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Numerical Simulations of Chemically Dissipative MHD Mixed Convective Non-Newtonian Nanofluid Stagnation Point Flow over an Inclined Stretching Sheet with Thermal Radiation Effects

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ABSTRACT

The study of non-Newtonian nanofluid stagnation point flow over an inclined stretching sheet with thermal radiation effects aims to understand how the fluid's non-Newtonian behavior, nanoparticles, the inclined sheet, and thermal radiation affect velocity profiles, temperature distribution, shear stress, and heat transfer rates. It might be used in materials processing, chemical engineering, and energy systems, where understanding fluid behavior in complicated settings is essential for process optimization and system efficiency. The flow problem is reflected in a set of partial differential equations (PDEs) that serve as the governing equations. After appropriate reformatting into Ordinary Differential Equations (ODEs). Mathematica's NDSolve technique is implemented to do a numerical treatment of the dimensionless equations once they have been translated. The upsides of this strategy lie in its ability to automatically track errors and select the best algorithm. Various dimensionless parameters effects on velocity, temperature, and nanoparticle concentration have been studied, and the results are graphically shown. These include the Casson parameter, Brownian motion and thermophoresis, chemical reaction parameter, thermal radiation, viscous dissipation, and mixed convection parameter. The Casson parameter slows down the velocity and speeds up the distributions of temperature and concentration. The skin friction coefficient increases rapidly with increasing tilt and thermophoretic impact amplitudes. The insights were cross-referenced with previous inquiries in order to validate their veracity. All indications are that it complies rigorously and is highly accurate.

1. Introduction

Traditional heat transfer fluids may be converted into nanofluids by dispersing and keeping a suspension of nanoparticles with typical lengths of 1 to 100 mm in solution. Ingenious scientists and technicians have shown that even a small number of guest nanoparticles may significantly enhance the thermal properties of the underlying fluids. Some nanofluids have a nonlinear relationship between thermal conductivity and concentration, an extremely high thermal conductivity that varies

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strongly with temperature and particle size, and a tripling of the critical heat flux at small particle concentrations of the order of 10 ppm. These are only a few of the many ways in which their thermal properties stand out. Nanofluids are of tremendous scientific interest because to their distinct thermal transport processes, which go beyond the fundamental constraints of the already available macroscopic prototypes of suspensions. Nanofluids research has the potential to pave the way for the development of novel nanotechnology-based cooling agents with potential medical and scientific uses. Therefore, studying nanofluids has become an exciting new area of study Choi [1]. In this context, "heat transfer" refers to the transmission of thermal energy. Heat transfer is used often in the human body for a variety of functions. In this context, "heat transfer" refers to the transmission of thermal energy. Cengel *et al.,* [2] looked at the role of temperature regulation in dehydration, refrigeration, and cooking. Heat transfer is used in today's modern structures. According to research by Krishna *et al.,* [3], a proper thermal management system must be devised to remove the produced heat from the electrical equipment for reliable operation. Nanofluid coolants are very effective in microelectronics because of their high thermal conductivity and heat transfer coefficient. Nanofluids are superior heat conductors than conventional fluids, opening the door to smaller and lighter radiators, pumps, and other equipment without sacrificing engine performance. This is especially helpful for big vehicles like semis and trucks, where space is at a premium. Since less pumping power is required, lighter automobiles with improved fuel economy may be manufactured using nanofluids with higher heat transfer coefficients. With an eye towards their use in thermal management and conversion, the experimental study presented in Murshed *et al.,* [4] examines the natural convection heat transfer capabilities of nanofluids with a wide range of forms. Bhatti *et al.,* [5] investigated the motion of diamond and silica nanoparticles in a water-based hybrid nanofluid across an exponentially elastic surface. Efemwenkiekie *et al.,* [6] evaluate the radiator performance of distilled water (DW), a combination of ethylene glycol and DW (EG-DW), aluminium sulphate nanofluid, magnesium sulphate, and their hybrids. Benedict *et al.,* [7] contrasted purified water and heat transfer nanofluids with mono or hybrid metal oxides and CNC generated from plants.

A non-Newtonian fluid model is crucial to increase production and better understand how fluids travel through modern industrial materials. There are a variety of applications for fluid motion that deviates from Newtonian physics. The connection between shear stress and the rate of shear strain is nonlinear if the fluid is not Newtonian. Under modest shear stresses, non-Newtonian fluids behave like elastomeric solids and do not flow. Numerous non-Newtonian fluid models have been put up to explain this behaviour, but the Casson fluid is still regarded as one of the most important of its kind. Casson fluids are shear-thinning liquids with infinite viscosity at zero shear, a yield stress below which no flow occurs, and infinitesimal viscosity at infinite shear. Biological, chemical, medical, metallic, and engineering disciplines are just some of the many that make use of Casson fluids. The food industry isn't the only one that uses Casson fluid; the drilling industry does, too. Casson fluid is used in medications, china clay, paints, synthetic lubricants, biological fluids such synovial fluids, sewage sludge, jelly, tomato sauce, honey, soup, and blood due to its plasma, fibrinogen, and protein content. For years, researchers have examined many conditions affect industrial Casson fluid flow. The Casson fluid model was used by Mukhopadhyay *et al.,* [8] to examine the behaviour of a non-Newtonian fluid in unstable two-dimensional flow across a stretched surface maintained at a constant surface temperature. Bioprocessing requires gas-liquid mass transfer, particularly with non-Newtonian fluids. Nino *et al.,* [9] recorded bubble diameters in a Rushton turbine-stirred bioreactor to numerically compute and experimentally quantify bubble sizes at different axial locations. In Newtonian (water) and non-Newtonian (0.4% CMC) fluids, viscosity affected bubble dispersion calculations. Mixed convection of Casson and oldroyd-B fluids over a stretched, linearly stratified sheet was investigated by Algehyne *et al.,* [10]. The MHD impacts of thermal radiation, chemical reaction, and heat source were investigated by Bejawada *et al.,*[11] utilising a Forchheimer porous medium and a nonlinear inclined stretching surface with velocity slip. Casson fluid flow, RK technique, shot methods, and MATLAB are utilised to discover numerical solutions. Through the use of a magnetic dipole and a twofold stratification in a Casson hybrid nanofluid flow, Ahmad *et al.,* [12] studied heat and mass transfer in an expanding cylinder. By synthesising magnetic and nonmagnetic nanoparticles into Newtonian (water) and non-Newtonian (sodium alginate) convective base fluids, Ayesha Shaukat *et al.,*[13] studied numerical heat transfer in convective boundary layer flow with Casson nanofluid, which combines the captivating effects of nonlinear thermal radiation and magnetic field embedded in a porous medium. The unsteady radiative two-dimensional stagnation point flow of a Casson fluid along a stretching and contracting sheet was investigated by Khan *et al.,* [14], who looked at the effects of mixed convection, convective state, and slip condition. Guadagli *et al.,* [15] studied a symmetric channel Casson flow using the lubrication approximation. The effects of heat sinks and chemical processes on magnetic fluids were investigated by Saeed *et al.,* [16]. This study examines the effects of temperature and concentration dependence on an incompressible magnetic Casson fluid in Darcy's medium inside a porous-surfaced plate subject to generalised boundary conditions. A Casson-Williamson reactive nanofluid species is simulated by Ogunseye *et al.,* [17] in a moving vertical medium. The exothermic reaction of a viscoplastic nanofluid substance in a cylindrical system is modelled using a generalised Arrhenius kinetic. Viscoelastic fluids are produced by Casson and Williamson fluids. Using the Casson fluid model, Gupta *et al.,* [18] looked at how nanoparticles impact blood convection current initiation.

