

# MHD Casson Fluid Flow in Stagnation-Point over an Inclined Porous Surface

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ARTICLE INFO	ABSTRACT
Article history: Received 30 June 2023 Received in revised form 25 July 2023 Accepted 21 August 2023 Available online 1 January 2024	Motivated by the need to comprehend and optimize complex fluid flow phenomena in various engineering and industrial applications, this paper investigates the magnetohydrodynamic (MHD) Casson fluid flow characteristics in the vicinity of a stagnation point over an inclined porous surface. The study addresses the interplay of permeability, viscous dissipation, buoyancy, and volumetric heat source, chemical reaction of the diffusion species, thermal slip, and obliqueness at the bounding surface. The governing equations are transformed into a dimensionless form using appropriate similarity transformations. The resulting nonlinear ordinary differential equations are solved numerically using the fourth-order Runge-Kutta method, coupled with the shooting
Keywords:	technique as coded into the bvp4c solver of MATLAB 2021a. Findings from this study show
MHD; Stagnation Point; Viscous	that instability arises due to reduced velocity at low permeability, and Biot number
dissipation; chemical reaction;	enhances the Newtonian cooling at the surface, a requirement for the design of heat
Thermal radiation	exchangers.

#### 1. Introduction

The significance of the non-Newtonian fluids flow in engineering and industrial applications, especially in chemical processes, has been lately piqued. Due to the fact that the exact horizontal or vertical surfaces are not easy to identify in some industrial processes, it becomes necessary to pay some attention to the study of inclined surfaces. Moreover, a little adjustment in the angle of inclination is enough to bring about the required result in these industrial processes. In the industrial process of forming glass, casting metals, and spinning fibres, stretching of surfaces is necessitated. Crane [1] obtained a solution for an incompressible fluid across a stretching surface. Mohapatra *et al.,* [2, 3] have examined the effect of visco-elastic fluid flow on a deformable surface. Oke and Mutuku [4] carried out a comprehensive review of the Eyring-Powell flow in the existence of a magnetic field (MF) and stretching surface under convective heating. Parida *et al.,* [5] have explored the impact of viscous dissipative nanofluid flow over a deformable sheet. They noticed that

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strengthening of chemical reactiveness leads to reduced fluid motion and mass concentration. In solar thermal collectors and refrigeration, the simultaneous study of heat transfer and mass transport in MHD flow on stretched surfaces finds significance.

Many scholars work on fluid models such as the viscoelastic, Casson fluid, Oldroyd fluid and nanofluid models [6 - 18]. Shehzad *et al.*, [19] expanded Rahman's [20] work to the Jeffrey fluid model. Swain and Senapati [21] investigated mass transport on MHD flow past a vertical plate that started spontaneously. Chaim [22] and Abel *et al.*, [23] studied dissipative flow in an MHD flow over a sheet stretching. Thorough research on the nature of dissipative flows in various geometrical surface structures is also a subject of interest [24 - 31]. Importantly, the outcomes show that chemical reaction has a significant effect on flows. The study of heat transport along an inclined porous surface is crucial in various industries such as paper production, glass manufacturing, plastic film production, wire drawing, and more. Juma *et al.*, [32] explored the MHD Williamson fluid flow across a rotating inclined plane.

This research presents a novel investigation into the magnetohydrodynamic (MHD) Casson fluid flow near a stagnation point over an inclined porous surface, addressing a significant research gap in the field of fluid dynamics. The originality of this paper lies in its comprehensive exploration of complex fluid flow phenomena, including the intricate interplay of permeability, viscous dissipation, buoyancy effects, volumetric heat source, chemical reactions of diffusing species, thermal slip, and surface inclination. While existing literature has provided foundational insights into MHD flows and porous media, our study advances the field by uniquely combining these factors. The practical significance of this research is underscored by its relevance to heat exchanger design and performance optimization in various industrial processes. By employing numerical techniques and innovative mathematical transformations, we bridge the existing knowledge gap, providing a clear roadmap for further research in this domain and offering valuable insights into fluid dynamics, which have critical implications for engineering and industrial applications. This work not only enhances our understanding of complex fluid flow but also serves as a foundation for future investigations in optimizing fluid dynamics in porous media and industrial systems.

