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MHD Stagnation Point Flow of Micropolar Fluid over a Stretching/ Shrinking Sheet

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ABSTRACT

In this article, the stagnation point flow of a micropolar fluid on a stretching/shrinking sheet has been discussed subject to the assumption of velocity slip. Similarity transformation is used to transform the modelled Partial Differential Equations (PDEs) into a system of Ordinary Differential Equations (ODEs). The numerical results have been found by the shooting technique along with Adams Moulton method of order four. The obtained numerical results are compared with the help of Fortran Language program and compared with the earlier published results and excellent validation of the present numerical results has been achieved for the local Nusselt number. Finally, the numerical results are presented with discussion of the effects of different physical parameters.

1. Introduction

Stagnation point refers to the location in the flow field when the fluid velocity is zero. In the subject of fluid dynamics, the study of viscous, incompressible fluid passing through a permeable plate or sheet is crucial. Because of its wide range of applications in the manufacturing sectors, research on the stagnation point flow of an incompressible fluid across a permeable sheet has gained prominence in recent decades.

Some of the most common uses include fan-cooled electrical devices, atomic receptacles cooling for the length of an emergency power outage, solar receivers, and so on. Hiemenz [15] was the first to examine two-dimensional (2D) stagnation point flow, while Eckert [11] expanded this problem by including the energy equation to obtain an accurate answer. As a result, Mahapatra and Gupta [14], Ishak *et al.,* [2], and Hayat *et al.,* [29] investigated the effects of heat transmission in stagnation point across a permeable plate.

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The effect of slip condition gives an interesting result for different fluids. Sharma *et al.,* [27] investigated the slip effect of the heat transfer due to stretching sheet on a CuO water nanofluid. A new model effect of second order slip velocity was introduced by Wu [34]. Wang *et al.,* [6] extended the article of Wu [34] by considering the slip effect of stagnation point flow on a heated vertical plate. Fang *et al.,* [8] investigated the second order velocity slip effect on the viscous flow due to a stretching sheet. Nandeppanaver *et al.,* [19] discussed the heat transfer and second order slip flow due to a stretching sheet. Deissler [7], Rosca and Pop [22] and Turkyilmazoglu [32] investigated the second order velocity slip effect, under different physical conditions.

Many researchers found interest in the study of micropolar fluid for the different geometries. Erigen [12] was the first one who investigated micropolar fluid. Ariman *et al.,* [30] theoretically investigated micropolar fluids and their applications. Ishak *et al.,* [1] discussed the stagnation point flow of a micropolar fluid in a two-dimensional boundary layer flow of mixed convection on a stretching sheet. Bhargava *et al.,* [24] numerically investigated the solutions of micro-polar transport due to a non-linear stretching sheet. Rees and Pop [23] theoretically discussed free convection from a vertical at plate in a micropolar fluid. Nazar *et al.,* [25]. Sajid *et al.,* [28] analyzed the stretching flow with a general slip consition.

System involving chemical reactions are completed. Batcha Srisailam *et al.,* [5] analyzed the effect of viscous dissipation and chemical reaction on the flow of MHD nanofluid over a stretching sheet. Lim Yeou Jiann *et al.,* [18], the effects of Wu's velocity slip and Smoluchowski's temperature slip are taken into consideration. Thirupathi *et al.,* [31] presented a numerical investigation for the magnetohydrodynamics (MHD) stagnation point Casson nanofluid flow towards stretching surface with velocity slip and convective boundary condition. Abu Bakar, Shahirah *et al.,* [3], investigated the mixed convection boundary layer flow over a permeable surface embedded in a porous medium, filled with a nanofluid and subjected to thermal radiation, magnetohydrodynamics (MHD) and internal heat generation. E. N. Maral *et al.,* [10] studied on peristaltic transport of menthol electrolytes altered utilizing an external electric field which contributes to the creation of a net surface charge attracting counter ions and repels co ions from the menthol based nanofluid.

Recently Noreen Sher Akbar *et al.,* [21] focused on the viscous flow of cu-water/Methanol suspended nanofluids towards a 3D stretching sheet. Faisal Z. Duraihem *et al.,* [13] analysed the impact of thermal stratification and medium porosity on gravity-coerced transport of hybrid carbon nanotubes. E.N. Maraj *et al.,* [9] study on oscillatory pressure-driven MHD flow of a hybrid nanofluid in a vertical rotating channel. Khalid Y. Ghailan *et al.,* [17] study of unsteady peristaltic flow across a channel with finite width and porous medium. Javaria Akram *et al.,* [16] investigation emphasizes the fluid flow analysis and the heat transfer characteristics of 10 W40-based titanium dioxide nanofluid subject to electroosmotic forces and the peristaltic propulsion in a curved microchannel. Javaria Akram and Noreen Sher Akbar *et al.,* [4] analysis is conducted for a theoretical examination of the fluid flow characteristics and heat transferred by the nanoparticle enhanced drilling muds flowing through drilling pipes under various physical conditions.

To the best of Authors' knowledge, no information is available on the effect of magnetic parameter on the stagnation point flow of a micro fluid over a stretching/shrinking sheet. The present work is aims to fill the gap in the existing literature. Therefore, in the present paper, we consider the MHD stagnation point flow over a stretching/shrinking sheet placed in a micropolar fluid with second order slip condition. We shall apply Shooting technique along with Adams – Moulton method of order four to solve the similarity equations obtained from the governing boundary layer equations with the help of similarity transformation. The structure of the present paper is as follows: The problem formulation and quantity of physical interest are presented in Section 2. Adams – Moulton method for the proposed problem is presented in Section 3. In Section 4 results and discussion are reported whereas Section 6 is reserved for concluding remarks.

