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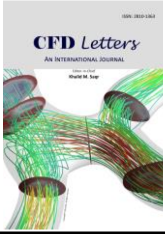

Numerical Analysis of Boundary Layer Flow and Heat Transfer over a Shrinking Cylinder

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ABSTRACT

The investigation concerning the movement of the viscous fluid and heat transfer rate when the cylinder shrinks is carried out. The role of curvature parameter and effect of cylinder parameter on skin friction and heat transfer rate were studied. The existence of a dual solution is observed and subsequently identifies which solution is stable. By using the similarity transformation, the boundary layer equations are transformed into nonlinear ordinary differential equations which are then solved numerically using bvp4c in MATLAB. The results for the skin friction coefficient, temperature gradient coefficient as well as velocity and temperature profiles are presented graphically. The effects of mass suction parameter and curvature parameter on flow and heat transfer characteristics are also presented. It is found that the dual solution existed when mass suction parameter greater or equal to 2. Cylinder surface diminished the performance of skin friction coefficient and heat transfer rate. The stability analysis result is performed to verify that the upper branch solution is stable and physically meaningful.

1. Introduction

The flow over a shrinking sheet has attracted many researchers due to its interesting physical characteristics and applications in processes for example on a rising or shrinking balloon. From our investigation, Wang [1] was first who studied this unusual type of flow. After couples of years, Miklavčič and Wang [2] investigated the properties of the flow due to a shrinking sheet with suction. Hayat *et al.,* [3] discussed on the analytic solution of MHD flow of a second-grade fluid over a shrinking sheet. After that, Wang [4] studied the flow towards a shrinking sheet near the stagnation point. Fang *et al.,* [5] studied the unsteady viscous flow over a continuously shrinking surface with mass transfer. Ishak *et al.,* [6] investigated the flow due to shrinking sheet in a micropolar fluid near stagnation area. Bhattacharyya and Layek [7] considered the boundary layer flow and heat transfer over a shrinking sheet with thermal radiation with presence of suction and injection through the wall of the sheet. Then, Bachok *et al.,* [8-10] investigated the boundary layer flow over a stretching/shrinking sheet and exponentially stretching/shrinking sheet in a nanofluid. Bhattacharyya

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and Vajravelu [11] and Bhattacharyya [12] again considered the flow near stagnation point and heat transfer over an exponentially shrinking sheet. Then, the mass transfer with chemical reaction past a stretching/shrinking cylinder near stagnation point was studied by Najib *et al.,* [13]. The industrial applications of the cylinder are numerous. The cylinder is used as boiler shells, pressure tanks, pipes and in other low-pressure processing equipment. Besides that, submarine hulls and also certain air plane components are the common examples of real practices of cylinder.

Couple decades before, the existence of multiple solutions of boundary layer flow has been found. The method of finding dual solutions and analyzing stability is practically importance to those interested in engineering analysis. The BVP4C method by MATLAB software is performed to analyze which solution is stable and physically meaningful. Hence, such study is very useful for determination of the treatment of the fluid flow problems with multiple solutions. Merkin [14] was first pioneer who introducing the stability analysis. By implementing a stability analysis Merkin [14], Weidman *et al.,* [15] considered the simultaneous effects of normal transpirations through and tangential movement of a semi-infinite plate on self-similar boundary layer flow beneath a uniform free stream. Merill *et al.,* [16] studied the unsteady mixed convection boundary layer flow near stagnation point on a vertical surface embedded in a porous medium. The study of characteristics of fluid flow and heat transfer over a permeable shrinking sheet has been done by Ishak [17]. Then, a stability analysis of MHD flow and heat transfer near stagnation point with effects of viscous dissipation, Joule heating and partial velocity slip was analyzed by Yasin *et al.,* [18]. Recently, the investigation on stability analysis of unsteady stagnation-point flow and heat transfer over a stretching/shrinking sheet immersed in nanofluid in the presence of slip velocity have been done by Dzulkifli *et al.,* [19]. From all investigation regards to stability analysis, researchers conclude that only the first solution is stable and can be realized physically.

Although lots of investigations investigating fluid flow in modern liquids along a cylinder (see Refs. [20-24]), however we believe investigating fluid flow and heat transfer rate in viscous fluid is necessary to be a measuring stick of how effective fluid flow and heat transfer rate in such fluid. As mention earlier, there are many modern fluids have been introduced in investigating the boundary layer flow for example nanofluid, Casson nanofluid, micropolar fluid, Williamson nanofluid as well hybrid nanofluid whereby the research papers could be found in Refs. [25-34]. Apart from that, many approaches have been done regards to this boundary layer flow study such as study the flow behavior analytically, by using metaheuristic algorithm, applied Arrhenius kinetics to study the suspension of nanoparticles and motile gyrotactic microorganisms in boundary layer and many more, see Refs. [35-40].

Motivated from the above-mentioned researches and applications, the aim of this paper is to investigate the behavior of the boundary layer flow due to a shrinking cylinder. Although many studies on boundary layer flow over shrinking cylinder have been conducted however we found that there is a significant gap in term of researches findings where such flow over a shrinking cylinder in viscous fluid have not been carried out. We believe that this such study should be conducted and hence the numerical results obtained should be a measuring data regards to the flow in viscous fluid. Further, the comparison table or data between viscous fluid and modern fluid (such as nanofluid, hybrid nanofluid and etc.) can be made to find out the extent of effectiveness of the fluids. Thus, the numerical study will be carried out to obtain the dual solutions due to the presence of mass suction in boundary layer. Regards from the dual solutions obtained, the stability analysis will be performed to verify and proof that the only the first solution is stable and can be realized physically.

