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Numerical Solution of Heat Transfer Flow of Casson Hybrid Nanofluid over Vertical Stretching Sheet with Magnetic Field Effect

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ABSTRACT

In this study, heat transfer flow of Casson hybrid nanofluid over vertical stretching sheet with magnetic field effect is studied. Copper oxide and graphite oxide in methanol are considered as the hybrid nanoparticles. This analysis of study points out several aspects of flow physically which is related to transportation of heat. The conservation of partial differential equations (PDEs) are taken place into non-linear coupled ordinary differential equations (ODEs) by using transformation of dimensionless variables, and solution of these ODEs is performed using Chebyshev differential quadrature method (CDQM). Effects of many involved parameters like M, β , χ , a/c, denotes the magnetic parameter, Casson parameter, the nanoparticles volume fraction and ratio parameter are shown through tables for skin friction and Nusselt number. Moreover, it is presented as sketching on temperature and velocity profiles briefly. Results revealed that the Casson hybrid nanofluid has highest value when <1 for Nusselt number, skin friction, and temperature profile. Moreover, it is lowest value when >1 for velocity profile and Nusselt number. It also is observed that, when the magnetic field and Cason parameter increase, this leads to an increase in the temperature profile and a decrease in the Nusselt number and the velocity profile. A comparison study is also made which compares the present result with published data is executed and findings meet with them.

1. Introduction

It is hard to imagine humans living in the current century without heat transfer. Most heat transfer processes in thermal systems are accomplished using heat transfer fluids. The application of conventional heat transfer fluids such as water, methanol, ethylene glycol, and engine oil, due to their low thermal conductivities is limited [1]. To upgrade the energy efficiency of systems in engineering applications the conventional heat transfer fluids were replaced with nanofluids (NFs) [2]. nanofluids are suspension of small solid particles of metallic or non-metallic, called nanoparticle in the aforementioned base fluids [3]. By the higher thermal conductivity of nanoparticle, the application of nanofluid improves the heat transfer. Aydin and Guru [4] reviewed the nanofluids in terms of their preparation, stability, properties, and thermal performance. Although nanofluids are

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made by dispersing one type of nanoparticle into the base fluid, the thermal features of nanofluids could be boosted using the hybrid nanofluids.

Hybrid nanofluids are the mixture of two or more two dissimilar nanoparticles in the base fluid. To achieve the desired heat exchange rate, appropriate nanoparticle types with specific weight percent are employed. The hybrid nanofluids found large applications in different aspects of human life such as transportation [5], defense [6], microfluidics [7], etc.; in this way, the concept of hybrid nanofluids has been the subject of numerous numerical and experimental investigations.

A state-of-the-art review on hybrid nanofluid, their preparation methods, the influencing factors on their performance and their fields of application was presented by Salman *et al.,* [8]. The water-based hybrid nanofluid of Graphite and Silicon dioxide was prepared by Dalkılıç *et al.,* [9] and its viscosity in different NP volume fractions and temperatures was measured. The viscosity increment by the volume fraction increase and temperature decrease was demonstrated; also, the highest viscosity increment was obtained in the case of 2% volume fraction and 15^{o}C temperature which was equal to 36.12%. Several research studies on the flow of hybrid nanofluids have been analyzed in the pertinent literature; see [10-12]

