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Effect of Gravity Modulation on the Stability Analysis of Viscoelastic Dielectric Liquids

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ABSTRACT

The linear and non-linear analysis of convection in viscoelastic dielectric liquids is presented in the paper. The viscoelastic equation of state is the upper convected Jeffrey model, also known as Oldroyd-B model. The amplitude equations which are the Khayat-Lorenz model for the dielectric liquids is derived with the aid of minimal mode double Fourier series. A modified method of Venezian is applied on the linearized amplitude equations to obtain a correction to the threshold eigenvalues that determine the onset of convection. The non- linear amplitude equations are non-autonomous due to modulation of gravity. Hence the numerical computation is performed using the "ode" function in Scilab, a free and open-source software which uses the LSODA solver. The heat transfer is quantified using the average Nusselt number where the average is computed using Simpsons $\left(\frac{3}{8}\right)^{th}$ rule. The effect of different parameters and viscoelastic models on heat transfer is discussed.

1. Introduction

Dielectric liquids owing to its high electrical resistance, finds its application in many fields. Some of them are stated by Bhavya *et al.*, [1]. The applications of viscoelastic liquids in engineering and industry have resulted in significant research in the field. Control of heat transfer through convection in liquids is an important industrial application.

The linear and non-linear theory of Rayleigh–Bénard convection of viscoelastic liquids has been reported by Green [2], Eltayeb [3], Rosenblat [4], Malashetty and Swamy [5] and references therein. A comparative study on the onset of oscillatory convection in different viscoelastic liquids was reported by Siddheshwar *et al.*, [6] and concluded that Maxwell liquid to be the most unstable liquid. Alhushaybari and Uddin [7] discussed the absolute instability and convection under the effect of g-jitter for a viscoelastic liquid jet. Recent work by Ewis [8] discussed about the natural convection in viscoelastic liquids and study of heat transfer. Shawky *et al.*, [9] studied the impact of heat and mass transfer on a non-Newtonian nanofluid.

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The early work of Stiles [10] discusses the convection in dielectric liquid in the presence of electric field. Siddheshwar and Pranesh [11] studied the delayed convection under the action of gravity modulation in a weakly electrically conducting magnetic fluid. Siddheshwar and Abraham [12] have reported the influence of time-periodic body force on convective instabilityin a dielectric liquid. Temperature and electric dominance on heat transport in a Newtonian dielectric liquid with field-dependent viscosity was reported by Siddheshwar *et al*., [13].

The onset of convection influenced by AC electric field and viscoelasticity has been reported by Takashima and Ghosh [14] and Othman [15]. The study by Agrait and Castellanos [16] highlights the influence of an AC field on convection in viscoelastic dielectric liquid. Othman and Sweilam [17] discussed the internal heating effects on convective instability of a viscoelastic dielectric liquid. The stabilizing effect of rotation on convection in a dielectric liquid of the viscoelastic type was reported by Othman [18].

The presence of vertical vibration results in time-periodic modulations of the body force and is termed as g-jitter or gravity modulation. The control of heat transfer externally(modulation) has been reported by Gresho and Sani [19], Wadih *et al*., [20], Malashetty and Padmavathi [21], Gaikwad and Irfana [22], Bhadauria and Kiran [23], Swamy *et al*., [24] and Siddheshwar *et al*., [25]. A study on thermal instability under the influence of temperature/gravity modulation fora rotating viscous fluid has been reported by Bhadauria *et al*., [26]. The recent work by Kiran [27] throws light on the influence of gravity modulation in heat and mass transfer for a viscoelastic fluid layer. A nonlinear stability analysis was performed by Gaikwad and Rangdal [28] on non-Newtonian fluids under the influence of gravity modulation.

However, the above-mentioned studies do not address the impact of g-jitter on convective instability in dielectric liquids of viscoelastic type. The paper presents the onset and post onset convective regimes with the consideration g-jitter and viscoelasticity.

2. Mathematical Formulation

An infinite horizontal layer of dielectric liquid is confined between the boundaries separated by distance 'h' units. The boundaries are free of tangential stresses and maintained at constant temperature. The lower boundary is maintained at higher temperature than the upper boundary. The system is subjected to time-periodic body force and hence $\vec{g} = g_0\{1 + \beta\, cos(\omega t)\}\hat{k}$, where g_0 is the unmodulated acceleration due to gravity, β is the amplitude and ω is the frequency of modulation. An external alternating current (AC) field is applied to the system in an upward direction. The physical representation of the problem considered is illustrated in Figure 1.

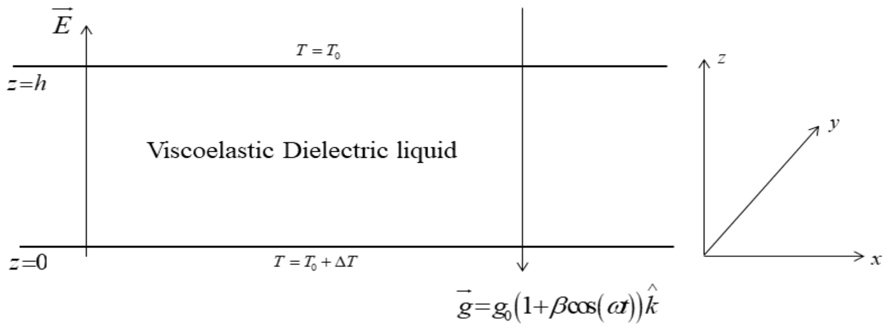

Fig. 1. Schematic of the flow configuration

The equations governing the physical phenomena under consideration are as stated by Siddheshwar and Radhakrishnan [29]:

$$\nabla.\vec{q} = 0, \tag{1}$$

$$\rho_0\left[\frac{\partial\vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right] = -\nabla p - \rho g_0(1 + \beta\cos\omega t)\hat{k} + (\vec{P}.\nabla)\vec{E} + \nabla.\underset{\sim}{T}, \tag{2}$$

$$\rho_0 C_{VE}\left[\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T\right] = \kappa_1\nabla^2 T, \tag{3}$$

$$\rho = \rho_0[1 - \alpha(T - T_0)], \tag{4}$$

where $\vec{q} = (u, v, w)$- velocity, t – time, p – pressure, $\vec{g} = (0,0,-g)$- acceleration due to gravity, ρ_0- reference density, ρ – density, $\vec{P}$ - dielectric polarization, $\vec{E}$ - electric field, $\underset{\sim}{T}$ - liquid stress tensor, C_{VE} - specific heat capacity, T – temperature, κ_1 - thermal conductivity, α - coefficient of thermal expansion.

Viscoelastic equation of state for the upper - convected Jeffrey model is given by

$$\left(1 + \lambda_1\frac{\partial}{\partial t}\right)\underset{\sim}{T} = \left(1 + \lambda_2\frac{\partial}{\partial t}\right)[\mu(\nabla\vec{q} + \nabla\vec{q}^T)], \tag{5}$$

where λ_1- stress relaxation time, λ_2 - strain retardation time and μ – Viscosity.