## 2. Mathematical Formulation

A 2-D MHD Casson fluid free-convective incompressible flow towards an inclined nonlinear porous expanding sheet is studied (see Figure 1). Inclination of the surface at angle  $\alpha$  ( $0 \le \alpha \le \pi/2$ ). The flow is in the upper region y > 0 under a normal uniform MF of strength $B(x) = \frac{B_0}{3\sqrt{x}}$ . The surface is undergoing a nonlinear stretching with velocity  $u_w(x) = c\sqrt[3]{x}$ , where a stretching intensity is. The flow arises in fluids near a stagnation point attributable to relations of gravity and density difference down to diffusion of chemical species, thermal energy diffusion, and stretching of the inclined sheet. The temperature T and concentration C have values  $T_w$  and  $C_w$  precisely on the surface. The characteristics of thermal radiation are investigated through convective heating involving temperature  $T_f$  and a heat exchange factor  $h_f$  that is inversely proportional to x. However, the temperature  $T_{\infty}$  and mass partition  $C_{\infty}$  for nano-liquids at the free stream are as illustrated in Figure 1. The mathematical representation of the Casson fluid is expressed as follows [33]:

$$\tau_{ij} = \begin{cases} 2\left(\mu_b + \left(\frac{P_y^2}{2\pi_0}\right)^{\frac{1}{2}}\right) e_{ij}, & \pi_0 > \pi_c \\ \\ 2\left(\mu_b + \left(\frac{P_y^2}{2\pi_c}\right)^{\frac{1}{2}}\right) e_{ij}, & \pi_0 < \pi_c \end{cases}$$
(1)

where  $\tau_{ij}$  represents the  $(i, j)^{th}$  stress tensor component,  $p_y$  indicates fluid's yield stress,  $\mu_B$  is synthetic absolute fluids' viscosity, and  $\pi$  is the component of the deformation rate multiplied by itself, which is defined as  $\pi_0 = e_{ij}e_{ij}$ , and  $e_{ij}$  is  $(i, j)^{th}$  deformation rate, and  $\pi_c$  is considered as the critical value of  $\pi_0$ .



Fig. 1. Geometry of the problem

The governing equations for flow separation and Boussinesq approximation were specified by Eq. (2) to Eq. (5), which are based on Reddy *et al.*, [33]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \left[\frac{\sigma B_0^2}{\rho} + \frac{v}{K}\right]u + \left[g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty)\right]\cos(a)$$
(3)

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C p}\frac{\partial^2 T}{\partial y^2} - \frac{\mu}{\rho C p}\left(1 + \frac{1}{\beta}\right)\left(\frac{\partial u}{\partial y}\right)^2 - \frac{1}{\rho C p}\frac{\partial q_r}{\partial y} + \frac{Q_1}{\rho C p}(T - T_\infty)$$
(4)

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_M \frac{\partial^2 C}{\partial y^2} - Kr'(C - C_{\infty})$$
(5)

with the boundary conditions:

$$\begin{cases} u = u_w(x), v = v_w, -k\frac{\partial T}{\partial y} = h_f(x)(T_w - T), C = C_\infty + bx, \text{ at } y = 0\\ u \to 0, T \to T_\infty, C \to C_\infty \text{ as } y \to \infty \end{cases}$$
(6)

where velocity in x and y- planes are u and v, K is the porous medium specification,  $\beta$  is Casson fluid parameter, g is gravity acceleration, Kr is chemical reaction parameterv is fluid kinematic viscosity,  $\rho$  is fluid density,  $\sigma$  is electrical conductivity, a is angle of inclination, Cp is specific heat, DM is molecular diffusion,  $q_r$  is radiative heat flux, and Q<sub>1</sub> is heat absorption.