Casson fluid is a non-Newtonian fluid that acts like an elastic solid in which no motion occurs with a low yield stress (see [13]). It's suitable for heating or cooling operations due to its efficient impact on the energy transmission rate, which qualifies it for utilization in many applications relevant to food processing, metallurgy, drilling, and bioengineering operations. Casson's model was effective in modeling the flow of pigment suspensions in lithographic polishes employed during the production of printing ink [14]. Casson fluid is also capable of effectively describing the flow characteristics of numerous polymers widely [15]. Furthermore, experiments conducted on blood have demonstrated that blood can act as a Cassone fluid, especially when the shear stress is low and the flow-through small blood vessels [16-19]. Human blood, honey, jelly, tomato sauce, custard, toothpaste, starch suspensions, foams, molten cosmetics, yogurt, and nail polish are familiar examples of this fluid. Casson [14]. was the first to address the Casson fluid model (rheological model). Recently, Mustafa *et al.,* [20] reported that the velocity is a decreasing function of the dimensionless time but the temperature is an increasing function of it, and that raising the Casson parameter boosts shear stress and heat transfer. Mukhopadhyay *et al.,* [21] utilized the shooting method to investigate the unsteady flow of a Casson liquid and energy transmission on a stretching surface. Khalid *et al.,* [22] addressed the magneto-free convection of a Casson fluid from an oscillating vertical plate in a porous medium. Animasaun *et al.,* [23] demonstrated that the growth in the values of the Cason parameter causes velocity curves augmentation and temperature curves reduction. EL-Kabeir *et al.,* [24] investigated the effect of chemical reactions on the mixed convection flow of Casson liquid over a sphere. Makinde *et al.,* [25] revealed that there is a significant relationship between the effect of the Lorentz force and the flow of Casson fluid, with the natural impact of this force occurring when the surface thickness is low, and the exact opposite of this influence occurring when the surface thickness increases. Thumma *et al.,* [26] observed that raising the Casson parameter values considerably reduces velocity curves in their investigation of the magneto-3D flow of non-Newtonian Casson liquid with thermal radiation For more reading, see the following articles [19, 27-30].

The implementation of magnetic field or magneto-hydrodynamic (MHD) could improve the heat transfer in many engineering applications. Alfven [31] introduced the concept of MHD fluid flow and described the MHD waves, which are known as Alfven waves. Rahbari *et al.,* [32] performed the analytical and numerical study of the MHD flow in a parallel plate channel. The effect of the De, Ha, Re and Pr numbers on the velocity and temperature fields was investigated. It was shown that increasing the Ha number reduces in the velocity values while increasing the De number does not alter the velocity field significantly. Atif *et al.,* [33] investigated the transient MHD squeezed flow having variable thermal conductivity with thermal radiation over the sensor surface. The nonlinear ordinary differential equations are implemented using the shooting technique. The sensitivity analysis was performed to reveal the effect of the magnetic and Weissenberg number, squeezed flow index, power-law index, thermal radiation parameter and Pr number on velocity and temperature profiles. It was found that the permeable velocity, the Weissenberg number and the power-law index has decreasing effect on the velocity profile. Recently, many researchers presented in MHD convective Casson nanofluid over different types of geometry, see the following papers [34-39].

The main focus of the present study is to use a numerical method namely: Chebyshev differential quadrature method to find the solutions of MHD flow of Casson hybrid nanofluid over vertical stretching sheet for two different oxide nanoparticles GO and CuO with methanol base fluid methanol. There is very limited work about Casson hybrid nanofluid with this fact of magnetic field effect on free convection flow. Impact of Casson parameter, ratio parameter, magnetic parameter and all other parameters which are involved in study on skin friction, Nusselt number, velocity and temperature is discussed. Based on the literature studies, this topic is never been consider by anyone previously, thus the results provided from this investigation are new.

2. Mathematical Modeling

Consider a 2-D flow of free convection and heat transfer for two different oxide nanoparticles near a stagnation point caused by vertical stretching sheet. Copper oxide (CuO) and graphite oxide (GO) are chosen as nanoparticles with methanol (CH_3OH) as base fluid. The vertical plate placed along *y* - horizontal axis and *x* - perpendicular on it, and flow is incompressible as showed in Figure 1. Uniform temperature of the flow is T_w and T_∞ is to be noted as ambient temperature of the fluid, and assumed that $\left(T_w > T_\infty\right)$, initially flow is running with nonlinear velocity $\left(U_w(x) = cx\right)$, here a parameters c is related to stretching surface . For the orthogonal fluid flow over the surface with $\left(U_e\left(x\right) = ax\right)$ here a is stretching velocity Now focusing the made assumption we move toward mathematical formulation.