2. Methodology
2.1 Problem Formulation

Consider a boundary layer flow and heat transfer over a shrinking cylinder with radius R placed in an incompressible viscous fluid of constant temperature T_w. It is assumed that the shrinking velocity is in the forms $u_w(x) = cx/L$, where $c < 0$ is shrinking constant, x is the coordinate measured along the cylinder and L is the characteristics length. The boundary layer equations are

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(rv) = 0 \tag{1}$$
$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) \tag{2}$$
$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \alpha\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) \tag{3}$$

where x and r are coordinates measured along the surface of the cylinder and in the radial direction, respectively, with u and v being the corresponding velocity components. Further, T is the temperature in the boundary layer, ν is the kinematic viscosity coefficient and α is the thermal diffusivity. The boundary conditions are

$$u = -u_w(x), \quad v = v_w, \quad T = T_w \text{ at } r = R$$
$$u \to 0, \quad T \to T_\infty \text{ as } r \to \infty \tag{4}$$

The similarity solutions of Eqs. (1) – (3), , subject to the boundary conditions (4), given by

$$\eta = \frac{r^2 - R^2}{2R}\left(\frac{c}{\nu L}\right)^{1/2}, \qquad \psi = \left(\frac{\nu c}{L}\right)^{\frac{1}{2}} xRf(\eta), \qquad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{5}$$

where η is the similarity variable, ψ is the stream function defined as $u = r^{-1}\frac{\partial\psi}{\partial r}$ and $v = -r^{-1}\frac{\partial\psi}{\partial x}$, which identically satisfies Eq. (1). By defining η in this form, the boundary conditions at $r = R$ reduce to the boundary conditions at $\eta = 0$, which is more convenient for numerical computations.

Substituting Eq. (5) into Eqs. (2) and (3), the following nonlinear ordinary differential e quations as follows:

$$(1 + 2\gamma\eta)f''' + 2\gamma f'' + ff'' - f'^2 = 0 \tag{6}$$
$$(1 + 2\gamma\eta)\theta'' + 2\gamma\theta' + Pr(f\theta' - f'\theta) = 0 \tag{7}$$

subjected to the boundary conditions (4) which become (see Najib *et al.,* [41])

$$f(0) = S, \quad f'(0) = -1, \quad \theta(0) = 1,$$
$$f'(\infty) \to 0, \quad \theta(\infty) \to 0, \tag{8}$$

where $Pr = \frac{\nu}{\alpha}$ is the Prandtl number and γ is the curvature parameter defined as

$$\gamma = \left(\frac{\nu L}{cR^2}\right)^{1/2}, \tag{9}$$

and $S = v_w r/R\sqrt{\frac{\nu c}{L}}$ is the mass transfer parameter with $S > 0$ ($v_w < 0$) corresponds mass suction and $S < 0$ ($v_w > 0$) corresponds to the mass injection.

The main physical quantities of interest are the value of $f''(0)$, being a measure of the skin friction, and the temperature gradient $-\theta'(0)$. Our main aim is to find how the values of $f''(0)$ and $-\theta'(0)$ vary in terms of parameters S, γ and Pr.

2.2 Method of Solutions

The ordinary differential equations (6) and (7) subject to boundary condition (8) are solved computationally using bvp4c solver in MATLAB. The numerical results were obtained by following the steps below.

a) Reduce Eqs. (6) – (7) and the boundary condition (8) to first order system by introducing new dependent variables as follow:
   i. $f = y(1)$,
   ii. $f' = y(2)$,
   iii. $f'' = y(3)$,
   iv. $\theta = y(4)$,
   v. $\theta' = y(5)$.
b) Define the ordinary differential equations in MATLAB by writing a 1×5 matrix which contains f, f', f'', θ and θ' in terms of the dependent variables in step (i).
c) Define the boundary condition (8) in MATLAB in terms of dependent variables in step (i) by letting the right side of the equations becomes 0.
d) Set up the initial guesses and boundary layer thickness for the first and second solutions, respectively, to obtain the profiles that satisfy the boundary condition (8) asymptotically.

2.3 Stability Analysis

In order to perform a stability analysis, we consider the unsteady problem. Eq. (1) holds, while Eqs. (2) and (3) are replaced by

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) \quad (10)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \alpha\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) \quad (11)$$

where t denotes the time. Based on the variables Eq. (5), the following new dimensionless variables are introduced:

$$\eta = \frac{r^2 - R^2}{2R}\left(\frac{c}{\nu L}\right)^{1/2}, \quad \psi = \left(\frac{\nu c}{L}\right)^{1/2} xRf(\eta,\tau), \quad \theta(\eta,\tau) = \frac{T - T_\infty}{T_w - T_\infty}, \; \tau = \frac{ct}{L} \quad (12)$$