By taking into account the Rosseland dispersion for the  $q_r$  [29], it is possible to minimise the viability condition Eq. (4), and we get.

$$q_r = -\frac{4\sigma * \partial T^4}{3k * \partial y} \tag{7}$$

where  $\sigma^*$  is Stefan-Boltzmann consistent and  $k^*$  is the mean ingestion.

For a very small temperature in the stream, Taylor's method calls for expanding  $T^4$  around T and avoiding higher terms, we get:

$$T^4 = 4T_{\infty}^3 T - 3T_{\infty}^4 \tag{8}$$

Heat flux can be estimated as

$$\frac{\partial q_r}{\partial y} = -\frac{16T_{\infty}^3 \sigma * \partial^2 T}{3k * \partial y^2}$$
(9)

Using Eq. (9) in energy Eq. (4), one can get:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C p}\left(1 + \frac{1}{\beta}\right)\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{\rho C p}\left(\frac{16T_{\infty}^3\sigma^*}{3k*}\right)\frac{\partial^2 T}{\partial y^2} + \frac{Q_1}{\rho C p}\left(T - T_{\infty}\right)$$
(10)

The below-mentioned similarity transformation reduces Eq. (2), (3), (10), and Eq. (5):

$$\eta = cv^{\frac{1}{2}}yx^{-\frac{1}{3}}, \psi = x^{\frac{2}{3}}c^{\frac{1}{2}}v^{\frac{1}{2}}f(\eta)$$
  

$$\theta(\eta) = \frac{T - T_{\infty}}{T_{w} - T_{\infty}}, \phi(\eta) = \frac{C - C_{\infty}}{C_{w} - C_{\infty}}$$
(11)

The Eq. (3), (5), and Eq. (10) are represented in a dimensionless form by utilizing similarity transformation Eq. (11) for a nonlinear ODE system:

$$(1+\beta^{-1})f''' + \frac{2}{3}ff'' + \left(M + \frac{1}{K}\right)f' + \lambda(\theta + N\phi)\cos(a) = 0$$
(12)

$$\left(1 + \frac{1}{\beta}\right)Ecf''^{2} + \frac{2}{3}f\theta' - \theta f' + \frac{1}{Pr(1 + Nr)''} + PrQ = 0$$
(13)

$$\phi'' - ScKr\phi + \frac{2}{3}Sc(f\phi' - f'\phi) = 0$$
(14)

The appropriate dimensionless version of boundary conditions is given by:

$$f = S, f' = 1, \theta' = -Bi(1 - \theta), \phi = 1 \text{ at } \eta = 0$$
  
$$f' \to 0, \theta \to 0, \phi \to 0 \quad \eta \to \infty$$
(15)

The dimensionless constants are

$$M = \frac{\sigma B_0^2}{\rho c}, K = \frac{K_1 c}{\nu x^{\frac{2}{3}}}, \lambda_T = \frac{g \beta_T (T_w - T_\infty)}{c^2 x^{-\frac{1}{3}}}, Re_x = \frac{u_w x}{\nu}, \lambda_C = \frac{g \beta_C (C_w - C_\infty)}{c^2 x^{-\frac{1}{3}}}, Se_x = \frac{u_w x}{\nu}, \lambda_C = \frac{g \beta_C (C_w - C_\infty)}{c^2 x^{-\frac{1}{3}}}, Se_x = \frac{16\sigma^2 T_w^3}{c^2 x^{-\frac{1}{3}}}, Se_x = \frac{16\sigma^2 T_w^3}{c^2 v^2}, Se_x = \frac{h_f x^{\frac{1}{3}} \sqrt{v}}{kc^{\frac{1}{2}}}, Se_x = \frac{16\sigma^2 T_w^3}{c^2 v^2}, Se_x = \frac{h_f x^{\frac{1}{3}} \sqrt{v}}{kc^{\frac{1}{2}}}, Se_x = \frac{16\sigma^2 T_w^3}{c^2 v^2}, Se_x = \frac{16\sigma^2 T_w^$$