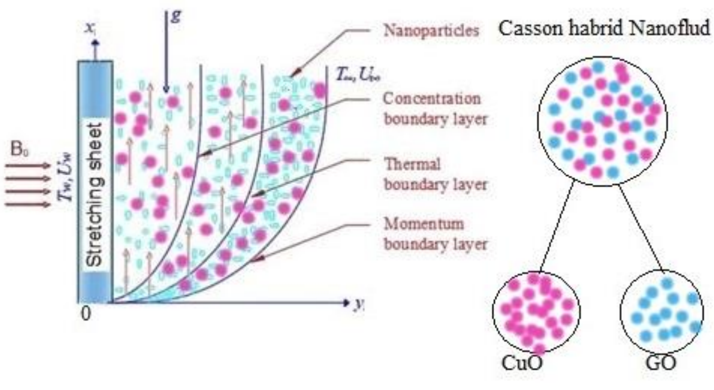

Fig. 1. Geometry of problem

Continuity, momentum, and energy equations with boundary conditions (BCs) are see([40] and [41])

$$\left(u\right)_x + \left(v\right)_y = 0, \tag{1}$$

$$u(u)_x + v(u)_y = U_e(U_e)_x + v_{hnf}\left(1+\frac{1}{\beta}\right)(u)_{yy} - \frac{\sigma_{hnf}B_0^2}{\rho_{hnf}}(U_e - u) \tag{2}$$

$$u(T)_x + v(T)_y = \alpha_{hnf}(T)_{yy}, \tag{3}$$

Along BCs

$$u = U_w(x) = cx,\ T = T_w(x) = T_\infty + bx,\ v = 0 \text{ at } y = 0;$$

$$u = U_e(x) = ax,\ T = T_\infty \text{ at } y \to \infty; \tag{4}$$

where β is Casson parameter and χ_1, χ_2 is nanoparticles volume fractions. All other symbols and quantities are given in nomenclature of the hybrid nanofluid. β_{hnf}, ρ_{hnf}, μ_{hnf}, α_{hnf}, $(\rho c_p)_{hnf}$, k_{hnf} and σ_{hnf} are coefficient of thermal expansion, density, viscosity thermal diffusivity, heat capacity, thermal conductivity and electrical conductivity of hybrid nanofluid, which are defined by as (see [42, 43])

$$\begin{aligned}
&\beta_{hnf} = \left[(1-\chi_1)\beta_f + \chi_1\beta_{s1}\right](1-\chi_2) + \chi_2\beta_{s2}\\
&\rho_{hnf} = \left[(1-\chi_1)\rho_f + \chi_1\rho_{s1}\right](1-\chi_2) + \chi_2\rho_{s2},\\
&\mu_{hnf} = \frac{\mu_f}{(1-\chi_1)^{2.5}(1-\chi_2)^{2.5}},\\
&(\rho c_p)_{hnf} = (1-\chi_2)\left[(1-\chi_1)(\rho c_p)_f + \chi_1(\rho c_p)_{s1}\right] + \chi_2(\rho c_p)_{s2},\\
&\frac{k_{hnf}}{k_f} = \frac{(k_{s2}+2k_f) - 2\chi_2(k_f - k_{s2})}{(k_{s_2}+2k_f) + \chi_2(k_f - k_{s2})},\ \text{where } \frac{k_f}{k_f} = \frac{(k_{s1}+2k_f) - 2\chi_1(k_f - k_{s1})}{(k_{s_1}+2k_f) + \chi_1(k_f - k_{s1})},\\
&\alpha_{hnf} = \frac{k_{hff}}{(\rho c_p)_{hff}},\\
&\frac{\sigma_{hnf}}{\sigma_f} = 1 + \frac{3\left[\frac{\chi_1\sigma_1 + \chi_2\sigma_2}{\sigma_f} - (\chi_1+\chi_2)\right]}{\left[\frac{\chi_1\sigma_1 + \chi_2\sigma_2}{\sigma_f} + 2\right] - \left[\frac{\chi_1\sigma_1 + \chi_2\sigma_2}{\sigma_f} - (\chi_1+\chi_2)\right]}
\end{aligned} \tag{5}$$

Suitable and appropriate transformation are; see([36,37])

$$\eta = y\sqrt{c/v_f},\quad \psi = xF(\eta)\sqrt{cv_f},\quad \theta(\eta) = \left[\frac{T - T_\infty}{T_w - T_\infty}\right]. \tag{6}$$

ψ describes the locus of flow so it is called stream function.

$\left(\frac{\partial\psi}{\partial y}, -\frac{\partial\psi}{\partial x}\right)=(u, v)$ are velocity component and θ the dimensionless temperature.

