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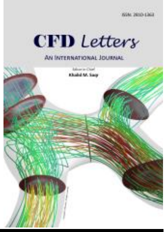

Characteristics of MHD Jeffery Fluid Past an Inclined Vertical Porous Plate

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ABSTRACT

This paper is concerned with the study of an unsteady, MHD natural convective boundary layer flow of a viscous, incompressible and electrically conducting, non-Newtonian Jeffery fluid over a semi-infinite vertically inclined permeable moving plate embedded in a porous medium in the presence of thermal radiation, heat absorption and thermal diffusion, heat and mass transfer. . The permeability of the porous medium and the suction velocity are considered to be an exponentially decreasing function of time. The fundamental governing equations for this investigation are solved numerically using the perturbation technique. The results are presented graphically and in tabular form for various controlling parameters. The behavior of different physical parameters is shown graphically. The numerical values of Skin friction, Nusselt number, and Sherwood number are presented in a tabular form. Obtained outcomes are compared with earlier studies in the special case and strong agreement is noted. From graphical representation, it is concluded that velocity and temperature distribution increases with the mixed convection parameter and buoyancy force parameter. An increasing value of magnetic field parameter, slip parameter, and Jeffery parameter tends to reduced velocity and also raising the values of Prandtl number, radiation parameter and heat absorption parameter tends to downfallen temperature profiles. This study may be useful in several industrial applications, for example, polymer production, manufacturing of ceramics or glassware and food processing, and so forth.

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1. Introduction

Now-a-days, the examination of non-Newtonian liquid flows has become renowned for the reason of numerous applications in sciences and technical fields. Inspiration of researchers in these fluids is owed too many applications, precisely in food products, biological material and chemical material. Obviously, all non-Newtonian fluids are subjected to their effects happening within shear that are not specified by a single essential relationship.

Consequently, few non- Newtonian fluids are instructed for the discussion relating to their many features. Non-Newtonian fluids are conferred through three chief classifications rate, differential and integral. The existing data witnesses that many studies given to the flow of different classes of many type materials. It is owed to focus that in several materials like shear and standard stresses are explained clearly in terms of velocity component. But, it is often not correct for two and three dimensional flows containing different classes of fluids. Currently, using Jeffrey fluid has grown in popularity for the modelling. It has a special influence of reduction and delay times. Few investigations show flow of Jeffrey fluid can be expressed. The authors [1, 2] have studied the nonlinear radially shrinking sheet through a permeable medium with MHD Jeffrey fluid. The examination reveals that the boundary layer thickness is studied in either stretching or shrinking sheet cases. Hayat *et al.,* [3] investigated their research by considering the instantaneous influence of melting heat and inside heat production in stagnation point flow of Jeffrey fluid with variable thickness on a nonlinear stretching surface. Rana *et al.,* [4] suggested a three-dimensional Couette flow of Jeffrey fluid alongside periodic suction. Imran *et al.,* [5] projected Jeffrey fluid in the field of thermo diffusion; thermal radiation effects with chemical reaction using first-order evenly distributed heat flux. Vasu *et al.,* [6] examined free convective heat transfer in Jeffrey fluid with suspended nanoparticles and Cattaneo–Christov heat flux. Das *et al.,* [7] examined Jeffrey fluid (MHD) flow on a stretching sheet that has a slip surface. Quasim *et al.,* [8] discussed heat and mass transfer in a Jeffrey fluid over a stretching sheet with heat source/sink. Sreenadh *et al.,* [9] investigated peristaltic pumping of a fluid using power-law with Jeffrey fluid at the ending path with permeable walls. Zeeshan *et al.,* [10] examined heat transfer analysis of Jeffery fluid flow over a stretching sheet with suction/injection and magnetic dipole effect. Kohilavani Naganthran *et al.,* [11] investigated Effects of heat generation/absorption in the Jefrey fluid past a permeable stretching/shrinking disc. Lu *et al.,* [12] suggested upshot of chemical species and nonlinear thermal radiation on Oldroyd-B Nano fluid flow past a bi-directional stretched surface with heat absorption in a porous media. Suleman *et al.,* [13] studied entropy analysis of 3D non-Newtonian MHD Nano fluid flow with nonlinear thermal radiation past over exponential stretched surface.

The radiation is the method used to study the heat transfer of material through a magnetic environment. The radiative convective flows may occur in almost all industrial processes. For instance, heating and cooling chambers, fuel combustion energy processes evaporation from massive open water reservoirs, etc. The term radiative plays a substantial role in the field of heat and mass transfer studies of a problem. Raju *et al.,* [14] discussed the unsteady MHD mixed convection flow of Jeffrey fluid past over a radiating inclined permeable moving plate in the presence of thermophoresis heat generation and chemical reaction. Cui *et al.,* [15] examined the effect of convection and radiation of heat transfer upon cooling performance over radiative panel. The unsteady MHD free convective Couette flow among vertical permeable plates through thermal radiation discussed by Jha *et al.,* [16]. Patel [17] examined Thermal radiation effects on MHD flow with heat and mass transfer of micropolar fluid between two vertical walls. Patel [18] reported Heat and mass transfer in MHD Casson fluid flow past over an oscillating vertical plate embedded in porous medium with ramped wall temperature. Effects of cross diffusion and heat generation on mixed convective MHD flow of

Casson fluid through porous medium with non-linear thermal radiation discussed by Patel [19]. Hsiao [20] presented that radiative and viscous dissipation impacts on an electrically MHD heat transfer thermal system by means of Maxwell fluid. The authors [21, 22] explained that the radiation and natural convection flow through with thermal radiation and mass transfer past a moving perpendicular permeable plate. Raghunath *et al.,* [23] reported heat and mass transfer on an unsteady MHD flow through porous medium between two porous vertical plates. MHD Casson fluid flow through a vertical plate discussed by Parandhama *et al.,* [24]. Srinivasacharya *et al.,* [25] reported the radiation and chemical reaction effects upon mixed convection heat and mass transfer across a perpendicular plate inwards power-law fluid concentrated permeable medium. Pattnaik *et al.,* [26] observed mass transfer and radiation effects upon MHD flow through a permeable medium past an exponentially accelerated disposed plate on flexible temperature. Luo *et al.,* [27] contribute the influences of thermal radiation on MHD flow and heat transfer in a cubic cavity. Obulesu *et al.,* [28] discussed radiation absorption effect on MHD dissipative fluid past a vertical porous plate embedded in porous media. Murthy *et al.,* [29] developed heat and mass transfer effects on MHD natural convective flow through a vertical permeable plate with thermal radiation and hall current. Sandya *et al.,* [30] have proposed radiation and chemical reaction effects on MHD Casson fluid flow past a semi-infinite vertical moving porous plate.

