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The Effects of an Aligned Magnetic Field on Nanofluid Flow with Newtonian Heating

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ABSTRACT

Magnetic field involvement can influence heat absorption in electrically conducting fluid flow, which is useful in the control of features of final products in industries such as radiation therapy, aeronautics, and MHD generators. Consequently, this study investigates the effect of an aligned magnetic field on nanofluid flow over a stretching sheet with the boundary condition of Newtonian heating. Steady nanofluid flow with copper as chosen nanoparticles and water as conventional base fluid is considered. The problem is governed by a system of nonlinear boundary layer equations with appropriate boundary conditions which are then transformed into non-dimensionless equations using an appropriate similarity transformation. A numerical approach known as Keller-Box method is used to solve the transformed governing equations. The numerical solutions obtained are presented graphically in the form of velocity and temperature profiles for different values of aligned angle of magnetic field, Newtonian heating parameter, Prandtl number and nanoparticles volume fraction. A significant increase in aligned angle results in a decrease in fluid velocity, but an increase in temperature profile of nanofluid flow. The increment in the Newtonian heating parameter as well as the nanoparticle volume fraction also causes the temperature to increase.

1. Introduction

Nanofluid has received a huge attention for its enhanced thermal properties. The term nanofluid was first introduced by Choi [1] which contains solid nanoparticles with average particles sizes of 1-100 nanometer in conventional heat transfer fluids such as water, ethylene glycol and oil. The combination of two substances is likely to produce a heat transfer medium that behaves as a fluid but has the thermal properties of metal [2]. Based on previous studies, it is found that as solid volume fraction increases, the effect of heat transfer capacity of base fluid is more pronounced. Therefore, to explore more on the effects of flow thermophysical parameters such as nanoparticle volume fraction, Mabood *et al.,* [3] proposed a mathematical model using Tiwari-Das model to study the

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magnetohydrodynamic (MHD) stagnation-point flow and heat transfer characteristics of an electrically conducting nanofluid. The effect of nanoparticle volume fraction toward the skin friction and heat transfer coefficients was studied by Dzulkifli *et al.,* [4]. Their findings showed that as the nanoparticles volume fraction in the base fluid increases, velocity slip between the fluid decreases, which resulting in the increasing of skin friction coefficient. Dutta *et al.,* [5] studied the effect of magnetic field on the mixed convection of Al_2O_3-H_2O nanofluid. The results showed that heat transfer in the presence of magnetic field is noticeable at a higher value of nanoparticle volume fraction. Recently, Rubaa'i *et al.,* [6] who studied the effect of non-uniform heat source/sink on the hybrid nanofluid mixed convection flow over a stretching sheet concluded the concentration of the chosen nanoparticles significantly contributed to the enhancement of heat transfer rate. Meanwhile, Low *et al.,* [7] investigated the heat transfer characteristics of dusty nanofluid over a moving plate in the presence of MHD with convective boundary condition. The comparative analysis between copper-oxide (CuO) and aluminium oxide (Al_2O_3) -water dusty nanofluid showed that the increasing of the volume fraction of nanoparticles and volume fraction in dust particles significantly enhanced the temperature profiles of the flow.

The involvement of aligned magnetic field may affect heat transfer process in the boundary layer flow for numerous fluid and geometry surface. When the aligned angle of magnetic field increases, the applied magnetic field strength will be weakened causing the efficiency of heat transfer to decrease. In a study by Udhayakumar *et al.,* [8], the influence of aligned magnetic field on incompressible and electrically conducting fluid is investigated. The results indicated that the applied magnetic field is useful to suppress the flow separation hence heat convection is controlled. Haq *et al.,* [9] analyzed the combined effects of the inclined magnetic field and velocity slip conditions. The results of the study indicated that aligning the magnetic field angle has a fundamental influence on controlling the magnetic field on the nanofluid flow. Ullah *et al.,* [10] focused on the numerical solutions on the flow of aligned magnetic field and heat transfer over stretching sheet with Newtonian heating. They found that the increment of aligned angle is to strengthen the magnetic field thus leading to the decreasing of the fluid velocity and increase of temperature profiles. In the following year, Ashwinkumar *et al.,* [11] investigated the heat and mass transfer characteristics of magnetic-nanofluid flow in the presence of aligned magnetic field. It is found that the flow and thermal transport phenomenon is more effective in the case of aligned magnetic field in comparison with transverse magnetic field. Khan *et al.,* [12] compared the different types of hybrid nanofluid flow along a stretched surface with the presence of aligned magnetic field, non-linear radiation, and suction effects. The authors concluded that suspension of multiple solid nanoparticles in the composition of conventional base fluids provides a better rate of heat transfer and limits the friction drag. Kumar *et al.,* [13] have analyzed the Blasius and Rayleigh-Stokes hybrid nanofluid under aligned magnetic field with the presence of ohmic heating effect.