The electric field equations for a dielectric liquid in the presence of AC electric field, takes the form:

$$\nabla.\vec{D} = 0, \nabla\times\vec{E} = 0, \tag{6}$$

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P}, \vec{P} = \varepsilon_0(\varepsilon_r - 1)\vec{E}, \tag{7}$$

$$\varepsilon_r = {\varepsilon_r}^0 - e(T - T_0), \tag{8}$$

where $\vec{D}$ - electric displacement, ε_0 – electric permittivity, ε_r- relative permittivity and e – positive free charge.

Boundary conditions on velocity, temperature and electric field are given by:

$$T = T_0 + \Delta T \quad \text{at } z = 0, \tag{9}$$

$$T = T_0 \qquad \text{at } z = h, \tag{10}$$

$$\vec{q} = 0 \text{ at } z = 0 \text{ and } z = h, \tag{11}$$

$$(\vec{D}_{internal} - \vec{D}_{external}).\vec{n} = 0 \text{ at } z = 0 \text{ and } z = h, \tag{12}$$

$$(\vec{E}_{internal} - \vec{E}_{external})\times\vec{n} = 0 \text{ at } z = 0 \text{ and } z = h. \tag{13}$$

Operating $\left(1 + \lambda_1\frac{\partial}{\partial t}\right)$ on Eq. (2) and using Eq. (5), we get

$$\rho_0\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\frac{\partial\vec{q}}{\partial t}+(\vec{q}.\nabla)\vec{q}\right]=\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[-\nabla p-\rho g_0(1+\beta\cos(\omega t))\hat{k}+(\vec{P}.\nabla)\vec{E}\right]$$
$$+\left(1+\lambda_2\frac{\partial}{\partial t}\right)[\nabla^2\vec{q}]. \quad (14)$$

2.1 Conduction State

The conduction state is a motionless state and all the physical parameters are varying only along the z-axis and hence all the physical quantities are assumed to be functions of z. In view of this, the quiescent state solutions are:

$$\vec{q}_b=(0,0,0), \quad (15)$$

$$\rho_b(z)=\rho_0\left[1-\alpha\left(1-\frac{z}{h}\right)\Delta T\right], \quad (16)$$

$$T_b(z)=T_o+\left(1-\frac{z}{h}\right)\Delta T, \quad (17)$$

$$E_b(z)=\frac{E_0(1+\chi_e)}{\varepsilon_{rb}}, \quad (18)$$

$$D_b(z)=\varepsilon_0\varepsilon_{rb}E_b(z), \quad (19)$$

$$P_b(z)=\varepsilon_0E_0(1+\chi_e)\left[1-\frac{1}{\varepsilon_{rb}}\right], \quad (20)$$

where χ_e - electric susceptibility and $\varepsilon_{rb}=(1+\chi_e)+\frac{e\Delta T}{h}(z-h)$,

3. Stability Analysis

The system in the quiescent state is superposed with infinitesimally small disturbances. Hence all the physical quantities f are expressed as $f=f_b+f'$ where f_b is the conduction state solution given by Eq. (15) to Eq. (20). As a result,

$$P_1{}'=-e\,\varepsilon_0\,E_1{}'T'+\varepsilon_0\chi_eE_1{}', \quad (21)$$

$$P_3{}'=-e\,\varepsilon_0\,E_3{}'T'+\varepsilon_0\chi_eE_3{}'-e\varepsilon_0T'E_0, \quad (22)$$

under the assumption $e\Delta T<<(1+\chi_e)$.

The Eq. (1) to Eq. (4) are nondimensionalized using the characteristic quantities as follows:

$$t^*=\frac{t'}{h^2/\kappa},\ x^*=\frac{x}{h},\ z^*=\frac{z}{h},\ T^*=\frac{T-T_0}{\Delta T},\ \omega^*=\frac{h^2}{\kappa}\omega',\ \psi^*=\frac{\psi}{\kappa},\ \phi^*=\frac{\phi'(1+\chi_e)}{eE_0\Delta Th},\ \Lambda_1=\frac{\lambda_1\kappa}{h^2}\text{ and }\Lambda_2=\frac{\lambda_2\kappa}{h^2}.$$

The configuration considered renders the flow to be two-dimensional and hence $\frac{\partial}{\partial y}=0$. In view of this and Eq. (1), we get $\vec{q}=\nabla\times\psi(x,z)\hat{j}$ where $\psi(x,z)$ is the stream function. The resulting dimensionless equations are obtained by introducing the electric potential ϕ. Further, the $\hat{j}$

component of curl of Eq. (14) can be decomposed as explained by Siddheshwar *et al.*, [6] and are obtained as:

$$\frac{1}{Pr}\frac{\partial}{\partial t}(\nabla^2\psi)=\frac{1}{Pr}\frac{\partial(\psi,\nabla^2\psi)}{\partial(x,z)}-[R(1+\beta\cos(\omega t))+RL]\frac{\partial T}{\partial x}+RL\frac{\partial\left(T,\frac{\partial\phi}{\partial z}\right)}{\partial(x,z)}$$
$$+RL\frac{\partial^2\phi}{\partial x\partial z}+M+\Lambda\nabla^4\psi, \tag{23}$$

$$-\Lambda_1\frac{\partial M}{\partial t}=M-\nabla^4\psi+\Lambda\nabla^4\psi. \tag{24}$$

Eq. (3) and Eq. (6) reduces to the following:

$$\frac{\partial T}{\partial t}=\nabla^2 T-\frac{\partial\psi}{\partial x}+\frac{\partial(\psi,T)}{\partial(x,z)}, \tag{25}$$

$$0=\nabla^2\phi-\frac{\partial T}{\partial z}. \tag{26}$$

Eq. (23) to Eq. (26) are solved subject to

$$\psi=\nabla^2\psi=T=\frac{\partial\phi}{\partial z}=0 \text{ at } z=0,1. \tag{27}$$

where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}$.

In the process of non-dimensionalization, we obtain the following dimensionless parameters: the Prandtl number $Pr=\frac{\mu}{\rho_0\kappa}$, thermal Rayleigh number $R=\frac{\rho_0\alpha g\Delta T h^3}{\mu\kappa}$, electric number $L=\frac{\varepsilon_0 e^2(E_0)^2\Delta T}{\alpha g\rho_0(1+\chi_e)h}$ as defined by Siddheshwar and Radhakrishnan [29], the non-dimensional stress relaxation parameter $\Lambda_1=\frac{\lambda_1\kappa}{h^2}$, non-dimensional strain retardation parameter $\Lambda_2=\frac{\lambda_2\kappa}{h^2}$ and $\Lambda=\frac{\Lambda_2}{\Lambda_1}$.