Thus, Eqs. (2) and (3) can be written as

$$(1 + 2\gamma\eta)\frac{\partial^3 f}{\partial \eta^3} + 2\gamma\frac{\partial^2 f}{\partial \eta^2} + f\frac{\partial^2 f}{\partial \eta^2} - \left(\frac{\partial f}{\partial \eta}\right)^2 - \frac{\partial^2 f}{\partial \eta \partial \tau} = 0 \quad (13)$$

$$(1 + 2\gamma\eta)\frac{\partial^2 \theta}{\partial \eta^2} + 2\gamma\frac{\partial \theta}{\partial \eta} + Pr\left(f\frac{\partial \theta}{\partial \eta} - \frac{\partial f}{\partial \eta}\theta\right) - Pr\frac{\partial \theta}{\partial \tau} = 0 \quad (14)$$

and are subjected to the boundary conditions

$$f(0,\tau) = S, \quad \frac{\partial f}{\partial \eta}(0,\tau) = -1, \quad \theta(0,\tau) = 1,$$
$$\frac{\partial f}{\partial \eta}(\infty,\tau) \to 0, \quad \theta(\infty,\tau) \to 0 \tag{15}$$

To test the stability of the steady flow solution $f(\eta) = f_0(\eta)$ and $\theta(\eta) = \theta_0(\eta)$ satisfying the boundary value problem Eqs. (1) – (4), we write

$$f(\eta,\tau) = f_0(\eta) + e^{-\lambda\tau}F(\eta,\tau),$$
$$\theta(\eta,\tau) = \theta_0(\eta) + e^{-\lambda\tau}G(\eta,\tau), \tag{16}$$

where λ is an unknown eigenvalue, and $F(\eta,\tau)$ and $G(\eta,\tau)$ are small relative to $f_0(\eta)$ and $\theta_0(\eta)$. Solutions of the eigenvalue problem Eqs. (13) – (15) give an infinite set of eigenvalues $\lambda_1 < \lambda_2 < \ldots$; if the smallest eigenvalue is negative, there is an initial growth of disturbances and the flow is unstable but when λ_1 is positive, there is an initial decay and the flow is stable. Introducing Eq. (16) into Eqs. (13) and (14), the following linearized problem are obtained

$$(1+2\gamma\eta)\frac{\partial^3 F}{\partial \eta^3} + 2\gamma\frac{\partial^2 F}{\partial \eta^2} + f_0\frac{\partial^2 F}{\partial \eta^2} + f_0''F - 2f_0'\frac{\partial F}{\partial \eta} + \lambda\frac{\partial F}{\partial \eta} - \frac{\partial^2 F}{\partial \eta \partial \tau} = 0 \tag{17}$$
$$(1+2\gamma\eta)\frac{\partial^2 G}{\partial \eta^2} + 2\gamma\frac{\partial G}{\partial \eta} + Pr\, f_0\frac{\partial G}{\partial \eta} + Pr\,\theta_0'F + Pr\,\lambda G - \frac{\partial G}{\partial \tau} = 0 \tag{18}$$

along with the boundary conditions

$$F(0,\tau) = 0, \quad \frac{\partial F}{\partial \eta}(0,\tau) = 0, \quad G(0,\tau) = 0,$$
$$\frac{\partial F}{\partial \eta}(\infty,\tau) \to 0, \quad G(\infty,\tau) \to 0 \tag{19}$$

The solutions $f(\eta) = f_0(\eta)$ and $\theta(\eta) = \theta_0(\eta)$ of the steady equations Eqs. (6) and (7) are obtained by setting $\tau = 0$. Hence $F(\eta) = F_0(\eta)$ and $G(\eta) = G_0(\eta)$ in Eqs. (17) and (18) identify initial growth or decay of the solution Eq. (16). In this respect, we have to solve the linear eigenvalue problem

$$(1+2\gamma\eta)F_0''' + 2\gamma F_0'' + f_0F_0'' + f_0''F_0 - 2f_0'F_0' + \lambda F_0' = 0 \tag{20}$$
$$(1+2\gamma\eta)G_0'' + 2\gamma G_0' + Pr\, f_0\, G_0' + Pr\,\theta_0'F_0 + Pr\,\lambda\, G_0 = 0 \tag{21}$$

along with the boundary conditions

$$F_0(0) = 0, \quad F_0'(0) = 0, \quad G_0(0) = 0,$$
$$F_0'(\infty) \to 0, \quad G_0(\infty) \to 0 \tag{22}$$

It should be stated that for particular values of Pr and λ, the stability of the corresponding steady flow solutions $f_0(\eta)$ and $\theta_0(\eta)$ are determined by the smallest eigenvalue λ. As it has been suggested by Harris *et al.,* [42], the range of possible eigenvalues can be determined by relaxing a boundary condition on $F_0(\eta)$ or $G_0(\eta)$. For the present problem, we relax the condition that $F_0'(\eta) \to 0$ as

$\eta \to \infty$ and for a fixed value of λ we solve the system Eqs. (20) – (22) along with the new boundary condition $F_0''(0) = 1$.

3. Results

Numerical solutions to the ordinary differential equations from Eqs. (6) and (7) with the boundary conditions (8) are obtained using bvp4c in MATLAB. The different initial guess values for skin friction coefficient $f''(0)$ and the temperature gradient $-\theta'(0)$ are chosen, where all profiles satisfy the far field boundary conditions (8) asymptotically but with different shapes that illustrated in graphs, while the values S_c for different values of γ are given in Table 1. From the table we indicate that the values of S_c increased as γ increases.