2. Mathematical Modeling

Consider a steady, electrically conducting, two-dimensional stagnation point flow of an incompressible micropolar fluid on a stretching/shrinking sheet with the assumption of slip velocity effect. Assume that $u_e\left(x\right)=ax$ be the free stream velocity and $u_w\left(x\right)=bx$ be the stretching/shrinking velocity respectively, where a and b are some real constants. The flow configuration and axes system are depicted in Figure 1. The length of the sheet is taken along the $x-$axis whereas $y-$axis taken normal to the sheet.

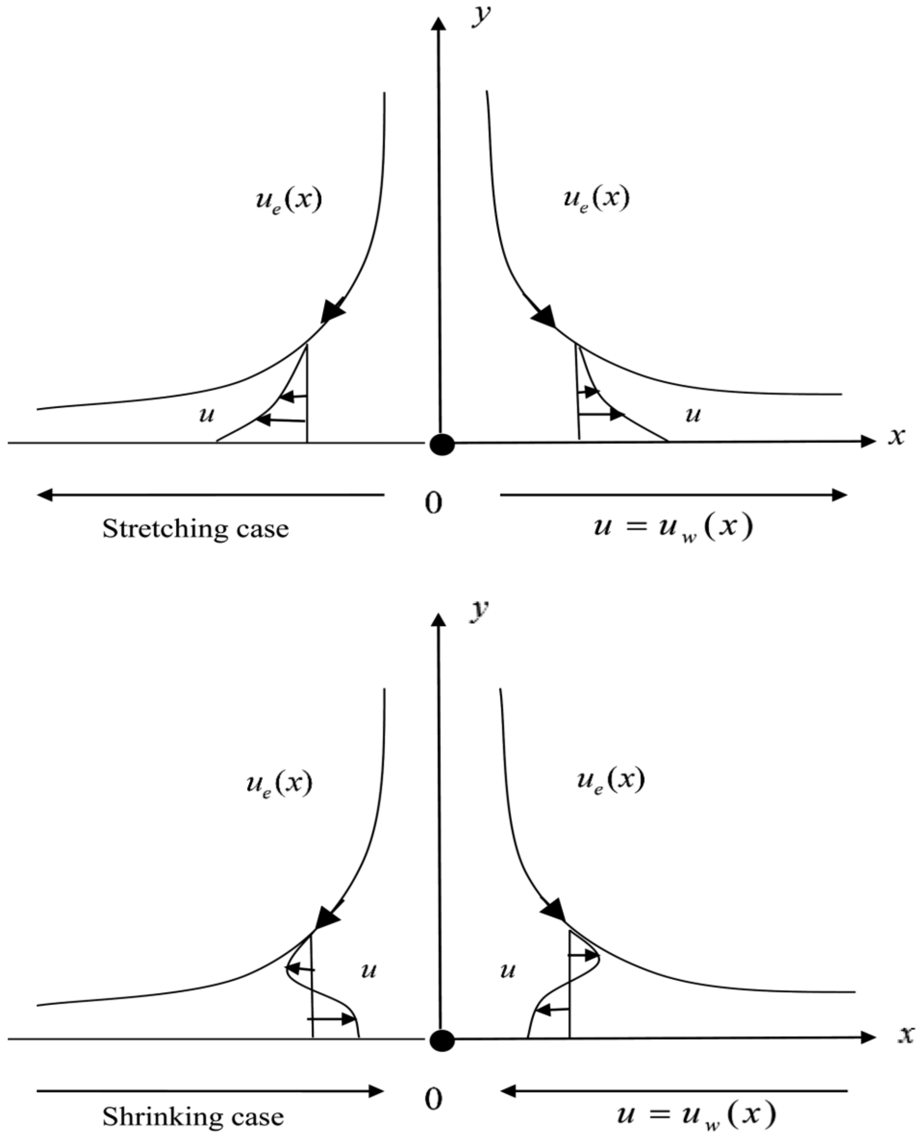

Fig. 1. Geometry of the problem

For stretching sheet $b>0$ and for shrinking sheet $b<0$. The mathematical model of the flow, presented by Sharma *et al.,* [26] is as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial x}u+\frac{\partial u}{\partial y}v=u_e\frac{\partial u_e}{\partial x}+\left(\frac{\mu+k}{\rho}\right)\frac{\partial^2 u}{\partial y^2}+\frac{k}{\rho}\frac{\partial N}{\partial y}-\frac{\sigma B_0^2}{\rho}\left(u-u_e\right) \tag{2}$$

$$\rho j\left(\frac{\partial N}{\partial x}u+\frac{\partial N}{\partial y}v\right)=\left(\mu+\frac{k}{2}\right)j\frac{\partial^2 N}{\partial y^2}-k\left(2N+\frac{\partial u}{\partial y}\right), \tag{3}$$

Where the velocity components has been represented by u and v respectively. Dynamic viscosity is denoted by μ, microrotation viscosity by k, fluid density by ρ, micro inertia density by j and component of microrotation is denoted by N. The boundary conditions of the above equations are given as [31]

$$\left.\begin{aligned}&v=0,\quad u=u_w\left(x\right)+u_{slip},\quad N=-n\frac{\partial u}{\partial y}\quad at\quad y=0,\\&u=u_e\left(x\right),\quad N\to 0\qquad as\quad y\to\infty,\end{aligned}\right\} \tag{4}$$

Where $u_e\left(x\right)$, u_{slip} and $u_w\left(x\right)$ represent the free steam velocity, slip velocity and stretching /shrinking velocity.

In the boundary conditions $n=$ constant with $0\le n\le 1$. The boundary condition with $n=0=$ no slip condition, which requires that fluid particles closest to the solid boundary stick to it. The boundary condition with $n\ne 0$ (i.e., microrotation is equal to the fluid velocity at the boundary) implies that the neighbourhood of a grid boundary.