Using all requirements (5) and (6) in (1)-(3) PDEs are transformed into following ordinary equations

$$\frac{\rho_f}{\rho_{hnf}}\frac{1}{(1-\chi_1)^{2.5}(1-\chi_2)^{2.5}}\left(1+\frac{1}{\beta}\right)F'''+FF''-F'^2+\left(\frac{a}{c}\right)^2-\frac{\sigma_{hnf}}{\sigma_f}\frac{\rho_f}{\rho_{hnf}}M\left(\frac{a}{c}-F'\right)=0 \tag{7}$$

$$\frac{1}{\Pr}\frac{k_{hff}/k_f}{(1-\chi_2)\left[(1-\chi_1)\rho_f+\chi_1(\rho c_p)_{s1}/(\rho c_p)_f\right]+\chi_2(\rho c_p)_{s2}/(\rho c_p)_f}\theta''+F\theta'-F'\theta=0 \tag{8}$$

subject to

$$F(0)=0,\ F'(0)=1,\ \theta(0)=1$$

$$F'(\infty)\to\frac{a}{c},\ \theta'(\infty)\to 0 \text{ as } \eta\to\infty \tag{9}$$

where $M=\left(\frac{\sigma_f B_0^2}{\rho_f U_w}\right)$ is the magnetic parameters, $\frac{a}{c}$ is the velocity ratio parameter. and $\Pr=\frac{v_f}{\alpha_f}$ is Prandtl numbers.

Finally, for aspect of practical concern, the quantities are mentioned as coefficient of drag force usually termed as local skin friction coefficient and rate of heat transference play vital role practical point of view.

Drag force the skin friction coefficient C_f and the local Nusselt number Nu which are given by see ([40] and [41])

$$C_f=\frac{\tau_w}{\rho_{nf}U_w^2} \quad Nu=\frac{xq_w}{k_f(T_w-T_\infty)} \tag{10}$$

where

$$\tau_w=\mu_{hnf}\left(u_y\right)\Big|_{y=0} \quad ,q_w=-k_{hnf}\left(T_y\right)\Big|_{y=0}, \tag{11}$$

Putting their requirements in (10) so

$$C_f\,\mathrm{Re}^{1/2}=\left(1+\frac{1}{\beta}\right)\frac{1}{(1-\chi_1)^{2.5}(1-\chi_2)^{2.5}}F''(0),\ Nu\,\mathrm{Re}^{-1/2}=-\frac{k_{hnf}}{k_f}\theta'(0). \tag{12}$$

Here Re is Reynolds number

3. Numerical Solution (Chebyshev Differential Quadrature Method (CDQM))

The boundary layer problem in Eq. (7)– (9) has a solution characterized by a rapid change at the left end boundary point. If the grid points are concentrated in the layer, this problem can be solved effectively. The Chebyshev differential quadrature method (CDQM) ensures that the boundary layer contains more collocation points. For more information on the method's spectral accuracy and convergence. It is well known that when solving boundary layer problems, the CDQM enhanced with an adaptive grid transformation [42, 44-46] produces more accurate results than the standard CDQM.

We used the CDQM to solve the ODE system (7)– (9), with the Lagrange interpolation polynomial as our based functions over the transformed Chebyshev–Gauss– Lobatto grid points.

$$\gamma_j = -0.5\left[\gamma_0 \sinh\left(\frac{N}{2}\left(1-\cos\frac{j\omega}{K_{\bar{\eta}_\infty}-1}\right)\right)\right]\bar{\gamma}_\infty \tag{13}$$

where $j = 0,1,\ldots, K_{\bar{\eta}_\infty}-1$, $N = \sinh^{-1}(2/\vartheta_0)$, ϑ_0 is the width of boundary layer, $\bar{\gamma}_\infty$ is the initial estimation of γ_∞ and $K_{\bar{\eta}_\infty}$ is the number of grid points over $\gamma \in [0,\bar{\gamma}_\infty]$. Applying the CDQM on system (7) and (8) we have: (see [42])

$$\begin{aligned} p_i = \tilde{A}\sum_{j=0}^{K_{\bar{\eta}_\infty}-1}\left(w_{i,j}^{(3)}F_j\right) + F_j\sum_{j=0}^{K_{\bar{\eta}_\infty}-1}\left(w_{i,j}^{(2)}F_j\right) - \left(\sum_{j=0}^{K_{\bar{\eta}_\infty}-1}\left(w_{i,j}^{(1)}F_j\right)\right)^2 \\ +\left(\frac{a}{c}\right)^2 - \frac{\sigma_{hnf}}{\sigma_f}\frac{\rho_f}{\rho_{hnf}}M\left(\frac{a}{c} - \sum_{j=0}^{K_{\bar{\eta}_\infty}-1}\left(w_{i,j}^{(1)}F_j\right)\right) = 0 \end{aligned} \tag{14}$$