Newtonian heating is a thermal boundary condition that greatly influences heat transfer characteristics. In this heating system, the heat transfer rate from the bounding surface with a finite heat capacity is proportional to the local surface temperature. It is defined as the process in which internal resistance is negligible compared to surface resistance. Merkin [14] was the first to discover that Newtonian heating can be set up in the convection flow from the surface. He found the solution both analytically and numerically near the leading edge and the full solution along the whole plate for free convection boundary layer over vertical surfaces respectively. Since then, researchers have shown their interest in Newtonian heating instead of constant surface temperature. A numerical analysis reported by Kamran and Wiwatanapataphee [15] who investigated the incompressible mixed convective micropolar fluid flow discovered that the fluid temperature increases with the increasing Newtonian heating parameter, thus causing the thermal boundary layer thickness to increase. Another study was done by Kumar [16] focused on the effect of Newtonian heating/cooling on electrically conducting flow of free convection by considering induced magnetic field. The result showed that temperature field profiles are to increase with Newtonian heating effect. Recently, Habib *et al.,* [17] numerically investigated the non-Newtonian Jeffrey MHD fluid flow with boundary condition of Newtonian heating with the influence of viscous dissipation over an exponential stretching sheet.

Motivated from the above literature, the present study aims to examine the impact of Newtonian heating with aligned magnetic field effect on the nanofluid flow past a stretching sheet by using the nanofluid model as studied by Tiwari and Das [18], which incorporates the effect of solid nanoparticle volume fraction. The governing partial differential equations are then transformed into a set of coupled dimensionless ordinary differential equations by using appropriate similarity transformation. The transformed governing equations are then solved numerically using an implicit finite difference scheme known as Keller-box method.

2. Mathematical Formulation

The two-dimensional, incompressible nanofluid flow over a stretching sheet in the presence of Newtonian heating (NH) is considered. Copper (Cu) nanoparticles is chosen, and the problem is governed by the nanofluid model proposed by Tiwari and Das [18]. The aligned magnetic field is set to the flow with an acute angle, α_1. The governing boundary layer equations are given previous study by Arifin *et al.,* [19],

$$\frac{\partial u}{\partial x}+\frac{dv}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma}{\rho_{nf}}uB_0{}^2\sin^2\alpha_1, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\left(\frac{\partial^2 T}{\partial y^2}\right), \tag{3}$$

Subject to the boundary conditions,

$$\begin{gathered} u=u_w(x)=ax, T=T_w=T_\infty+cx, \frac{\partial T}{\partial y}=-h_sT(\mathrm{NH}) \text{ at } y=0 \\ u\to 0, T\to T_\infty, \text{ as } y\to\infty, \end{gathered} \tag{4}$$

where u and v denote the velocity components in *x*-direction and *y*-direction, respectively, μ_{nf} is the dynamic viscosity of the nanofluid, ρ_{nf} is the density of the nanofluid, σ_{nf} is the electrical conductivity, B_0 is the magnetic field strength, α_1 us the inclined angle, T is the fluid temperature and α_{nf} is the thermal diffusivity. $u_w(x)$ is the velocity of the stretching surface with *a* being a positive constant, T_∞ is the ambient temperature and h_s is the heat transfer parameter. Under the assumption that water as the base fluid and nanoparticles are in thermal equilibrium and in a no-slip condition, the following nanofluid expressions are introduced [20, 21].

