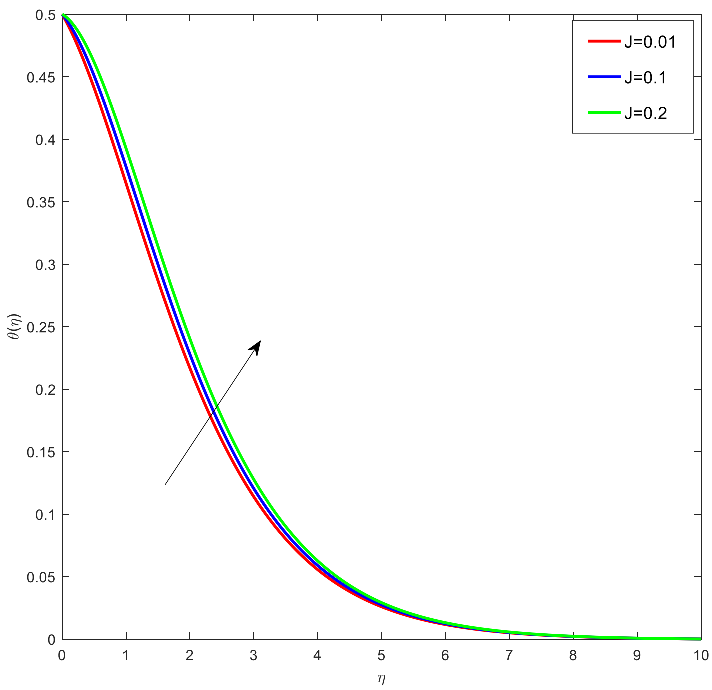

Fig. 12. Temperature graphs of Joule heating parameter

Figure 13 depicts concentration profiles of Schmidt number. For higher observations of Schmidt number, viscosity enhances. So, the concentration of fluid falls down.

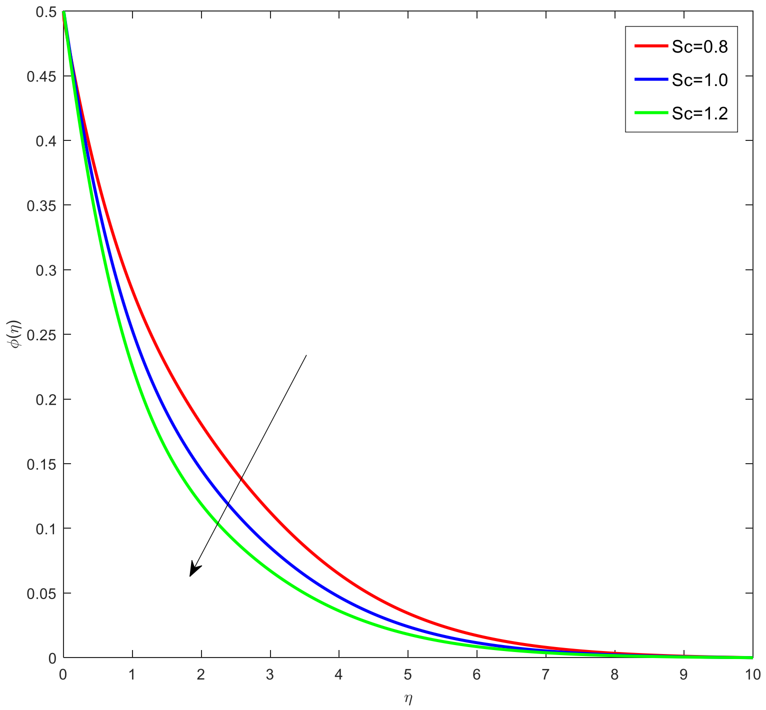

Fig. 13. Concentration graphs of Schmidt number

The concentration boundary layer thickness grows for incremental thermophoresis parameter measurements, whereas the marginal layer width decreases for incremental mass transfer rate observations. They demonstrate that as concentration rises in the border layer, the mass transfer rate declines, as seen in Figure 14.

