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Investigation of Magneto Hydrodynamics Properties of Reiner–Philippoff Nanofluid with Gyrotactic Microorganism in a Porous Medium

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ABSTRACT

Nanofluids have many potential applications in engineering, medicine, and biotechnology due to their enhanced thermal, electrical, and optical properties. However, the flow and heat transfer characteristics of nanofluids are influenced by various factors, such as the type and size of nanoparticles, the base fluid, the magnetic field, the radiation, the chemical reaction, and the presence of microorganisms. Therefore, it is important to study the effects of these factors on the nanofluid flow and heat transfer using mathematical models and numerical methods. One of the mathematical models that can describe the nanofluid flow is the Reiner-Philippoff model, which is a classical non-Newtonian fluid model that accounts for the shear-thinning behaviour of some fluids. The Reiner-Philippoff model has been used to study the nanofluid flow over a stretching sheet, which is a simplified model of many industrial processes involving stretching or shrinking surfaces. However, most of the previous studies have neglected the effects of the Arrhenius reaction, thermal radiation, viscous dissipation, and bio-convection on the nanofluid flow over a stretching sheet. The objective of this paper is to fill this gap by conducting a numerical investigation of the effects of the Arrhenius reaction, thermal radiation, viscous dissipation, and bio-convection on a Reiner-Philippoff nanofluid of MHD flow through a stretching sheet. This also considers the effects of thermophoresis and Brownian motion, which are two mechanisms that govern the transport of nanoparticles in nanofluids. The article utilized a similarity transformation to reduce the governing partial differential equations into ordinary differential equations, which are then solved by using the MATLAB computational tool bvp4c technique. The paper also employs a hybrid numerical solution method using Runge-Kutta fourth order with a shooting technique and an optimization technique using the Bayesian regularization method for Runge-Kutta to improve the accuracy of the prediction outcomes. The main finding of this paper is that the Arrhenius reaction, thermal radiation, viscous dissipation, and bio-convection have significant effects on the velocity, temperature, concentration, and motile microorganism profiles of the nanofluid flow over a stretching sheet. The paper also discusses how these effects can be controlled by varying the relevant parameters. This provides graphical results for the profiles of velocity, temperature, concentration, and motile microorganisms for different values of these parameters. The study also compares its results with some existing results in the literature and finds good agreement.

Keywords:
Non-Newtonian fluid; Reiner-Philippoff Nanofluid; Bioconvection; Gyrotactic Microorganism; Thermal Radiation; MHD; Bayesian Regularization; Fourth Order Runge-Kutta; MATLAB

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1. Introduction

Due to its many applications in science and engineering, the study of non-Newtonian fluids has drawn a lot of attention from researchers. It is challenging to establish a single relation that fully explains the behaviour of non-Newtonian fluids [1] because of the intricate interaction between stress and shear rate strain. Shear-thinning (Pseudo-plastic), shear-thinning (dilatant), and Newtonian behaviour are all characteristics of the Reiner-Philippoff fluid. Power-law, Sisko, and Prandtl-Eyring are some more fluids that fall under the same category. Fluids such as Sutterby and Powell-Eyring, etc. [2]. Gangadhar [3] considered the use of thermal radiation and micropolar Ferrofluid on a stretchy sheet. When a Sutterby fluid impinges on an extendible surface, along with viscous and Ohmic dissipation, Nawaz [4] considered the effects of the nanoparticles and observed that the power-law index of the velocity field increases as a result. Jyothi [5] examined the characterization of gold nanoparticles on Sisko fluid moving over a slippery extendable surface. Eyring-Powell fluid embedded with aluminium oxide nanoparticles and observed that thermal boundary thickness escalates by a magnification in the radiation parameter.