Table 1
Variations of S_c for different values of γ

γ	Present Work
0	2.000000
0.2	2.276154
0.4	2.522534

The variations of skin friction coefficient and heat transfer rate with S are shown in Figures 1 and 2. The figures indicate that there exist two solutions when $S > S_c$ and subsequently for $S < S_c$ there are no solutions exist. As can be seen from Figures 1 and 2, the curvature parameter γ significantly affects the skin friction coefficient as well as the heat transfer rate at the surface. The increment of γ reduced the domain of the solution. The interpretation regards from these results are the skin friction and heat transfer rate at the surface diminished as γ increased. Therefore, the flow behavior and heat transfer performed well on flat surface $(\gamma = 0)$ compare to cylinder surface $(\gamma > 0)$.

Figures 3 to 7 displays the variations of velocity and temperature profiles within the boundary layer for different values of S, γ and *Pr.* It is evident from these figures that all curves approach the far field boundary conditions asymptotically. As we can see in these figures, the boundary layer thickness for the second solutions is always thicker than the first solutions. These velocity and temperature profiles support the existence of dual nature of the solutions shown in Figures 1 and 2.

Figures 3 and 4 indicate the velocity and temperature profiles for some values of suction parameter, S. As shown, the increment of S caused to reduce the momentum and thermal boundary layer thickness and consequently accelerates the boundary layer separation. Therefore, the acceleration of boundary layer separation increased the skin friction coefficient and heat transfer rate at the cylinder surface. Thus, the interpretation from Figures 3 and 4 tallies with the findings in Figures 1 and 2.