The three primary physical qualities taken into consideration are the friction force (Cf), Nusselt number (Nu), and Sherwood number (Sh):

$$Cf = \frac{\mu}{\rho u_w^2} \left(\frac{\partial u}{\partial y}\right)_{y=0}, Nu = \frac{x}{(T_w - T_\infty)} \left(\frac{\partial T}{\partial y}\right)_{y=0}, Sh = \frac{x}{(C_w - C_\infty)} \left(\frac{\partial C}{\partial y}\right)_{y=0}$$
(17)

Substituting Eq. (11) into Eq. (17) to obtain the final dimensional form:

$$C_f = \sqrt{Re_x} f''(0), Nu = -\sqrt{Re_x} \theta'(0), Sh = -\sqrt{Re_x} \phi'(0)$$
(18)

#### 3. Method of Solution

Eq. (12) - (14) alongside the conditions Eq. (15) are solved by shooting method along with BVP4C with the help of MATLAB 2017a numerically. Eq. (8) - (10) have been converted to a set of 1storder ODE by taking following substitutions:

$$f = h_1, f' = h_2, f'' = h_3, \theta = h_4, \theta' = h_5, \phi = h_6, \phi' = h_7$$
(19)

The transformed equations are as follows:

$$\begin{split} h_{3}' &= \frac{\left[\left(M + \frac{1}{K}\right)h_{2} - \frac{2}{3}h_{1}h_{3} + \frac{1}{3}h_{2}^{2} - \lambda\cos(a)\left(h_{4} + Nh_{6}\right)\right]}{\left(1 + \frac{1}{\beta}\right)} \\ h_{5}' &= \frac{Pr\left(-Qh_{4} - \frac{2}{3}h_{1}h_{5} + h_{1}h_{4} - Ech_{3}^{2}\right)}{1 + Nr} , \\ h_{7}' &= KrSch_{6} - \frac{2}{3}Sch_{1}h_{7} + Sch_{2}h_{6} , \end{split}$$

with;

$$h_1 = S, h_2 = 1, h_5 = -Bi(1 - h_4), h_6 = 1$$
 at  $\eta = 0$ .  
 $h_2 \to 0, h_4 \to 0, h_6 \to 0$ as $\eta \to \infty$ 

The initial guess values to unknowns  $h_3$ ,  $h_5$  and  $h_7$  are provided at  $\eta = 0$ . The numerical procedure adopts a uniform step length  $\delta = 0.001$  and an error tolerance of 0.001. See Oke [34] for more numerical methods.

## 4. Validation

It is important to validate the solutions obtained in this study to ascertain the reliability of the results. To do this, we compare our results with the results of Rashidi *et al.*, [35] and Thumma *et al.*, [36]. The first table (Table 1) is obtained by setting Pr = 2, Q = 0, a = 0,  $Bi \rightarrow \infty$ , Kr = 0, Ec = 0,  $K \rightarrow \infty$ , M = 0, Nr = 0 in our study so that it coincides with that of Rashidi *et al.*, [35] Thumma *et al.*, [36] and the second table (Table 2) is obtained by setting Nr = 0.5, Sc = 0.78, Kr = 0, S = 0.2,  $a = \frac{\pi}{4}$ , Ec = 0,  $K \rightarrow \infty$ , Pr = 0.71, M = 1 for varying values of Q. Both tables have shown that there is a good agreement between our results and the existing literature. Hence, we can rely on the solution produced from the numerical approach.

Table 1							
$Pr = 2, Q = 0, a = 0, Bi \rightarrow \infty, Kr = 0, Ec = 0, K \rightarrow \infty, M = 0, Nr = 0$							
S	-f''(0)						
	Rashidi et al., [35]	Thumma <i>et al.,</i> [36]	Presents results				
-0.5	0.8736447	0.8736784	0.873635				
0	0.6776563	0.6776562	0.677635				
0.5	0.5188901	0.5188997	0.518856				

#### Table 2

 $Nr = 0.5, Sc = 0.78, Kr = 0, S = 0.2, a = \frac{\pi}{4}, Ec = 0, K \to \infty, Pr = 0.71, M = 1$ 