The stream function   identically satisfies the continuity equation. Mathematically,

$$u=\frac{\partial\psi}{\partial y},\qquad v=\frac{\partial\psi}{\partial x}. \tag{5}$$

Now, introduce the following similarity variables from [26],

$$\left.\begin{aligned}&\psi=\sqrt{\nu x u_e\left(x\right)}f\left(\eta\right)=\sqrt{a\nu}xf\left(\eta\right),\\&\eta=\sqrt{\frac{u_e\left(x\right)}{\nu x}}y=\sqrt{\frac{a}{\nu}}y,\\&N=u_e\left(x\right)\sqrt{\frac{u_e\left(x\right)}{\nu x}},\\&h\left(\eta\right)=a\sqrt{\frac{a}{\nu}}xh\left(\eta\right)\end{aligned}\right\} \tag{6}$$

Where the stream function is represented by ψ and the kinematic viscosity is represented by ν.

Thus, the dimensionless form of the mathematical model of the present problem is:

$$\left(1-f^{'2}\right)+\left(1+K\right)f^{'''}+ff^{''}+Kh^{'}-M\left(f^{'}-1\right)=0, \tag{7}$$

$$\left(1+\frac{K}{2}\right)h^{''}+fh^{'}=f^{'}h+K\left(2h+f^{''}\right) \tag{8}$$

along with BCs:

$$f(0)=0,\ f^{'}\left(0\right)=\varepsilon+\lambda f^{''}\left(0\right)+\delta f^{'''}\left(0\right),h\left(0\right)=-nf^{''}\left(0\right), \tag{9}$$

$$f^{'}\left(\eta\right)\rightarrow 1,h\left(\eta\right)\rightarrow 0,\text{as }\eta\rightarrow\infty, \tag{10}$$

In the above equations, the micropolar parameter by $K=\frac{k}{\mu}=\frac{k}{\nu\rho}$ the stretching/shrinking rate has been represented by $\varepsilon=\frac{b}{a}$ the first order slip represented by $\lambda=A\sqrt{\frac{a}{\nu}}$ and the second order slip by $\delta=B\frac{a}{\nu}$ where A and B have the following formulations [7]

$$\left.\begin{aligned} A&=\frac{2}{3}\left(\frac{3-\alpha l^{3}}{\alpha}-\frac{3}{2}\frac{1-l^{2}}{K_{n}}\right)\lambda,\\ B&=-\frac{1}{4}\left[l^{4}+\frac{2}{K_{n}^{2}}\left(1+l^{2}\right)\right]\lambda^{2}.\end{aligned}\right\} \tag{11}$$

3. Solution Methodology

The numerical solution of the mathematical model in the form of non-linear differential equations Eq. (7) – Eq. (8) along with the boundary conditions Eq. (9) – Eq. (10) was reported by Sharma *et al.,* [26]. They opted for the finite-difference method for the numerical solution of the above model. In the present section, shooting technique with Adams Moulton method has been proposed to reproduce the same solution. The Adams Moulton method of order four and the Newton's method for solving the non-linear algebraic equations, are the main components of the shooting method. Let us re-write Eq. (7) – Eq. (8) as:

$$f^{'''}=-\frac{1}{\left(1+K\right)}\left[ff^{''}+\left(1-f^{'2}\right)+Kh^{'}+M\left(f^{'}-1\right)\right], \tag{12}$$

$$h^{''}=\frac{2}{(2+K)}\left[fh^{'}-f^{'}h-K\left(2h+f^{''}\right)\right]. \tag{13}$$

Use the notations to construct a system of first order ODEs:

$$f = y_1, f^{'} = y_2, f^{''} = y_3, h = y_4, h^{'} = y_5 \tag{14}$$

By using the notations Eq. (14), we have the following IVP:

$$\left.\begin{aligned}
&y_1^{'} = y_2, && y_1(0) = 0,\\
&y_2^{'} = y_3, && y_2(0) = s,\\
&y_3^{1} = -\frac{1}{(1+K)}\left[y_1 y_3 + \left(1-y_2^2\right) + K y_5 + M\left(y_2 - 1\right)\right], &&\\
&\qquad y_3(0) = \frac{1}{\lambda}\left[s - \varepsilon + \delta\left(\frac{1}{1+K}\right)\left(\left[\left(1-s^2\right) + Kt\right]\right)\right], &&\\
&y_4^{'} = y_5, \qquad y_4(0) = -\frac{n}{\lambda}\left[s - \varepsilon + \delta\left(\frac{1}{1+K}\right)\left(\left[\left(1-s^2\right) + Kt\right]\right)\right], &&\\
&y_5^{1} = \frac{2}{(2+K)}\left[-y_1 y_5 + y_2 y_4 + K(2y_4 + y_3)\right], && y_5(0)) = t.
\end{aligned}\right\} \tag{15}$$

In order to get the approximate numerical results, the problem's domain is considered to be bounded i.e., $\left[0, \eta_\infty\right]$, where η_∞ is chosen to be an appropriate finite positive real number so that the variation in the result for $\eta > \eta_\infty$ is ignorable. In Eq. (15), the missing initial conditions s and t are to be chosen such that.

$$y_2\left(\eta_\infty, s, t\right) - 1 = 0, y_4\left(\eta_\infty, s, t\right) = 0. \tag{16}$$

To start the iterative process, choose $s = s_0$, and $t = t_0$. To the values of s, t Newton's iterative scheme has been used.

$$\begin{pmatrix} s_{n+1} \\ t_{n+1} \end{pmatrix} = \begin{pmatrix} s_n \\ t_n \end{pmatrix} - \begin{pmatrix} \dfrac{\partial y_2}{\partial s} & \dfrac{\partial y_2}{\partial t} \\ \dfrac{\partial y_4}{\partial s} & \dfrac{\partial y_4}{\partial s} \end{pmatrix}^{-1}_{(s_n, t_n)} \begin{pmatrix} y_2\left(\left(\eta_\infty, s_n, t_n\right) - 1\right) \\ y_4\left(\left(\eta_\infty, s_n, t_n\right)\right) \end{pmatrix} \tag{17}$$