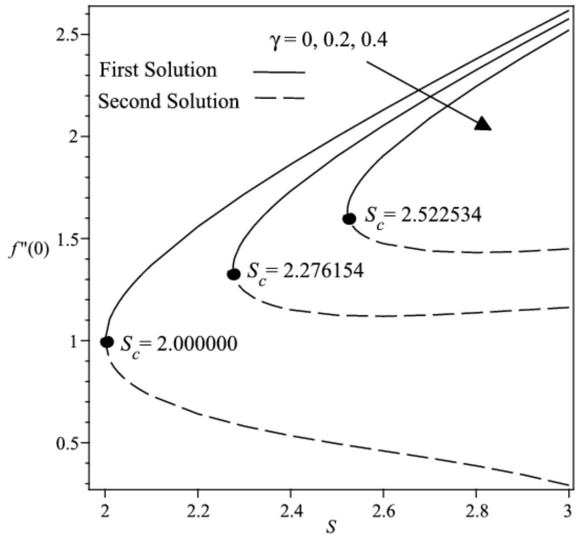

Fig. 1. Skin friction coefficient $f''(0)$ for various values of γ

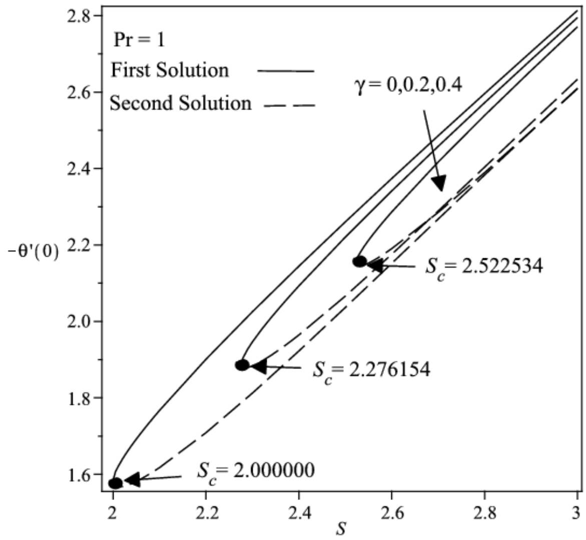

Fig. 2. Temperature gradient $-\theta'(0)$ for various values of *S*

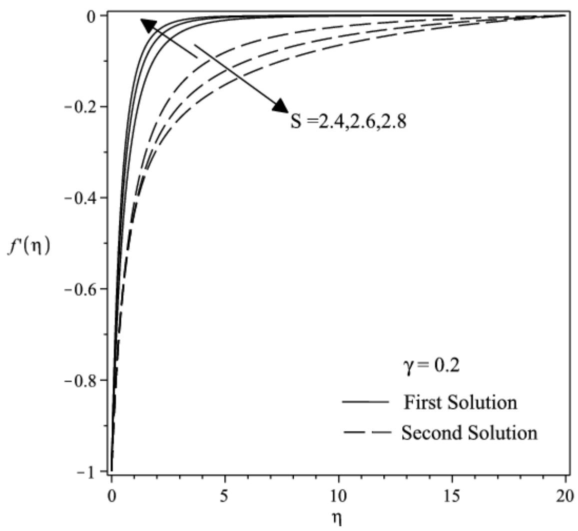

Fig. 3. Dual velocity profiles $f'(\eta)$ for various values of *S*

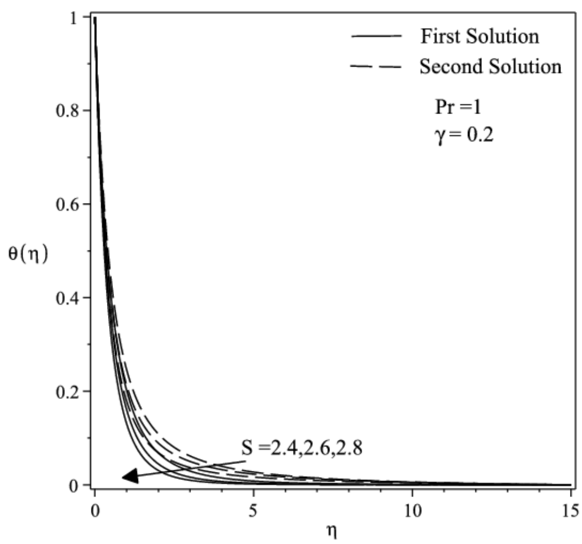

Fig. 4. Dual temperature profiles $\theta(\eta)$ for various values of *S*

Figures 5 and 6 indicate the velocity and temperature profiles for some values of curvature parameter, γ. As shown, the momentum and thermal boundary layer thickness increases as γ increases. Consequently, delays the boundary layer separation. The increment of γ Interprets the reduction in length of radius of cylinder. Physically, as the length of radius of the cylinder become smaller, it increases the contact area of the cylinder with the fluid. Therefore, it decreased the skin friction coefficient and heat transfer rate at the cylinder surface as γ increases. The interpretation from Figures 5 and 6 tallies with the findings in Figs. 1 and 2.

Figure 7 depict the temperature profiles for different value of Pr. As shown, the thermal boundary layer thickness decreased as Pr increased. Higher Pr has lower thermal conductivity which leads to reduce conduction and thermal boundary layer thickness. This implies an increase of heat transfer rate at the surface.

Regards to the dual solutions obtained, a stability analysis is performed using bvp4c in MATLAB to verify the stability of the solutions. To find the unknown eigenvalue λ in Eq. (16) we need to use the linear eigenvalue problem Eqs. (20) and (21) subjected to the boundary condition Eq. (22). The flow is claimed to be stable as the smallest eigenvalue λ is positive whereby there is a disturbance in the initial decay. On the other hand, the flow is claimed to be unstable as the smallest eigenvalue λ is negative. This such claim is made due to the initial growth of disturbance in the flow. The smallest eigenvalue λ can be seen in Table 2 for selected values of S and different values of γ. Mathematically, though two solutions exist only the stable solution is physically meaningful and can be realized physically.