$$q_i = \tilde{B}\sum_{j=0}^{K_{\bar{\eta}_\infty}-1}\left(w_{i,j}^{(2)}\theta_j\right) + \sum_{j=0}^{K_{\bar{\eta}_\infty}-1}\left(w_{i,j}^{(1)}F_j\theta_j\right) - \sum_{j=0}^{K_{\bar{\eta}_\infty}-1}\left(w_{i,j}^{(1)}\theta_j F_j\right) = 0 \tag{15}$$

The boundary condition become

$$\begin{aligned} &F'(0) = \sum_{j=0}^{K_{\bar{\eta}_\infty}-1}\left(w_{0,j}^{(1)}F_j\right) = 1, \quad F(0) = \sum_{j=0}^{K_{\bar{\eta}_\infty}-1}\left(I_{0,j}F_j\right) = 0, \quad \theta(0) = \sum_{j=0}^{K_{\bar{\eta}_\infty}-1}\left(I_{0,j}\theta_j\right) = 1 \\ &F'(\infty) = \sum_{j=0}^{K_{\bar{\eta}_\infty}-1}\left(w_{K_{\bar{\eta}_\infty}-1,j}^{(1)}F_j\right) = \frac{a}{c}, \; \theta'(\infty) = \sum_{j=0}^{K_{\bar{\eta}_\infty}-1}\left(w_{K_{\bar{\eta}_\infty}-1,j}^{(1)}\theta_j\right) = 0 \end{aligned} \tag{16}$$

where $\tilde{A} = \frac{\rho_f}{\rho_{hnf}}\frac{1}{(1-\chi_1)^{2.5}(1-\chi_2)^{2.5}}\left(1+\frac{1}{\beta}\right)$ and

$\tilde{B} = \frac{1}{\Pr}\frac{k_{hff}/k_f}{(1-\chi_2)\left[(1-\chi_1)\rho_f + \chi_1(\rho c_p)_{s1}/(\rho c_p)_f\right] + \chi_2(\rho c_p)_{s2}/(\rho c_p)_f}$, $i = 0,1,\ldots, K_{\bar{\eta}_\infty}-1$, $w_{i,j}^{(s)}$ and

$s=1,2,3$ are the *j* elements of the *i* row of the s^{th} derivatives of the Chebyshev differentiation matrices $I_{0,j}$ is identity matrix of order *j*.

Thus, the system of Eq. (7) and (8) with boundary conditions (9) are transformed into a non-linear algebraic system of equations written as vector form.

$$Q=\begin{bmatrix} \sum_{j=0}^{K_{\bar{\eta}_\infty}-1}\left(w_{0,j}^{(1)}F_j\right)-1 \\ \sum_{j=0}^{K_{\bar{\eta}_\infty}-1}\left(I_{0,j}F_j\right) \\ \left[p_i\right]_{i=2}^{K_{\bar{\eta}_\infty}-1} \\ \sum_{j=0}^{K_{\bar{\eta}_\infty}-1}\left(I_{0,j}\theta_j\right)-1 \\ \sum_{j=0}^{K_{\bar{\eta}_\infty}-1}\left(w_{K_{\bar{\eta}_\infty}-1,j}^{(1)}F_j\right)-\frac{a}{c} \\ \left[q_i\right]_{i=2}^{K_{\bar{\eta}_\infty}-1} \\ \sum_{j=0}^{K_{\bar{\eta}_\infty}-1}\left(w_{K_{\bar{\eta}_\infty}-1,j}^{(1)}\theta_j\right) \end{bmatrix}=0 \tag{17}$$