To implement the Newton's scheme, consider the following notations:

$$\frac{\partial y_1}{\partial s} = y_6, \frac{\partial y_2}{\partial s} = y_7, \ldots\ldots\ldots\ldots\ldots\ldots \frac{\partial y_5}{\partial s} = y_{10},$$
$$\frac{\partial y_1}{\partial t} = y_{11}, \frac{\partial y_2}{\partial t} = y_{12}, \ldots\ldots\ldots\ldots\ldots\ldots \frac{\partial y_5}{\partial t} = y_{15}.$$

Differentiating Eq. (15), first w.r.t. s and then w.r.t. t, we get the following fifteen first order ODEs along with the associated initial conditions.

$$
\left.\begin{aligned}
&y_6' = y_7, && y_6(0) = 0,\\
&y_7' = y_8, && y_7(0) = 1,\\
&y_3^1 = -\frac{1}{(1+K)}\left[y_6 y_3 + y_1 y_8 - 2y_2 y_7 + K y_{10} + M\, y_9\right], && y_8(0) = \frac{1}{\lambda}\left[1-\left(\frac{2\delta s}{1+K}\right)\right],\\
&y_9' = y_{10}, && y_9(0) = -\frac{n}{\lambda}\left[1-\frac{2\delta s}{1+K}\right],\\
&y_{10}^1 = \frac{2}{(2+K)}\left[-y_6 y_5 + y_7 y_4 + y_2 y_9 - y_1 y_{10} + K(2y_9 + y_8)\right], && y_{10}(0)) = 0,\\
&y_{11}' = y_{12}, && y_{11}(0)) = 0,\\
&y_{12}' = y_{13}, && y_{12}(0)) = 0,\\
&y_{13}^1 = -\frac{1}{(1+K)}\left[y_{11} y_{13} + y_1 y_{13} - 2y_2 y_{12} + K y_{15} + M\, y_{11}\right], && y_{13}(0) = \frac{1}{\lambda}\left[\left(\frac{2\delta s}{1+K}\right)\right],\\
&y_{14}' = y_{15}, && y_{14}(0) = -\frac{n}{\lambda}\left[\frac{2\delta s}{1+K}\right],\\
&y_{15}^1 = \frac{2}{(2+K)}\left[-y_{11} y_5 + y_{12} y_4 + y_2 y_{14} - y_1 y_{15} + K(2y_{14} + y_{13})\right], && y_{15}(0)) = 1
\end{aligned}\right\} \tag{18}
$$

Next, the IVP in the form of fifteen first order ODEs given in Eq. (15) and Eq. (18) is solved by the fourth order Adams Moulton method and the Newton's method. If for a sufficiently small ε^*,

$$
\max\left\{\left|y_2\left(\eta_\infty, s_n, t_n\right) - 1\right|, \left|y_4\left(\eta_\infty, s_n, t_n\right) - 1\right|\right\} > \varepsilon^* \tag{19}
$$

The guessed values of s and t are updated by the Newton's iterative scheme:

$$
\begin{pmatrix} s_{n+1} \\ t_{n+1} \end{pmatrix} = \begin{pmatrix} s_n \\ t_n \end{pmatrix} - \begin{pmatrix} y_7 & y_{12} \\ y_9 & y_{14} \end{pmatrix}^{-1}_{(s_n, t_n)} \begin{pmatrix} y_2\left(\left(\eta_\infty, s_n, t_n\right) - 1\right) \\ y_4\left(\left(\eta_\infty, s_n, t_n\right)\right) \end{pmatrix} \tag{20}
$$

The iterative process is repeated until the following criteria is met.

$$
\max\left\{\left|y_2\left(\eta_\infty, s_n, t_n\right) - 1\right|, \left|y_4\left(\eta_\infty, s_n, t_n\right) - 1\right|\right\} < \varepsilon^* \tag{21}
$$

4. Results and Discussion

The main objective of the present section is to study the effect of different physical parameters like K (micro-polar parameter), λ (the first order slip parameter), ε (the stretching/shrinking rate), δ (the second order slip parameter) on the velocity and micro-rotation profiles. The present results have been compared with the previous results of Wang [33] and Bachok *et al.,* [20] for different values of the stretching/shrinking rate ε in Table 1 which are in good agreement. Wang [33] and Bachok *et al.,* [20] have discussed the stagnation point flow towards a stretching/shrinking sheet.

Table 1
Comparison of $f^{''}(0)$ for different values of ε when $\lambda = 0, \delta = 0, K = 0$, and $n = 0.5$

Values of ε	Wang [33]	Bachok *et al.,* [20]	[26]	Current results
2.0	-1.88731	-1.8873066	-1.88730667	-1.88730627
1.0	0	0	0	0
0.5	0.713300	0.7132949	0.71329496	0.71525570
0.0	1.232588	1.2325877	1.23258765	1.23257700
-0.25	1.402240	1.4022408	1.40224081	1.40224872
-0.5	1.495670	1.4956698	1.49566977	1.49566265
-1.0	1.328820	1.3288170	1.32881688	1.32881259
-1.2	0.554300	0.9324730	0.93247336	0.93247167
-1.2465	--	0.5842956	0.58428274	0.58428643

The impact of the first order slip λ on the velocity profile is presented in Figure 2. By increasing the values of the λ, the velocity profile is increased. Physically, when a slip occurs, the velocity of flow near the sheet is no longer equal to the stretching velocity of the sheet.

The variations in the micro-rotation profile for the λ are demonstrated in Figure 3 and 4. An opposite flow behavior is determined with the first and second solution. The thickness of boundary layer is deceased in the first solution and increases in the second solution.