3.1 Khayat-Lorenz Model

The normal mode analysis is performed by solving Eq. (23) to Eq. (26) through periodic form solutions given below which satisfies Eq. (27).

$$\psi(x,z,t)=A(t)\sin(kx)\sin(\pi z), \tag{28}$$

$$T(x,z,t)=B(t)\cos(kx)\sin(\pi z)+C(t)\sin(2\pi z), \tag{29}$$

$$M(x,z,t)=D(t)\sin(kx)\sin(\pi z), \tag{30}$$

$$\phi(x,z,t)=E(t)\cos(kx)\cos(\pi z)+F\cos(2\pi z). \tag{31}$$

Projecting the equations Eq. (28) to Eq. (31) onto the equations Eq. (23) to Eq. (26) and applying the orthogonality conditions over the domain $(x,z)\in\left[0,\frac{2\pi}{k}\right]\times[0,1]$ yields the system of non-autonomous differential equations given by:

$$\frac{1}{Pr}\frac{dA}{dt} = -\Lambda A\eta^2 + \frac{RkB}{\eta^4}[\eta^2 + k^2L] + \frac{RL\pi k^3}{\eta^4}BC + \frac{Rk\beta\cos(\omega t)}{\eta^2}B - \frac{D}{\eta^2}, \tag{32}$$

$$\frac{dB}{dt} = kA - \eta^2B + k\pi AC, \tag{33}$$

$$\frac{dC}{dt} = -4\pi^2C - \frac{k\pi AB}{2}, \tag{34}$$

$$\frac{dD}{dt} = \frac{1-\Lambda}{\Lambda_1}\eta^4A - \frac{D}{\Lambda_1}, \tag{35}$$

where $\eta^2 = \pi^2 + k^2$.

The above differential equations do not contain the amplitudes and F explicitly as equation Eq. (26) is independent of time. Eq. (26) results in $E = \frac{-\pi B}{\eta^2}$ and $F = -\frac{C}{2\pi}$. The Khayat-Lorenz model for dielectric liquids Eq. (32) to Eq. (35) is then suitably scaled by using the transformations $\tau = \eta^2 t$, $X = \frac{Ak\pi}{\sqrt{2}\eta^2}$, $Y = \frac{B\pi r}{\sqrt{2}}$, $Z = -\pi rC$ and $N = \frac{k\pi D}{\sqrt{2}(1-\Lambda)\eta^6}$ where $r = \frac{R}{R_S}$, $R_S = \frac{\eta^8}{k^2[\eta^2+k^2L]}$, $M_L = \frac{\pi Lk^2}{\eta^2+k^2L}$ and $\Omega = \frac{\omega}{\eta^2}$. Thus, the scaled Khayat-Lorenz model for dielectric liquid is:

$$\frac{dX}{d\tau} = Pr\left[-\Lambda X + Y\left(1 - \frac{M_LZ}{\pi r}\right) + \frac{\eta^2}{\eta^2+k^2L}\beta\cos(\Omega\tau)\,Y - (1-\Lambda)N\right], \tag{36}$$

$$\frac{dY}{d\tau} = rX - Y - XZ, \tag{37}$$

$$\frac{dZ}{d\tau} = -\frac{4\pi^2}{\eta^2}Z + XY, \tag{38}$$

$$\frac{dN}{d\tau} = \frac{1}{\Lambda_1\eta^2}(X - N). \tag{39}$$

3.2 Linear Stability Analysis

The linear stability analysis is performed on Eq. (36) to Eq. (39) by ignoring the non-linear terms. Hence, we obtain

$$\frac{dX}{d\tau} = -\Lambda\,Pr\,X + Pr\,Y\left[1 + \frac{\eta^2}{\eta^2+k^2L}\beta\,cos(\Omega\tau)\right] - (1-\Lambda)NPr, \tag{40}$$

$$\frac{dY}{d\tau} = rX - Y, \tag{41}$$

$$\frac{dZ}{d\tau} = -\frac{4\pi^2Z}{\eta^2}, \tag{42}$$

$$\frac{dN}{d\tau} = \frac{1}{\Lambda_1\eta^2}(X - N). \tag{43}$$

3.2.1 Limiting case

The linear system of equations under the particular case of

$$\left|\frac{\eta^2}{\eta^2+k^2L}\beta\, cos(\Omega\tau)\right| \ll 1, \tag{44}$$

reduces to Eq. (45) to Eq. (48).

$$\frac{dX}{d\tau} = -\Lambda\, Pr\, X + Pr\, Y - (1-\Lambda)NPr, \tag{45}$$

$$\frac{dY}{d\tau} = rX - Y, \tag{46}$$

$$\frac{dZ}{d\tau} = -\frac{4\pi^2 Z}{\eta^2}, \tag{47}$$

$$\frac{dN}{d\tau} = \frac{1}{\Lambda_1\eta^2}(X-N). \tag{48}$$

The above mathematical system is a system of linear equations representing the unmodulated case and hence can be resolved analytically. The linear solution is obtained by substituting Eq. (46) and Eq. (48) in Eq. (45) to obtain:

$$\left[D^3 + D^2\left(\frac{1}{\Lambda_1\eta^2} + \Lambda Pr + 1\right) + D\left(\frac{1}{\Lambda_1\eta^2} + \Lambda Pr - rPr + \frac{Pr}{\Lambda_1\eta^2}\right) - \left(\frac{rPr}{\Lambda_1\eta^2} - \frac{Pr}{\Lambda_1\eta^2}\right)\right]X = 0, \tag{49}$$

where $D = \frac{d}{d\tau}$ is the differential operator.

The solution of Eq. (45) to Eq. (48) is given by

$$X(\tau) = c_1 e^{m_1\tau} + c_2 e^{m_2\tau} + c_3 e^{m_3\tau} \tag{50}$$

$$Y(\tau) = c_4 e^{-\tau} + r\left[\frac{c_1 e^{m_1\tau}}{m_1+1} + \frac{c_2 e^{m_2\tau}}{m_2+1} + \frac{c_3 e^{m_3\tau}}{m_3+1}\right] \tag{51}$$

$$Z(\tau) = c_5 e^{\frac{-4\pi^2\tau}{\eta^2}} \tag{52}$$

$$N(\tau) = c_6 e^{-\frac{\tau}{\Lambda_1\eta^2}} + \frac{1}{\Lambda_1\eta^2}\left[\frac{c_1 e^{m_1\tau}}{m_1+\frac{1}{\Lambda_1\eta^2}} + \frac{c_2 e^{m_2\tau}}{m_2+\frac{1}{\Lambda_1\eta^2}} + \frac{c_3 e^{m_3\tau}}{m_3+\frac{1}{\Lambda_1\eta^2}}\right] \tag{53}$$

where m_1, m_2 and m_3 are the roots of the differential Eq. (49) and c_1, c_2, c_3, c_4, c_5 and c_6 are arbitrary constants which are obtained by evaluating the boundary conditions given by Eq. (73).

It is interesting to observe that $\left|\frac{\eta^2}{\eta^2+k^2L}\beta\, cos(\Omega\tau)\right| \ll 1$ provided $\beta \ll \frac{\eta^2+k^2L}{\eta^2}$. On estimating the right-hand side of the inequality, it is found that it is of order $O(10^{-2})$ and Eq. (44) is the condition under which the modulation effect is negligible and g_0 is the effect of nonlinear terms.