This system was solved by MATLAB using an initial profile that satisfies the boundary conditions (9) written by: see [42]

$$F(\eta)=2-e^{-\eta}(1+\eta),\ \ \theta(\eta)=e^{-\eta} \tag{18}$$

4. Result and Discussion

In this part, the numerical results obtained using the (CDQM) are shown to study the effect of the magneto-free convection flow of Casson hybrid nanofluid (CuO +GO/ methanol) over vertical stretching sheet. Table 1 present the thermo-physical properties of methanol, Copper oxide (CuO) and graphite oxide (GO). To ensure the efficiency of the present method, the results that we obtained are compared with the results in Table 2 which is found that very good agreement between the results that Mahapatra and Gupta [47], Nazar *et al.,* [40] and Ishak *et al.,* [41], for the special case *M* = 0, $\beta\to\infty$ and different values of *a/c*

Exploration of result related to physical quantities are evaluated through Tables 3-4 by using the (CDQM). Impact of magnetic parameter M , ratio parameter a/c and Casson parameter β *,* and as well as the nanoparticles volume fractions χ_1, χ_2 with (CuO+GO)/CH_3OH and (GO)/CH_3OH on skin friction $C_f\,\mathrm{Re}^{1/2}$ and Nusselt number $Nu\,\mathrm{Re}^{-1/2}$ are marked in detailed by Table 3 and Table 4. It is noted that for increasing value of β and a/c increment is seen in skin friction coefficient, and growing value of *M* opposite conduct is found in Table 3. Table 4 discloses that for greater values of *M* and β the values of $Nu\,\mathrm{Re}^{-1/2}$ gets down but increases in ratio parameter a/c in both cases of Casson hybrid nanofluid and Casson nanofluid. It is noticeable from these tables, that hybrid Casson nanofluids have higher values than Casson nanofluids in both values of $C_f \mathrm{Re}^{1/2}$ and $Nu\,\mathrm{Re}^{-1/2}$ for all parameter values except a/c >1 the opposite occurs in $C_f \mathrm{Re}^{1/2}$, due to shear thickness reflecting values.

Table 1
Thermo-physical properties of methanol and metals nanoparticles [48, 49]

Materials	ρ	C_p	k	$B\times10^{-5}$	σ
CH_3OH (base fluid)	792	2545	0.2035	1.49	0.5×10^{-6}
Copper oxide (CuO)	6320	531.8	76.5	1.8	59.6×10^{6}
Graphene (GO)	1800	717	5000	28.4	1.1×10^{-5}

Table2
Compared value of a/c on $f''(0)$ when M =0 and $\beta\to\infty$

a/c	Mahapatra and Gupta [47]	Nazar *et al.,* [40]	Ishak *et al.,* [41]	Present
0.1	-0.9694	-0.9694	-0.9694	-0.9693
0.2	-0.9181	-0.9181	-0.9181	-0.9181
0.5	-0.6673	-0.6673	-0.6673	-0.6672
2	2.0175	2.0175	2.0175	2.0174
3	4.7293	4.7293	4.7293	4.7292

Table 3
Influence of M, β, and a/c on $C_f \mathrm{Re}^{1/2}$

M	β	a/c	$(CuO+GO)/CH_3OH$	$(GO)/CH_3OH$
1	4	0.5	-0.7749	-1.0499
2			-1.2194	-1.5787
5			-2.7753	-3.4994
2	0.01	0.5	-11.9587	-18.9363
	0.1		-3.6178	-4.7106
	0.5		-1.8890	-2.4458
2	4	0.5	-1.2194	-1.5787
		1.5	1.3222	1.7435
		2	2.7415	3.6407

Table 4
Influence of M, β, and a/c on $Nu\,\mathrm{Re}^{-1/2}$

M	β	a/c	$(CuO+GO)+ CH_3OH$	$(GO)+ CH_3OH$
1	4	0.5	4.4455	3.8548
2			4.2836	3.7641
5			3.9695	3.5251
2	0.01	0.5	4.7955	3.9842
	0.1		4.6323	3.9600
	0.5		4.4383	3.8595
2	4	0.5	4.2836	3.7641
		1.5	5.3865	4.3549
		2	5.8070	4.6028

In order to achieve the insight of physical behaviour, numerical calculations have been utilized using the methodology which describes in here for different values of the β , M , and a/c . For impact of all these parameters profiles sketching (2-7) are mentioned.