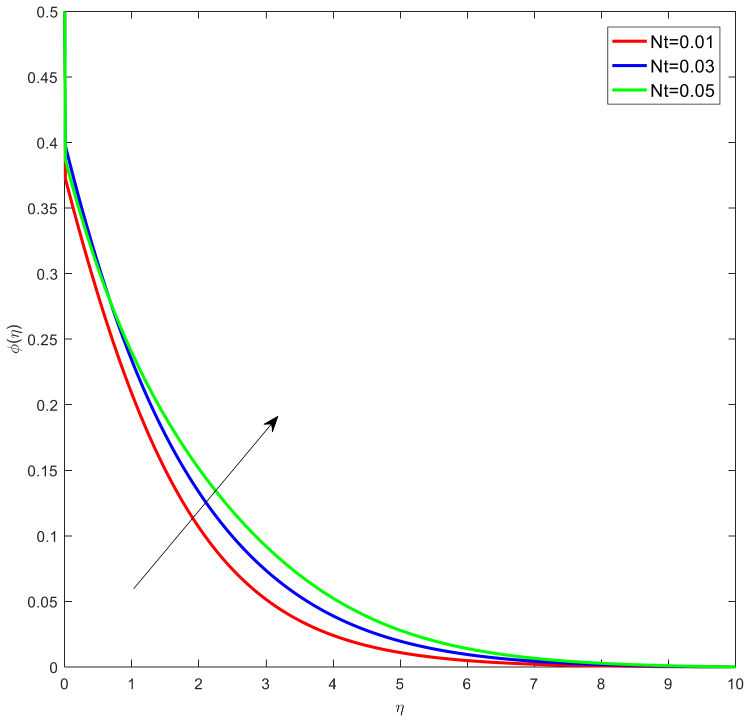

Fig. 14. Concentration graphs of Nt

Increasing Brownian diffusion parameter, decrement in concentration profiles is noted due to random movement of nanoparticles more heat will be produced so concentration of fluid decreases depicted in Figure 15.

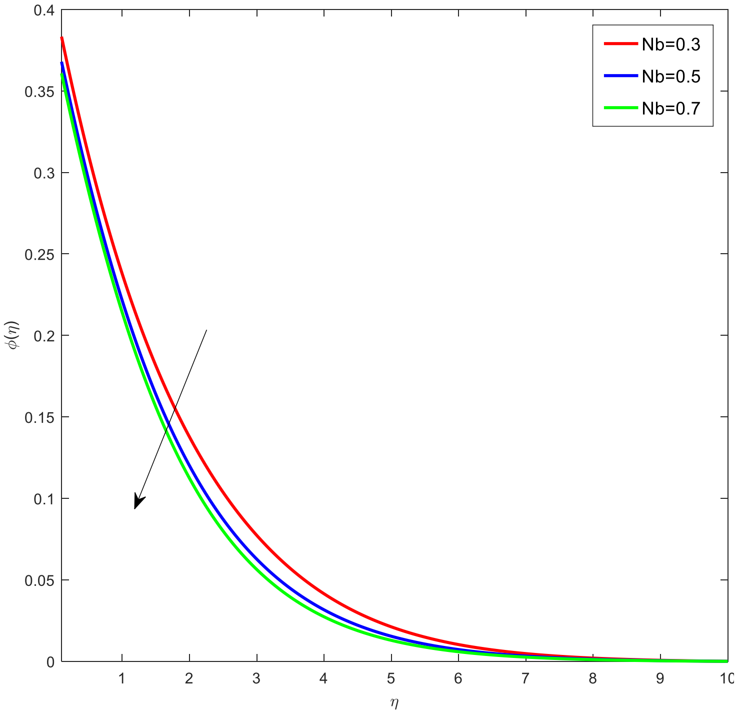

Fig. 15. Concentration graphs of Nb

Profiles of Chemical reaction parameter for the concentration are drawn in Figure16.