The flow through a porous media could be a most interesting subject and it became emerging analysis for the explanation of heat and mass transfer in a wet medium which is incredibly related to nature and will even be used in many technical processes. Babu *et al.,* [31] have proposed the heat and mass transfer of MHD flow characteristics of a Non-Newtonian fluid over an infinite perpendicular porous plate using the principle of Hall effects. Tripathy *et al.,* [32] have demonstrated the chemical characteristics of magneto hydrodynamics over a stirring perpendicular plate within porous medium. Haq *et al.,* [33] have analyzed MHD expected convection flow supplement in a flat hollow occupied through a permeable media. Patel [34] reported Cross diffusion and heat generation effects on mixed convection stagnation point MHD Carreau fluid flow in a porous medium. Effects of Magnetic field, thermo-diffusion and hall current on Casson fluid flow past an oscillating plate in porous medium studied by Patel [35]. Ibrahim and Makinde [36] considered the influence of radiation over the chemically countering MHD margin level flow characteristics of heat and mass transfer passing through a porous plate. Pattnaik *et al.,* [37] discussed the influence of Soret and Dufour with the hall current over instable hydro magnetic flow earlier in a vertical porous plate. Chamkha and Ahmed [38] have suggested similar description for unstable MHD flow exactly about a stagnation idea of three dimensional porous frame over the heat and mass transfer substance reaction and the heat generation. Patel [39] have discussed heat and mass transfer effects on unsteady free convective MHD flow of a micro polar fluid between two vertical walls. Some authors studied [40-41] MHD Convective heat and mass transfer flow of a Newtonian fluid flow past a vertical porous plate with chemical reaction, radiation absorption, radiation and thermal diffusion." Guled [42] have studied the heat transfer effects of MHD slip flow with suction and injection and radiation over a shrinking sheet by optimal homotopy analysis method. Dharmaiah *et al.,* [43] discussed performance of magnetic dipole contribution on ferromagnetic non-Newtonian radiative MHD blood flow. Some authors studied [44-56] MHD Convective heat and mass transfer flow of a Newtonian fluid flow past a vertical porous plate with chemical reaction, radiation absorption, radiation and thermal diffusion.

In this paper, the author/researchers have studied the effects of various characteristics of heat and mass transfer on Jeffrey fluid in the presence of thermal radiation, heat generation, chemical reaction and thermal diffusion past an inclined plate. The objective of this study is to investigate analytically characteristics of MHD mixed convection flow of Jeffrey fluid past a radiating inclined permeable moving porous plate in the presence of heat absorption, Soret and chemical reaction. The uniqueness of this study is the consideration of well-known non-Newtonian fluid namely Jeffrey fluid past an inclined plate which is an extension of the work done by Ravikumar *et al.,* [41], who studied various flow characteristics of a Newtonian fluid. In the absence of Jeffrey parameter, the results of our study are in good agreement with the results of Ravikumar *et al.,* [41]. The study has importance in many metallurgical processes including magma flows, polymer and food processing, and blood flow in micro-circulatory system etc.

In applied mathematics, perturbation theory comprises methods for finding an approximate solution to a problem, by starting from the exact solution of a related, simpler problem. A critical feature of the technique is a middle step that breaks the problem into “solvable” and “perturbative” parts. In perturbation theory, the solution is expressed as a power series in a small parameter. The first term is the known solution to the solvable problem. Successive terms in the series at higher powers usually become smaller. An approximate “perturbation solution” is obtained by truncating the series, usually by keeping only the first two terms, the solution to the known problem and the “first-order” perturbation correction. Perturbation theory is used in a wide range of fields and reaches its most sophisticated and advanced forms in quantum field theory. Perturbation theory describes the use of this method in fluid mechanics. The field, in general, remains actively and heavily researched across multiple disciplines.

The present work is concerned with the aligned magnetic field, Soret effect, and Jeffery parameter on MHD convective heat and mass transfer flow of an unsteady viscous incompressible electrically conducting fluid past a semi-infinite inclined vertical porous plate in

Presence of chemical reaction and heat sink. The governing equations of motion are solved analytically by using the perturbation technique. In this study, we have generalized the work done by Ravi Kumar *et al.,* [41] by considering inclined vertical porous plates, aligned magnetic field, and Casson parameter. This study may be useful in several industrial applications, for example, polymer production, manufacturing of ceramics or glassware and food processing, and so forth.

2. Basic Equations

The equations governing the flow of a viscous incompressible and electrically conducting fluid in the presence of magnetic field are

Equation of continuity

$$\overline{\nabla} \cdot \overline{q} = 0$$

Momentum equation

$$\rho\left[\frac{\partial \overline{q}}{\partial t^*} + \left(\overline{q} \cdot \overline{\nabla}\right)\overline{q}\right] = -\overline{\nabla} p + \overline{J} \times \overline{B} + \rho \overline{g} + \mu\left(\frac{1}{1+\beta}\right)\nabla^2 \overline{q} - \frac{\mu}{k^*}\overline{q}$$

Ohm’s law

$$\overline{J} = \sigma\left[\overline{E} + \left(\overline{q} \times \overline{B}\right)\right]$$

Gauss’ law of magnetism

$$\overline{\nabla} \cdot \overline{B} = 0$$

Energy equation

$$\rho C_p \left[\frac{\partial T^*}{\partial t^*} + \left(\overline{q} \cdot \overline{\nabla}\right) T^* \right] = K \nabla^2 T^* - \nabla q_r^* - Q^* \left(T^* - T_\infty^*\right)$$

Species continuity equation

$$\frac{\partial C^*}{\partial t^*} + \left(\overline{q} \cdot \overline{\nabla}\right) C^* = D_M \nabla^2 C^* - K^* (C_\infty^* - C^*) + \frac{D_M K_T}{T_M} \nabla^2 T^*$$

All the physical quantities are defined in the Nomenclature.

3. Mathematical Formulation

We consider a two-dimensional unsteady free convection flow of an incompressible viscous fluidpast an infinite vertical porous plate (see Figure 1). In rectangular Cartesian coordinate system, we take the x-axis along theplate in the direction of flow and the y-axis normal to it. Further, the flow is considered in the presence of temperature gradient dependent heat source. In the analysis, the magnetic Reynolds number is taken to besmall so that the induced magnetic field is neglected. Likewise for small velocity, the viscous dissipation and Darcy's dissipation are neglected. The flow in the medium is entirely due to the buoyancy force caused by a temperature difference between the porous plate and the fluid. Under the above assumptions, the equations governing the conservation of mass (continuity), momentum, energy and concentration can be written as follows;

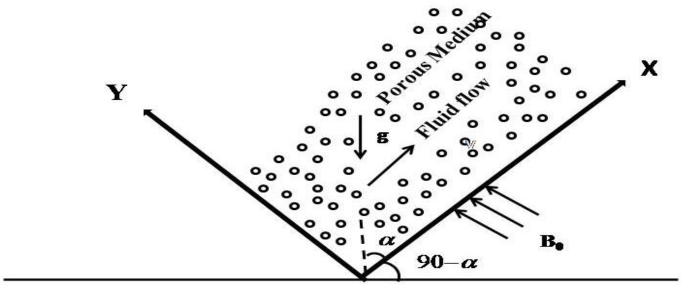

Fig. 1. Schematic diagram of the problem

$$\frac{\partial v*}{\partial y*} = 0 \tag{1}$$

$$\frac{\partial u*}{\partial t*} + V*\frac{\partial u*}{\partial y*} = \left(\frac{\vartheta}{1+\lambda}\right)\frac{\partial^2 u*}{\partial y*^2} + g\beta_T (T* - T_\infty^*)\cos\alpha + g\beta_c (C* - C_\infty^*)\cos\alpha - \frac{\sigma B_0^2}{\rho} u* - \frac{\vartheta u*}{K^*} \tag{2}$$

$$\frac{\partial T*}{\partial t*} + V*\frac{\partial T*}{\partial y*} = \frac{K_T}{\rho C_p}\frac{\partial^2 T*}{\partial y*^2} - \frac{1}{\rho C_p}\frac{\partial q_r*}{\partial y*} - \frac{Q_1}{\rho C_p}(T* - T_\infty^*) \tag{3}$$

$$\frac{\partial C*}{\partial t*} + V*\frac{\partial C*}{\partial y*} = D\frac{\partial^2 C*}{\partial y*^2} - K_c^*(C* - C_\infty^*) + D_1\frac{\partial^2 T*}{\partial y*^2} \quad (4)$$

The relevant boundary conditions are given as follows

$$\begin{aligned} &u* = L*(\frac{\partial u*}{\partial y*}), \quad T* = T_w^*, \quad C* = C_w^* \quad \text{at } y^* = 0 \\ &u* \to 0, \qquad T* \to T_\infty^*, \quad C* \to C_\infty^* \quad \text{as } y^* \to \infty \end{aligned} \quad (5)$$

Where $L_1 = (2-M_x)/M_x$)

The equation of continuity yields that $V^*$is either a constant or some function of time, hence it is assumed that

$V^* = -U_0(1+\varepsilon e^{-n^*t^*})$ (6)

The negative sign indicates that the suction velocity acts towards the plate.