Researchers are interested in the subject of stagnation point flow since it is significant to so many other areas of study, including science and industry. Numerous researchers are still working diligently on various facets of the stagnation-point flow. Due to its practical importance, flow near to the stagnation point has been the subject of intense research for more than a century. Fans used to cool electronic equipment, nuclear reactors in emergency shutdown, solar central receivers exposed to wind currents, and many other technological operations depend on hydrodynamic processes. Many researchers have studied the flow past a stretched sheet because of its potential use in industrial processes including metal spinning, plastic film drawing, glass blowing, crystal growth, and filament cooling. The presence of microorganisms considerably improves the stability of the fluid, which has several applications in biotechnology, bio-microsystems, and bio-nano cooling systems. Khan *et al.,* [19] talk about the physical properties of the MHD stagnation point flow of Williamson nanomaterial over a stretched surface. Mass and heat transfer velocities are modelled using the concept of Cattaneo-Christov Double Diffusion (CCDD). At the stagnation point of a Williamson fluid, Hamid *et al.,* [20] used the wavelet technique to examine the effect of thermophysical and Brownian motion

Three-dimensional Brownian diffusion and thermophoresis are included into the Buongiorno model to describe the flow of a viscoelastic nanofluid around a circular cylinder with a sinusoidal variation in radius, as reported by Ghasemian *et al.,* [21]. Neethu *et al.,* [22] investigated the effects of thermal radiation, heat generation, chemical reaction, porosity, and dissipation on bioconvective MHD hybrid nanofluid (TiO2 and Ag in water) flow over an exponentially expanding permeable surface. Besthapu *et al.,* [23] study the interaction between heat radiation and velocity slip across a convectively nonlinear stretched surface. Slipped effects with the porous medium are considered to minimise the drag reduction at the sheet's surface, and MHD effects are considered at the stagnation point flow of Casson nanofluid. Naganthran *et al.,* [24] investigated numerically and graphically the steady mixed convection flow in a Powell-Eyring fluid as it nears a stagnation point across a permeable, vertically stretching/shrinking flat plate.

Dissipation is the transformation of the kinetic energy of falling water into heat and sound. The erosive potential of running water on streambanks and riverbeds may be mitigated by installing various devices in streambeds. Due to its ubiquity in several industrial production processes, the problem of flow and heat transfer in two-dimensional boundary layers on a continually expanding surface, moving in an otherwise quiescent fluid medium, has attracted considerable attention in recent decades. Continuous casting is used in a wide variety of industries, including but not limited to: metal and polymer extrusion; metal spinning; glass fibre production; hot rolling; wire drawing; paper producing; drawing of plastic films. The effects of viscous dissipation and Joule heating on momentum and thermal transmission were computed for magnetohydrodynamic flow across an inclined plate by Das *et al.,* [25]. Khashi'ie *et al.,* [26] performed a series of numerical simulations to look at how MHD and viscous dissipation affect the radiative heat transfer of a Reiner-Philippoff fluid flow over a nonlinearly contracting sheet. Considering non-linear radiation, viscous dissipation, thermo-diffusion, and Dufour effects, Sharma *et al.,* [27] analyse the 2-D MHD flow of the Casson and Williamson motions through an expanding zone of varying thickness. Hussain *et al.,* [28] use numerical methods to model the lateral motion of a non-Newtonian fluid. Two-dimensional flow of a Tangent-Hyperbolic fluid in a homogeneous conduit with porous walls has been studied in the presence of transversely acting magnetic fields. With suction/blowing and viscous dissipation present, Banerjee *et al.,* [29] looked into the existence of a boundary layer in a diverging porous channel carrying a non-Newtonian Casson fluid with heat transfer. The study by Gudekote *et al.,* [30] investigates the combined effects of slip and inclination on peristaltic transport of Casson fluid in an elastic tube with porous walls. The magnetohydrodynamic flow through non-parallel porous walls or between two solid porous plates crossing at an angle is addressed analytically by Hamrelaine *et al.,* [31]. Bakar *et al.,* [32] looked at how a porous medium filled with a nanofluid, a permeable surface, magnetohydrodynamics (MHD), and internal heat production affected the flow of a mixed convection boundary layer the research by Rafique *et al.,* [33] examines how heat is transferred by nanofluids floating in various-sized carbon nanotubes across a stretched surface. Ali *et al.,* [34] investigated MHD flow of nanofluid over a moving thin needle with impacts of nanoparticle aggregation and viscous dissipation. Nanofluid flow over stretching/shrinking sheets has been studied by several researchers using a variety of theoretical approaches [35-39].

Makhdoum *et al.,* [35] studied the effect of an angled Lorentz force, viscous dissipation, and a permeable stretching surface on nanofluid flow is investigated Researchers [36-39] have shown interest in nanoparticles and nanofluids.

Energized by beforehand studies and real-world use of stagnation point flow, this study applies the Casson rheological model and viscous dissipation to the problem of a Casson fluid in a stretching surface.

Four novel aspects served as the basis for our current effort. The primary goal of this study was to simulate and analyse the two-dimensional chemically radiative MHD stagnation point flow of mixed convective non-Newtonian fluid. The second objective is to examine the dynamics of this flow over an inclined surface. The third step is to examine characteristics of the viscous dissipation, Brownian motion, and thermophoresis. As a fourth goal, we intend to use the NDSolve method to generate numerical solutions for the velocity, temperature, and concentration fields. Also, graphical analyses of the skin friction coefficient and the local Nusselt and Sherwood numbers have been performed. None of the previous works considered the analysis of the MHD stagnation point flow of mixed convective non-Newtonian fluid using NDSolve method. This paper is then an endeavour to fill this gap. As research in this area continues, it is likely that we will see even more applications for these techniques in the future.

2. Mathematical Formulation

We pictured a steady flow of Casson fluid in two dimensions, down an angled stretched sheet around the Magnetohydrodynamic (MHD) stagnation point (see Figure 1). The magnetic field B_0 occupies a plane parallel to the surface with a uniform intensity. The x-axis was thought to be parallel to both the stretching velocity $U_w(x) = ax$ and the free stream velocity $U_\infty(x) = bx$. Assuming angled vertical stretch sheet. T_w and C_w represent the temperature and concentration of the fluid, respectively, across a flat surface. As y tends to infinity at the stretch sheet, the concentration C_∞ and the wall temperature T_∞ of the surrounding fluid are connected. The magnetic Reynolds number is considered to be very small, and the pressure gradient and generated magnetic field are disregarded.

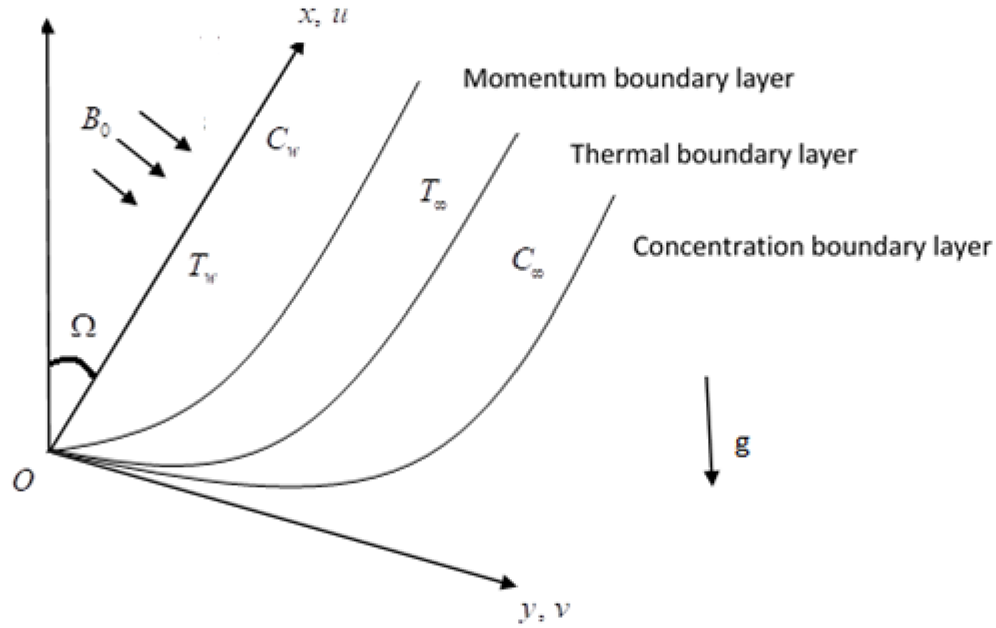

Fig. 1. Physical representation of the fluid flow

The boundary layer equations for continuity, momentum, energy, and concentration are