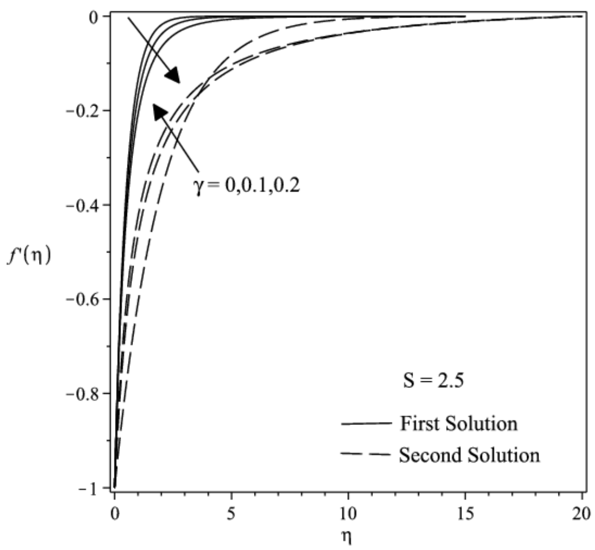

Fig. 5. Dual velocity profiles $f'(\eta)$ for various values of γ

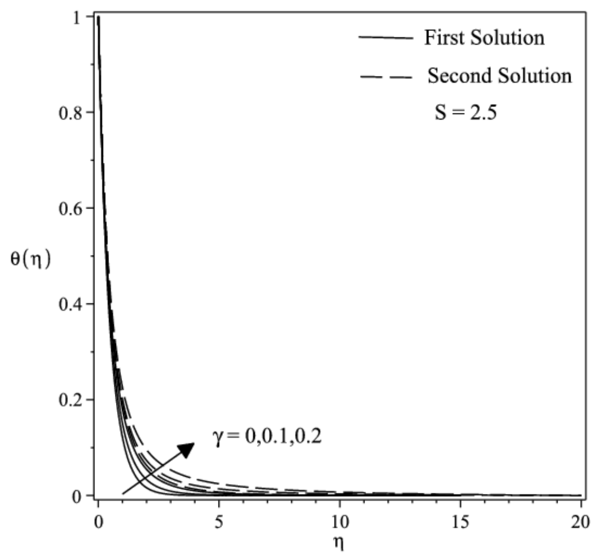

Fig. 6. Dual temperature profiles $\theta(\eta)$ for various values of γ

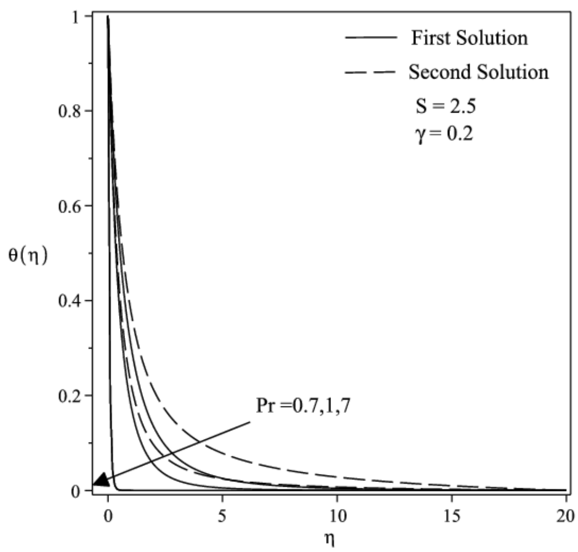

Fig. 7. Dual temperature profiles $\theta(\eta)$ for various values of Pr

Table 2
Smallest eigenvalues λ at selected values of S for γ = 0, 0.2 and 0.4

γ	S	First Solution	Second Solution
	2.01	0.1209	-0.1127
	2.05	0.2783	-0.2439
	2.1	0.4040	-0.3355
0	2.2	0.5956	-0.4545
	2.3	0.7574	-0.5371
	2.4	0.9073	-0.6004
	2.5	1.0527	-0.6812
	2.6	1.1970	-0.7370
	2.7	1.3426	-0.7898
	2.8	1.4903	-0.8413
	2.9	1.6410	-0.8926
	3.0	1.7952	-0.9443
	2.28	0.7150	-0.5094
	2.3	0.7422	-0.5224
	2.4	0.8738	-0.5793
	2.5	1.0005	-0.6263
0.2	2.6	1.1252	-0.7302
	2.7	1.2494	-0.7820
	2.8	1.3743	-0.8320
	2.9	1.5006	-0.8809
	3.0	1.6287	-0.9224
	2.53	1.0669	-0.6389
	2.55	1.0893	-0.6460
	2.6	1.1450	-0.6632
0.4	2.7	1.2563	-0.6949
	2.8	1.3678	-0.7240
	2.9	1.4800	-0.7515
	3.0	1.5935	-0.7783

4. Conclusions

The steady boundary layer flow and heat transfer over a shrinking cylinder was investigated. The numerical results were solved by using bvp4c in MATLAB. Therefore, the findings are summarized in points below.

i. The solutions (dual) only exist when $S \geq 2$.
ii. The increment of S reduces the domain of the solutions.
iii. The curvature parameter γ can be used as controller parameter to delay or accelerate boundary layer separation.
iv. The performance of skin friction coefficient and heat transfer rate better on the flat surface $(\gamma = 0)$.
v. The first solution is stable and physically meaningful, while the second solution is unstable and hence not physically meaningful.

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References

[1] Wang, C.Y. "Liquid film on an unsteady stretching sheet." *Quarterly of Applied Mathematics* 48, (1990): 601-610.

[2] Miklavčič, M., and C. Wang. "Viscous flow due to a shrinking sheet." *Quarterly of Applied Mathematics* 64, no. 2 (2006): 283-290.

[3] Hayat, T., Z. Abbas, and M. Sajid. "On the analytic solution of magnetohydrodynamic flow of a second grade fluid over a shrinking sheet." (2007): 1165-1171. https://doi.org/10.1115/1.2723820

[4] Wang, C. Y. "Stagnation flow towards a shrinking sheet." *International Journal of Non-Linear Mechanics* 43, no. 5 (2008): 377-382. https://doi.org/10.1016/j.ijnonlinmec.2007.12.021

[5] Tie-Gang, Fang, Zhang Ji, and Yao Shan-Shan. "Viscous flow over an unsteady shrinking sheet with mass transfer." *Chinese Physics Letters* 26, no. 1 (2009): 014703. https://doi.org/10.1088/0256-307X/26/1/014703

[6] Ishak, Anuar, Yian Yian Lok, and Ioan Pop. "Stagnation-point flow over a shrinking sheet in a micropolar fluid." *Chemical Engineering Communications* 197, no. 11 (2010): 1417-1427. https://doi.org/10.1080/00986441003626169

[7] Bhattacharyya, Krishnendu, and G. C. Layek. "Effects of suction/blowing on steady boundary layer stagnation-point flow and heat transfer towards a shrinking sheet with thermal radiation." *International Journal of Heat and Mass Transfer* 54, no. 1-3 (2011): 302-307. https://doi.org/10.1016/j.ijheatmasstransfer.2010.09.043

[8] Bachok, N., Ishak, A. and Pop, I. “Stagnation-point flow over a stretching/shrinking sheet in a nanofluid.” *Nanoscale Research Letters* 6, (2011): p. 623. https://doi.org/10.1186/1556-276X-6-623

[9] Bachok, Norfifah, Anuar Ishak, and Ioan Pop. "Boundary layer stagnation-point flow and heat transfer over an exponentially stretching/shrinking sheet in a nanofluid." *International Journal of Heat and Mass Transfer* 55, no. 25-26 (2012): 8122-8128. https://doi.org/10.1016/j.ijheatmasstransfer.2012.08.051

[10] Bachok, N., Ishak, A. and Pop, I. “Boundary layer stagnation-point flow toward a stretching/shrinking sheet in a nanofluid.” *ASME Journal of Heat Transfer* 135, (2013): ID 054501. https://doi.org/10.1115/1.4023303

[11] Bhattacharyya, Krishnendu, and Kuppalapalle Vajravelu. "Stagnation-point flow and heat transfer over an exponentially shrinking sheet." *Communications in Nonlinear Science and Numerical Simulation* 17, no. 7 (2012): 2728-2734. https://doi.org/10.1016/j.cnsns.2011.11.011