"Consider the fluid which is optically thin with a relatively low density and the radioactive heat flux is given by as follows."

$$\frac{\partial q_r*}{\partial y*} = 4I^*(T* - T_\infty^*) \quad (7)$$

The permeability of the porous medium in a non-dimensional form is considered as

$K^* = K_0^*(1+\varepsilon e^{-n^*t^*})$ (8)

On introducing the following non-dimensional quantities,

$$u = \frac{u*}{U_0}, y = \frac{U_0 y*}{\vartheta}, T = \frac{T* - T_\infty^*}{T_w^* - T_\infty^*}, C = \frac{C* - C_\infty^*}{C_w^* - C_\infty^*}, \Pr = \frac{\mu C_p}{K_T}, Sc = \frac{\vartheta}{D}, M = \frac{\sigma B_0^2 \vartheta}{\rho U_0^2},$$

$$Gr = \frac{\vartheta g \beta_T (T_w^* - T_\infty^*)}{U_0^3}, Gm = \frac{\vartheta g \beta_c^* (C_w^* - C_\infty^*)}{U_0^3}, \quad K = \frac{U_0^2 K_0^*}{\vartheta^2}, t = \frac{t* U_0^2}{4\vartheta}, h = \frac{U_0^2 L_1}{\vartheta},$$

$$K_C = \frac{\vartheta K_C^*}{U_0^2}, R = \frac{4I^*\vartheta}{\rho C_p v_0^2}, Q = \frac{Q_1 v}{U_0^2}, S_0 = \frac{D_1(T_w^* - T_\infty^*)}{\vartheta(C_w^* - C_\infty^*)}, R = \frac{4\vartheta n^*}{U_0^2}$$

The governing Eq. (1) to Eq. (4) can be rewritten in the non-dimensional form as follows

$$\frac{1}{4}\frac{\partial u}{\partial t} - (1+\varepsilon e^{-nt})\frac{\partial u}{\partial y} = (\frac{1}{1+\lambda})\frac{\partial^2 u}{\partial y^2} + Gr\cos\alpha\, T + Gm\cos\alpha\, C - M_1 u \quad (9)$$

$$\frac{\Pr}{4}\frac{\partial T}{\partial t} - \Pr(1+\varepsilon e^{-nt})\frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} - \Pr n_1 T \tag{10}$$

$$\frac{Sc}{4}\frac{\partial C}{\partial t} - Sc(1+\varepsilon e^{-nt})\frac{\partial C}{\partial y} = \frac{\partial^2 C}{\partial y^2} - ScK_C C + So\frac{\partial^2 T}{\partial y^2} \tag{11}$$

Where n_1=R+Q,

The corresponding boundary conditions are given by

$$\begin{aligned} &u = h(\frac{\partial u}{\partial y}), \quad T = 1, \quad C = 1, \quad at \quad y = 0 \\ &u \to 0, \quad T \to 0, \quad C \to 0 \quad as \quad y \to \infty \end{aligned} \tag{12}$$

4. Solution of the Problem

To tackle the partial differential Eq. (9), Eq. (10) and Eq. (11), we decrease them into conventional differential conditions. To get the arrangement we follow the methodology given by Gersten and Gross. In this manner the articulations for fluid velocity, temperature and focus are accepted in the accompanying structure.

$$\begin{aligned} U(y,t) &= F_0(y) + \varepsilon F_1(y)e^{-nt} \\ T(y,t) &= G_0(y) + \varepsilon G_1(y)e^{-nt} \\ C(y,t) &= H_0(y) + \varepsilon H_1(y)e^{-nt} \end{aligned} \tag{13}$$

Substituting the above expressions Eq. (13) in to Eq. (9), Eq. (10), Eq. (11) and equating the coefficient of ε^0, ε^1 (neglecting ε^2terms etc.,), we obtain the following set of ordinary differential equations

Zero order terms:

$$\frac{F_0''}{(1+\lambda)} + F_0' - \mathrm{M_1F_0} = -\mathrm{Gr}\cos\alpha\ G_0 - \mathrm{Gm}\cos\alpha\ H_0 \tag{14}$$

$$G_0'' + \Pr G_0' - \Pr n_1 \mathrm{G_0} = 0 \tag{15}$$

$$H_0'' + Sc\ H_0' - \mathrm{ScK}cH_0 = -\mathrm{S_0}\ \mathrm{Sc}\, G_0'' \tag{16}$$

First order terms:

$$\frac{F_1''}{(1+\lambda)} + F_1' + \mathrm{M_2F_1} = -\mathrm{Gr}\cos\alpha\, G_1 - \mathrm{Gm}\cos\alpha\, H_1 - F_0' \tag{17}$$

$$G_1'' + \Pr G_1' + (\frac{n\Pr}{4} - n_1 \Pr) G_1 = -\Pr G_0' \tag{18}$$

$$H_1'' + ScH_1' + Sc(\frac{n}{4} - Kc)H_1 = -S_0 ScG_1'' - ScH_0' \tag{19}$$

The corresponding boundary conditions Eq. (12) reduce to

$$\begin{aligned} &F_0 = h(\frac{\partial F_0}{\partial y}), F_1 = h(\frac{\partial F_1}{\partial y}),\ G_0 = 1,\ G_1 = 0, \quad H_0 = 1, H_1 = 0 \quad at \quad y = 0 \\ &F_0 \to 0, \quad F_1 \to 0, \quad G_0 \to 0, G_1 \to 0,\ H_0 \to 0, H_1 \to 0 \quad as \quad y \to \infty \end{aligned} \tag{20}$$

Solving Eq. (14) – (19) under the boundary conditions Eq. (20), the following solutions are obtained

$$G_0 = \exp(-l_1 y) \tag{21}$$

$$H_0 = b_1 \exp(-l_1 y) + b_2 \exp(-l_2 y) \tag{22}$$

$$F_0 = b_3 \exp(-l_1 y) + b_4 \exp(-m_2 y) + b_5 \exp(-l_3 y) \tag{23}$$

$$G_1 = b_6 \exp(-l_1 y) - b_6 \exp(-l_4 y) \tag{24}$$

$$\begin{aligned} H_1 = &\, b_7 \exp(-l_1 y) + b_8 \exp(-l_2 y) + b_9 \exp(-l_4 y) \\ &+ b_{10} \exp(-l_5 y) \end{aligned} \tag{25}$$

$$\begin{aligned} F_1 = &\, b_{11} \exp(-l_1 y) + b_{12} \exp(-l_2 y) + b_{13} \exp(-l_3 y) \\ &+ b_{14} \exp(-l_4 y) + b_{15} \exp(-l_5 y) + b_{16} \exp(-l_6 y) \end{aligned} \tag{26}$$