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_\infty \frac{dU_\infty}{dx} + v\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma_{nf}}{\rho_f} B_0^{\,2}\left(U_\infty - u\right) + \left[(1-C_\infty)(T-T_\infty)\beta - \frac{(\rho_{nf}-\rho_f)(C-C_\infty)}{\rho_p}\right] g\cos\Omega, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\tau\left[D_B\left(\frac{\partial T}{\partial x}\frac{\partial C}{\partial x}+\frac{\partial T}{\partial y}\frac{\partial C}{\partial y}\right)+\frac{D_T}{T_\infty}\left(\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2\right)\right]$$
$$+\frac{\upsilon}{\left(C_p\right)_f}\left(1+\frac{1}{\beta}\right)\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{\left(\rho C_p\right)_f}\frac{\partial q_r}{\partial y}, \quad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)-K^*\left(C-C_\infty\right) \quad (4)$$

Associated boundary conditions are

$$\left.\begin{aligned}&u=U_w\left(x\right)=ax,v=0,T=T_{w,}C=C_w \ \ at \ \ y=0,\\&u\to U_\infty\left(x\right)=bx,v=0,T\to T_\infty,C\to C_\infty \ \ at \ \ y\to\infty.\end{aligned}\right\} \quad (5)$$

The thermal conductivity is supposed to be linear in temperature. In radiative heat flux q_r the term T^4 is linearized about T_∞ with the help of the above Taylor series and is given as follows;

$$\frac{\partial q_r}{\partial y}=-\frac{16T_\infty{}^3\sigma^*}{3k^*}\frac{\partial^2 T}{\partial y^2},$$

By introducing the subsequent similarity transformation, we are able to create the governing equations from Eq. (6) to Eq. (8).

$$\psi=x\sqrt{(av)}f\left(\eta\right),\eta=\sqrt{\frac{a}{v}}y,\theta\left(\eta\right)=\frac{T-T_\infty}{T_w-T_\infty},\ \phi\left(\eta\right)=\frac{C-C_\infty}{C_w-C_\infty}.$$
$$u=\frac{\partial\psi}{\partial y},v=-\frac{\partial\psi}{\partial x}$$

$$\left(1+\frac{1}{\beta}\right)f'''+f\ f''-f'^2+\varepsilon^2+\lambda\left(\theta-N_r\phi\right)\cos\Omega-M\left(f'-\varepsilon\right)=0, \quad (6)$$

$$\left(1+\frac{4R}{3}\right)\theta''+\Pr\left[f\theta'+Nb\theta'\phi'+Nt\theta'^2+\left(1+\frac{1}{\beta}\right)Ec(f'')^2\right]=0, \quad (7)$$

$$\phi''+Le\left(f\phi'-C_r\phi\right)+\frac{Nt}{Nb}\theta''=0, \quad (8)$$

By corresponding boundary conditions are

$$\left.\begin{aligned}&f(0)=0,\ f'(0)=1,\ \theta(0)=1,\ \ \phi(0)=1,\\&f'(\infty)\to\varepsilon,\ \theta(\infty)\to 0,\ \phi(\infty)\to 0.\end{aligned}\right\} \quad (9)$$

$M = \dfrac{\sigma_{nf} B_0^{\ 2}}{\rho_f a}$, $Nb = \dfrac{\tau D_B \left(C_w - C_\infty\right)}{\nu}$, $Nt = \dfrac{\tau D_T \left(T_w - T_\infty\right)}{\nu T_\infty}$, $\Pr = \dfrac{\nu}{\alpha}$, $Gr = \dfrac{\beta_T g(T_w - T_\infty)}{\nu^2} x^3$, $Le = \dfrac{\nu}{D_B}$,

$C_r = \dfrac{\nu x K^*}{U_w}$, $Gr = \dfrac{(1-C_\infty)(T-T_\infty)\beta}{\nu T\infty}$, $Nr = \dfrac{(\sigma_{nf} - \rho)(C_w - C_\infty)}{\beta \rho_p \left(1-C_\infty\right)\left(T_w - T_\infty\right)}$, $R = \dfrac{4\sigma^* T_\infty^{\ 3}}{3K^* k}$, $\varepsilon = \dfrac{b}{a}$, $\lambda = \dfrac{Gr}{\mathrm{Re}^2}$,

$Ec = \dfrac{U_w^2}{C_p \left(T_w - T_\infty\right)}$.

The physical quantities Coefficient of Skin Friction, Local Nusselt Number, and Sherwood Number are provided as;

$$C_f = \frac{\tau_w}{\rho_f U_w^{\ 2}},\ Nu_x = \frac{x q_w}{k\left(T_w - T_\infty\right)},\ Sh_x = \frac{x q_m}{D_B (C_w - C_\infty)} \tag{10}$$

τ_w is the stress with the stretched surface, q_w is the wall heat flux, q_m is the mass heat flux, and k is the thermal conductivity which are stated as;

$$\tau_w = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0},\ q_w = -\alpha \left(\frac{\partial T}{\partial y}\right)_{y=0},\ q_m = -D_B \left(\frac{\partial C}{\partial y}\right)_{y=0}. \tag{11}$$

Using variables with no dimensions, we get the following in Eq. (12)

$$\sqrt{\mathrm{Re}_x} C_f = -\left(1 + \frac{1}{\beta}\right) f''(0),\ \frac{Nu_x}{\sqrt{\mathrm{Re}_x}} = -\left(1 + \frac{4}{3} R\right)\theta'(0),\ \frac{Sh_x}{\sqrt{\mathrm{Re}_x}} = -\phi'(0) \tag{12}$$

3. Methodology for Solution

Using the Mathematica tool NDSolve, we have solved the system of non-dimensional ordinary differential Eq. (6) through Eq. (8) subject to boundary conditions Eq. (9). For evidence of the numerical scheme's preciseness, the latest results of $-\theta'\left(0\right)$ are assessed against the outcomes of outcomes of Rudraswamy *et al.,* [40] and Gupta *et al.,* [41] in Table 1.

Table 1
Comparison of the numerical values of $-\theta'\left(0\right)$ for different values of $\Pr$ when $Le = 10$ and in the absence of remaining parameters

$\Pr$	Rudraswamy *et al.,* [40]	Gupta *et al.,* [41]	Present outcomes
0.2	0.1691	0.1691382	0.169124
0.7	0.4539	0.4538682	0.453917
2.0	0.9112	0.9113432	0.911358
7	1.8953	1.8954124	1.895403
20	3.3539	3.3538714	3.353901

4. Results and Discussion

While most of the figures become available by running through a set of values for a specific parameter, others remain fixed at a single value during the whole simulation such as:

$$\beta = 1.0, M = 0.2, Nb = Nt = \varepsilon = 0.1, R = 0.5, \lambda = 0.2, Nr = Ec = 0.2, C_r = 0.5, Le = 2.0,$$
$$\Omega = 45^o, \Pr = 2.0.$$

The rheological properties of non-Newtonian fluids, notably Casson fluids, is described by the Casson parameter. The fluid becomes more viscous and less likely to flow as the Casson parameter is increased. The flow near the stretched sheet may experience considerable changes in velocity and boundary layer thickness characteristics. Figures 2 to 4 illustrate the effect of β the Casson constraint on velocities, temperatures, and concentrations. It has been observed that the velocity profile decreases across a wide range of β values. The reason for this unexpected behavior is that increasing β increases fluid viscosity, which in turn decreases the yield stress and therefore the thickness of the momentum barrier layer. This results in a thinner momentum boundary layer. However, the temperature and concentration components are showing the opposite tendency.