[12] Bhattacharyya, Krishnendu. "Boundary layer flow and heat transfer over an exponentially shrinking sheet." *Chinese Physics Letters* 28, no. 7 (2011): 074701. https://doi.org/10.1088/0256-307X/28/7/074701

[13] Najib, N., Bachok, N., Arifin, N. M. and Ishak, A. “Stagnation point flow and mass transfer with chemical reaction past a stretching/shrinking cylinder.” *Scientific Reports* 4, (2014): ID 4178. https://doi.org/10.1038/srep04178

[14] Merkin, J. H. "On dual solutions occurring in mixed convection in a porous medium." *Journal of engineering Mathematics* 20, no. 2 (1986): 171-179. https://doi.org/10.1007/BF00042775

[15] Weidman, P. D., D. G. Kubitschek, and A. M. J. Davis. "The effect of transpiration on self-similar boundary layer flow over moving surfaces." *International journal of engineering science* 44, no. 11-12 (2006): 730-737. https://doi.org/10.1016/j.ijengsci.2006.04.005

[16] Merrill, Keith, Matthew Beauchesne, Joseph Previte, Joseph Paullet, and Patrick Weidman. "Final steady flow near a stagnation point on a vertical surface in a porous medium." *International journal of heat and mass transfer* 49, no. 23-24 (2006): 4681-4686. http://dx.doi.org/10.1016/j.ijheatmasstransfer.2006.02.056

[17] Ishak, A. “Flow and heat transfer over a shrinking sheet: a stability analysis.” *International Journal of Mechanical, Aerospace, Industrial and Mechatronics Engineering* 8, no. 5, (2014): 905-909. https://doi.org/10.5281/zenodo.1092443

[18] Yasin, M. H. M., Ishak, A. and Pop, I. “MHD stagnation-point flow and heat transfer with effects of viscous dissipation, Joule heating and partial velocity slip.” *Scientific Reports* 5, (2015): ID 17848. https://doi.org/10.1038/srep17848

[19] Dzulkifli, Nor Fadhilah, Norfifah Bachok, Nor Azizah Yacob, Ioan Pop, Norihan Arifin, and Haliza Rosali. "Stability Solution of Unsteady Stagnation-Point Flow and Heat Transfer over a Stretching/Shrinking Sheet in Nanofluid with Slip Velocity Effect." *CFD Letters* 14, no. 1 (2022): 66-86. https://doi.org/10.37934/cfdl.14.1.6686

[20] Abu Bakar, Shahirah, Norihan Md Arifin, Fadzilah Md Ali, Norfifah Bachok, Roslinda Nazar, and Ioan Pop. "A stability analysis on mixed convection boundary layer flow along a permeable vertical cylinder in a porous medium filled with a nanofluid and thermal radiation." *Applied Sciences* 8, no. 4 (2018): 483. https://doi.org/10.3390/app8040483

[21] Khashi'ie, Najiyah Safwa, Iskandar Waini, Nurul Amira Zainal, Khairum Hamzah, and Abdul Rahman Mohd Kasim. "Hybrid nanofluid flow past a shrinking cylinder with prescribed surface heat flux." *Symmetry* 12, no. 9 (2020): 1493. https://doi.org/10.3390/sym12091493

[22] Waini, I., Ishak, A. and Pop, I. "Hybrid nanofluid flow on a shrinking cylinder with prescribed surface heat flux." International Journal of Numerical Methods for Heat & Fluid Flow 31, no. 6 (2020): 1987-2004. https://doi.org/10.1108/HFF-07-2020-0470

[23] Waini, Iskandar, Anuar Ishak, and Ioan Pop. "Hybrid nanofluid flow towards a stagnation point on a stretching/shrinking cylinder." *Scientific Reports* 10, no. 1 (2020): 1-12. https://doi.org/10.1038/s41598-020-66126-2

[24] Zeeshan, Ahmad, Obaid Ullah Mehmood, Fazle Mabood, and Faris Alzahrani. "Numerical analysis of hydromagnetic transport of Casson nanofluid over permeable linearly stretched cylinder with Arrhenius activation energy." *International Communications in Heat and Mass Transfer* 130 (2022): 105736. https://doi.org/10.1016/j.icheatmasstransfer.2021.105736

[25] Uddin, Ziya, and Manoj Kumar. "Hall and ion-slip effect on MHD boundary layer flow of a micro polar fluid past a wedge." *Scientia Iranica* 20, no. 3 (2013): 467-476. https://doi.org/10.1016/j.scient.2013.02.013

[26] Mabood, Fazle, Sami Ullah Khan, and Iskander Tlili. "Numerical simulations for swimming of gyrotactic microorganisms with Williamson nanofluid featuring Wu's slip, activation energy and variable thermal conductivity." *Applied Nanoscience* (2020): 1-14. https://doi.org/10.1007/s13204-020-01548-y

[27] Bhatti, M. M., M. B. Arain, A. Zeeshan, R. Ellahi, and M. H. Doranehgard. "Swimming of Gyrotactic Microorganism in MHD Williamson nanofluid flow between rotating circular plates embedded in porous medium: Application of thermal energy storage." *Journal of Energy Storage* 45 (2022): 103511. https://doi.org/10.1016/j.est.2021.103511