Substituting Eq. (21) – (26) in Eq. (13) we obtain the velocity temperature and concentration field

$$\begin{aligned} U(y,t) = &\, b_3 \exp(-l_1 y) + b_4 \exp(-m_2 y) + b_5 \exp(-l_3 y) \\ &+ \varepsilon (b_{11} \exp(-l_1 y) + b_{12} \exp(-l_2 y) + b_{13} \exp(-l_3 y) \\ &+ b_{14} \exp(-l_4 y) + b_{15} \exp(-l_5 y) + b_{16} \exp(-l_6 y)) e^{-nt} \end{aligned} \tag{27}$$

$$T(y,t) = \exp(-l_1 y) + \varepsilon (b_6 \exp(-l_1 y) - b_6 \exp(-l_4 y)) e^{-nt} \tag{28}$$

$$\begin{aligned} C(y,t) = &\, b_1 \exp(-l_1 y) + b_2 \exp(-l_2 y) \\ &+ \varepsilon (b_7 \exp(-l_1 y) + b_8 \exp(-l_2 y) + b_9 \exp(-l_4 y) + b_{10} \exp(-l_5 y)) e^{-nt} \end{aligned} \tag{29}$$

Skin Friction

The non-dimensional skin friction at the surface is given by;

$$\tau = \left(\frac{\partial U}{\partial y}\right)_{y=0}$$

$$\tau = -(l_1 b_3 + l_2 b_4 + l_3 b_5) - \varepsilon(l_1 b_{11} + l_2 b_{12} + l_3 b_{13} + l_4 b_{14} + l_5 b_{15} + l_6 b_{16}) e^{-nt} \quad (30)$$

Rate of Heat Transfer

The rate of heat transfer in terms of the Nusselt number is given by;

$$Nu = -\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

$$Nu = l_1 - \varepsilon(l_4 b_6 - l_1 b_6) e^{-nt} \quad (31)$$

Rate of Mass Transfer

The rate of mass transfer on the wall in terms of Sherwood number is given by;

$$Sh = -\left(\frac{\partial C}{\partial y}\right)_{y=0}$$

$$Sh = (l_1 b_1 + l_2 b_2) + \varepsilon(l_1 b_7 + l_2 b_8 + l_4 b_9 + l_5 b_{10}) e^{-nt} \quad (32)$$

5. Results and Discussion

To assess the physical depth of the problem, the effects of various parameters such as the slip parameter h, Grashof number Gr, magnetic parameter M, permeability of porous medium K, heat source parameter Q, radiation parameter R, chemical reaction parameter Kc, modified Grashof number Gm, Jeffery parameter λ, inclined angle α and Schmidt number Sc on velocity distribution, temperature distribution and concentration distribution are studied in Figure 2 to 14, while keeping the other parameters as constants.

Figure 2 depicts the velocity profiles with the variations in h. It is observed that the significance of the velocity is high near the plate and thereafter it decreases and reaches to the stationery position at the other side of the plate. As expected the velocity increases with an increase in h.

Figure 3 and Figure 4 exhibits the effect of Grashof number for heat and mass transfer on the velocity profile with other parameters are fixed. The Grashof number for heat transfer signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is an enhancement in the velocity due to the rising of thermal buoyancy force. Also, as *Gr* increases, the peak values of the velocity increases rapidly near the porous plate and then decays smoothly to the free stream velocity. The modified Grashof number defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases and the peak value is more distinctive due to increase in the species buoyancy force. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach the free stream value. It is noticed that the velocity increases with increasing values of the Grashof number for mass transfer.

The effect of Hartmann number on the velocity is shown in Figure 5. The velocity decreases with an increase in the Hartmann number. It is because that the application of transverse magnetic field will result a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. Also, the boundary layer thickness decreases with an increase in the Hartmann number.

Figure 6 shows the effect of the permeability of the porous medium parameter on the velocity distribution. As shown, the velocity is increasing with the increasing dimensionless porous medium parameter. Physically, this result can be achieved when the holes of the porous medium may be neglected. Figure 7 demonstrates the influence of angle of inclination parameter on velocity profiles. From this figure, it is observed that the velocity profiles are decreasing with increasing values of angle of inclination parameter. In Figure 8, it is observed at nearest to the plate the velocity increases and at the farthest of the plate the velocity gets reduced under the influence of the Jeffery parameter λ.

In Figure 9 depicts the effect of Prandtl number on the temperature field. It is observed that an increase in the Prandtl number leads to decrease in the temperature field, because, either increases of kinematic viscosity or decrease of thermal conductivity leads to increase in the value of Prandtl number. Hence temperature decreases with increasing of Prandtl number. Figure 10 tells the influence of thermal radiation conduction on the temperature. It is cleared that temperature is decrease when R is increase. Figure 11 illustrate the influence of the heat absorption coefficient Q on the temperature profiles at t = 1.0, respectively. Physically speaking, the presence of heat absorption (thermal sink) effects has the tendency to reduce the fluid temperature. This causes the thermal buoyancy effects to decrease resulting in a net reduction in the fluid temperature. This behavior clearly obvious from Figure 11, in temperature distributions decrease as Q increases. It is also observed that the thermal (temperature) boundary layer decrease as the heat absorption effects increase.

Influence of Schmidt number on concentration is shown in Figure 12, from this figure it is noticed that concentration decreases with an increase in Schmidt number. Because, Schmidt number is a dimensionless number defined as the ratio of momentum diffusivity and mass diffusivity, and is used to characterize fluid flows in which there are simultaneous momentum and mass diffusion convection processes. Therefore concentration boundary layer decreases with an increase in Schmidt number. Figure 13 display the effects of the chemical reaction parameteron the concentration profiles at t = 1. As the chemical reaction parameter “Kc” increases, the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid concentration. This behavior is clearly seen in Figure 13. Figure 14 displays the influence of Soret number on concentration profile. From this figure, we may conclude that the concentration profile enhances with an increase in the Soret number. This is due to the fact that an increase in Soret number causes for concentration boundary layers.