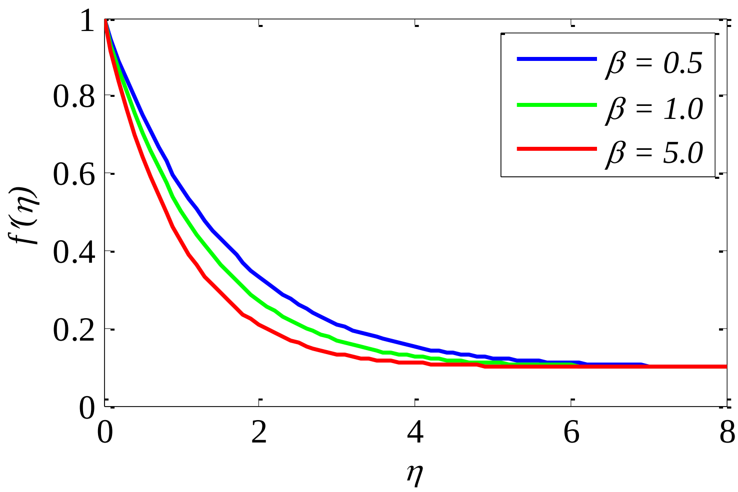

Fig. 2. Effect of β on velocity profile

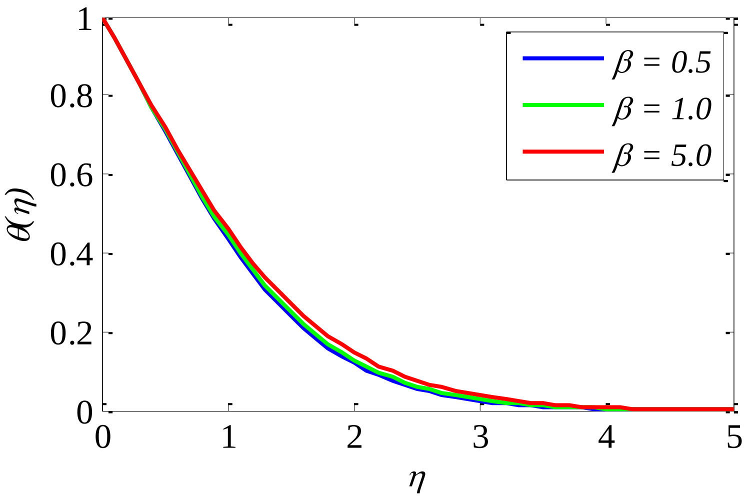

Fig. 3. Effect of β on temperature profile

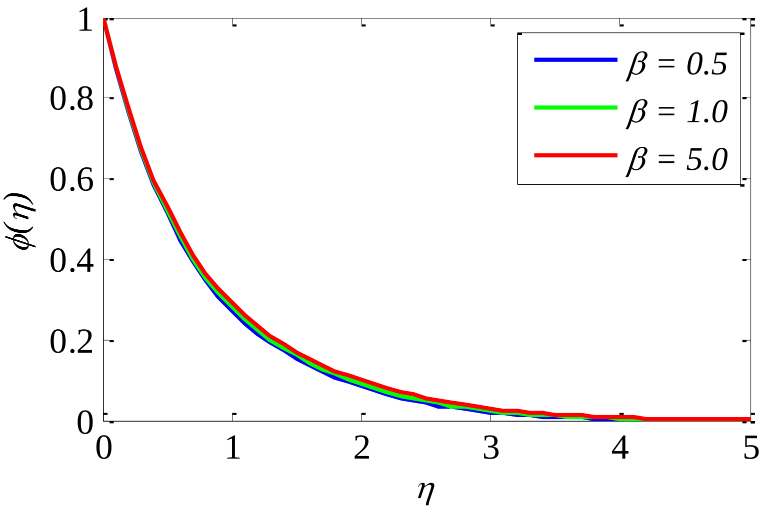

Fig. 4. Effect of β on concentration profile

A magnetic field's existence may influence how an electrically conducting fluid behaves. Strong magnetic fields have the ability to stop fluid velocity and lessen turbulence, which may dampen the flow. The fluid's temperature and velocity characteristics may be affected by magnetic fields. As seen in Figure 5, the Lorentz force exerted by a magnetic field M slows the velocity profile when it encounters obstacles in its route. However, contrasting results have been witnessed for temperature and concentration in physical phenomena. The effects of both temperature and concentration can be reduced by increasing the parameters shown in Figures 6 and 7.