Figure 5 and Figure 6 demonstrate the impact of the second order slip parameter δ on the velocity profile. Figure 5 indicates that by increasing δ, the velocity profile is increased. Figure 6 represents that by increasing δ, the velocity profile is reduced.

The variations in the microrotation profile for different values of the second order velocity slip δ are demonstrated in Figure 7 and Figure 8. It shows that the microrotation profile is initially increased as δ is increased for the first solution and microrotation profile is decreased as δ is increased for the second solution.

The variations in the velocity profile for micropolar parameter K are demonstrated in Figure 9 and Figure 10. By increasing the values of the micropolar fluid K, the velocity field is reduced in both the first and the second solution. It is evident from Figure 9 and Figure 10 that all curves approach the far field boundary conditions asymptotically.

The variations in the microrotation profile for micropolar parameter K are demonstrated in Figure 11 and Figure 12. From these graphs, it can be observed that increasing the micropolar K, the velocity field is reduced in the lower half of the surface whereas it is enhanced in the upper half. The velocity is going to reduce initially with the mounting values of the micropolar K. The boundary layer thickness is increased in both the first and the second solution.

Figure 13 illustrates the changes in the velocity profile at various magnetic field strengths. As the magnetic field strength increased, the velocity profile increased. This phenomenon is a result of the magnetic field that enhances fluid motion within the boundary layer.

Figure 14 shows the variation in the micro-rotation profile for different estimations of the magnetic field M. By increasing M, the micro-rotation is increased. Thus, the boundary layer thickness is decreased.