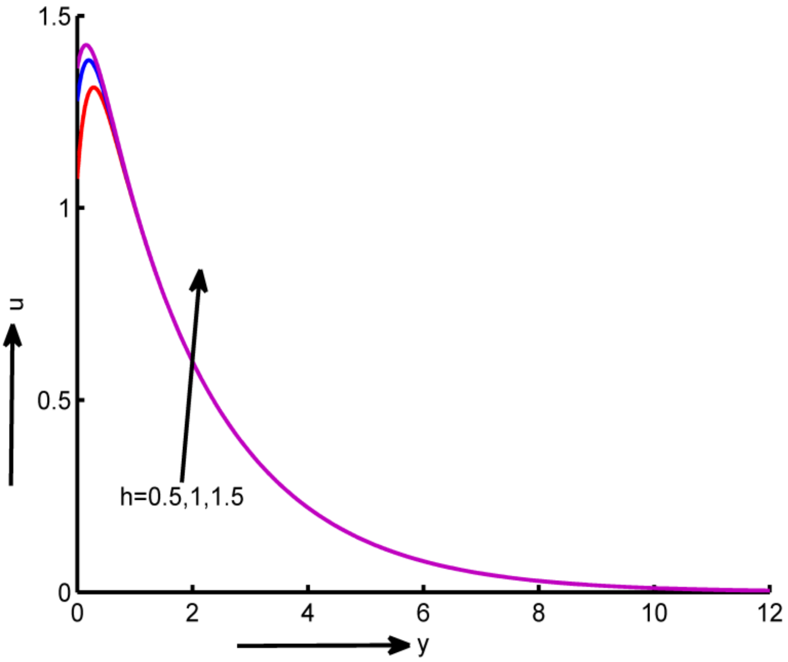

Fig. 2. Effects of slip parameter (h) on velocity

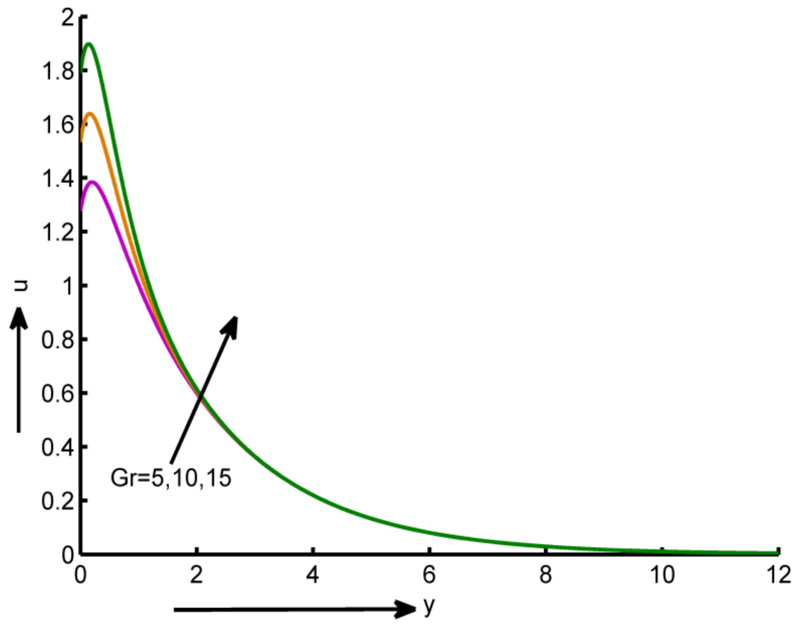

Fig. 3. Effects of the Grashof number (Gr) on velocity

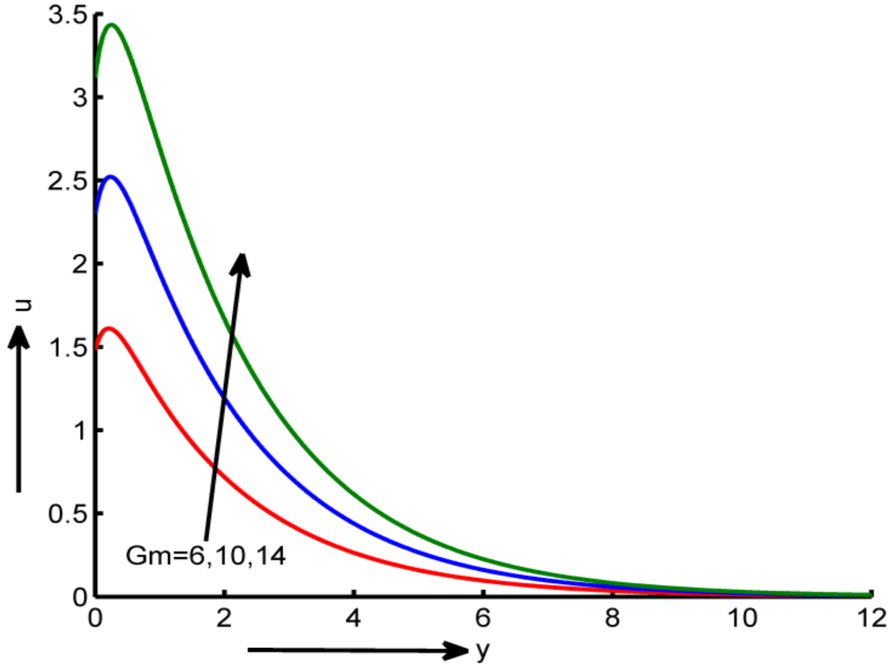

Fig. 4. Effects of modified Grashof number (Gm) on velocity

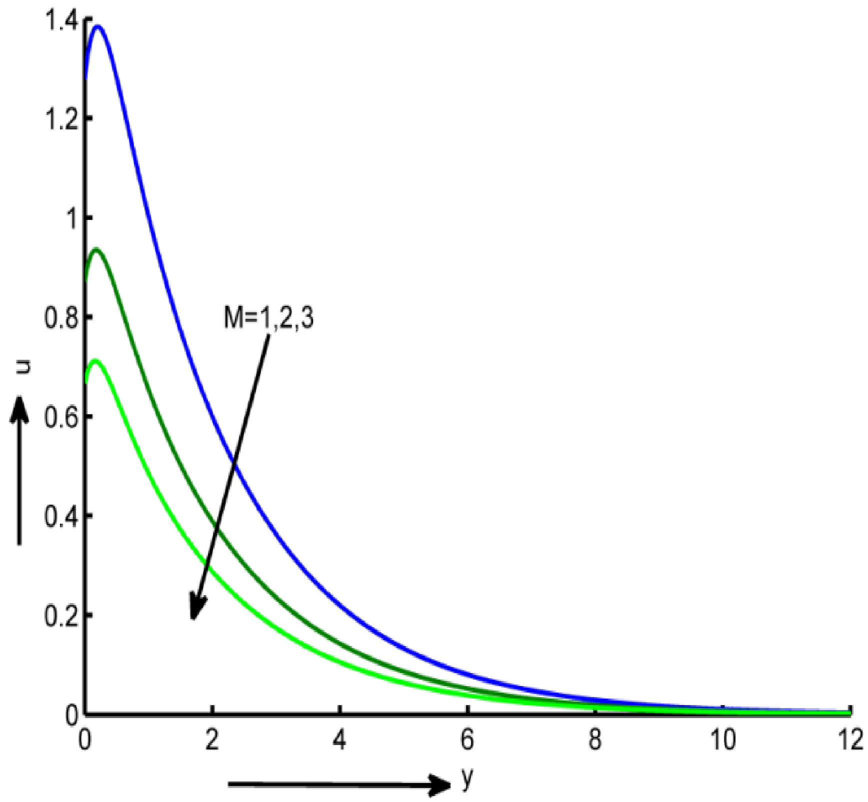

Fig. 5. Effects of magnetic parameter (M) on velocity

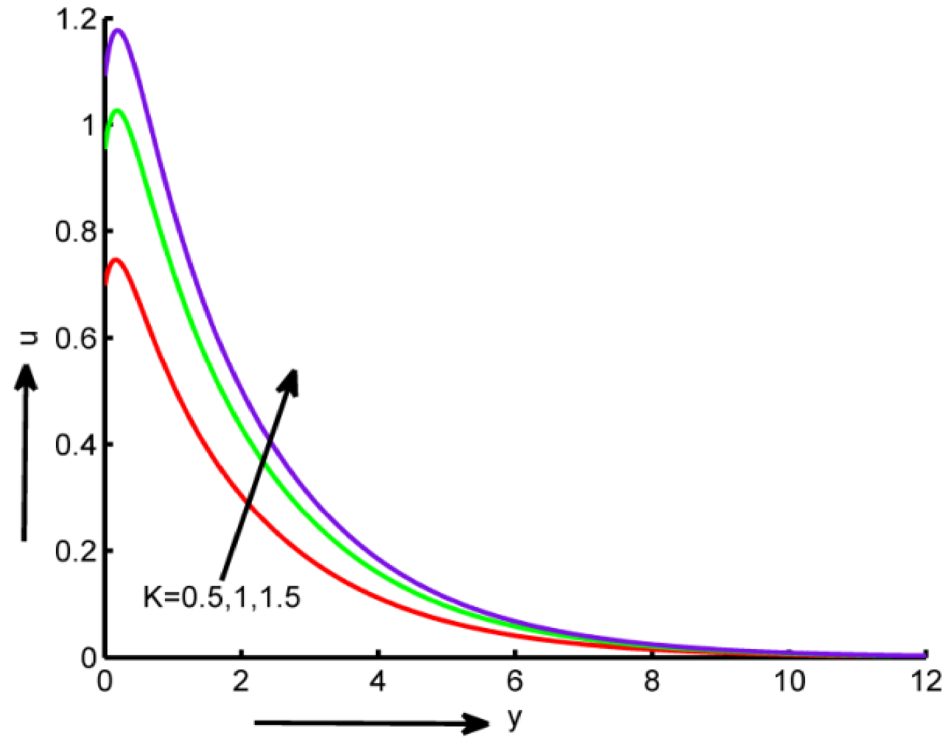

Fig. 6. Effects of permeability parameter (K) on velocity

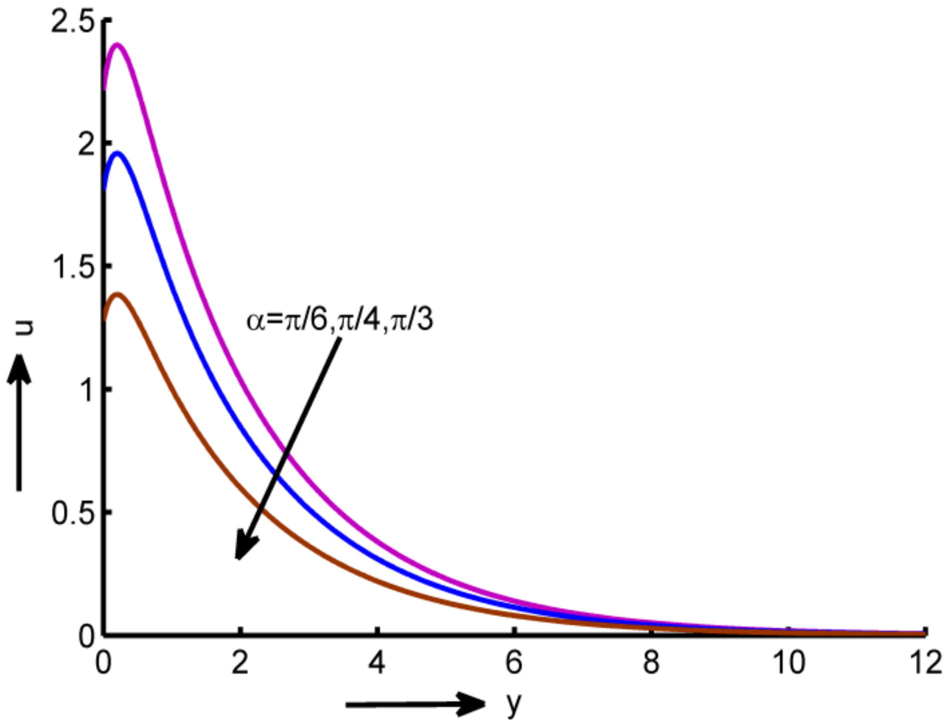

Fig. 7. Effects of inclined angle (α) on velocity

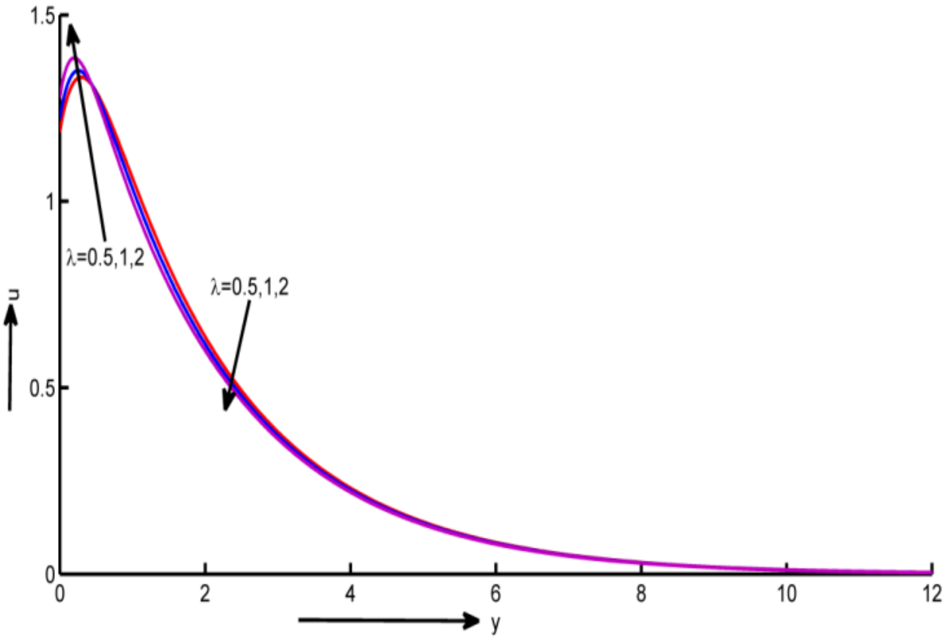

Fig. 8. Effects of Jeffery parameter (λ) on velocity

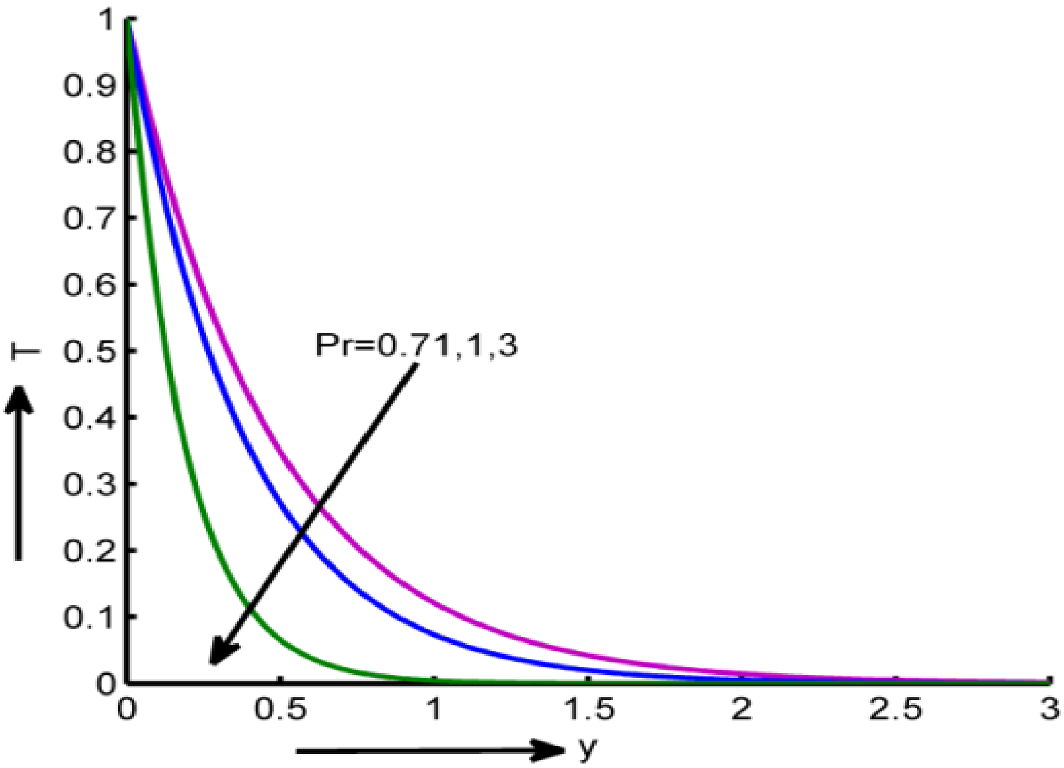

Fig. 9. Effect of the Prandtl number (Pr) on temperature

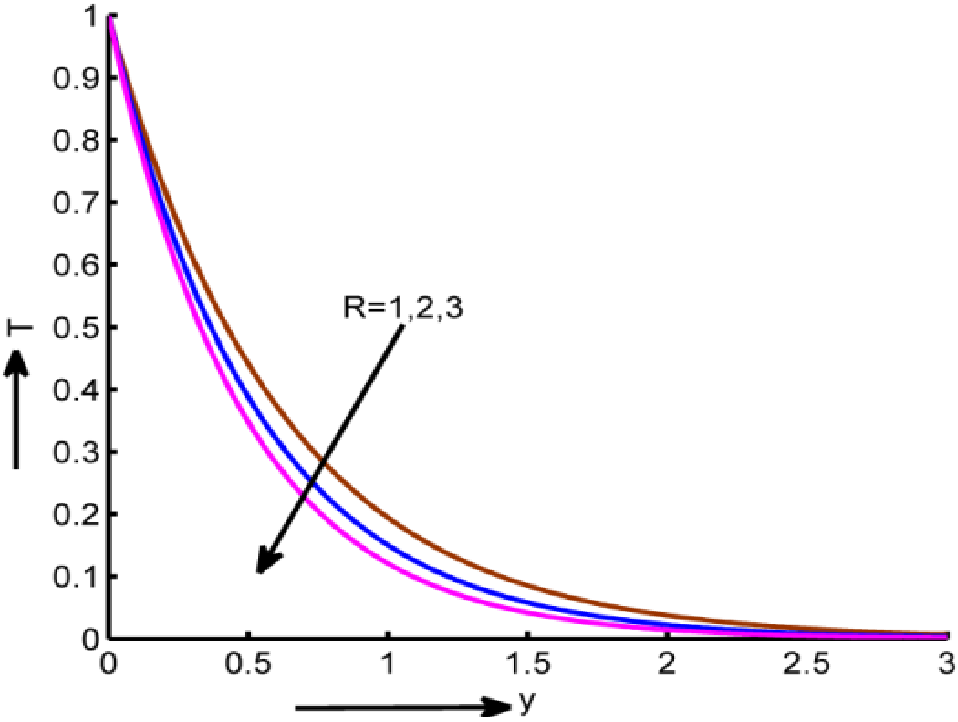

Fig. 10. Effects of radiation parameter (R) on temperature

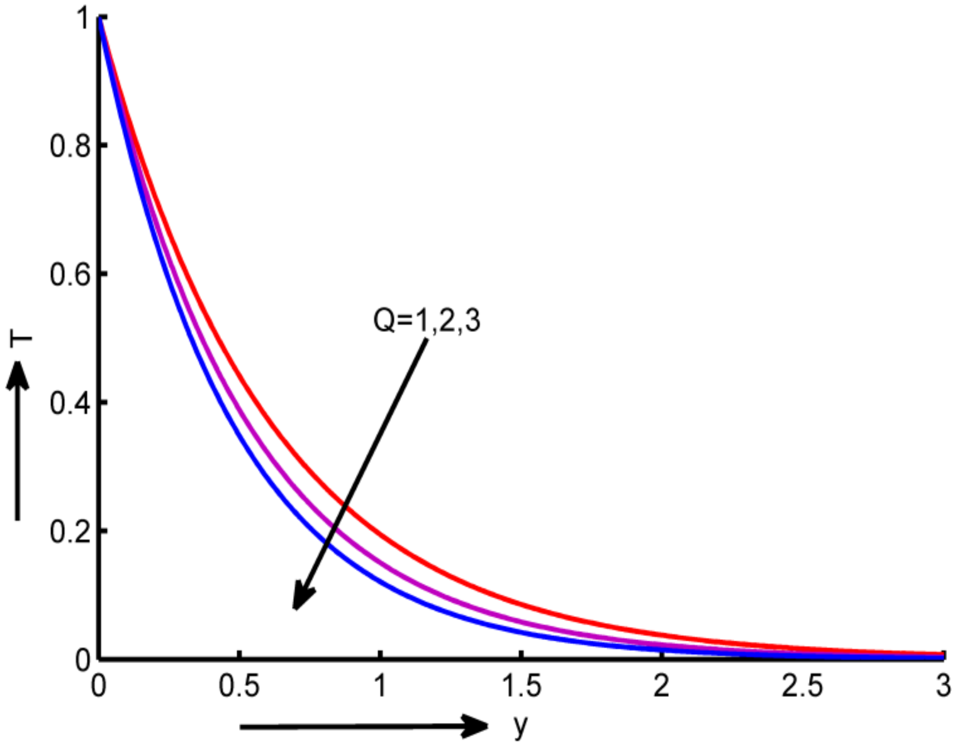

Fig. 11. Effects of Heat source parameter (Q) on temperature

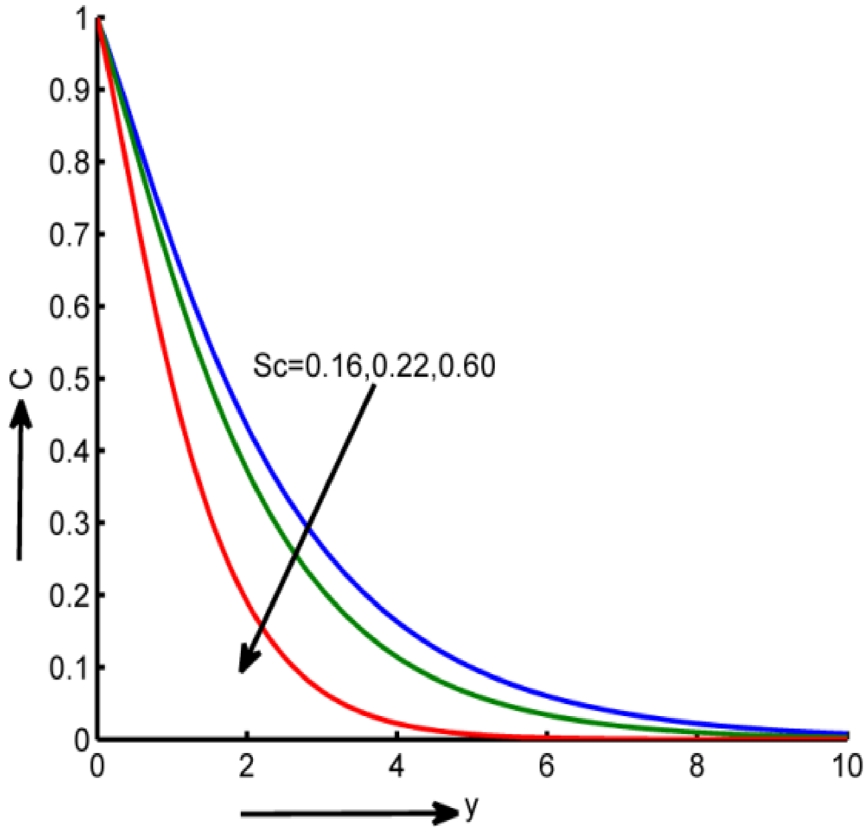

Fig. 12. Effects of Schmidt number (Sc) on concentration

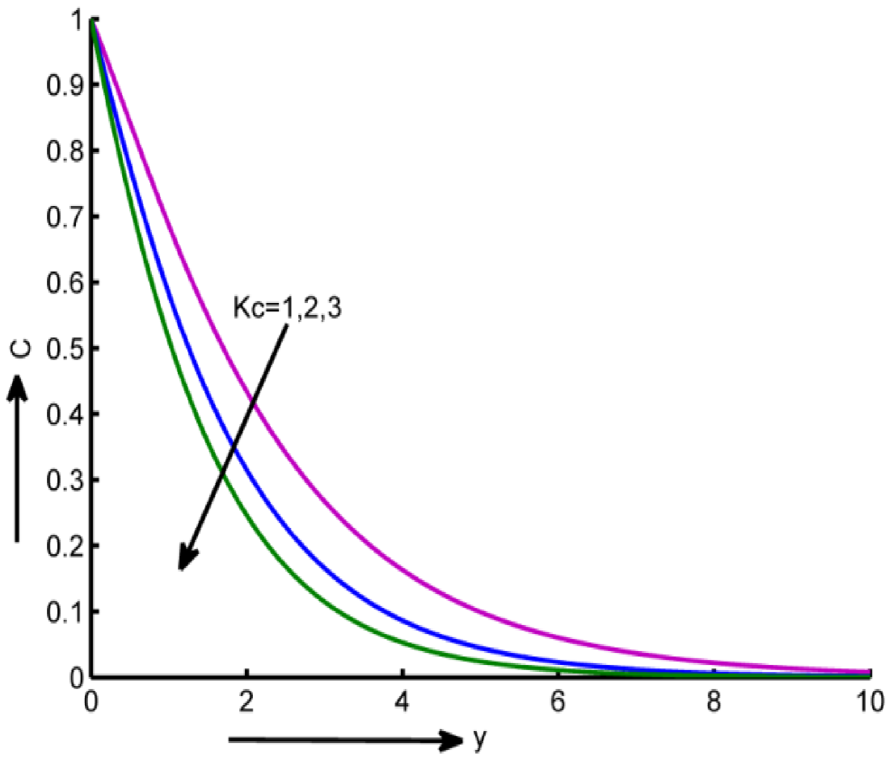

Fig. 13. Effects of chemical reaction parameter (Kc) on concentration

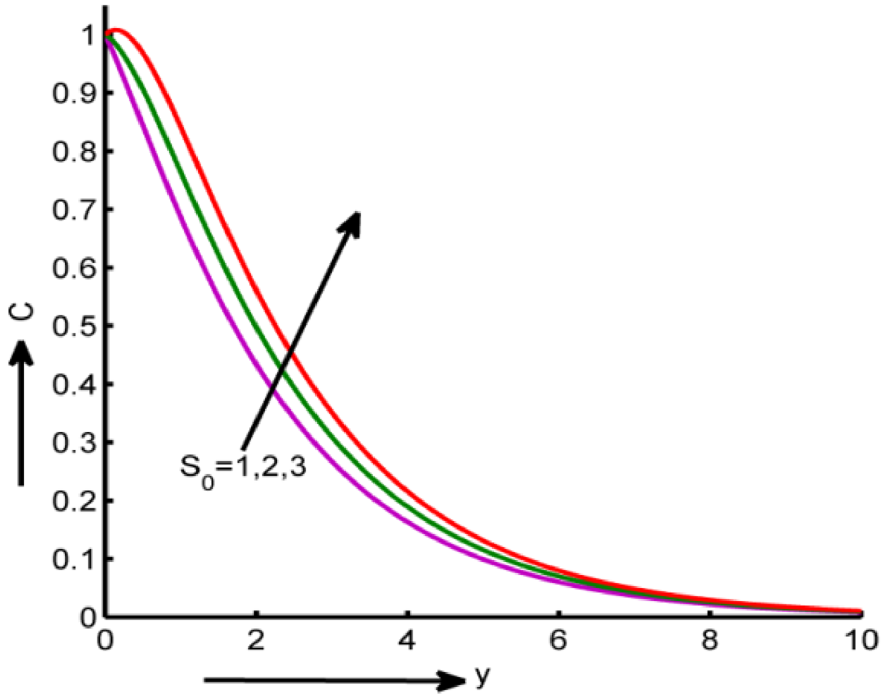