Figure 2 shows the variation in *M* (magnetic parameter) on the $\theta(\eta)$ profile for both the (CuO + GO) Casson hybrid nanofluid and the GO Casson nanofluid. Based on the sketch, it is discovered that $\theta(\eta)$ is boosted by *M* is rising credit. The magnetic parameter of the Casson hybrid nanofluid are inversely related. As a result, intensification in *M* reduces fluid density, causing temperature increases. Figure 3 depicts the effect of the *M* vs $F'(\eta)$ profile are same fluids in Figure 2. This figure shows that as the strength of *M* increases, the fluid flow for both the nanofluid and the hybrid nanofluid decreases gradually. This occurs as a result of the generation of a resistive Lorentz force. The opposing force increases as *M* is credit rises, counteracting the fluids motion. Both the CuO + GO Casson hybrid nanofluid and the GO Casson nanofluid exhibit this decreasing trend in $F'(\eta)$.

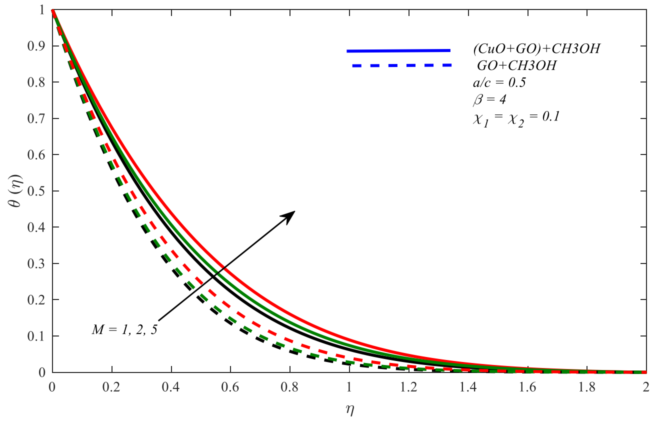

Fig. 2. The effect of *M* on $\theta(\eta)$

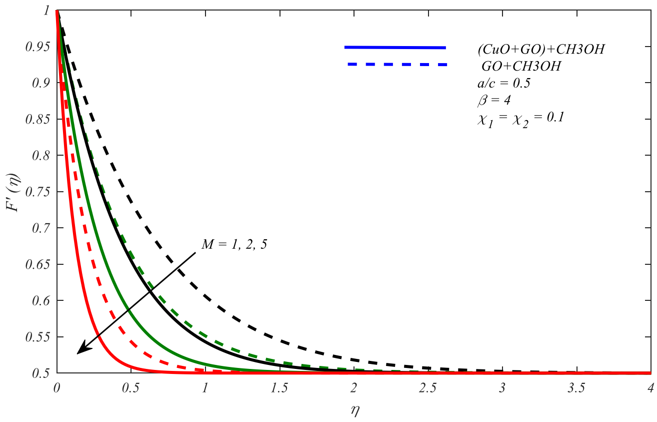

Fig. 3. The effect of *M* on $F'(\eta)$

The effect of Casson parameter on temperature and velocity profiles of the Casson hybrid nanofluid and Casson nanofluid are shown on Figure 4 and 5. For increasing value of β the temperature increasing but the velocity of fluid decreasing because stretchiness of sheet reduces the speed of fluid elements. The reason for this behaviour is that increasing the estimation of β improves the elasticity of hybrid nanofluids, causing the hybrid nanofluid to become more viscous. Physically, in such a case, the thickness of the momentum boundary layer decreases as increases in β .

