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Numerical Analysis Study of Chemically Radiative MHD Williamson Nanofluid Flow over an Inclined Surface with Heat Source

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ABSTRACT

The design and optimization of systems such as nuclear reactors, solar collectors, and thermal power plants may benefit from an understanding of the behaviour of nanofluids with chemical processes, thermal radiation, and magnetic fields over inclined surfaces with heat sources.This review looks at the magnetohydrodynamic (MHD) flow of a Williamson nanofluid over an inclinable stretched sheet and the impact of thermal radiation, heat source, and chemical processes. The outcomes of the generation of heat or absorption, as well as thermal radiation, are all factored into account in the energy equation. On the other hand, the mass transport equation also takes into account chemical interactions.The similarity substitution serves to turn the governed partial differential equations for velocity, temperature, and concentration into ordinary differential equations, which are then numerically resolved with Mathematica's NDSolve program. Changes in temperature, concentration, and dimensionless velocity as a function of various factors are graphically touched upon. The temperature diminishes while the Prandtl number accelerates as the thermal boundary layer thins and the viscosity enrichment. The temperature contour develops along with the magnetic field strength. When all the parameters were compared for a particular case, a very good association was discovered. Depending on the precise conclusions and understandings drawn from the investigation of chemically radiative MHD nanofluid flow over inclined surfaces with a heat source, the applications may range greatly. It's important to remember that such research often aids in the creation of more effective and environmentally friendly solutions in a variety of sectors.

1. Introduction

In recognition of their remarkable physical and chemical properties, nanofluids find widespread use in industry. The vastly improved thermal characteristics of nanofluids have garnered a lot of scientific interest recently. In addition to their use in electronics, suspensions of metal nanoparticles are being produced for medical applications like cancer therapy. In the late 1990s, Choi and Eastman [1] developed nanofluid with the aim of enhancing heat transmission when incorporated into a base fluid such as water, ethylene glycol, or oil. There has been an additional explosion of embryonic

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development beneath these fluids. For instance, the exceptional thermal conductivity of nanoliquids affords a tremendous possibility for boosting the effectiveness of solar systems. Hayat *et al.,* [2] simulated nanofluid collisions as they rotated across a Riga plate using computer simulations. In the evaluation of the expanding sheet for the flow of nanofluid, Hayat *et al.,* [3] also considered consequences of Joule heating and Brownian motion. Nanoparticles and Nanofluids are of great importance and so many researchers have done a lot of work in this field [4-10]

Scholars of hydrodynamics from all over the world have lately shifted their gaze to the immense technical breakthroughs in the usage of non-Newtonian fluids. One of the most fundamental non-Newtonian fluids, the Williamson fluid, shares many characteristics with polymeric solutions, including a drop in viscosity due to rising shear stress rates. That also necessitates that the effective viscosity of a Williamson fluid, which is simply infinite at rest and zero as the shear rate arrives at infinity, should fall endlessly as the shear rate grows. Williamson created the Williamson model [11] in 1929, and Subbarayudu *et al.,* [12] explored time-dependent flow characterization of the Williamson fluid with radiative blood flow against a wedge. In lieu of the work of Lyubimova *et al.,* [13], which emphasised the stability of quasi-equilibrium states and supercritical regimes of thermal vibrational convection of Williamson fluid in zero gravity, the work of Hamid *et al.,* [14] looked at multiple types of solutions for MHD transient flow of Williamson nanofluids with convective heat transport. Lot of work has been done involving Williamson fluid [15-17]

Thermal radiation is ubiquitous in the natural world. All things with finite temperatures emit thermal radiation because of the thermally induced motion of particles and quasiparticles. Several of these items are highlighted here because of their importance to the field of energy. Sunlight from the 6000 K surface of the sun is the most important renewable energy source. Whether it's an incandescent light bulb with filaments heated to 3000 K or our own body at roughly 310 K, everything we come into contact with in our daily lives produces thermal radiation. As a result, thermal radiation provides access to any heat source. For energy conversion, a low-temperature heat sink is as crucial as a high-temperature heat source from a thermodynamic point of view. Thermal radiation is a measurable heat transfer mechanism in nature. There has been remarkable development as seen in [18-24] in the study of the fundamental theory and applications of thermal radiation since the turn of the 20th century, owing to the activities of thermo science and thermal engineering. A majority of these endeavours have emphasised on the macroscopic level; however, with the rapid advancement of nanoscience and technology, electromagnetic radiation on the micro- and nano-levels has emerged as a new frontier. Lee *et al.,* [25] studied synthesis of coherent heat from monolithic photonic crystals.

Radiation's impact on boundary layer flow is significant in many fields of engineering and physics, including furnace aspiration, polymer processing, and space gas cooled nuclear reactors, glass production, and transportation technologies. Backwash from linear and nonlinear radiation deposits on a stagnancy point on a stretching surface was studied by Hussain *et al.,* [26]. The effect of thermal radiation afterglow on heat transfer on a pad sheet was studied by Cortell [27]. Many researchers have studied thermal radiation and heat transfer in fluids as seen in Ref. [28-32].