3.2.2 Correction Rayleigh number

Eq. (40) to Eq. (43) is a system of linear equations representing the modulated case. Eq. (42) does not have terms which are coupled with any of the amplitudes and therefore can be solved independently. However, the equations Eq. (40), Eq. (41) and Eq. (43) are solved by expanding the amplitudes $X, Y, N$and the scaled Rayleigh number r in terms of amplitude β as given below:

$$X = x_0 + \beta x_1 + \beta^2 x_2 + \ldots\ldots..,\tag{54}$$

$$Y = y_0 + \beta y_1 + \beta^2 y_2 + \ldots\ldots..,\tag{55}$$

$$N = n_0 + \beta n_1 + \beta^2 n_2 + \ldots\ldots..,\tag{56}$$

$$r = r_0 + \beta r_1 + \beta^2 r_2 + \ldots\ldots\ldots\tag{57}$$

Substituting the Eq. (54) to Eq. (57) in Eq. (40) to Eq. (43), we get the following equations by equating the like powers of β.

$$O(\beta^0): IW_0 = 0,\tag{58}$$

$$O(\beta^1): IW_1 = [R_{21} \quad R_{22} \quad R_{23}]^T,\tag{59}$$

$$O(\beta^2): I\,W_2 = [R_{31} \quad R_{32} \quad R_{33}]^T,\tag{60}$$

$$I = \begin{bmatrix} -\frac{d}{d\tau} - \Lambda Pr & Pr & -(1-\Lambda)Pr \\ r_0 & -1 - \frac{d}{d\tau} & 0 \\ \frac{1}{\Lambda_1 \eta^2} & 0 & -\frac{1}{\Lambda_1 \eta^2} - \frac{d}{d\tau} \end{bmatrix},\tag{61}$$

where I and W_i $(i = 0,1,2)$are the operators while T represents the transpose.

$$R_{21} = \frac{-Pr\,\eta^2}{\eta^2 + k^2 L}(\cos\Omega\,\tau) y_0; \qquad R_{22} = -r_1 x_0; \qquad R_{23} = 0,\tag{62}$$

$$R_{31} = \frac{-Pr\,\eta^2}{\eta^2 + k^2 L}(\cos\Omega\,\tau) y_1; \qquad R_{32} = -r_2 x_0 - r_1 x_1; \qquad R_{33} = 0.\tag{63}$$

Typically, for the stationary mode of convection, $\frac{d}{d\tau} = 0$. Therefore, the marginal stability analysis at $O(\beta^0)$ yields

$$W_0 = \begin{bmatrix} x_0 \\ y_0 \\ n_0 \end{bmatrix} = \begin{bmatrix} x_0 \\ r_0 x_0 \\ x_0 \end{bmatrix},\tag{64}$$

with $r_0 = 1$. The operator $I(\tau)$ is rewritten in terms of frequency following Venezian [30] as given below:

$$I(\Omega) = \begin{bmatrix} -i\Omega - \Lambda Pr & Pr & -(1-\Lambda)Pr \\ r_0 & -1 - i\Omega & 0 \\ \frac{1}{\Lambda_1 \eta^2} & 0 & -\frac{1}{\Lambda_1 \eta^2} - i\Omega \end{bmatrix}\tag{65}$$

Substitution of Eq. (65) in Eq. (59) results in

$$(-i\Omega - \Lambda Pr)x_1 + Pr\, y_1 - (1-\Lambda)\, Pr\, n_1 = \frac{-Pr\,\eta^2 y_0}{\eta^2 + k^2 L}\overline{cos(\Omega\tau)}, \tag{66}$$

$$r_0 x_1 - (1 + i\Omega)y_1 = -r x_0, \tag{67}$$

$$\frac{x_1}{\Lambda_1 \eta^2} - \left(\frac{1}{\Lambda_1 \eta^2} + i\Omega\right) n_1 = 0. \tag{68}$$

where the bar over $\cos(\Omega\tau)$ is the time-average over $\left[0, \frac{2\pi}{\Omega}\right]$.

In view of Venezian [30], $r_1 = 0$ and hence

$$x_1 = \frac{Pr\,\eta^2}{(\eta^2 + k^2 L)\left(-i\Omega - \Lambda\, Pr + \frac{Pr}{1+i\Omega} - \frac{(1-\Lambda)Pr}{i\Omega}\right)}\overline{\cos(\Omega\tau)};\ y_1 = \frac{x_1}{1+i\Omega} \text{ and } n_1 = \frac{\frac{x_1}{\Lambda_1 \eta^2}}{\left(\frac{1}{\Lambda_1 \eta^2} + i\Omega\right)}. \tag{69}$$

The Fredholm solvability condition as stated in Siddheshwar *et al.*, [31] yields $R_{31}\hat{x}_0 + R_{32}\hat{y}_0 + R_{33}\hat{n}_0 = 0$ where $\hat{x}_0$, $\hat{y}_0$ and $\hat{n}_0$ are the adjoint solutions of system.

$$\begin{bmatrix} -\Lambda_1 Pr & Pr & -(1-\Lambda)Pr \\ r_0 & -1 & 0 \\ \frac{1}{\Lambda_1 \eta^2} & 0 & -\frac{1}{\Lambda_1 \eta^2} \end{bmatrix}^T \begin{bmatrix} x_0 \\ y_0 \\ n_0 \end{bmatrix} = 0. \tag{70}$$

Thus, $\hat{y}_0 = Pr\,\hat{x}_0$ and $\hat{n}_0 = -\Lambda_1 \eta^2 (1-\Lambda)\, Pr\,\hat{x}_0$ where $\hat{x}_0 = 1$. Substituting of $\hat{x}_0$, $\hat{y}_0$ and $\hat{n}_0$ in the solvability condition results in

$$r_2 = Re\left[\frac{-\eta^2 \overline{(cos(\Omega\tau))} x_1}{(\eta^2 + k^2 L)(1 + i\Omega)}\right], \tag{71}$$

where Re denotes the real part. Ignoring terms of $O > \beta^2$ in Eq. (57), we get

$$r = r_0 + \beta^2 r_2. \tag{72}$$

In view of equation Eq. (72), $r_2$is a correction to the Rayleigh number. The critical values of r_2 for different parameters are obtained using $r_{2c} = r_2(k_c)$ following Venezian [30].

3.3 Non-Linear Analysis

The non-linear study primarily concentrates on the estimation of heat transport. The scaled Khayat-Lorenz model for dielectric liquid given by Eq. (36) to Eq. (39) is solved numerically subjected to the initial condition

$$\big(X(0), Y(0), Z(0), N(0)\big) = (3{,}3{,}3{,}3). \tag{73}$$

The numerical solutions thus obtained are used to calculate the Nusselt number, the ratio of heat transfer through convection to heat transfer through conduction. Hence

$$Nu = 1 + \left[\frac{\frac{k}{2\pi}\int_0^{2\pi/k} \frac{\partial T}{\partial z}\, dx}{\frac{k}{2\pi}\int_0^{2\pi/k} \frac{\partial T_b}{\partial z}\, dx} \right]_{z=0} = 1 - 2\pi C = 1 + \frac{2}{r} Z. \tag{74}$$

Further, the average Nusselt number is computed as

$$\overline{Nu(\tau)} = \frac{1}{b-a}\int_a^b \left(1 + \frac{2}{r} Z\right) d\tau. \tag{75}$$

4. Results and Discussion

The current study deals with the study of linear and non-linear stability analysis in a dielectric liquid under the influence of gravity modulation and an AC field. The liquid properties are characterized by the dimensionless parameters L and Pr whereas the effect of electric field is manifested by the electric number L. The influence of g-jitter is regulated through the modulation frequency, Ω and the amplitude, β. The impact of the above parameters on the onset of convection and heat transfer for Jeffrey, Newtonian and Maxwell dielectric liquid has been discussed. The presence of gravity modulation renders system of differential equations Eq. (40) to Eq. (43) to be a non-autonomous system of differential equations. Hence, they are solved numerically using the "ode" function in Scilab, a free and open-source software. The default setup in the"ode" function uses the LSODA solver from the ODE pack. Heat transfer is quantified by the average Nusselt number and computed using Simpson's $\left(\frac{3}{8}\right)^{th}$ rule for evaluating Eq. (75). The domain for the parameters defining the viscoelasticity as stated by Melson *et al.,* [32] is given by Table 1.