$$\alpha_{nf}=\frac{k_{nf}}{(\rho c_p)_{nf}},\mu_{nf}=\frac{\mu_f}{(1-\varphi)^{2.5}},\rho_{nf}=(1-\varphi)\rho_f+\varphi\rho_s,$$
$$(\rho c_p)_{nf}=(1-\varphi)(\rho c_p)_f+\varphi(\rho c_p)_s,\frac{k_{nf}}{k_f}=\frac{(k_s+2k_f)-2\varphi(k_f-k_s)}{(k_s+2k_f)+\varphi(k_f-k_s)}, \tag{5}$$

Where μ_f the viscosity of the fluid fraction is, φ is the nanoparticle volume fraction, ρ_f is the reference density of the fluid fraction, ρ_s is the reference density of the solid fraction. k_{nf} is the thermal conductivity of nanofluid, c_p is the specific heat at constant pressure, k_f is the thermal conductivity of the fluid fraction, k_s is the thermal conductivity of the solid volume fraction. The thermophysical characteristics of nanofluid with respect to thermal conductivity and viscosity models, as well as the analytical model, as given by Eq. (5) have been discussed in detail by Khanafer and Vafai [22].

The following similarity transformation are adopted from studies by Arifin *et al.,* [19] and Rawi *et al.,* [21], given as follows,

$$\eta=\left(\frac{a}{\upsilon_f}\right)^{\frac{1}{2}}y,\psi=\left(a\upsilon_f\right)^{\frac{1}{2}}xf(\eta),\theta(\eta)=\frac{T-T_\infty}{T_\infty}(\text{NH}), \tag{6}$$

Where the stream function, ψ defined as $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$. By using nanofluid expressions Eq. (5) and similarity transformation Eq. (6), the system of partial differential Eq. (1) to Eq. (3) are transformed into,

$$\frac{1}{(1-\varphi)^{2.5}}f'''(\eta)+\left((1-\varphi)+\varphi\frac{\rho_s}{\rho_f}\right)f(\eta)f''(\eta)-\left((1-\varphi)+\varphi\frac{\rho_s}{\rho_f}\right)f'^2(\eta)-Mf'(\eta)\sin^2\alpha_1=0, \tag{7}$$

$$\theta''(\eta)+\Pr\frac{k_{nf}}{k_f}f(\eta)\theta'(\eta)=0, \tag{8}$$

with the transformed boundary conditions,

$$\begin{aligned}&f'(\eta)=1,\theta(\eta)=1,\theta'(\eta)=-\gamma\left[1+\theta(\eta)\right]\text{ at }\eta=0\\&f'(\eta)\to0,\theta(\eta)\to0\text{ as }\eta\to0,\end{aligned} \tag{9}$$

Where $M=\frac{\sigma B_0^{\ 2}}{a\rho_f}$ is the magnetic field parameter, $\Pr=\frac{\nu_f}{\alpha_f}$ is the Prandtl number and $\gamma=h_s\sqrt{\frac{\nu_f}{a}}$ is the conjugate parameter for Newtonian heating. The thermophysical properties of the chosen nanoparticles and base fluid can be found in Table 1 [20].

Table 1
Base fluid and nanoparticle thermophysical characteristics

Properties	Water	Copper
$\rho\ (kgm^{-3})$	997.1	8933
$C_p(\ Jkg^{-1}K^{-1})$	4179	385
$k\ (\ Wm^{-1}K^{-1})$	0.613	401
Pr	6.2	-

3. Results and Discussion

The transformed governing equations are subjected to the boundary conditions which highlighted the influence of Newtonian heating are solved numerically using Keller-Box method. The use of this method has been found to be very effective in solving nonlinear parabolic problems [23-25]. The graphical results are presented in the form of velocity and temperature profiles for the parameters of interest. Figure 1 presents the influence of Prandtl number, Pr on nanofluid through the temperature profile. It can be observed that, an increase in Pr decreases the thermal boundary layer thickness and generally, decreases the temperature profile.

Figure 2 presents the effect of various values of Newtonian heating parameter, γ on temperature profile. It is evident that the temperature increases with increasing values of Newtonian heating parameter. Since the momentum equation is independent of this parameter, the velocity profile remains unchanged for different values of γ . From Figure 3, it can be examined that with the increase of aligned angle of magnetic field cause the fluid's temperature profile to increase as well. This is due to the strengthening of applied magnetic field when aligned angle is raised. On the contrary, the velocity distribution is seen to be declining as the angle of aligned magnetic field increases in Figure 4. This is because the increment in the aligned angle strengthens the magnetic flux that acts opposite to the flow causing the Lorentz force to increase which results in the deceleration of fluid flow [26].