Extraction of crude oil, production of polymers, and food processing are just a few examples of the many engineering and industrial operations that rely on non-Newtonian fluids. The Casson fluid, the Jeffrey fluid, the Maxwell fluid, the Johnson-Segalman fluid, the Ostwald-Waele Power Law fluid, and the micropolar fluid are only a few of the many non-Newtonian fluid models categorised by different physical characteristics. Any fluid that, under different circumstances, changes microscopically from its original structure with micromotion, whether or not there is an external impact force, is the subject of the study of micropolar fluid dynamics. As a baseline of a non-Newtonian liquid, the pseudoplastic liquid is the more intriguing fluids owing to all the ways it may be used in construction and manufacturing. The challenge, though, lies in understanding how such liquids behave.The Williamson liquid model was developed for this situation in order to effectively capture the intended behaviour. In-depth analyses of the Williamson nanofluid's flow properties when taking into account modified geometries have been provided by Hayat *et al.,* [33]. Numerous researchers investigated numerous facets of Newtonian fluid under diverse conditions as can be observed in Ref. [34-45].

This paper is organized into five sections: (i) introduction (ii) mathematical formulation (iii) methodology for the solution (iv) results and discussions and (v) conclusions. As far as we are aware, relatively little research has been undertaken to explore how chemical reaction and Brownian motion affect the flow of a Williamson nanofluid towards an angled stretching surface. Therefore, we undertook our research with a view to fill this void in the literature; our findings may potentially prove useful in the investigation of brain and breast cancer malignancies. Four novel aspects served as the basis for our current effort. The primary goal of this study was to simulate and analyse the two-dimensional Williamson nanofluid flow along an inclined stretched surface. The second objective is to examine the dynamics of this flow over an inclined surface. The third step is to examine characteristics of the chemical reaction, Brownian motion, and thermophoresis. As a fourth goal, we intend to use the NDSolve method to generate numerical solutions for the velocity, temperature, and concentration fields. The impact among various physically relevant parameters on the distributions of temperature, velocity, and concentration, as computed numerically and portrayed graphically.

2. Mathematical Formulation

The flow characteristics of a Williamson nanofluid on an inclined surface will be investigated in this section. When stretching a sheet in a vertical direction, the angle Ω is taken into account. The fluid flows as a result of an additional sheet being stretched at a rate of $'c'$. The effects of thermal radiation, heat generation or absorption and chemical reaction are being considered. T_w and C_w are used to represent the concentration and wall temperature, respectively, while u_w stands for the wall velocity. Additionally, as y approaches infinity T_∞, C_∞ exhibits temperature and concentration of nanoparticles, as depicted in Figure 1. Further, the flow simulation as well as the coordinate scheme may be perceived in Figure 1. The mathematical formulation for the Cauchy stress tensor has been provided in Nadeem and Hussian [46] for the Williamson fluid, and it reads as follows:

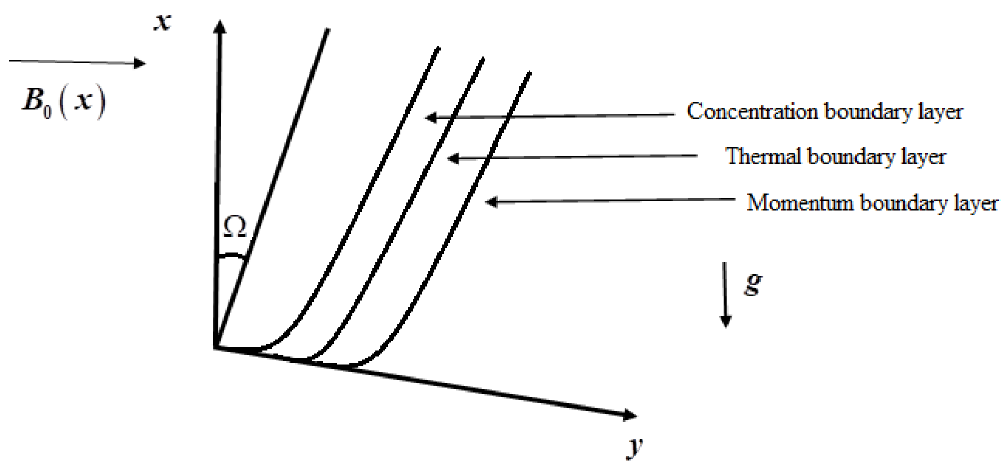

Fig. 1. Physical structure with coordinates

$$s = -pI + \tau \tag{1}$$

$$\tau = \left[\mu_\infty + \frac{(\mu_0 - \mu_\infty)}{1 - \Gamma \gamma^*} \right] A_1 \tag{2}$$

Where τ denote the extra stress tensor, A_1 stands for the first Rivlin-Erickson tensor, μ_0 stands for the limiting viscosity at zero shear rate, and μ_∞ stands for the limiting viscosity at infinite shear rate. From [46] we have;

$$\tau = \mu_0 [1 + \Gamma \gamma^*] A_1 \tag{3}$$

The mathematical formulation of the flow for this model is [46]

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{4}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \sqrt{2}\nu\Gamma \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} + g\left[\beta_t (T - T_\infty) - \beta_c (C - C_\infty)\right]\cos\Omega - \frac{\sigma B_0{}^2}{\rho_f} u \tag{5}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left[D_B \frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2 \right] + \frac{Q_0}{(\rho C_p)_f}(T - T_\infty) - \frac{1}{(\rho C_p)_f}\frac{\partial q_r}{\partial y} \tag{6}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} - K^*(C - C_\infty) \tag{7}$$