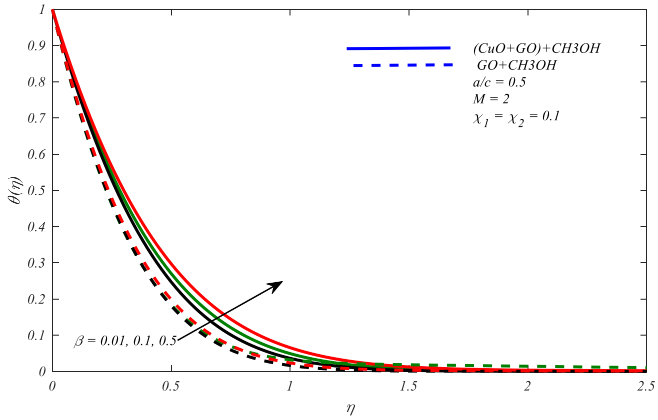

Fig. 4. The effect of β on $\theta(\eta)$

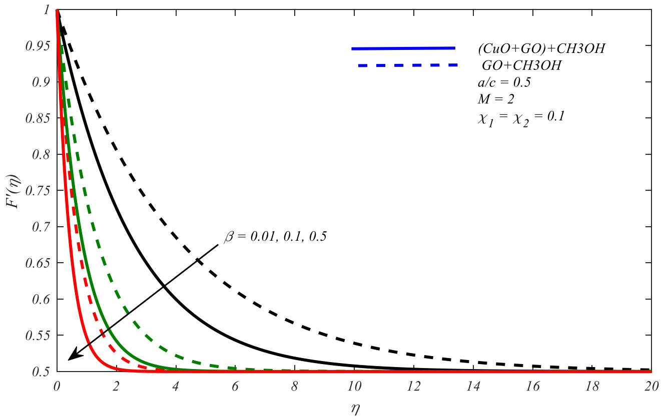

Fig. 5. The effect of β on $F'(\eta)$

The behavior of ratio parameter *a/c* on temperature profiles $\theta(\eta)$ and velocity profiles $F'(\eta)$ as shown in Figure 6 and 7. From Figure 6, it is clear that $\theta(\eta)$ is enhanced by decreasing the value of a/c The rising effect is larger when the Casson hybrid nanofluid is used when compared to the Casson nanofluid. However, Figure 7 depicts this fact in the opposite case, where increasing a/c leads to an increase in velocity profiles $F'(\eta)$ and the Casson hybrid nanofluid has the highest value when $a/c > 1$, but the lowest value when a/c <1.

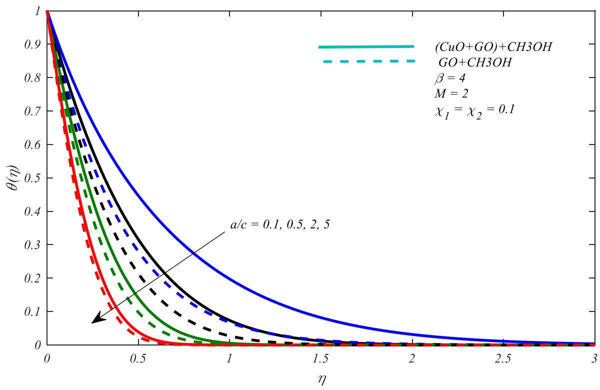

Fig. 6. The effect of a/c on $\theta(\eta)$

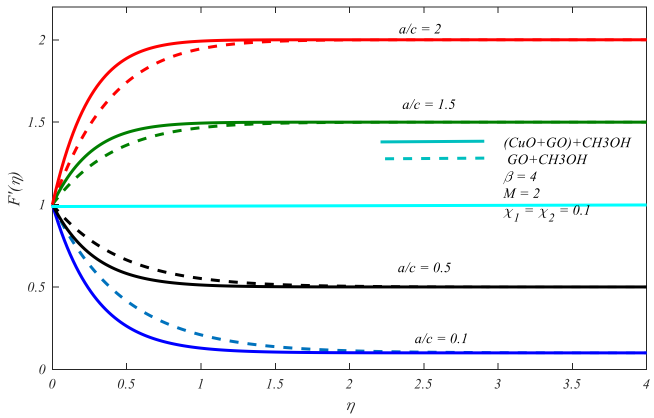

Fig. 7. The effect of a/c on $F'(\eta)$

5. Conclusion

Prime gist of present study is to investigate the impact magnetohydrodynamics (MHD) on Casson hybrid nanofluid over vertical stretching sheet. The governing conservation of (PDE) equations are taken into non-linear coupled ODEs by execution of transformation of dimensionless variables, we conclude the following

i. Increasing in the magnetic field leads to increases in the temperature profile and decreases in skin friction, Nusselt number and velocity profile.
ii. Increasing in Casson parameter leads to increases in the temperature profile and skin friction, but decreases in Nusselt number and velocity profile.
iii. Increasing in ratio parameters leads increases in skin friction, Nusselt number and velocity profile, but decreases in temperature profile.
iv. Casson hybrid nanofluid has highest value when a/c <1 for Nusselt number, skin friction, and temperature profile. Moreover, it is lowest value when a/c >1 for velocity profile and Nusselt number.

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