4									
		Thumma <i>et al.,</i> [36	Presents results						
Q	-f''(0)	- heta'(0)	$-\phi'(0)$	-f''(0)	$-\theta'(0)$	$-\phi'(0)$			
0.5	-0.6339719	3.4526318	1.3346567	-0.6339679	3.4526252	1.3346542			
1	-0.6450595	4.8635878	1.3333765	-0.6450569	4.8635755	1.3333651			
1.5	-0.6505565	5.9179405	1.3328534	-0.6505479	5.9179358	1.3328473			

## 5. Results and Discussion

The current work's major objective is to examine the influence of heat and mass transportation on the motion of MHD stagnation-point flow in an inclined stretching porous sheet with variable properties. By combining Runge-Kutta and gunshot procedures, the transformed Eq. (12) through Eq. (14) and the boundary conditions they relate to in Eq. (15) have been mathematically solved. Graphs are utilized to illustrate the behaviour of flow, heat, and mass transport characteristics. These graphs include all flow parameter contributions like Casson parameter ( $\beta$ ), magnetic specification (M), suction or injection parameter (S), Radiation parameter (Nr), heat supply parameter (Q), and others.

Figure 2 shows how the Casson parameter ( $\beta$ ) impacts the velocity profile. As the  $\beta$  value rises, it is seen that the velocity distributions get smaller. This is because as  $\beta$  increases, the yield stress of the Casson fluid parameter decays increasing the rate of plastic dynamic fluid flow. The velocity is reduced when the plastic's absolute viscosity rises, causing fluid flow resistance. Figures 3–4 illustrate the importance of the Schmidt number (Sc) and the chemical reaction parameter (Kr) on the concentration proportion. The concentration profiles degenerate when both Sc and Kr are present in significant amounts. Increasing chemical reaction increases the usage of the reactants thereby reducing the concentration. Figure 5 exhibits the response of dimensionless velocity profiles to M. Strengthening M values promotes diminution in fluid flow. Physically, M corresponds to Lorentz force as a consequence of which greater values of M enhance Lorentz force and this force is one type of resistive force acting against the motion of the fluid therefore fluid velocity decreases. The impact of the porosity parameter (K) on the velocity profile is seen in Figure 6. A decrease in fluid velocity contour is observed in response to an increase in K. The fluid penetration is extremely slow due to the Casson nanoparticles' random mixing and the imposed magnetic field's strength. Physically, the porous media allows fluid particles to travel within the boundary layer, but the yield exhibiting the fluid's plastic dynamic viscosity slows their motion. Figure 7 exhibits the velocity distribution on  $\alpha$ . In each of the figures, the velocity profiles decline asymptotically to zero at the free stream. It can be observed that increasing the angle of inclination has an influence(a). Raising the angle, velocity decreases.

Positive *Bi* physically supports Newtonian cooling, and its effects on the temperature profiles are shown in Figure 8. As a result of the increased Newtonian cooling, heat transfers from the wall into the fluid, and an increase in Bi raises the temperature. It is also noticed that at a value of Bi=0.1, the temperature is quite low because of the solid surface's poor conductivity. The effect of the electromagnetic radiation variable Nr on the temperature distributions is highlighted in Figure 9. The fluid temperature seems to rise with a high Nr value. Thermal engineering can benefit from this as it helps the fluid's thermal condition. Figure 10 shows that the heat source parameter raises the flow temperature. Subsequently, the thermal boundary layer thickens. Thermal power tends to increase as the heat source increases. The effect of the Eckert number on temperature profiles is shown in Figure 11. As Ec increases, the temperature increases. Clearly, *Ec* denotes the intensity of Ohmic heating, and a higher *Ec* indicates more Ohmic heating strength. As a result, heat dissipation improves, and the fluid temperature rises.