Let us consider the conditions as

$$\begin{aligned} &u = u_w(x) = cx, v = 0, T = T_w, C = C_w \quad at \ \ y = 0, \\ &u \to u_\infty(x) = 0, v \to 0, T \to T_\infty, C \to C_\infty \ \ as \ \ y \to \infty. \end{aligned} \tag{8}$$

The stream function is given as

$$u = \frac{\partial \psi}{\partial y}, \ v = -\frac{\partial \psi}{\partial x}, \tag{9}$$

Using appropriate similarity transformations such as

$$u = cxf(\eta), v = -\sqrt{c\nu}\, f(\eta), \eta = y\sqrt{\frac{c}{\nu}}, \ \ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \ \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}. \tag{10}$$

The solution to the system of Eq. (4) - Eq. (7) was obtained after switching Eq. (10) as

$$f''' + ff'' - f'^2 + \lambda f'' f''' + (Gr_x\theta + Gc_x\phi)\cos\Omega - Mf' = 0 \quad (11)$$

$$\left(\frac{1}{\Pr}\right)\left(1+\frac{4R}{3}\right)\theta'' + f\theta' + Q\theta + Nb\phi'\theta' + Nt\theta'^2 = 0, \quad (12)$$

$$\phi'' + Le\Pr f\phi' + \frac{Nt}{Nb}\theta'' - Le\Pr C_r\phi = 0 \quad (13)$$

Where

$$\begin{aligned} &M = \frac{\sigma B_0^{\ 2}}{a\rho_f}, Nt = \frac{\tau D_T(T_w - T_\infty)}{\nu T_\infty},\ Gr_x = \frac{g\beta_t(T_w - T_\infty)x^{-1}}{a^2}, Re_x = \frac{u_w(x)x}{\nu}, R = \frac{4T_\infty^{\ 3}\sigma^*}{k_f k^*} \\ &Gc_x = \frac{g\beta_c(C_w - C_\infty)x^{-1}}{a^2}, Q = \frac{Q_0}{a\rho C_p}, \lambda = \Gamma x\sqrt{\frac{2c^3}{\nu}}, Nb = \frac{\tau D_B(C_w - C_\infty)}{\nu}, C_r = \frac{K^*}{a} \end{aligned} \quad (14)$$

M is the magnetic parameter, Q is heat constraint climb, Nt is the thermophoresis parameter, Le is Lewis number, C_r is chemical reaction parameter

To find similarity solution, Gr_x and Gc_x should be independent of x. Hence, suppose that [47, 48].

$$\beta_t = nx^1, \beta_c = n_1 x^{1,} \quad (15)$$

Where n_1 and n denote constants, replacing Eq. (15) into the parameter Gr_x and, Gc_x we obtain

$$Gc = \frac{gn_1(C_w - C_\infty)}{a^2},\ Gr = \frac{gn(T_w - T_\infty)}{a^2} \quad (16)$$

Gr is thermal Grashof number, Gc is solutal buoyancy factor.

Eq. (8) is converted to

$$\begin{aligned} &f(\eta) = 0, f'(\eta) = 1, \theta(\eta) = 1, \phi(\eta) = 1 \ \ at\ \eta = 0, \\ &f'(\eta) \to 0, \theta(\eta) \to 0, \phi(\eta) \to 0,\ at\ \eta \to \infty. \end{aligned} \quad (17)$$

The physical parameters are defined as:

$C_f = \frac{\mathrm{T}_w}{\rho u_w^{\ 2}}$ is the skin-friction;

$N_u = \frac{xq_w}{k(T_w - T_\infty)}$ is the Nusselt number;

$Sh = \frac{xq_m}{D_B(C_w - C_\infty)}$ is the Sherwood number,

Where;

$$q_m = -D_B \frac{\partial C}{\partial y},$$
$$q_w = -\left(k + \frac{16\sigma^* T^3_\infty}{3k^*}\right)\frac{\partial T}{\partial y},$$
$$\tau_w = \mu\left[\frac{\partial u}{\partial y} + \frac{\Gamma}{2}\left(\frac{\partial u}{\partial y}\right)^2\right] \ at \ y = 0.$$

The related expressions of $-\theta'(0), -\phi'(0) \ and \ C_{fx}$ are defined as

$$-\left(1+\frac{4R}{3}\right)\theta'(0) = \frac{Nu_x}{\sqrt{\mathrm{Re}_x}}, -\phi'(0) = \frac{Sh_x}{\sqrt{\mathrm{Re}_x}}, C_{fx}\sqrt{\mathrm{Re}_x} = f''(0) + \frac{\lambda}{2}\left[f''(0)\right]^2.$$

3. Methodology for Solution

Mathematica's NDSolve package was used to find a solution to the system of non-dimensional standard differential Eq. (11) - Eq. (13) subject to boundary conditions in Eq. (14).For evidence of the numerical scheme's preciseness, the latest results of $-\theta'(0)$ and $-\phi'(0)$ are assessed against Khan and Pop [47] and Rafique and Alotaibi [49] outcomes in Table 1 and Table 2.