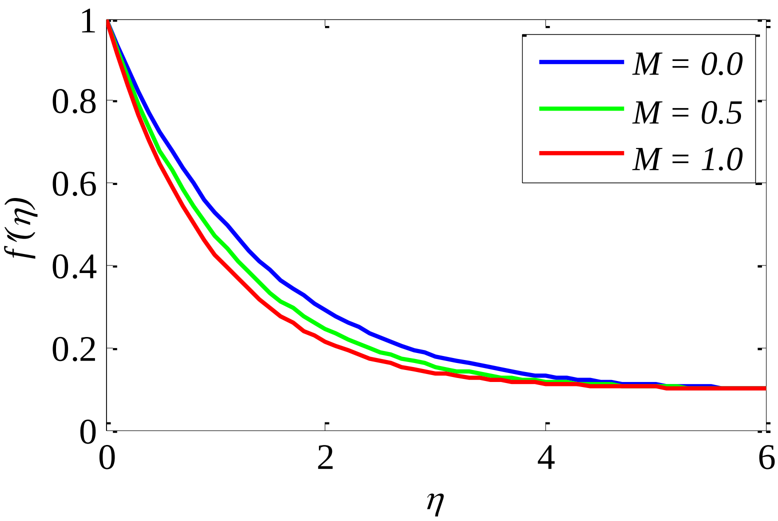

Fig. 5. Effect of M on velocity profile

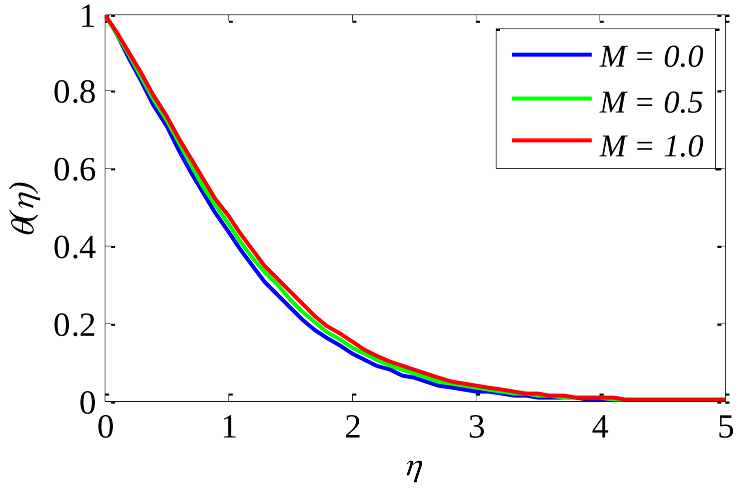

Fig. 6. Effect of M on temperature profile

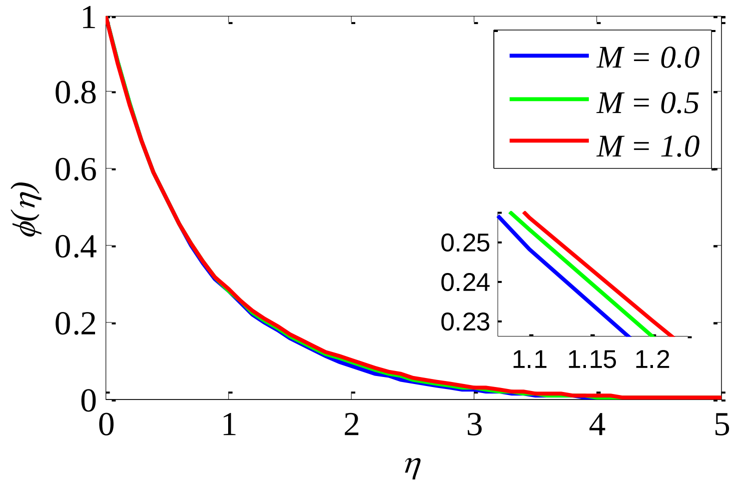

Fig. 7. Effect of M on concentration profile

A boundary layer forms in a flow when the free stream velocity is greater than the stretching velocity. However, a boundary layer with a downward-sloping fluid velocity from the surface to the layer's edge emerges when the stream velocity is less than the stretching velocity. This is given in Figure 8. In Figures 9 and 10 one can visualize that temperature and concentration decline with the raise of ε.

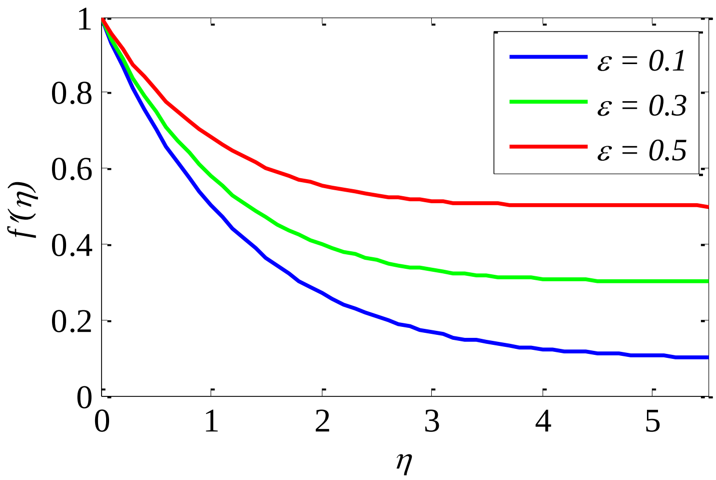

Fig. 8. Effect of ε on velocity profile

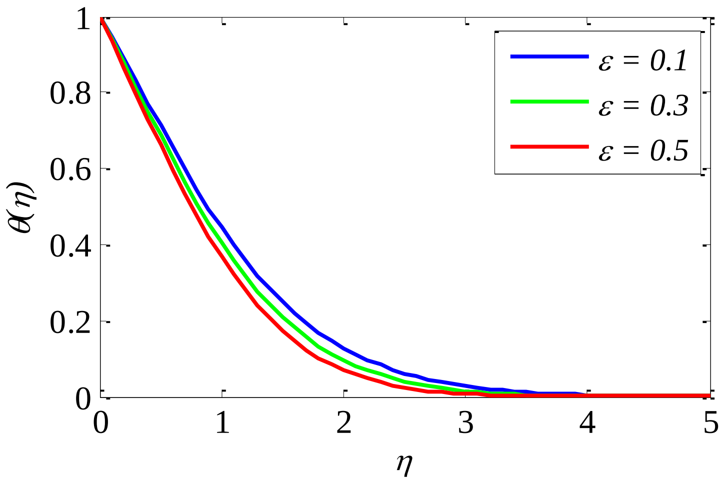

Fig. 9. Effect of ε on temperature profile

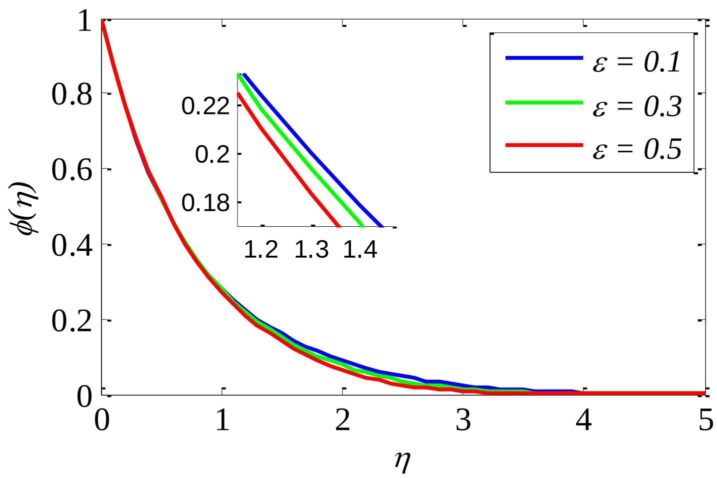

Fig. 10. Effect of ε on concentration profile

Figure 11 shows how mixed convection parameter λ governs velocity. λ promotes fluid velocity. The mixed convection parameter accelerates heat from the colder to the warmer surface near the boundary layer flow when the buoyancy force is larger than the inertia force, this improves fluid velocity.

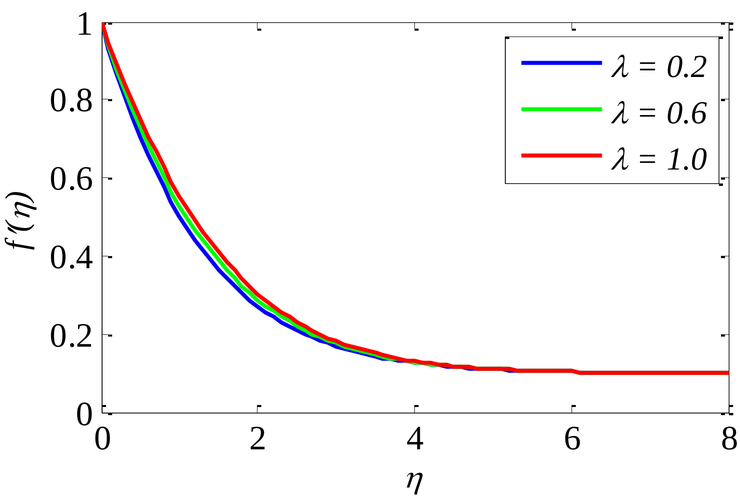

Fig. 11. Effect of λ on velocity profile

Buoyancy ratio parameter Nr implies a falling velocity profile. Buoyancy forces lessen velocity, providing this impact to occur. This is shown in Figure 12.

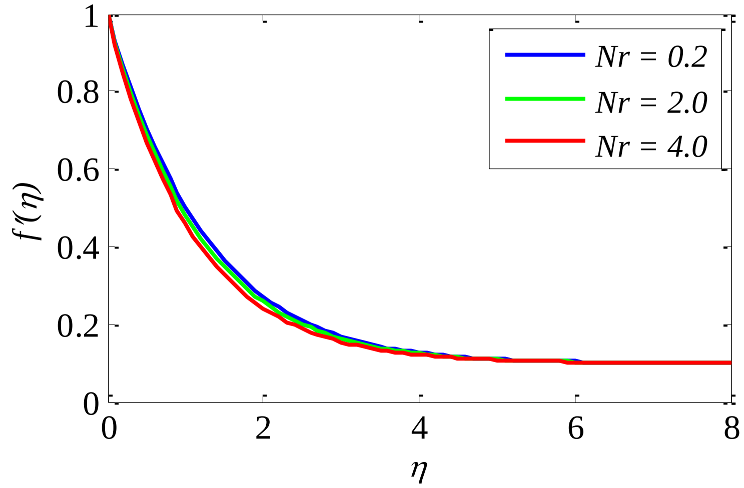

Fig. 12. Effect of Nr on velocity profile

Figure 13, clarifies and illustrates the impact of the inclination parameter Ω on the velocity outline. It is clear from seeing the graph that the velocity contour decreases with larger values of γ. Given the parameters, it stands to reason that the gravitational force acting on the flow will be greatest at $\Omega = 1^o$. This is the reason the sheet will be vertical at this moment. However, if $\Omega = 90^o$, the sheet will be horizontal, resulting in weaker bouncing forces and a subsequent decrease in velocity.