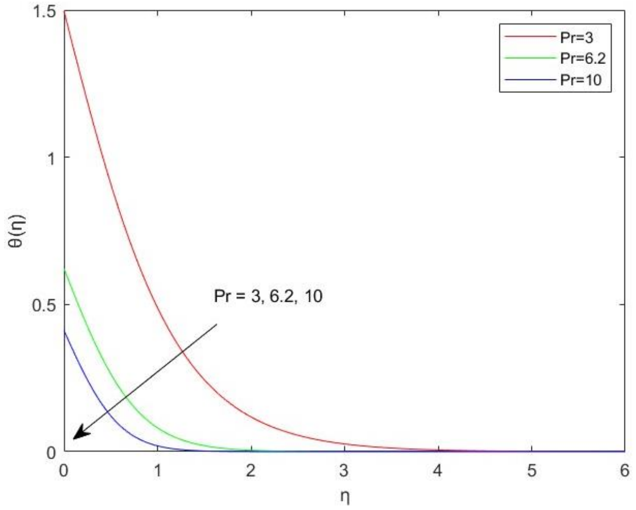

Fig. 1. Temperature profile for various values of Pr

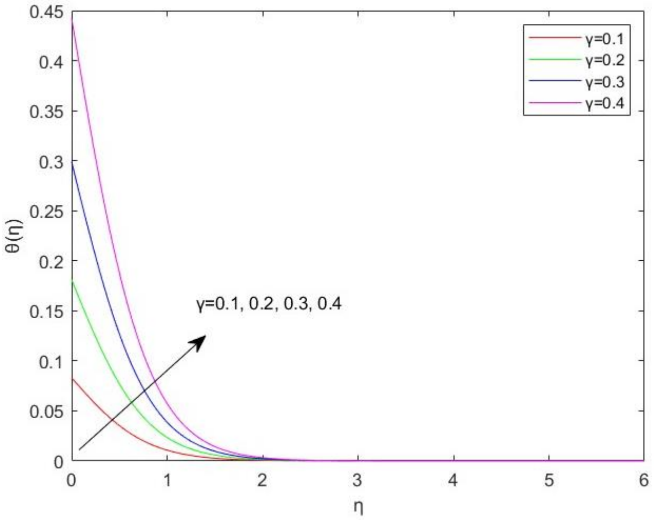

Fig. 2. Temperature profile for various values of γ

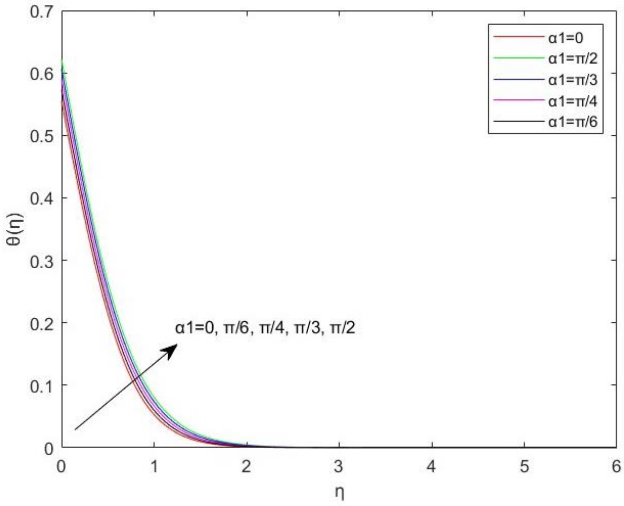

Fig. 3. Temperature profile for various values of α_1

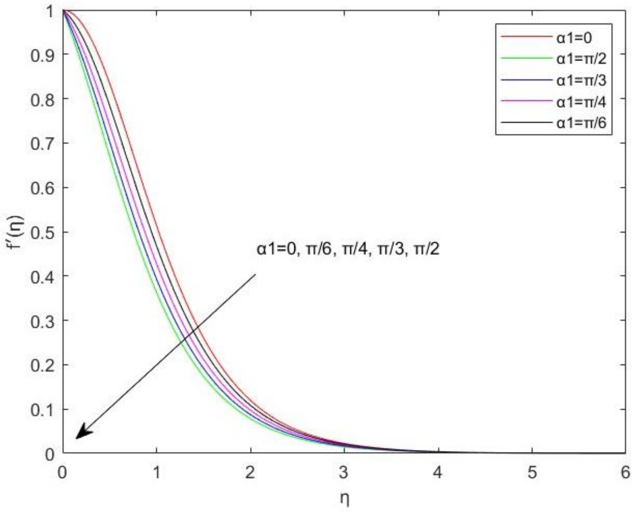

Fig. 4. Velocity profile for various values of α_1

Figure 5 illustrates the temperature profile for different values of nanoparticles volume fraction. For the increasing value of nanoparticles volume fraction, temperature profile also increases due to the rise in concentration of nanoparticles. This improves the kinetic energy in the base fluid which simultaneously improves the fluid temperature. Figure 6 presents the velocity profile for various values of nanoparticles volume fraction. Fluctuating behavior can be seen from this graph where it started to decrease with the increasing value of φ but then increase again.

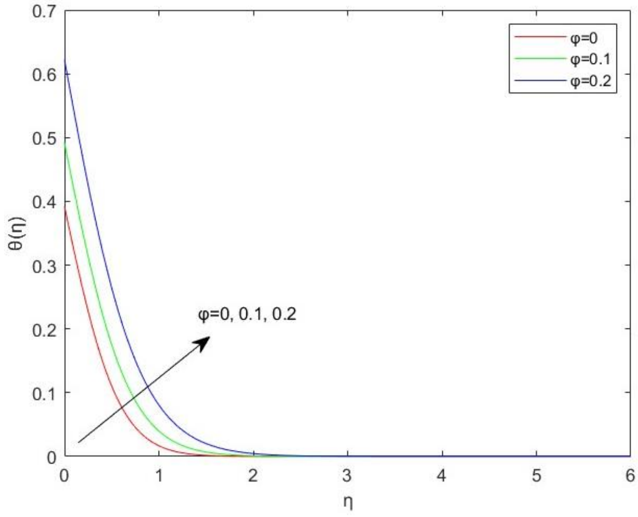