Fig. 14. Effects of the Soret number (S_0) on concentration

Table 1 show numerical values of skin-friction for several of Grashof number (Gr), modified Grashof number (Gm), Magnetic parameter (M), Porosity parameter (K) and slip parameter (h). From table 1, it is observed that the skin-friction increases with an increase in Grashof number (Gr), modified Grashof number (Gm), Porosity parameter (K) where as it decreases under the influence of magnetic parameter (M), slip parameter (h), Jeffery parameter (λ) and inclined angle (α).

Table 1
Skin friction

Gr	Gm	M	K	h	λ	α	τ
6							1.4396
10							1.7241
14							2.0085
	5						0.7633
	12						2.2731
	15						2.9201
		2					0.4306
		2.5					0.3404
		3					0.2745
			2				0.6786
			3				0.7633
			4				0.8123
				1			0.7633
				2			0.4201
				3			0.2898
					2		0.7633
					2.5		0.7517
					3		0.7417
						π/6	1.3221
						π/4	1.0795
						π/3	0.7633

Table 2 demonstrates the numerical values of Nusselt number (Nu) for different values of Prandtl number (Pr), Radiation parameter (R), Heat source parameter (Q). From table 2, we notice that the Nusselt number increases with an increase in Prandtl number, Radiation parameter and Heat source parameter.