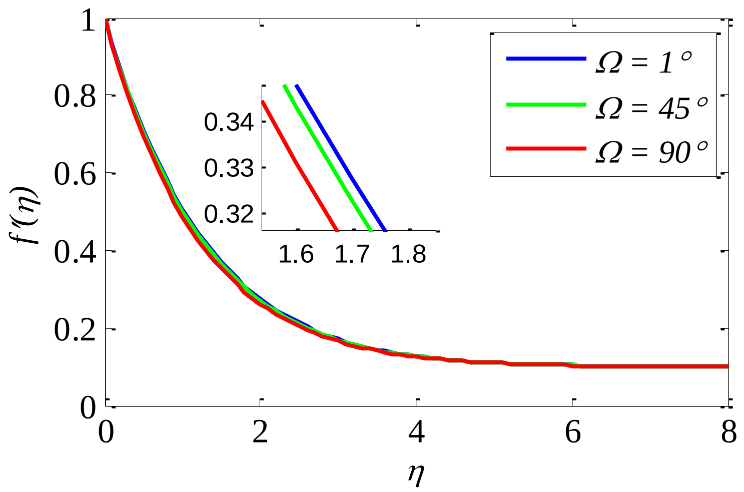

Fig. 13. Effect of Ω on velocity profile

Heat transfer characteristics in a non-Newtonian stagnation flow across an inclined stretched sheet are very sensitive to the Prandtl number. Figure 14 shows temperature dependence on Prandtl number Pr. This factor diminishes boundary layer thickness, which lowers boundary layer temperature.

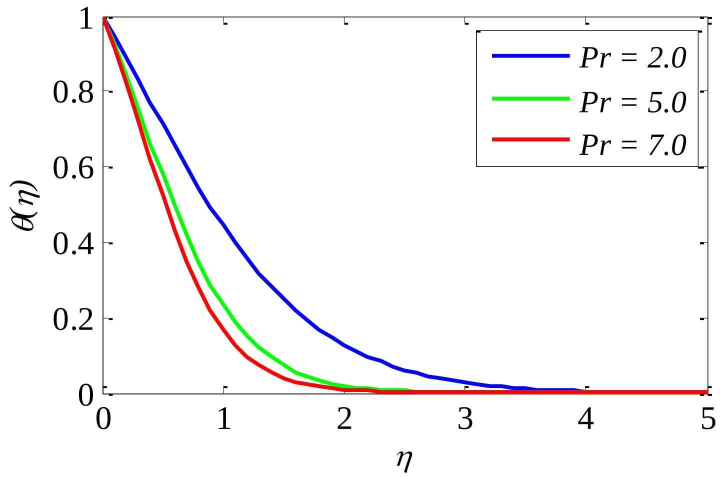

Fig. 14. Effect of $\Pr$ on temperature profile

Figure 15 emphasises how increasing the value of the radiation parameter R raises the temperature profile. When R is increased, the boundary layer's heat transfer rate is increased because more heat may be transmitted to the working fluid between the layers.

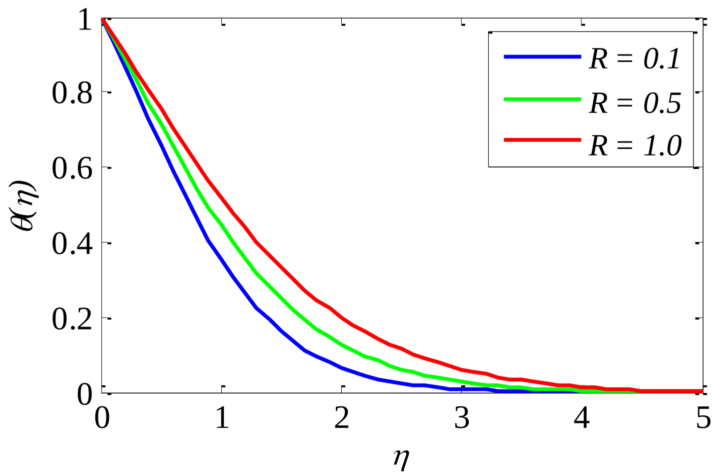

Fig. 15. Effect of R on temperature profile

The Eckert number can provide insights into how much kinetic energy is available to drive the flow compared to the energy required for thermal changes. Since the energy loss due to kinetic energy is large in comparison to the enthalpy difference in the thermal boundary layer, the heat equation predicts that the temperature of the Casson fluid would rise as the Eckert number Ec gets higher, which is apparent in Figure 16.

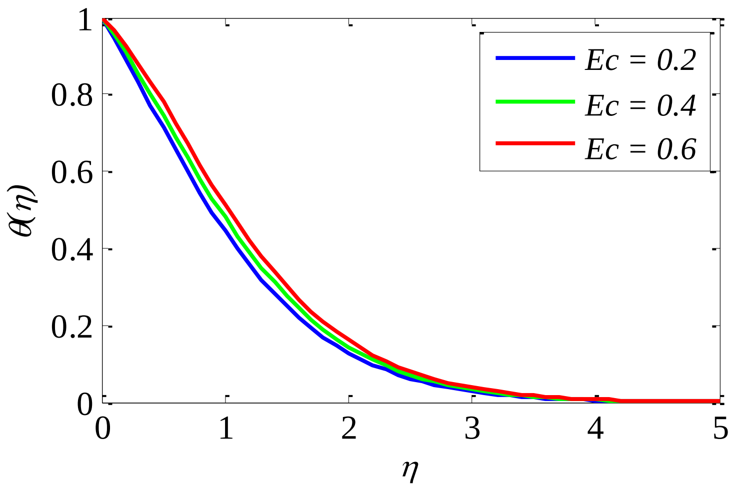

Fig. 16. Effect of Ec on temperature profile

The Brownian motion of nanoparticles is essential in nanofluids. The random motion of particles suspended in a fluid as a result of temperature fluctuations is known as Brownian motion. It has an impact on the nanofluid's viscosity and effective thermal conductivity. Higher Brownian factors make nanoparticles move more randomly, aiding in their dispersion. Figures 17 and 18 show how the Brownian motion factor Nb modifies temperature and concentration curves. The wider range of Nb boosts nanoparticle kinetic energy, allowing more particles to transition beyond the surface, resulting in higher temperatures but lessening concentration.