Table 1
Parametric domain for viscoelastic models

$\Lambda_1 = \Lambda_2$	Newtonian dielectric liquid
$\Lambda_1 > 0$and $\Lambda_2 = 0$	Maxwell dielectric liquid
$\Lambda_1 > 0$and $\Lambda_2 > 0$	Jeffrey dielectric liquid

The influence of g-jitter is expressed through the plots of frequency of modulation versus the critical scaled correction Rayleigh number, r_{2c} Figure 2 to Figure 7 for the Jeffrey, Newtonian and Maxwell dielectric liquids with variation of parameters L and Pr.

It is observed that for the Jeffrey dielectric liquid Figure 2 and Figure 3 and the Newtonian dielectric liquid Figure 4 and Figure 5 the correction scaled Rayleigh number remains positive throughout with the variation in parameters L, Pr and Ω.

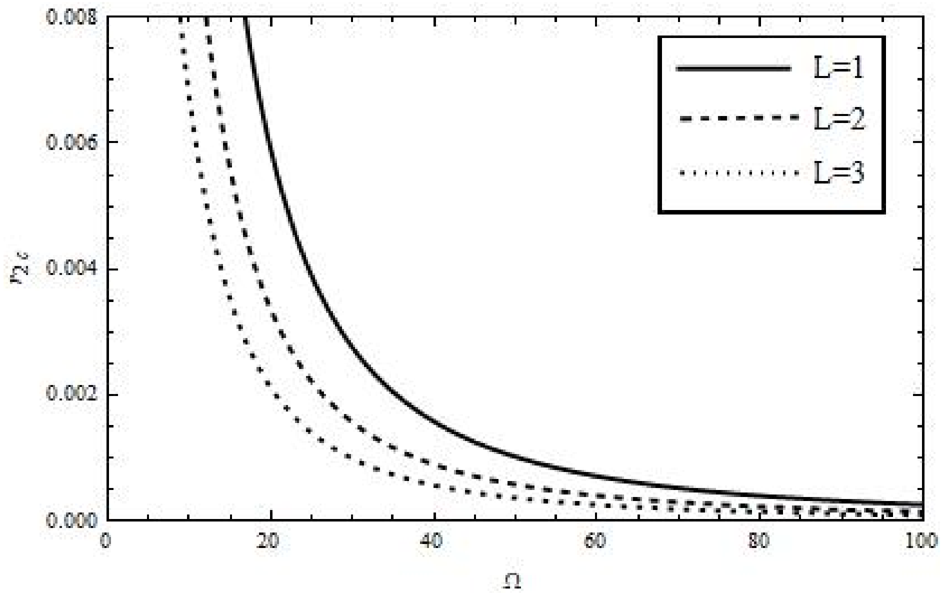

Fig. 2. Frequency $(\boldsymbol{\Omega})$ dependence of scaled correction Rayleigh number $\boldsymbol{r_{2c}}$ for Jeffrey dielectric liquid with variations in $\boldsymbol{L}$

This indicates that the modulation frequency stabilizes the system by delaying the onset of convection. The effect of frequency of modulation is significant at moderate values only. At very high values of Ω, r_{2c} tends to zero. Increasing values of L results in decreasing values of r_{2c} which indicates reducing supercritical region. This in turn indicates that the effect of modulation is insignificant at high values of L. An opposite effect is seen with an increase in Prandtl number, Pr that is, increasing Pr results in increasing r_{2c} hence an increase in the supercritical region which indicates that the system is more stable at higher values of Pr.

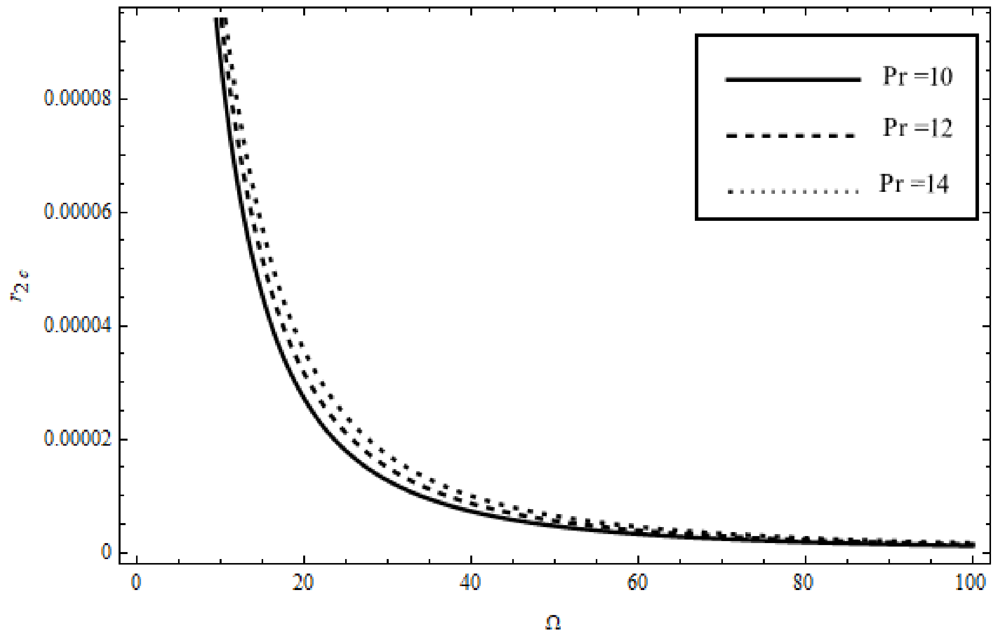

Fig. 3. Frequency $(\boldsymbol{\Omega})$ dependence of scaled correction Rayleigh number $\boldsymbol{r_{2c}}$ for Jeffrey dielectric liquid with variations in $\boldsymbol{Pr}$

In Figure 4 and Figure 5, the Newtonian dielectric liquid also shows the same result as noted in the Jeffrey dielectric liquid for increasing value of electric buoyancy, L whereas a mixed behavior is noticed with increase in Pr. For Newtonian dielectric liquid, increasing Pr results in thinning of supercritical region for $\Omega > 12$. However, the effect of Pr is opposite for $\Omega > 12$. A similar result is found in Siddheshwar and Kanchana [25] which states that with the increase in Pr, the critical scaled correction Rayleigh number decreases thereby destabilizing the system.