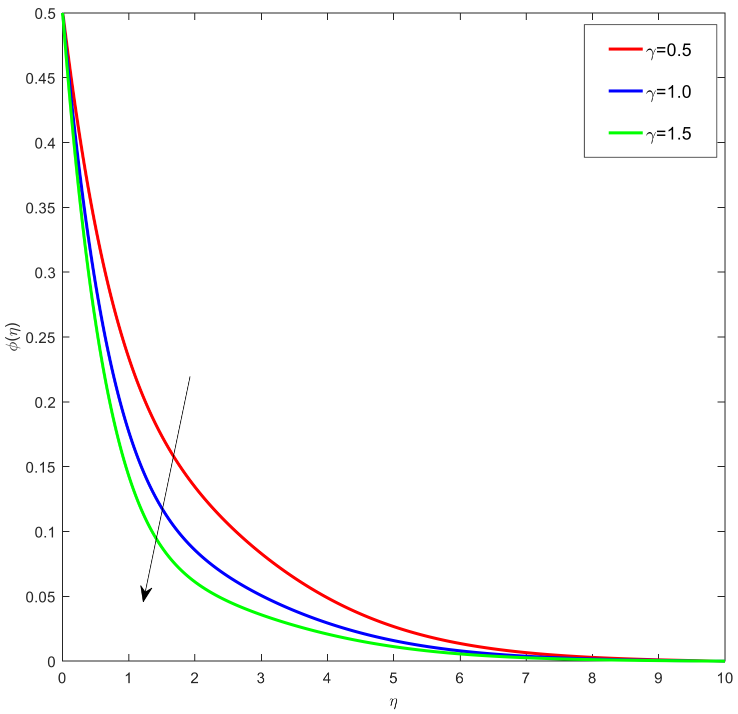

Fig. 16. Concentration g

For enhanced observations of chemical reaction parameter, reduction in solutal boundary layer thickness is noted consequently concentration profiles decreases. Increasing the inclination angle, the surface decrement in velocity is observed which is portrayed in Figure 17.

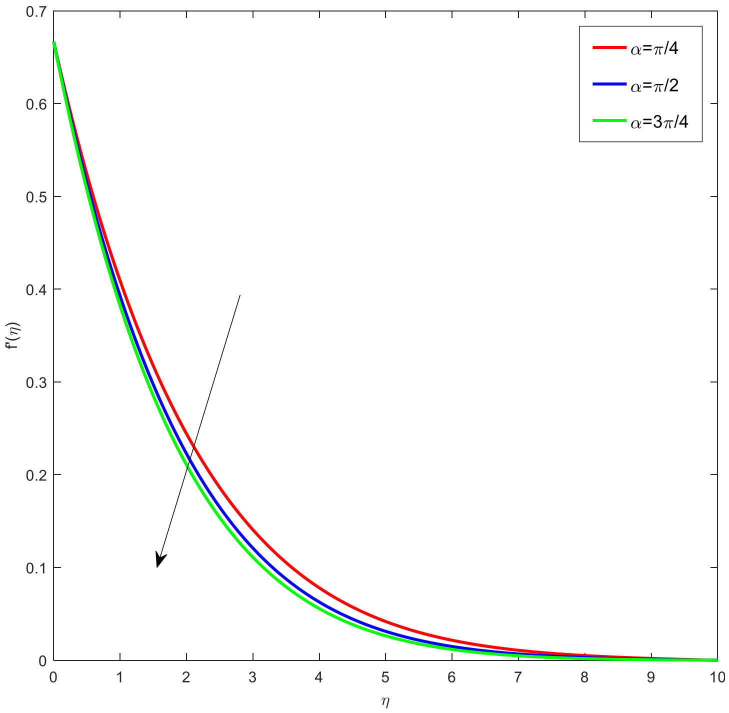

Fig. 17. Velocity graphs of Angle of inclination

Graphs of Chemical reaction parameter. For Progressive values of stretching sheet index parameter Nusselt number is calculated. The results are portrayed in table 1 and comparative study is made.

Table 1
Comparison of $-\theta'(0)$

N	Cortel [49]	Ullah *et al.,* [50]	Bezawada *et al.,* [48]	Present results
0.2	0.610262	0.6102	0.610203	0.610015
0.5	0.595277	0.5949	0.595204	0.595076
3	0.564472	0.5647	0.564670	0.564671
10	0.554960	0.5549	0.554890	0.554935

5. Conclusions

The present study, a two-dimensional Casson fluid stream over a slanted non-linear surface through Forchhiemer permeable medium is investigated. Flow is considered with the influences of MHD, thermal radiation, Joule heating, viscous dissipation, chemical reaction. The equations corresponding to the above problem are resolved by utilising Keller Box method. Numerical values are obtained and analysed the following observations are noted.

i. Velocity profiles reduce for progressive observations of Casson, Magnetic, angle of inclination, Forchhiemer number, and enhance for buoyancy parameter and permeability parameter.

ii. Temperature profiles increases for incremental observations of the Dofour parameter, Eckert number, Joule heating, and radiation parameters, the opposite trend is noted for the Prandtl number.
iii. Concentration profiles rise for the thermophoresis parameter and declines for the Schmidt number, Chemical reaction and Brownian diffusion constraints.
iv. Local Nusselt number decreases for increasing observations of stretching sheet index parameter.

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