Table 1
Contrast of $-\theta'(0)$ and $-\phi'(0)$ for different values of Nb and Nt with $\mathrm{Pr} = Le = 10,\ \Omega = \frac{\pi}{2}$ and in the absence of remaining parameters

Nb	Nt	Rafique and Alotaibi [49]		Khan and Pop [47]		Current outcomes	
		$-\theta'(0)$	$-\phi'(0)$	$-\theta'(0)$	$-\phi'(0)$	$-\theta'(0)$	$-\phi'(0)$
0.1	0.1	0.9524	2.1294	0.9524	2.1294	0.952363	2.129288
0.2	0.2	0.3654	2.5152	0.3654	2.5152	0.365358	2.515103
0.3	0.3	0.1355	2.6088	0.1355	2.6088	0.135517	2.608694
0.4	0.4	0.0495	2.6038	0.0495	2.6038	0.049467	2.603719
0.5	0.5	0.0179	2.5731	0.0179	2.5731	0.017924	2.572978

Table 2
Contrast of $C_{fx}(0), -\theta'(0)$ and $-\phi'(0)$ for different values of Nb, Nt, Ω & C_r when $M = 0.3, We = 0.1, \Pr = 6.5, R = 0.0, Q = 0.1$ & $Le = 5$

Nb	Nt	Ω	C_r	Rafique and Alotaibi [49]			Current outcomes		
				$C_{fx}(0)$	$-\theta'(0)$	$-\phi'(0)$	$C_{fx}(0)$	$-\theta'(0)$	$-\phi'(0)$
0.1	0.1	$\frac{\pi}{4}$	1.0	0.7823	0.9850	0.7456	0.782317	0.984991	0.745588
0.4	0.1	$\frac{\pi}{4}$	1.0	0.9576	0.2400	1.2491	0.957486	0.239911	1.249125
0.1	0.4	$\frac{\pi}{4}$	1.0	0.8236	0.5099	0.8885	0.82241	0.509874	0.888563
0.1	0.1	$\frac{\pi}{3}$	1.0	0.7619	1.0211	0.4417	0.761903	1.021062	0.441687
0.1	0.1	$\frac{\pi}{4}$	1.5	1.0816	0.9610	0.7143	1.081584	0.961012	0.714286

4. Results and Discussion

In this part, mathematical models of Williamson nano fluid, a non-Newtonian fluid, will be demonstrated during which it flows over an inclined surface. The outcomes of the generation of heat or absorption, as well as thermal radiation, are all factored into account in the energy equation. On the other hand, the mass transport equation also takes into account chemical interactions. The supported similarity transformation-converted differential equations are numerically resolved in Mathematica using NDSolve. While most of the figures become available by running through a set of values for a specific parameter, others remain fixed at a single value during the whole simulation such as:

$$M = 0.3, Nb = 0.5, Q = Gr = Nt = \lambda = R = 0.1, C_r = 0.5, Gc = 0.9, Le = 5.0, \Omega = 45^o, \Pr = 6.5.$$

A strong magnetic field can suppress fluid motion and reduce turbulence, which can have a damping effect on the flow. The implications of magnetic parameter M on the profiles are apparent in Figure 2 and Figure 3. In contrast to temperature, which goes up as M expands, velocity lowers. In reality, transport rates diminish according to rising M because the Lorentz force, which resists fluid motion, grows in magnitude with higher M .

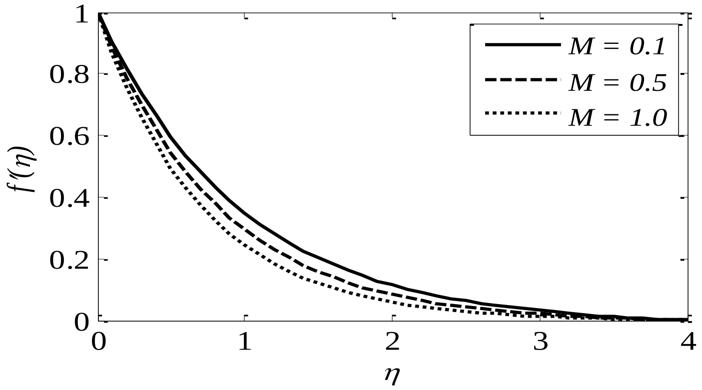

Fig. 2. Effect of M on velocity profile

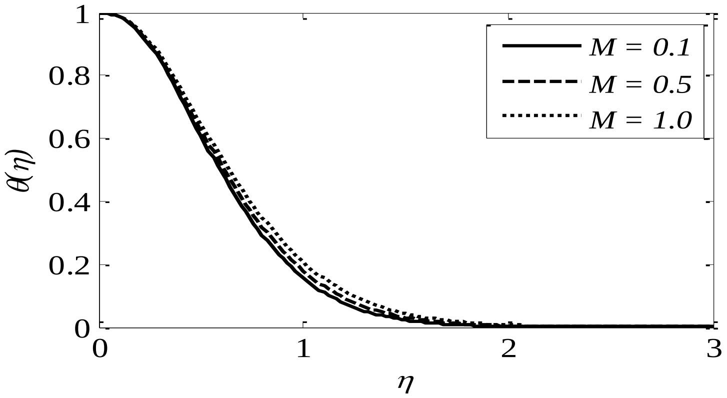

Fig. 3. Effect of M on temperature profile

The Grashof number may significantly affect the flow and heat transfer properties of a Williamson nanofluid, which is a nanofluid made up of solid nanoparticles scattered in a base fluid.The consequences of the thermal Grashof number Gr on velocity is seen in Figure 4. The velocity exactly improves when the number Gr is raised. This is because buoyancy influences the fluid particles as a result of gravitational force, which increases the fluid's velocity.