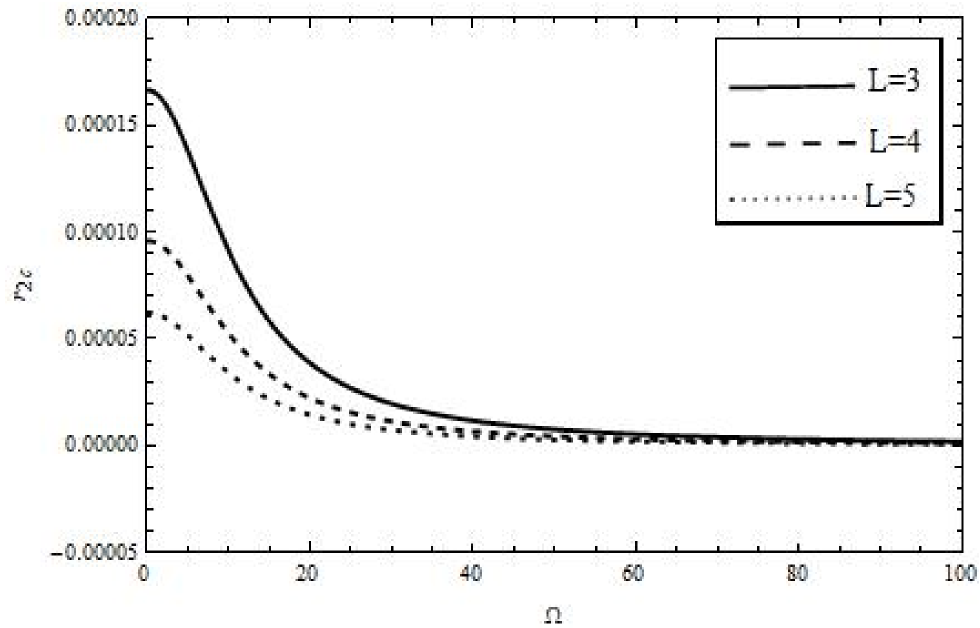

Fig. 4. Frequency ($\boldsymbol{\Omega}$) dependence of scaled correction Rayleigh number $\boldsymbol{r_{2c}}$ for Newtonian dielectric liquid with variations in $\boldsymbol{L}$

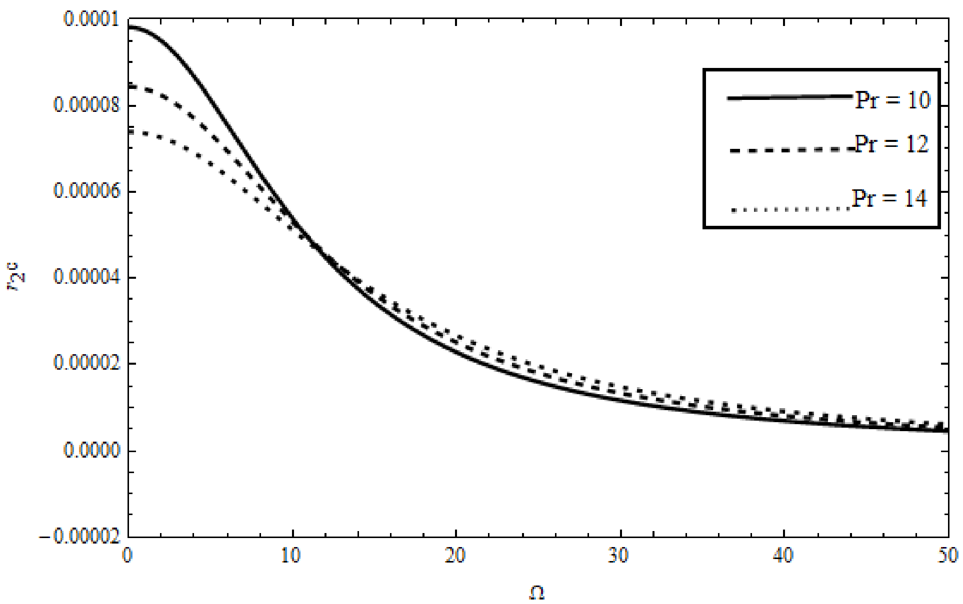

Fig. 5. Frequency ($\boldsymbol{\Omega}$) dependence of scaled correction Rayleigh number $\boldsymbol{r_{2c}}$ for Newtonian dielectric liquid with variations in $\boldsymbol{Pr}$

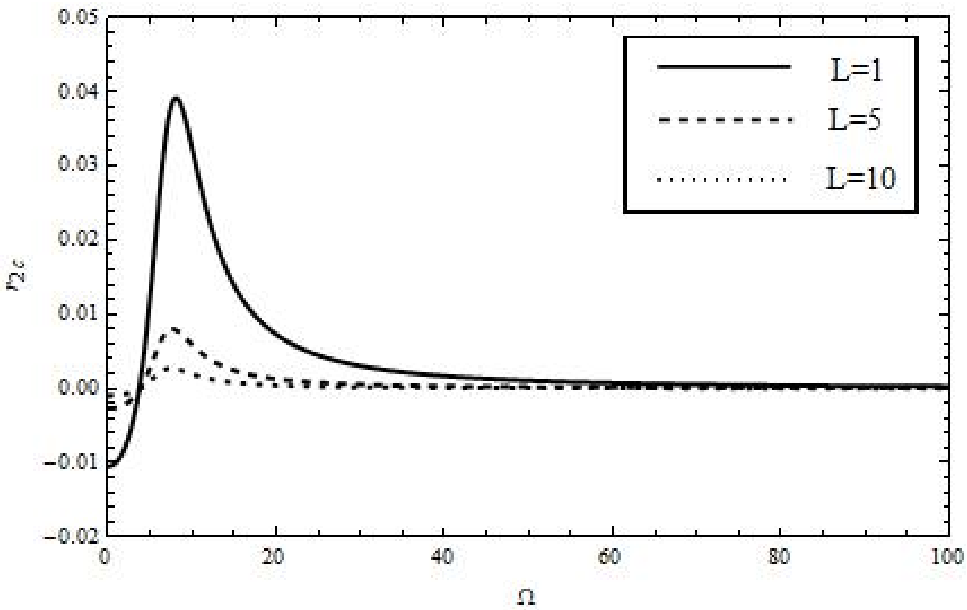

Fig. 6. Frequency $(\boldsymbol{\Omega})$ dependence of scaled correction Rayleigh number $\boldsymbol{r_{2c}}$ for Maxwell dielectric liquid with variations in $\boldsymbol{L}$