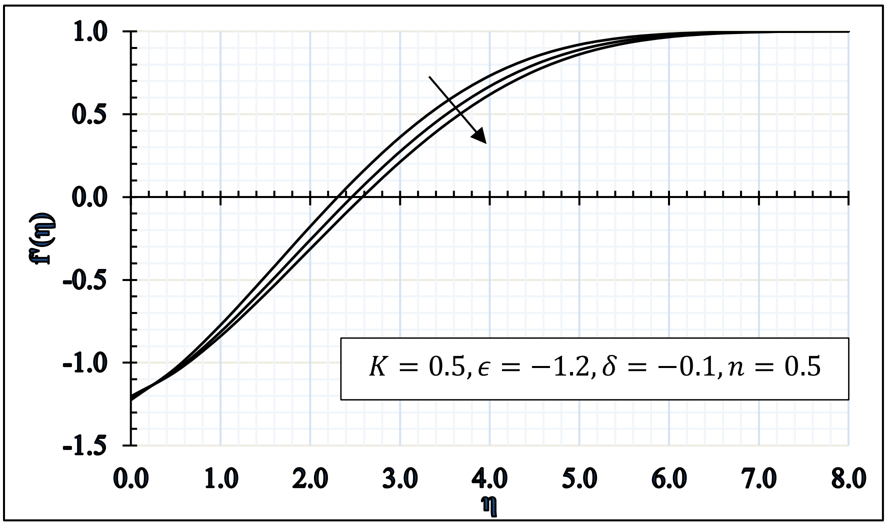

Fig. 2. Impact of $\lambda = 0.05, 0.1, 0.15$ on $f^{'}(\eta)$

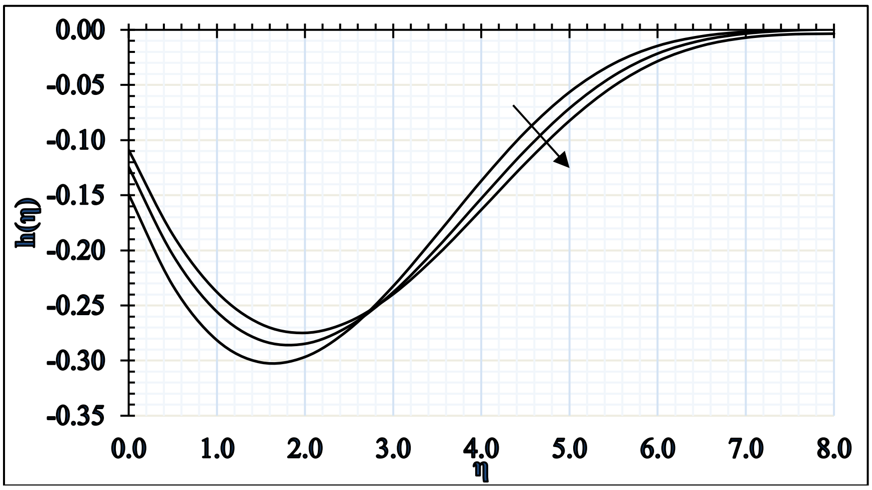

Fig. 3. Impact of $\lambda = 0.05, 0.1, 0.15$ on $h(\eta)$

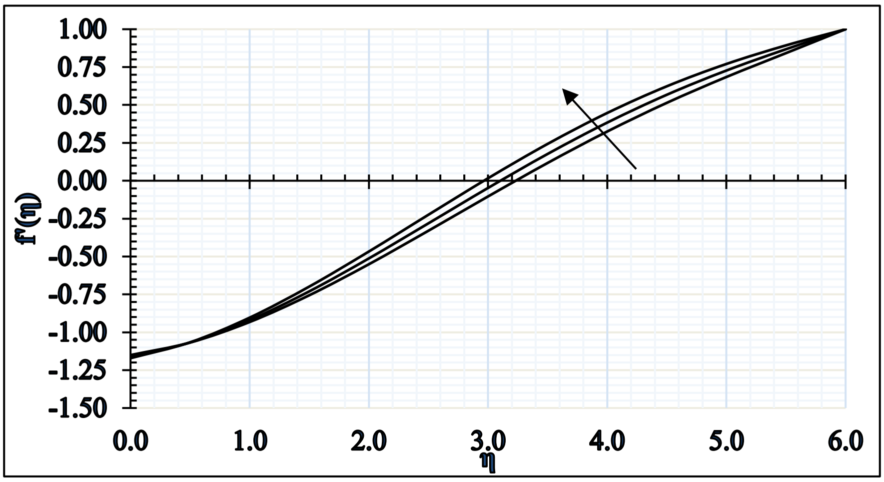

Fig. 4. Impact of $\delta = 0.05, 0.10, 0.15$ on $f^{'}(\eta)$

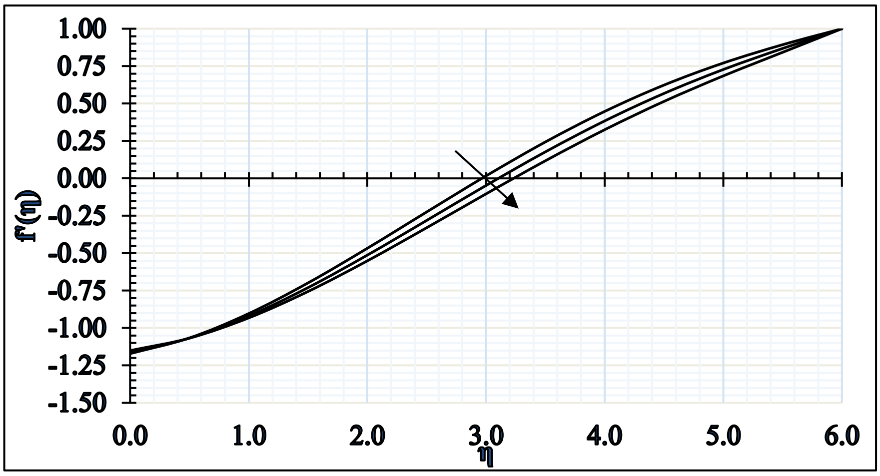

Fig. 5. Impact of $\delta = 0.05, 0.10, 0.15$ on $f^{'}(\eta)$

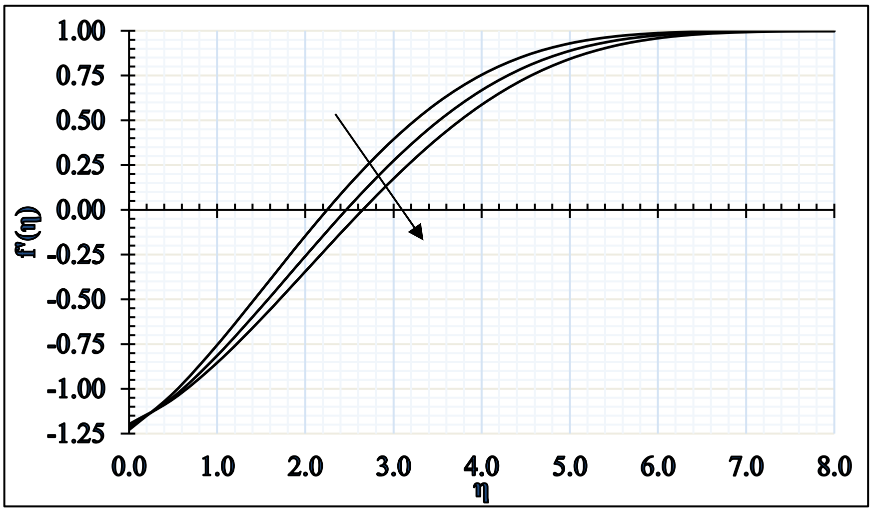

Fig. 6. Impact of $\delta = -0.05, -0.10, -0.15$ on $f^{'}(\eta)$

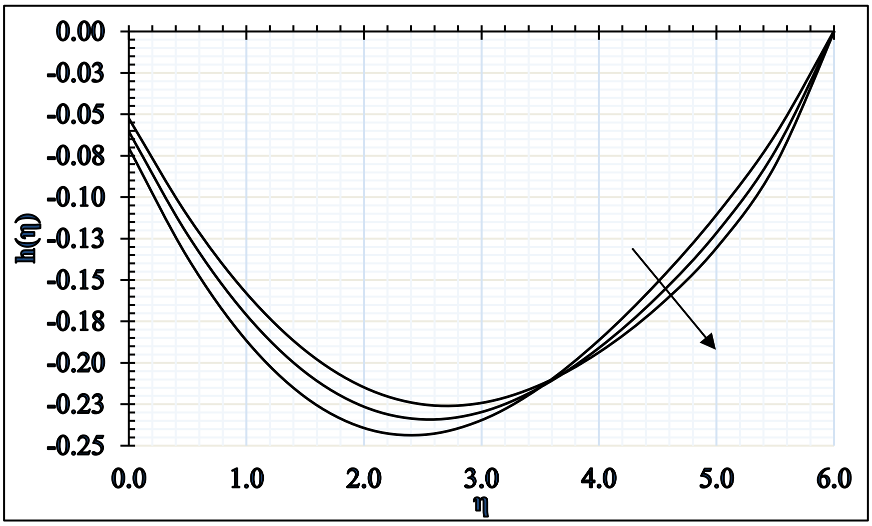

Fig. 7. Impact of $\delta = 0.05, 0.10, 0.15$ on $h(\eta)$

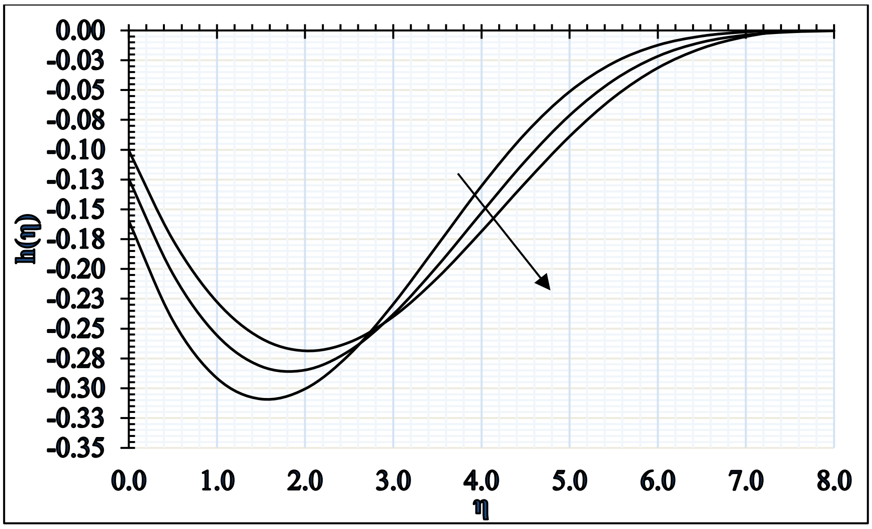

Fig. 8. Impact of $\delta = -0.05, -0.10, -0.15$ on $h(\eta)$

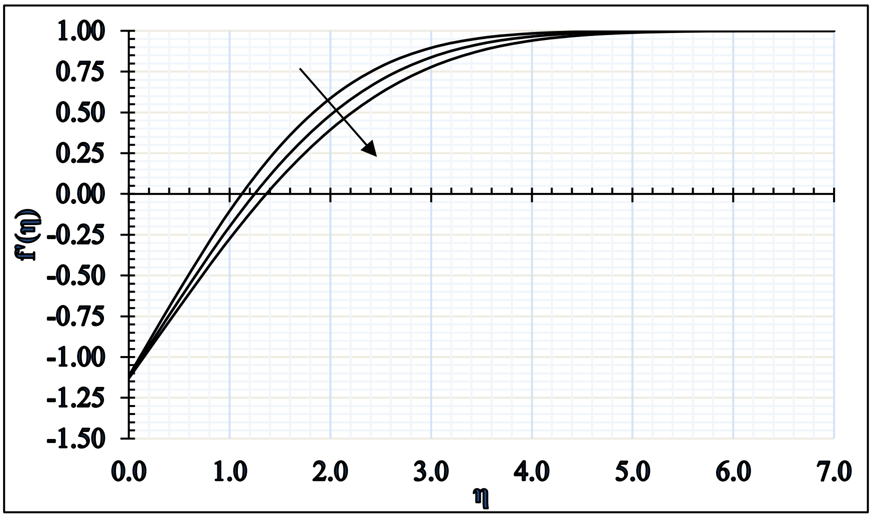