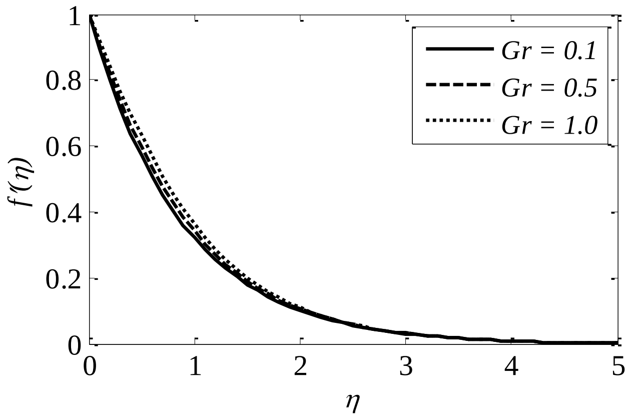

Fig. 4. Effect of Gr on velocity profile

The velocity profile is enhanced by raising the solutal buoyancy factor Gc, which is visible in Figure 5. In a physical sense, the buoyancy parameter Gc reduces the viscous forces that cause the velocity to increase.

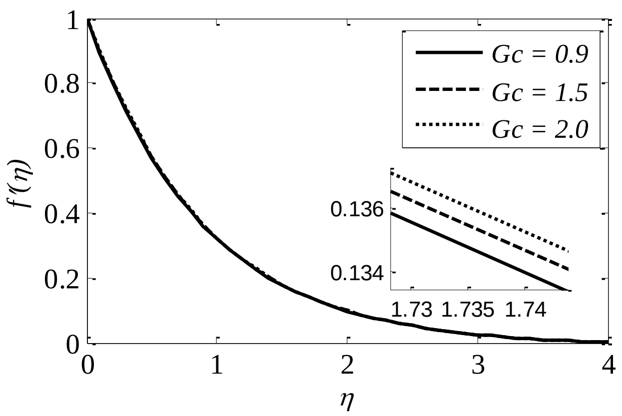

Fig. 5. Effect of Gc on velocity profile

Figure 6, clarifies and demonstrates the consequences of the inclination parameter Ω on the velocity outline. Looking at the figure makes it obvious that the velocity profile gets smaller as the values of γ increase. The constraints also suggest that the gravitational force acting on flow will be at its strongest when $\Omega = 0^o$. The sheet will be vertical at this time, explaining why. On the other hand, if $\Omega = 90^o$, the sheet will be horizontal, which will cause the bouncing forces to be less powerful, causing a drop in the velocity profile.

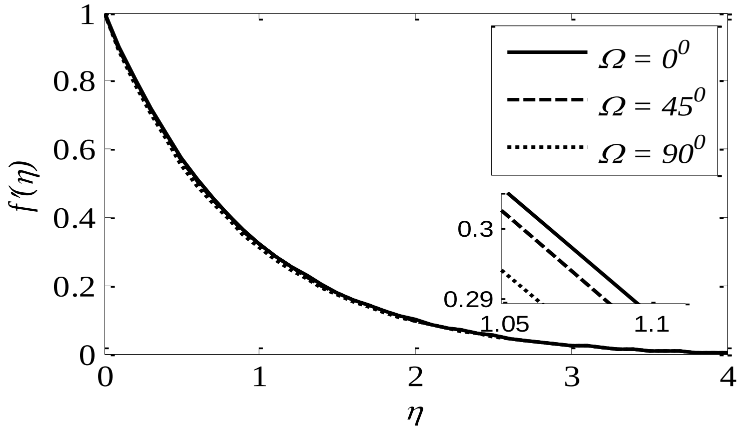

Fig. 6. Effect of Ω on velocity profile

Figure 7 displays the slowed velocity profile for Williamson parameter We . The reason for this is that the thickness of the boundary layer drops as the non-Newtonian Williamson parameter We grows.

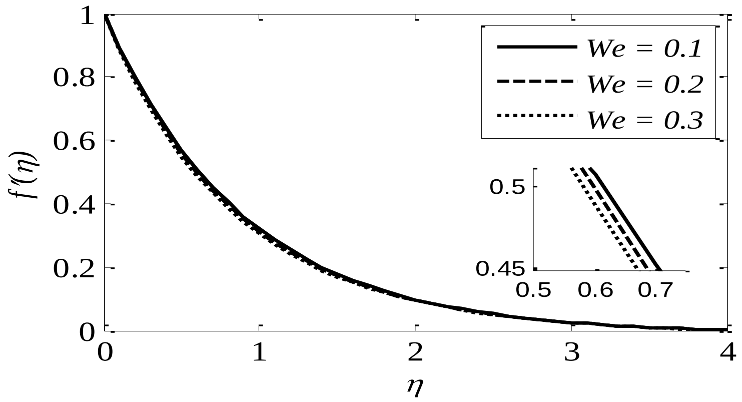

Fig. 7. Effect of We on velocity profile

The temperature distribution enlarges as the values of the heat generation Q constraint climb, reflected in Figure 8. As Q accelerates then the temperature of the thermal boundary layer raises and allowing heat to accumulate in the flow region.