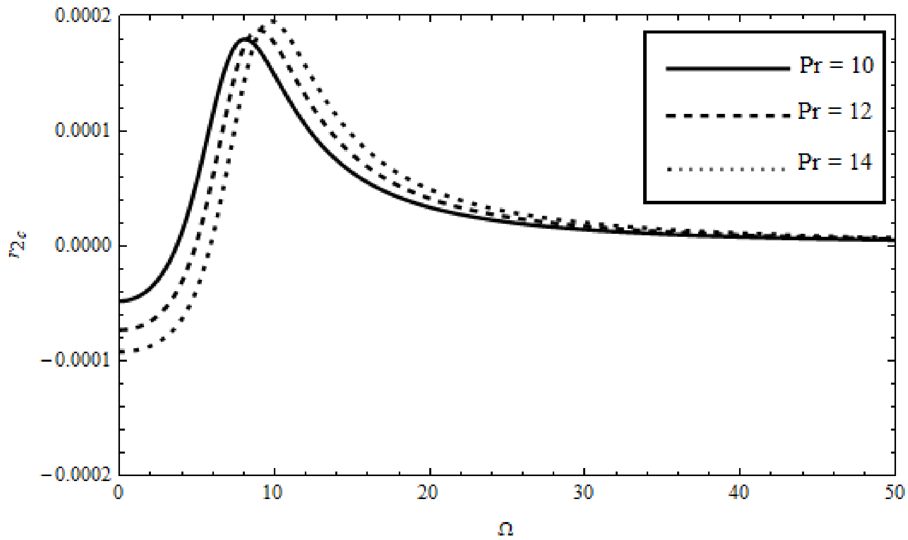

Fig. 7. Frequency $(\boldsymbol{\Omega})$ dependance of scaled correction Rayleigh number $\boldsymbol{r_{2c}}$ for Maxwell dielectric liquid with variations in $\boldsymbol{Pr}$

The Maxwell dielectric liquid shows different behaviour as compared to the Jeffrey and the Newtonian dielectric liquid. It is observed that, with the increase in frequency of modulation, the correction scaled Rayleigh number initially increases and then decreases for different values of L and a mixed behaviour is noted as we increase the Prandtl number, Pr. In a Maxwell's dielectric liquid, for $0 < \Omega < 3.86$, modulation advances the onset of convection. Further increase in Ω delays the onset of convection.

Now the discussion pertains to non-linear stability and heat transport. The solution of dynamical system of Eq. (36) to Eq. (39) show oscillatory behaviour which hampers the observations on the impact of different parameters on heat transport measured through Nusselt number, $Nu(\tau)$. Hence time averaged Nusselt number, $\overline{Nu(\tau)}$ is used to quantify heat transport.

Figure 8 shows that increasing values of electric buoyancy, L results in reduced heat transfer indicated by decreasing values of $\overline{Nu(\tau)}$. Heat transport decreases with increasing L and the Newtonian fluid shows the highest heat transfer and the Jeffrey fluid the least.

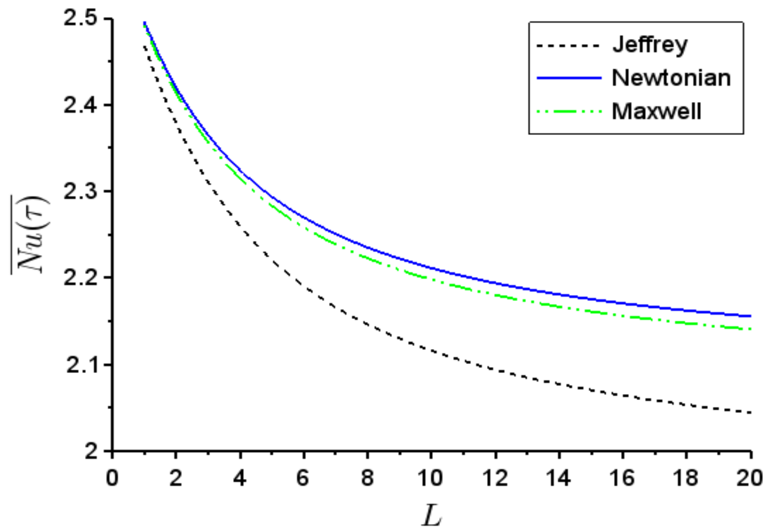

Fig. 1. Plot of $\overline{\boldsymbol{Nu}}$ versus $\boldsymbol{L}$ with $\boldsymbol{Pr} = \mathbf{10}$, $\boldsymbol{r} = \mathbf{5}$, $\boldsymbol{\beta} = \mathbf{0.2}$

Figure 9 shows the impact of Prandtl number, Pr on heat transport. A decrease in heat transfer is seen with an increase in Pr. On a comparative note, a similar result is observed that is the Newtonian fluid shows the highest heat transfer and the Jeffrey fluid the least.

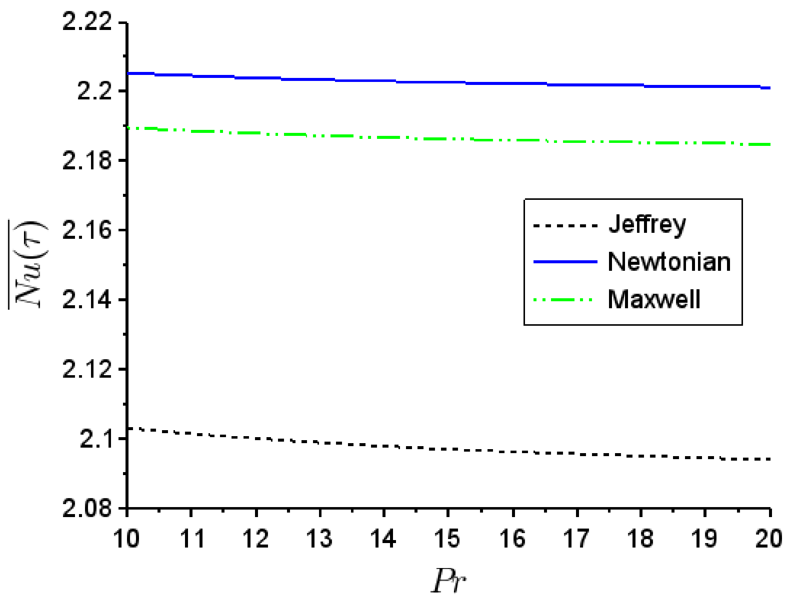

Fig. 9. Plot of $\overline{\boldsymbol{Nu}}$ versus $\boldsymbol{Pr}$ with $\boldsymbol{L} = \mathbf{10}$, $\boldsymbol{r} = \mathbf{5}$, $\boldsymbol{\beta} = \mathbf{0}$

In Figure 10, it is noticed that with an increase in the scaled Rayleigh number, the heat transfer gets enhanced. Newtonian fluid shows more heat transfer than the Jeffrey fluid and Maxwell fluid.