Comparisons of the skin friction coefficient for MHD Casson fluid free-convective stagnationpoint flow near an inclined non-linear stretching sheet surrounded by a permeable media are carried out to validate the contemporary effects and confirm the veracity of the existing evaluation. Concerning the findings of Rashidi *et al.*, [27], Table 1 compares the skin friction coefficient. The results shown in this table show that there is a good deal of consistency between them and that the skin friction coefficient reduces with the suction parameter's values increase. From Table 2, excellent agreement is between the results from Thumma *et al.*, [29] and our recent findings. The skin friction is increased, and the Sherwood number falls as a result of the heat source parameter Q, whereas the Nusselt number shows the opposite pattern. Table 3 shows the : Variation of physical parameters on Sf, Nu and Sh.

Variation of physical parameters on Sf, Nu and Sh												
Sc	S	Kr	М	К	α	Bi	Nr	Q	Ec	Sf	Nu	Sh
0.2	0.5	0.5	1	1	$\pi/4$	0	1	1	0	0.646698	0.07541	0.582091
0.3										0.659454	0.075865	0.695983
0.6										0.689958	0.077175	1.05006
0.8										0.701522	0.077728	1.22987
0.8	0.3	0.5	1	1	$\pi/4$	0	1	1	0	0.662524	0.073366	1.15831
	0.5									0.701522	0.07541	1.22987
	0.8									0.741482	0.077379	1.305353
	1									0.782189	0.079232	1.384657
0.8	0.5	0.3	1	1	$\pi/4$	0	1	1	0	0.881848	0.070377	1.099622
		0.5								0.887386	0.070583	1.205881
		0.8								0.891803	0.070847	1.300301
		1								0.895481	0.071202	1.386315
0.8	0.5	0.5	1	0.5	$\pi/4$	0	1	1	0	0.887386	0.055164	1.1584
			2							1.043394	0.060757	1.171517
			3							1.179926	0.065976	1.187023
			4							1.30248	0.070847	1.205881
0.8	0.5	0.5	1	0.3	$\pi/4$	0	1	1	0	0.701522	0.060757	1.171517
				0.5						0.767814	0.070847	1.205881
				0.8						0.887386	0.07392	1.221132
				1						1.179926	0.07541	1.22987
0.8	0.5	0.5	1	0.5	0	0	1	1	0	0.824009	0.061082	1.181243
					$\pi/6$					0.852647	0.070847	1.205881
					$\pi/4$					0.887386	0.071942	1.210419
					$\pi/2$					1.060038	0.072738	1.214082
0.8	0.5	0.5	1	0.5	0	0	1	1	0	0.829596	0.070847	1.205881
						0				0.84189	0.113268	1.210603
						0				0.85944	0.142587	1.2135
						0				0.887386	0.164293	1.2155
0.8	0.5	0.5	1	0.5	0	0	0	1	0	0.894326	0.073855	1.203368
							0			0.89616	0.074633	1.203864
							0			0.898346	0.075551	1.204272
							0			0.900987	0.076649	1.204613
0.8	0.5	0.5	1	0.5	0	0	0	0	0	0.901985	0.076526	1.199659
								0		0.910457	0.079859	1.200386
								0		0.915914	0.082037	1.201435
								0		0.919722	0.083581	1.203069
0.8	0.5	0.5	1	0.5	0	0	0	0	0	0.869909	0.063151	1.205881
									0.01	0.878418	0.066905	1.206031
									0.1	0.886546	0.070479	1.207476
									0.2	0.887386	0.070847	1.208978





Fig. 5. Velocity response to M



Fig. 7. Velocity response to angle of inclination



Fig. 9. Response of temperature to Nr



Fig. 11. Response of temperature to Ec

# 6. Conclusion

Below are the conclusion from this study:

- i. Oblique plate reduces the significance of medium permeability and MF causes a reduction in velocity.
- ii. Nonnegative Biot number leads to enhancement in fluid temperature.
- iii. Reduced permeability causes a drop in velocity at the wall, leading to instability in flow laminarity while high permeability raises the skin friction.
- iv. Velocity decreases with increasing Casson fluid flow.
- v. Due to the positivity of the heat transfer coefficient and mass transfer coefficient, the wall surface experiences cooling and a reduction in the solutal deposit. Thus, the conclusion finds application in heat exchanger design.

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