Fig. 9. Impact of $K = 0.05, 0.10, 0.15$ on $f^{'}(\eta)$

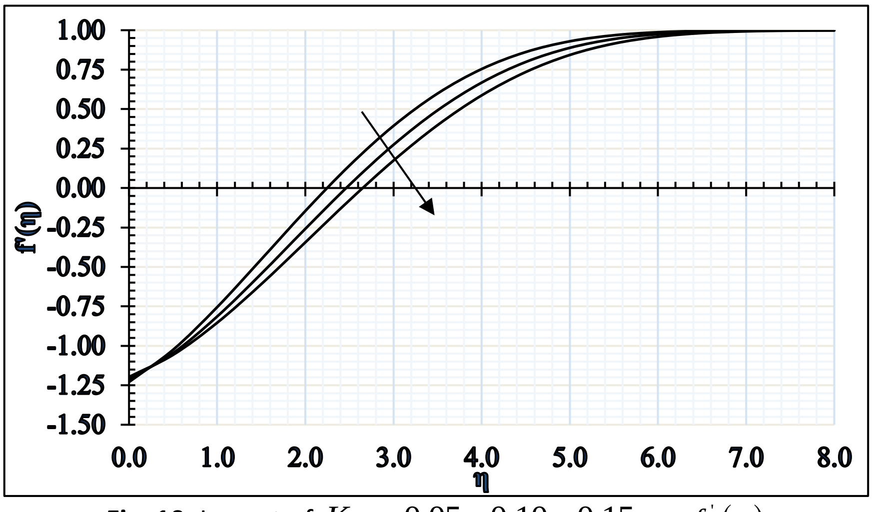

Fig. 10. Impact of $K = -0.05, -0.10, -0.15$ on $f^{'}(\eta)$

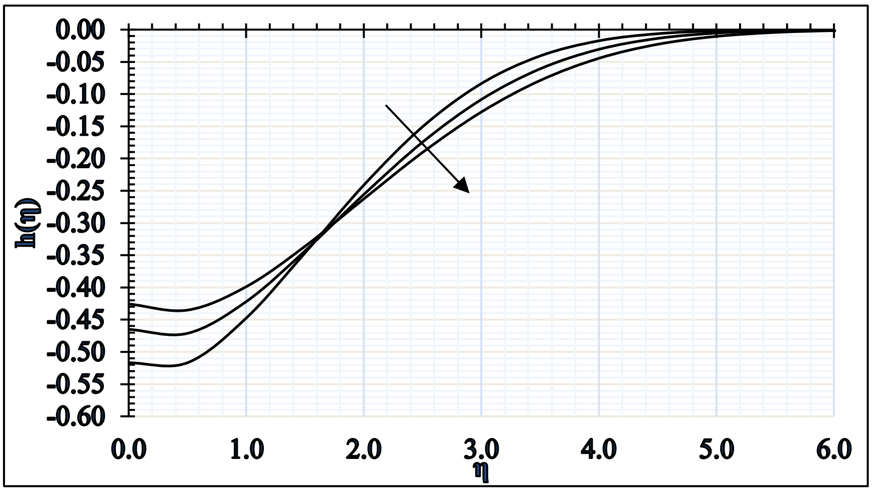

Fig. 11. Impact of $K = 0.05, 0.10, 0.15$ on $h(\eta)$

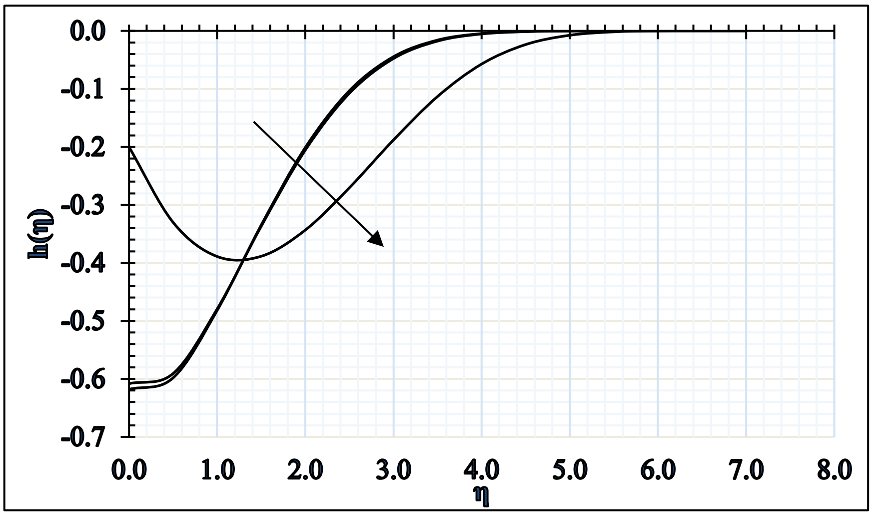

Fig. 12. Impact of $K = -0.05, -0.10, -0.15$ on $h(\eta)$

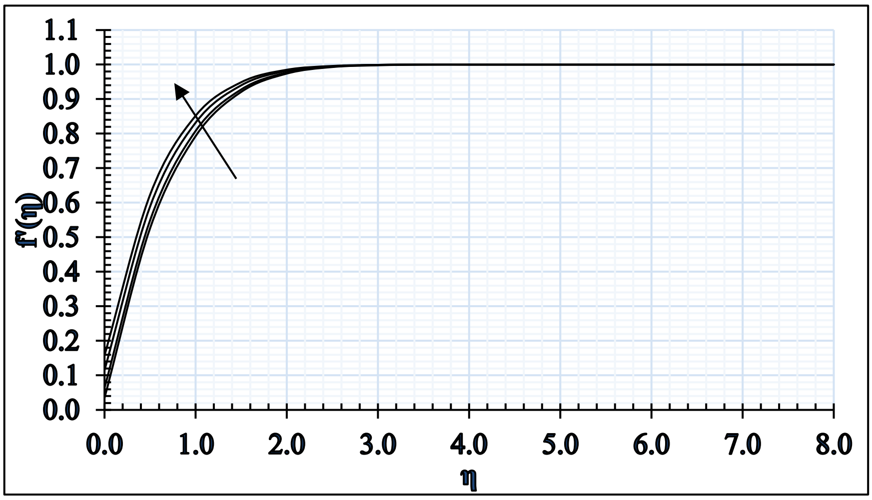

Fig. 13. Impact of $K = 0.05, 0.10, 0.15$ on $f^{'}(\eta)$

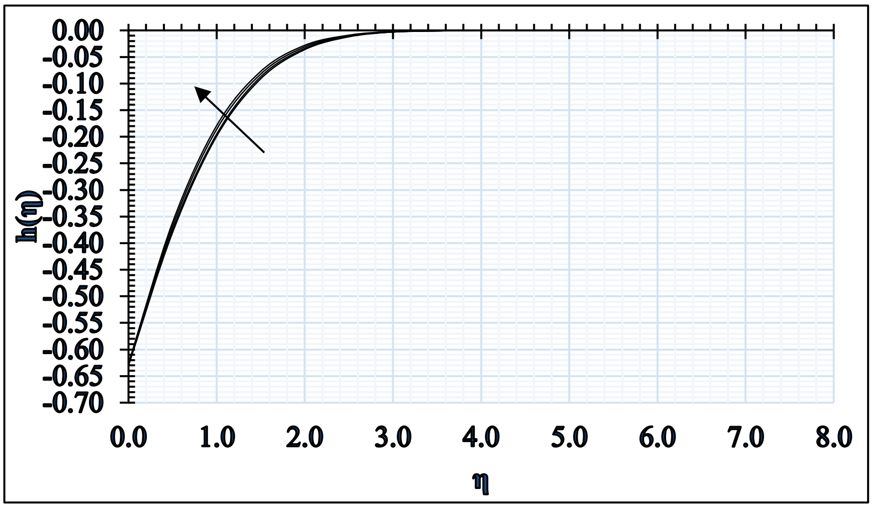

Fig. 14. Impact of $K = 0.05, 0.10, 0.15$ on $h(\eta)$

4. Conclusions

The governing equations for the 2D stagnation point flow of a viscous and incompressible fluid over a stretching/shrinking sheet with second order slip boundary condition and magnetic field were formulated. The resulting partial differential equations were transformed into a set of ordinary differential equations using the similarity transformations.

These equations are solved numerically using Shooting techniques with Adams Moulton method. The conclusions of the study are as follows:

Increasing the suction parameter, the velocity and microrotation profiles are increased. Due to an increase in the shrinking parameter, the velocity and micro-rotation profiles are decreased. Increasing the micropolar parameter, the velocity and micro-rotation profiles are decreased. By increasing magnetic field, the velocity and micro-rotation profiles are increased. This problem may be extended in many directions focusing on the fluid model of Jeffery, Tangent hyperbolic nanofluid.

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