Fig. 5. Temperature profile for various values of φ

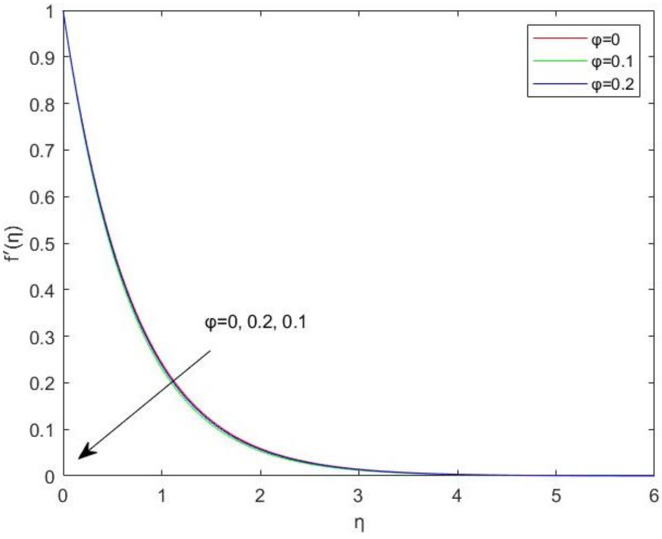

Fig. 6. Velocity profile for various values of φ

4. Conclusions

In this study, we studied and discussed in the details the influence of aligned magnetic field on the two-dimensional nanofluid flow over a stretching sheet with boundary condition of Newtonian heating by utilizing the importance of nanoparticles volume fraction. The transformed governing equations was solved using a finite difference method known as Keller-box method. Based on the numerical results, it can be concluded that, the adjustment of the aligned angle of magnetic field caused the temperature of the fluid flow to increase but depicted the contradict behaviour for the velocity of the fluid. Meanwhile, by increasing the Newtonian heating as well as the volume fraction of the chosen nanoparticles have raised the temperature of nanofluid in the heat transfer system.

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