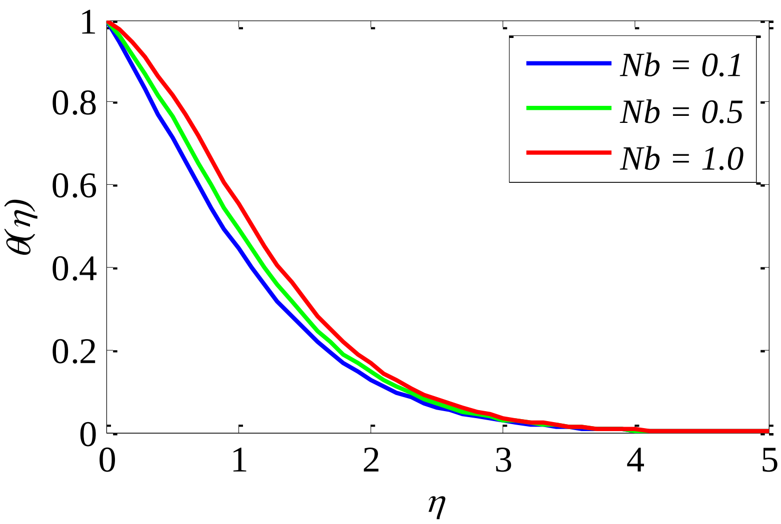

Fig. 17. Effect of Nb on temperature profile

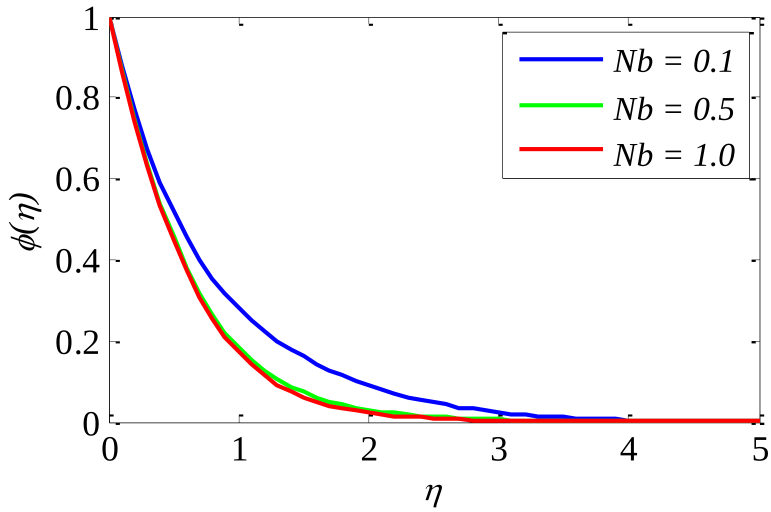

Fig. 18. Effect of Nb on concentration profile

Thermophoresis refers to the movement of particles in response to temperature gradients. This phenomena in nanofluids is quantified by the thermophoresis factor. The mobility of nanoparticles inside the fluid is affected by it. It may have an effect on the flow's nanoparticle concentration distribution. Heat transport may be impacted by the non-uniform particle distributions that result from thermophoresis. Thermophoresis factor Nt modifies temperature and concentration patterns in Figures 19 and 20. The thermophoresis force drives nanoparticles from hot to cold, evolving the temperature profile. Also Nt alters concentration profile similarly.

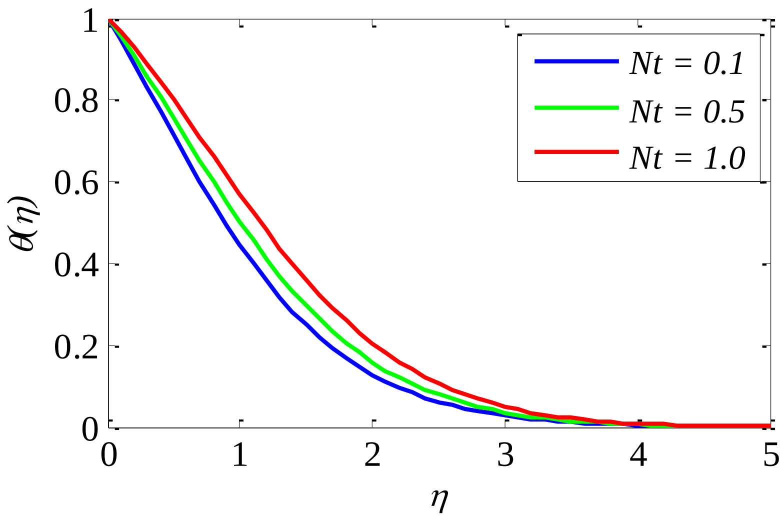

Fig. 19. Effect of Nt on temperature profile

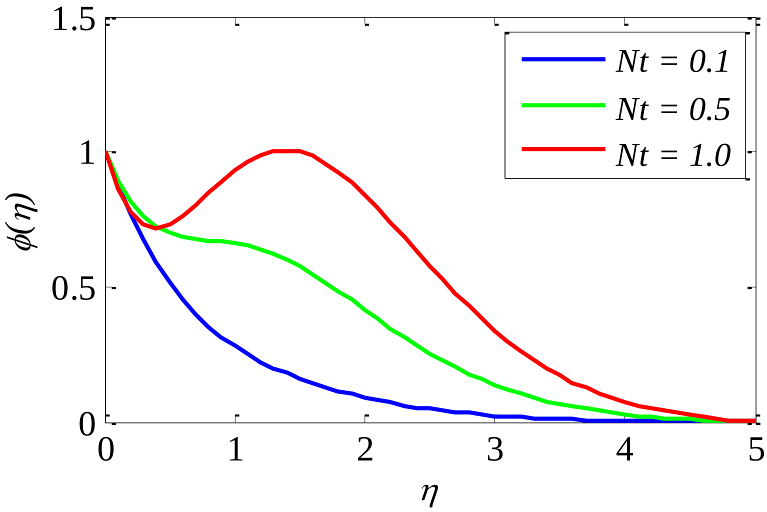

Fig. 20. Effect of Nt on concentration profile

The Lewis number quantifies the relative ease with which heat is transported through a nanofluid relative to the diffusion of mass (typically the mass of the nanoparticles) within the fluid. Understanding the Lewis number is important for optimizing the design of heat transfer systems that use nanofluids for a variety of applications, including cooling systems, heat exchangers, and thermal management. As the Lewis number increases, the concentration field weakens, and the boundary layer viscosity decreases as seen in Figure 21.

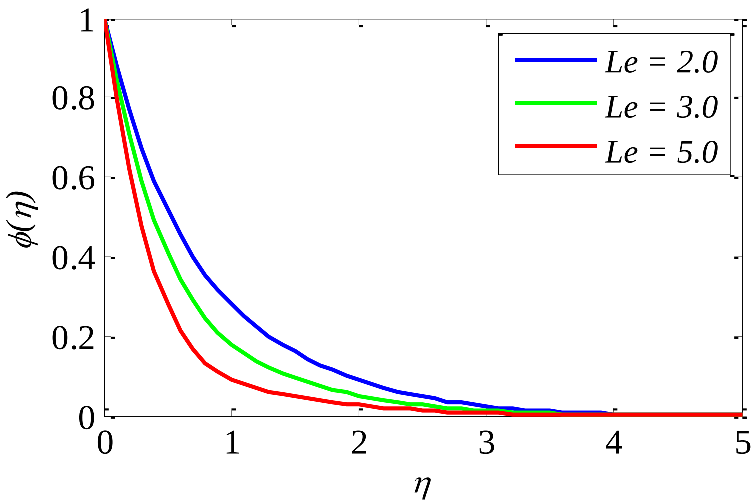

Fig. 21. Effect of Le on concentration profile

In Figure 22, we look into how the parameter Le in a chemical process alters the concentration of nanoparticles. As the repercussions of a chemical reaction grows the concentration profile diminishes. When a chemical reaction takes place, the rate of intermolecular mass transfer rises, resulting in a volume reduction of the nanoparticles.

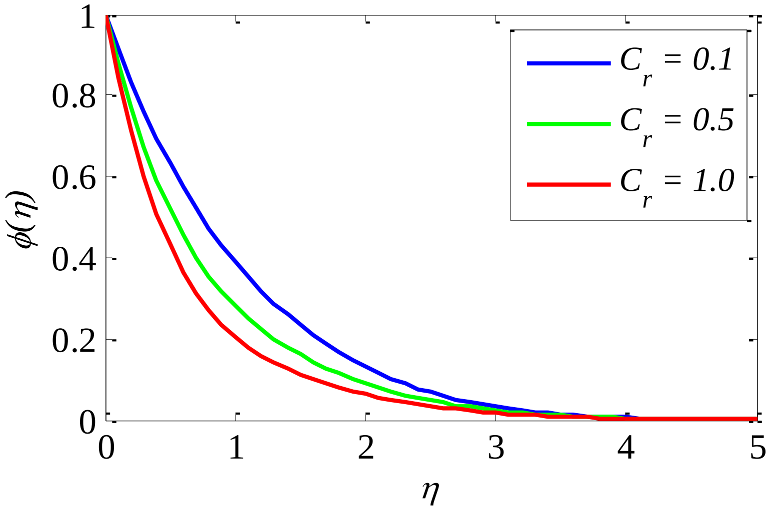

Fig. 22. Effect of C_r on concentration profile

The skin friction coefficient measures a fluid's drag or resistance on a solid surface as it flows across it. Nanoparticles in nanofluids may affect skin friction coefficients. The kind of nanoparticles, their concentration, and flow parameters of the nanofluid may affect these effects. Due to nanoparticles thermal conductivity, nanofluids often have better heat transmission. This increased heat transmission may also increase skin friction, particularly in thermally induced flows like natural convection.Higher velocity gradients near the wall owing to heat transfer increase skin friction. This increases frictional drag.