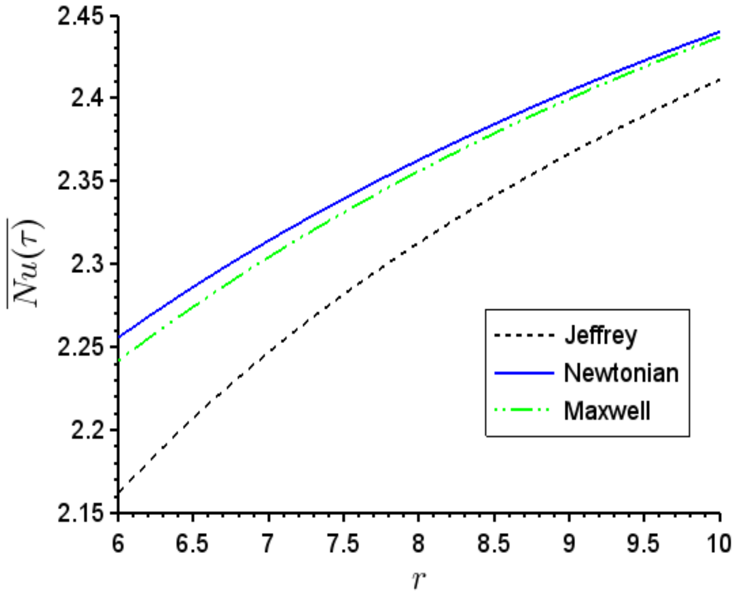

Fig. 10. Plot of $\overline{\boldsymbol{Nu}}$ versus $\boldsymbol{r}$ with $\boldsymbol{L} = \mathbf{10}$, $\boldsymbol{Pr} = \mathbf{10}$, $\boldsymbol{\beta} = \mathbf{0.2}$

Further, Figure 11 highlights the influence of stress relaxation parameter on heat transfer with and without modulation. An increasing value of Λ_1 results in reduced heat transport in the presence $(\beta \neq 0)$ as well as the absence $(\beta = 0)$of gravity modulation.

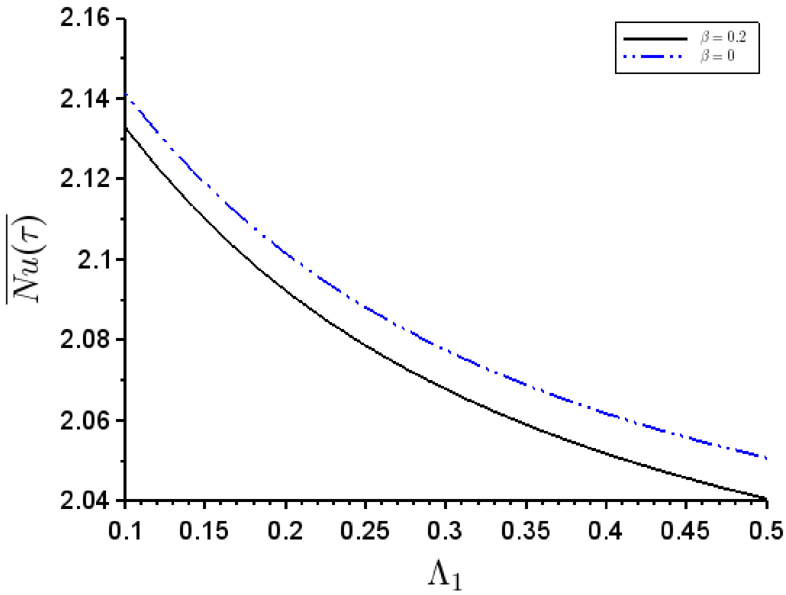

Fig. 11. Plot of $\overline{\boldsymbol{Nu}}$ versus $\boldsymbol{\Lambda_1}$ with $\boldsymbol{L} = \mathbf{10}$, $\boldsymbol{r} = \mathbf{5}$, $\boldsymbol{Pr} = \mathbf{10}$

A similar result is noted in Figure 12 for the elasticity ratio $\left(\Lambda = \frac{\Lambda_2}{\Lambda_1}\right)$. Heat transfer reduces with increasing values of elasticity ratio Λ or strain retardation parameter Λ_2. Thus, it can be inferred that the effect of modulation with an increase in viscoelastic parameters results in the reduced heat transfer.

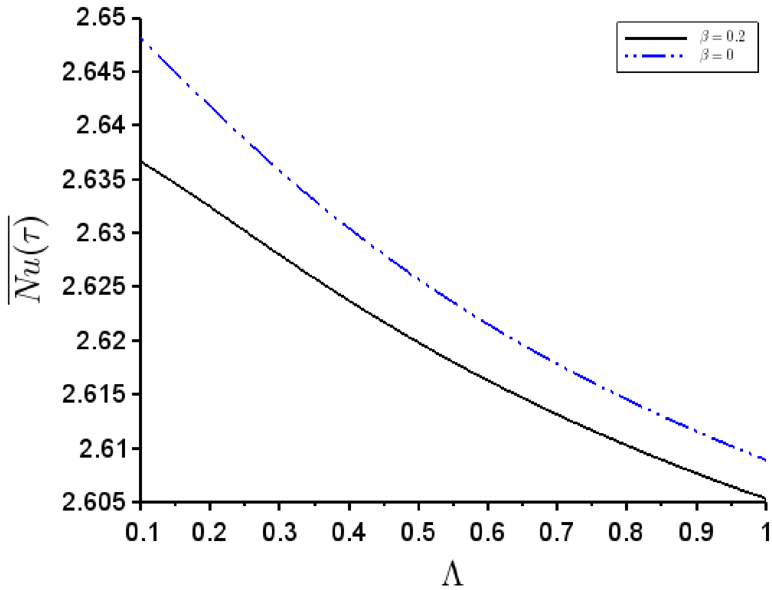

Fig. 12. Plot of $\overline{Nu}$ versus Λ with $L = 10$, $r = 5$, $Pr = 10$

5. Conclusion

The gravity modulation effect on convection and heat transfer in a dielectric liquid has been investigated. The linear stability analysis results in a correction scaled Rayleigh number which indicates the existence of supercritical/subcritical region. The non-linear theory results in a non-autonomous system of equations, the limiting case of which yields Lorenz-like model. The results of Jeffrey, Newtonian and Maxwell dielectric liquids are obtained as special cases.

i. The gravity modulation results in super-critical region in the case of Newtonian and Jeffrey dielectric liquid. In the Maxwell dielectric liquid, a sub-critical region is observed for $0 < \Omega < 3.86$ and a super-critical region for $\Omega > 3.86$.
ii. The impact of modulation frequency, Ω is observed for moderate values. At high values of Ω, the super-critical/sub-critical region almost vanishes.
iii. Increasing values of L leads to shrinking of the super-critical region in Jeffery and Newtonian dielectric liquid.
iv. A mixed behaviour is observed in r_{2c} for the variations in Pr.
v. The scaled correction Rayleigh number for dielectric liquids satisfy the relation
$$r_{2c}(Maxwell) > r_{2c}(Jeffrey) > r_{2c}(Newtonian)$$
vi. Heat transfer decreases with an increase in L.
vii. Prandtl number is not significantly influential on heat transport. A similar result reported in Melson *et al.*, [32] for $M_1 = L$, $M_3 = 0$ and $\beta = 0$.
viii. Heat transfer enhances with an increase in scaled Rayleigh number.
ix. Among the viscoelastic dielectric liquids, heat transfer capability is maximum in the Newtonian dielectric liquid and minimum in the Jeffery dielectric liquid.
x. The presence of modulation in the body force yields in reduced heat transfer.
xi. Increasing values of viscoelastic parameters results in reduced heat transfer.

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