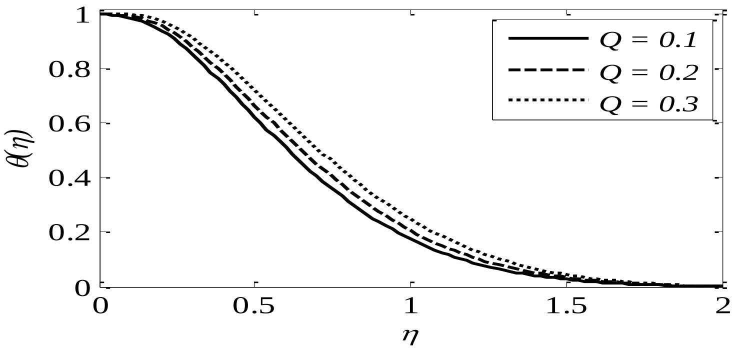

Fig. 8. Effect of Q on temperature profile

The temperature increases as the radiation parameter R does, as seen in Figure 9. A faster growth in fluid temperature comes from increases in surface heat transfer based on by greater radiation consumption.

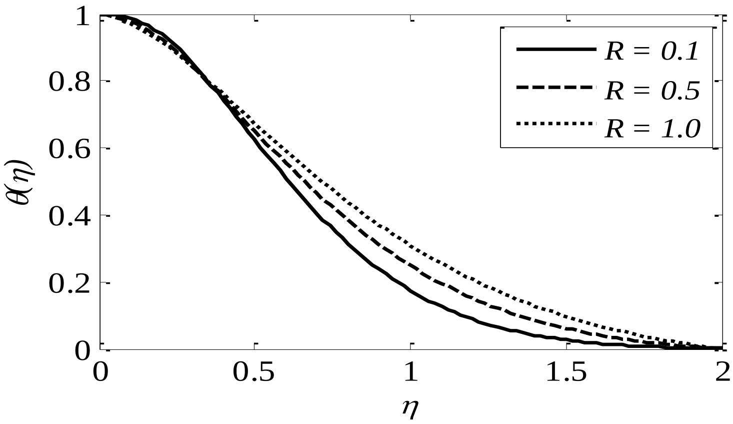

Fig. 9. Effect R on temperature profile

The Prandtl number affects fluid temperature near the stretched sheet. Different Prandtl numbers may change temperature profiles and flow thermal properties. The consequences of the Prandtl number $\Pr$ on the temperature and concentration profile are illustrated in Figure 10 and Figure 11 for a variety of $\Pr$ values. Working with larger amounts of $\Pr$ should have less thermal diffusion, so temperature and concentration profiles flatten out as the $\Pr$ value increases.

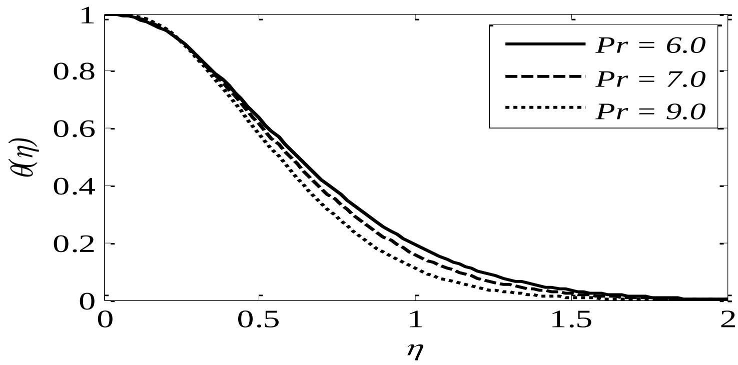

Fig. 10. Effect of $\Pr$ on temperature profile

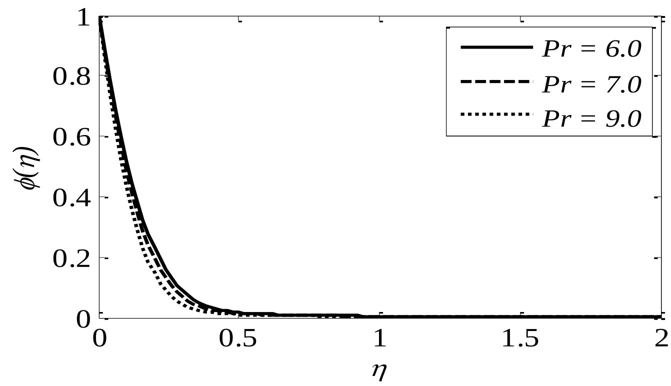

Fig. 11. Effect of $\Pr$ on concentration profile

Brownian motion is the random motion of dispersed particles in a fluid caused by thermal fluctuations. The Brownian factor influences the dispersion and motion of nanoparticles within the nanofluid. It influences the nanofluid's effective thermal conductivity and viscosity. Higher Brownian factors promote the dispersion of nanoparticles by enhancing their random motion. Figure 12 and Figure 13 exhibit the impact of Brownian motion's (Nb) on the temperature and concentration profiles. The temperature sketch for Nb develops along with it, but the concentration distribution shows an alternate style. The physical reason for the boundary layer heating up is the emerging Brownian motion, which is anticipated to transport nanoparticles from the expanding sheet to the stagnant liquid. There are therefore fewer nanoparticles to absorb as a result.