From Figure 23, higher the amplitudes of inclination and thermophoretic impacts leads to an acceleration in skin friction coefficient. Figure 24 shows that as the thermophoretic effect and the angle of inclination expand, heat exchange drops. Figure 25 shows mass exchange climbs with angle of inclination and reduces with thermophoretic effect.

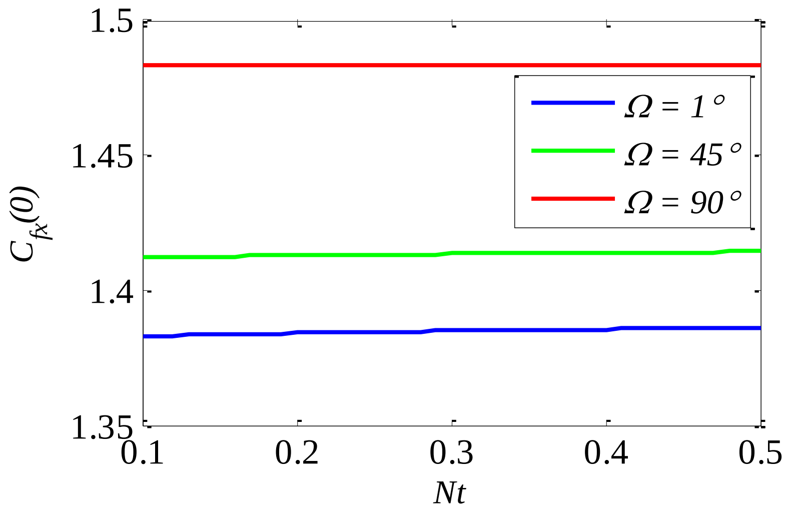

Fig. 23. Effect of Nt and Ω on skin friction coefficient

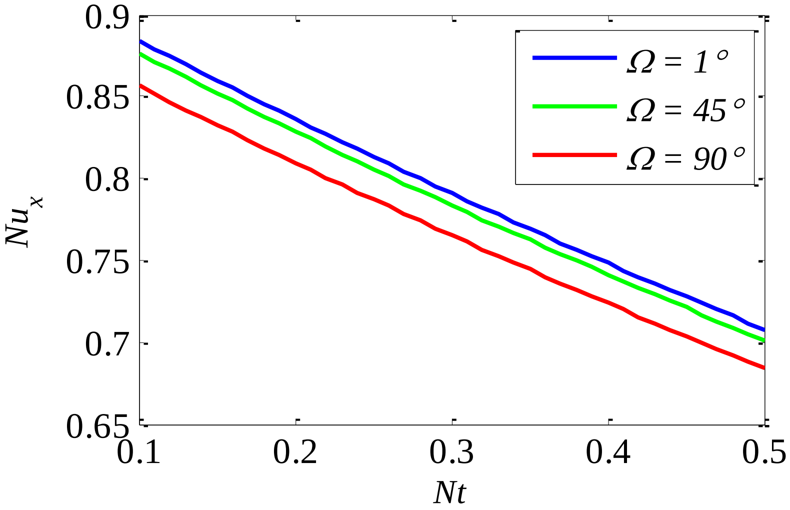

Fig. 24. Effect of Nt and Ω on reduced Nusselt number

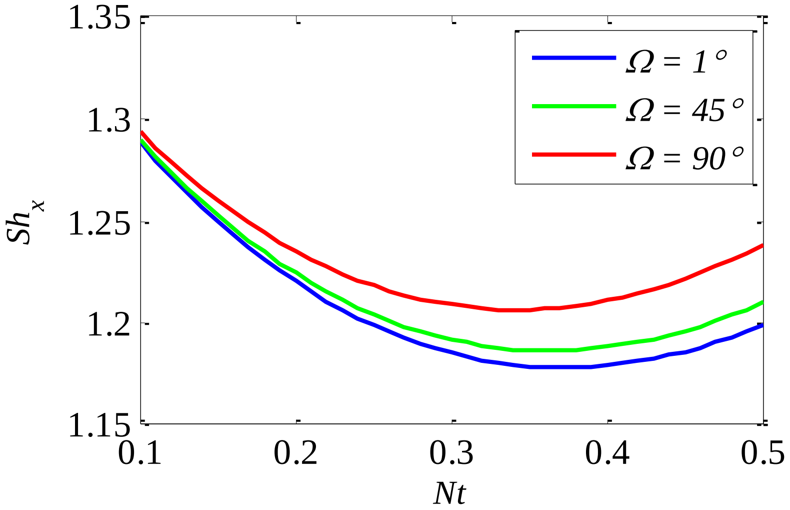

Fig. 25. Effect of Nt and Ω on Sh_x

5. Conclusion

The effects of chemical reaction, viscosity dissipation, and thermal radiation on the Brownian motion and thermophoresis diffusion of Casson nanofluid across an obliquely stretched surface are explored. The Casson parameter, magnetic field, Brownian factor, and thermophoresis factors are all critical parameters that can significantly influence the behavior of the nanofluid in the complex flow scenario. These parameters interact in intricate ways, making the analysis of such flows challenging and requiring advanced mathematical and computational techniques to model and predict their effects accurately.

The following are crucial points for the conclusion.

i. Velocity drops through the upsurge of β, whereas temperature and concentration
   a) gear up with β.
ii. Velocity top up with ε, λ and decline with M, Nr.
iii. Nb and Nt elevate boundary layer temperature. Nanoparticle concentration grows
   a) with Nt. Nanoparticle concentration goes down depending on the Nb.

iv. Temperature accelerates with Ec.
v. Skin friction coefficient enhances with Ω and Nt.
vi. Heat exchange rate reduces with Ω and Nt.

6. Scope and Future Work

The results obtained in this study will be used to analyse the heat and mass transport phenomena in many non-Newtonian fluid flow industrial applications. This work can be extended in future with some other geometries and physical conditions.

Non-Newtonian nanofluid stagnation point flow over an inclined stretched sheet with thermal radiation effects is a speciality in fluid dynamics and heat transfer. This study has engineering and scientific applications. These uses are possible:

Heat Exchanger Design:
Understand non-Newtonian nanofluid behaviour in stagnation point flows to develop more efficient heat exchangers.The information gathered may optimise heat transfer in industrial operations like cooling and HVAC systems.

Cooling Systems:
Electronic device cooling systems may use non-Newtonian nanofluids with improved heat transfer.The discovery may improve cooling systems for computers, power electronics, and other heat-sensitive equipment.

Manufacturing Methods:
Manufacturing operations may benefit from knowing non-Newtonian nanofluid fluid behaviour and heat transfer.It can boost polymer processing and metal casting efficiency.

Biomedical Engineering:
Non-Newtonian nanofluids may be used in biomedical engineering for heat treatmentand medication delivery.Understanding their flow and heat transfer behaviour helps create tailored medication delivery devices.

Conflict of interests
The authors declare that there is no conflict of interests regarding the publication of this paper.

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