Table 2
Nusselt number

Pr	R	Q	Nu
0.71			1.6369
1			2.0562
7			9.1141
	2		1.8937
	3		2.1128
	4		2.3072
		0.5	1.4864
		2	1.8937
		4	2.3072

Table 3 shows numerical values of Sherwood number (Sh) for the distinction values of Schmidt number (Sc), Chemical reaction parameter (Kc) and Soret parameter (S_0). It can be noticed from Table - 3 that the Sherwood number enhances with rising values of Schmidt number, chemical reaction parameter where as it decreases under the influence of Soret parameter. 1111222223

Table 3
Sherwood number

Sc	Kc	S_0	Sh
0.16			0.2619
0.22			0.2874
0.60			0.3580
	0.5		0.1331
	1		0.2619
	1.5		0.3602
		2	0.0442
		4	-0.3912
		6	-0.8266

Table 4 displays the validation of present results with that of published results of Ravikumar *et al.,* [41] by taking the influence of Prandtl number on Nusselt number in the absence of Jeffery parameter and angle of inclination. An excellent agreement is noticed in this comparison.

Table 4
Comparison of present results with published results

Pr	Nu Results of Ravikumar *et al.,* [41]	Nu present results
0.71	1.6369	1.635482
1	2.0562	2.045864
7	9.1141	9.113946

6. Conclusions

In this problem, the characteristics of MHD Jeffery fluid past an inclined vertical porous plate are investigated. In the analysis of the flow the following conclusions are made.

i. Fluid velocity increases with an increase in permeability parameter, Grashof number, modified Grashof number and Slip parameter where as it decreases under the influence of magnetic parameter and inclined angle.
ii. Fluid temperature decreases with rising values of Prandtl number, radiation parameter, and heat absorption parameter.
iii. Concentration decreases with an increase in the Schmidt number and chemical reaction Parameter and shows a reverse effect with an increase in the Soret effect.As significant acceleration in the seen in skin friction for increase permeability parameter.
iv. Grashofnumber modified Grashofnumberwhere as it decreases under the influence of magnetic parameter, slip parameter, Jeffery parameter and inclined angle.
v. The rate of heat transfer increases with an increase Prandtl number, heat absorption parameter, and radiation parameter.
vi. The rate of mass transfer accelerated with rising values of Schmidt number, chemical reaction parameter where as it decreases under the influence of Soret parameter.

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