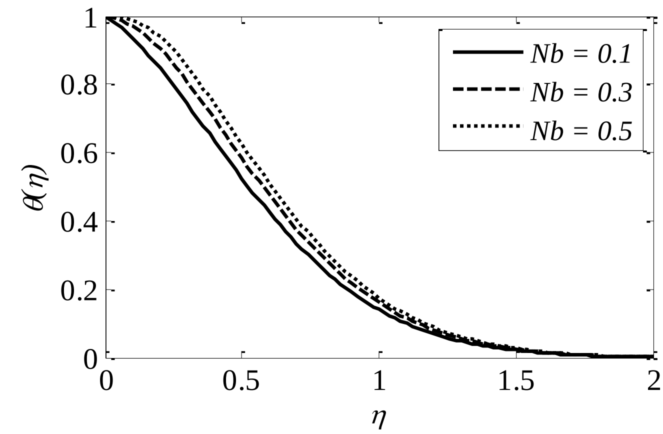

Fig. 12. Effect Nb on temperature profile

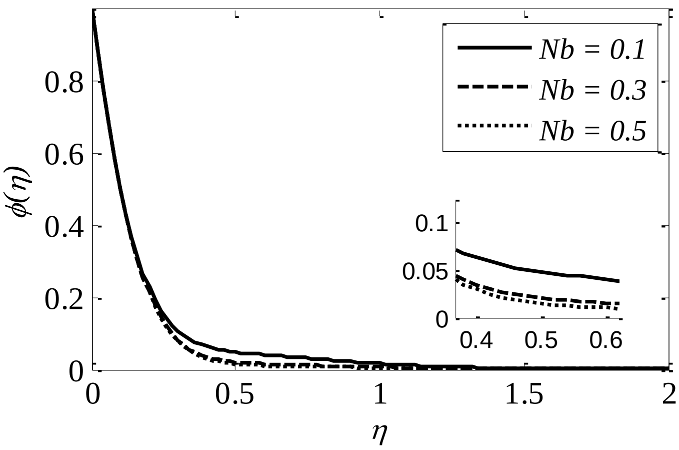

Fig. 13. Effect Nb on concentration profile

Thermophoresis refers to the movement of particles in response to temperature gradients. The thermophoresis factor quantifies this phenomenon in nanofluids. The thermophoresis factor plays a role in determining the movement of nanoparticles within the fluid. It can impact the concentration distribution of nanoparticles in the flow. Thermophoresis can lead to non-uniform particle distributions, which can affect heat transfer. Figure 14 and Figure 15 demonstrate how the thermophoresis parameter (Nt) influences the concentration and temperature profile. When Nt expands, a stronger thermophoretic force leads nanoparticles to migrate from warmer to cooler places as their concentration and temperature increase.

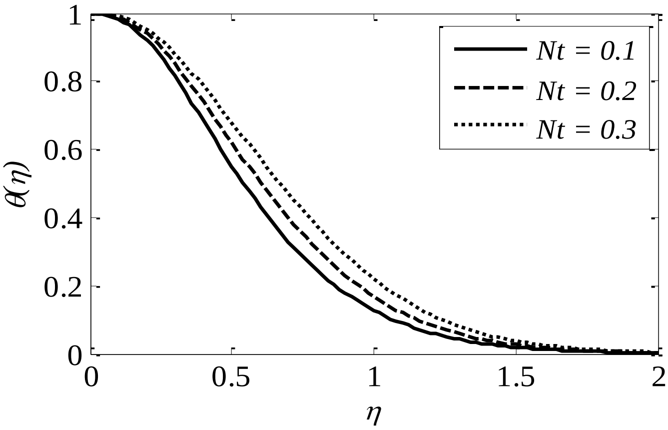

Fig. 14. Effect Nt on temperature profile

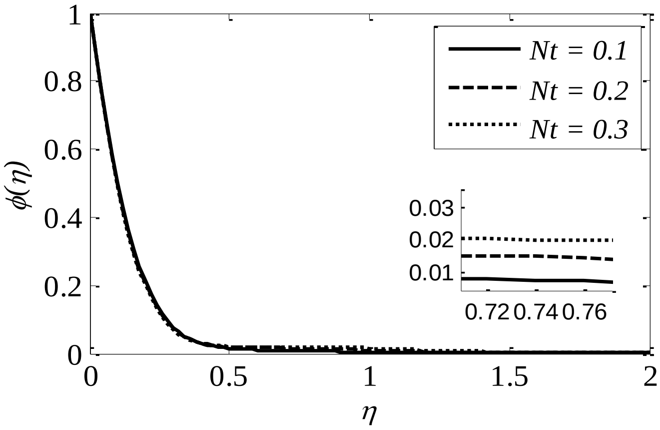

Fig. 15. Effect Nt on concentration profile

In nanofluids the Lewis number plays a role in characterizing the relative rates of heat transfer and mass transfer (diffusion) within the nanofluid. The Lewis number quantifies how readily heat is transported through the nanofluid compared to the diffusion of mass (typically the mass of the nanoparticles) within the fluid. The Lewis number can influence the distribution of nanoparticles within the fluid, which, in turn, affects mass transfer and particle dispersion. The thickness of the thermal boundary layer improves as the value of Lewis number Le gets higher, but it additionally results in a drop in the thickness of the concentration boundary layer. This is depicted in Figure 16.