[28] Wahid, Nur Syahirah, Norihan Md Arifin, Najiyah Safwa Khashi'ie, Ioan Pop, Norfifah Bachok, and Mohd Ezad Hafidz Hafidzuddin. "Flow and heat transfer of hybrid nanofluid induced by an exponentially stretching/shrinking curved surface." *Case Studies in Thermal Engineering* 25 (2021): 100982.https://doi.org/10.1016/j.csite.2021.100982

[29] Shamshuddin, M. D., Fazle Mabood, and O. Anwar Bég. "Thermomagnetic reactive ethylene glycol-metallic nanofluid transport from a convectively heated porous surface with Ohmic dissipation, heat source, thermophoresis and Brownian motion effects." *International Journal of Modelling and Simulation* (2021): 1-15. https://doi.org/10.1080/02286203.2021.1977531

[30] Uddin, Ziya, Korimerla Sai Vishwak, and Souad Harmand. "Numerical duality of MHD stagnation point flow and heat transfer of nanofluid past a shrinking/stretching sheet: Metaheuristic approach." *Chinese Journal of Physics* 73 (2021): 442-461. https://doi.org/10.1016/j.cjph.2021.07.018

[31] Zeeshan, A., M. B. Arain, M. M. Bhatti, F. Alzahrani, and O. Anwar Bég. "Radiative bioconvection nanofluid squeezing flow between rotating circular plates: Semi-numerical study with the DTM-Padé approach." *Modern Physics Letters B* 36, no. 03 (2022): 2150552. https://doi.org/10.1142/S0217984921505527

[32] Uddin, Ziya, and Souad Harmand. "Natural convection heat transfer of nanofluids along a vertical plate embedded in porous medium." *Nanoscale research letters* 8, no. 1 (2013): 1-19. https://doi.org/10.1186/1556-276X-8-64

[33] Uddin, Z., R. Asthana, M. Kumar Awasthi, and S. Gupta. "Steady MHD flow of nano-fluids over a rotating porous disk in the presence of heat generation/absorption: a numerical study using PSO." *J. Appl. Fluid Mech* 10, no. 3 (2017): 871-879. https://doi.org/10.18869/ACADPUB.JAFM.73.240.26650

[34] Uddin, Ziya, Manoj Kumar, and Souad Harmand. "Influence of thermal radiation and heat generation/absorption on MHD heat transfer flow of a micropolar fluid past a wedge considering hall and ion slip currents." *Thermal Science* 18, no. 2 (2014): 489-502. https://doi.org/10.2298/TSCI110712085U

[35] Mabood, F., Khan, W. A. and Ismail, A. I. M. "Analytical investigation for free convective flow of non-newtonian nanofluids flow in porous media with gyrotactic microorganisms." *Journal of Porous Media* 18, no. 7 (2015), 653-663. https://doi.org/10.1615/JPorMedia.v18.i7.10

[36] Uddin, Ziya, Rishi Asthana, Mukesh Kumar Awasthi, and Hamdy Hassan. "A metaheuristic approach for the comparative study of MHD flow of nano liquids in a semi-porous channel." *International Journal for Computational Methods in Engineering Science and Mechanics* 22, no. 3 (2021): 244-251. https://doi.org/10.1080/15502287.2021.1916700

[37] Arain, M. B., Bhatti, M. M., Zeeshan, A., Saeed, T., & Hobiny, A. "Analysis of arrhenius kinetics on multiphase flow between a pair of rotating circular plates." Mathematical Problems in Engineering, (2020), ID 2749105. https://doi.org/10.1155/2020/2749105
[38] Arain, Muhammad Bilal, Muhammad Mubashir Bhatti, Ahmad Zeeshan, and Faris Saeed Alzahrani. "Bioconvection reiner-rivlin nanofluid flow between rotating circular plates with induced magnetic effects, activation energy and squeezing phenomena." *Mathematics* 9, no. 17 (2021): 2139. https://doi.org/10.3390/math9172139
[39] Azam, M., T. Xu, F. Mabood, and M. Khan. "Non-linear radiative bioconvection flow of cross nano-material with gyrotatic microorganisms and activation energy." *International Communications in Heat and Mass Transfer* 127 (2021): 105530. https://doi.org/10.1016/j.icheatmasstransfer.2021.10553
[40] Shehzad, S. A., F. Mabood, A. Rauf, Mohsen Izadi, and F. M. Abbasi. "Rheological features of non-Newtonian nanofluids flows induced by stretchable rotating disk." *Physica Scripta* 96, no. 3 (2021): 035210. https://doi.org/10.1088/1402-4896/abd652
[41] Najib, N., Bachok, N. and Arifin, N. M. "Stability of Dual Solutions in Boundary Layer Flow and Heat Transfer over an Exponentially Shrinking Cylinder." Indian Journal of Science and Technology 9, no. 48 (2016): 6pp. doi: 10.17485/ijst/2016/v9i48/97740
[42] Harris, S.D., Ingham, D.B. and Pop, I. "Mixed Convection Boundary-Layer Flow Near the Stagnation Point on a Vertical Surface in a Porous Medium: Brinkman Model with Slip." *Transport Porous Media* 77, (2009): 267–285. doi:10.1007/s11242-008-9309-6