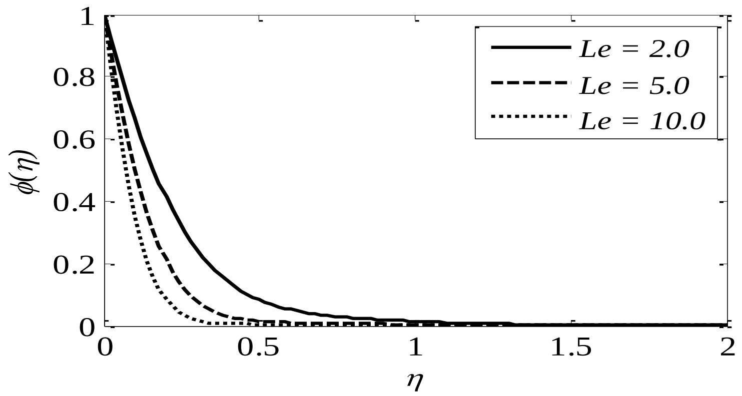

Fig. 16. Effect Le on concentration profile

Figure 17 demonstrates how a chemical reaction parameter (C_r) affects concentration profiles. It is understood that the concentration drops as the chemical reaction parameter improves.

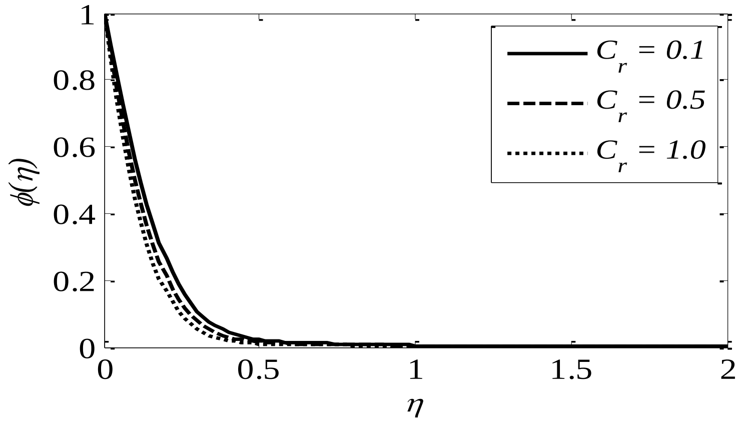

Fig. 17. Effect C_r on concentration profile

Clearly, when Ω and Nb go up, the heat transfer rate falls (see Figure 18).

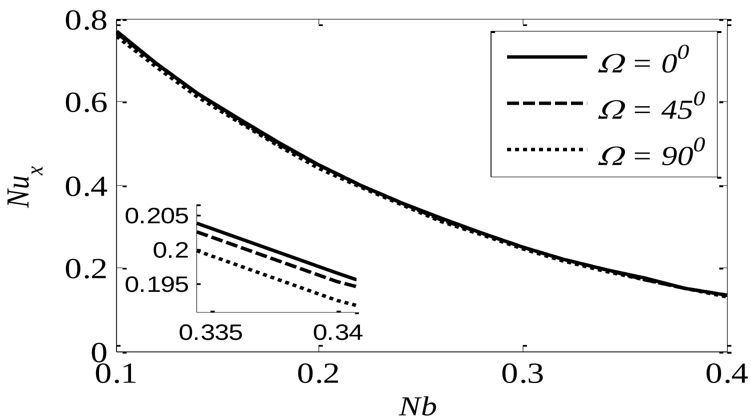

Fig. 18. Effect of Ω and Nb on Nu_x

5. Conclusions

Understanding nanofluid behaviour may improve aircraft engineering, notably heat shields for re-entry vehicles and spacecraft. Spacecraft design requires knowledge of magnetic field heat transfer and fluid movement. This study improves spaceship heat management and propulsion systems. MHD flows with heat sources may reveal magnetohydrodynamic generator performance. These generators may generate electricity from nanofluid kinetic energy. This study may affect renewable energy. In this study, the flow of a Williamson nanofluid on an inclined surface was examined. The ND Solve, a numerical method, was used to achieve the numerical results. The magnetic field is considered in this investigation to be parallel to the sheet. By taking into account inclined geometry, this article seeks to advance and explores the examination of the Williamson nano fluid flow. The following are significant results of the current analysis:

i. The fluid's velocity in the flow field is reduced by the Williamson parameter.
ii. The concentration profile falls as the potency of the chemical reaction develops.
iii. The velocity profile drops due to the inclination effect rise.
iv. As the buoyancy force effect becomes stronger, the velocity profile rises.
v. A rapid rise in the Prandtl number causes a flattening of the temperature and concentration profiles.
vi. As the heat generating constraint values grow, so does the spread of temperatures.
vii. The rate of heat flux lessens as the angle of inclination climbs.

Future guidance outcomes of the current study can help in future improvements where the heat effect of the heating system may be assessed by taking into account different non-Newtonian hybrid nanofuids (i.e., Carreau, second-grade, Casson, Maxwell, micro polar nanofuids, etc.). Additionally, impacts of porosity, as well as viscosity that is dependent on temperature together with magneto-slip flow, can be represented.

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