Thermal conductivity phenomena are improved by the presence of nanoparticles in a fluid. Nanofluids are well-known materials used in a variety of technical, industrial, and scientific fields [6]. They contain innovative thermal uses. Some dynamic applications in heating and cooling devices, nuclear heating and cooling devices, solar concerns, magnetic retention, astronomy and safety, automated operation, etc. are preceded by the thermophysical properties of nanofluids [7]. The nanoparticles are typically thought to be tiny metallic particles less than 100 nm in size. The nanofluids compensate for greater energy transmission than the conventional viscous liquids. Choi [8] was the first scientist to give the thermal perspective of nanofluids his direct attention. Because nanofluids have such vast industrial and medical uses, researchers from all around the world have been studying them. Mahanthesh [9] examined magneto nanofluids subjected to convective boundary conditions and a revolving disc. The most crucial models for comprehending the nature of these fluids are the Reiner-Philippoff models. A class of pseudoplastic/shear-thinning fluids includes the Reiner-Philippoff model. Regarding the boundary layer characteristics of the Reiner-Philippoff fluid, there are a limited number of studies in the literature. The science of electrically conducting fluid motion caused by applied magnetics forces is known as MHD (Magneto-hydrodynamics) [10]. Previous works focused on the MHD boundary layer flow problem of a nanofluid via a porous media over an exponentially extending sheet. In the presence of a magnetic field, thermal radiation, heat generation, and chemical reaction, unstable viscous nanofluid boundary-layer flow along a vertically stretched sheet is shown to transport heat and mass [11]. They discovered that the unsteady flow's velocity, temperature, and concentration profiles are different from their respective components in the steady-state flow scenario. Radiation is the emission or transmission of energy via a material medium, such as space, as waves or particles are taken from previous studies [12].

A phenomenon known as bio-convection occurs when smaller-scale physical rules cause a larger-scale occurrence. Microorganisms that are denser than water move upward during the bioconvection process, creating a top-heavy density stratification that frequently destabilises [13]. Different sorts of microorganisms include those that move under the influence of gravity (gravitaxis), rotate and spin (gyrotaxis), or move through oxygen (oxytaxis) (moving upward direction to take oxygen). The previous study [14] revealed an interesting phenomenon that the stability of nanofluid is enhanced by the addition of gyrotactic microorganisms. The problem of natural convection boundary layer flow around a vertical cone in porous media saturated by a nanofluid caused by gyrotactic microorganisms is affected by magnetic fields and microbes. Bioconvection concerns the suspensions of self-propelled microorganisms. The effect of small particles (that are heavier than water) on the stability of a suspension of motile gyrotactic microorganisms in a horizontal fluid layer of finite depth is essential. Nanofluid bioconvection is generated by the combined effects of buoyancy forces and magnetic fields on the interaction of motile microorganisms and nanoparticles [15]. Recently, a Computational investigation of Stefan blowing and multiple-slip effects on buoyancy-driven bioconvection nanofluid flow with microorganisms was studied. Many researchers have written on these fascinating phenomena in recent years. The functionality of bioconvection is established by an increase in the concentrations of motile microorganisms. They discovered that as the unsteadiness effect is improved, fluid velocity rises, while it is reduced with the viscoelastic term effect [16].

In a non-Newtonian Reiner-Philippoff fluid, Mohamed E. Nasr *et al.,* [17] proposed that the magnetohydrodynamics (MHD), the thermal energy, and the mass transport boundary layer flow characteristics are numerically explored. Non-linear radiation, Cattaneo-Christov double diffusions, convective conditions at the surface, and the species reaction concerning activation energy are all discussed in terms of energy and mass transfer. Using the right similarity variables, the specified governing system of partial differential equations (PDEs) is drained into a non-linear differential system. The determined flow equations have numerical solutions discovered for them. For both Newtonian and Reiner-Philippoff fluid examples, two-dimensional charts are used to show the flow field, temperature and species distributions, and rate of heat and mass transfers for the relevant parameters. The streamlining phenomenon is also mentioned in this article.

The numerical investigation of viscosity variation, thermal radiation, and Arrhenius reaction modifications on an electrically conducting nanofluid flow via a convectively heated surface reported by O.A. Famakinwa *et al.,* [18] is examined in this paper. The nonlinear coupled Ordinary Differential Equations produced from the mathematical model guiding the fluid flow are solved with the help of suitable similarity variables using the shooting strategy and the 4th-order Runge-Kutta formula in the built-in bvp4c software package of MATLAB. The slope of linear regression using the data point statistical tool is also introduced.

Taseer Muhammad *et al.,* [19] investigate the characteristics of the Jeffrey nanofluid flow with the influence of activation energy and motile microorganisms over a sheet is considered. The influence of the magnetic field is another important physical parameter in the flow analysis and has been regarded in this review. The remarkable properties of nanofluid are demonstrated by thermophoresis and Brownian motion characteristics. Thermophoresis has relevance in mass transport processes in much higher temperature gradient operating systems. An appropriate similarities transformation is utilized to make it convenient to partial differential equations into ordinary differential equations. The well-known shooting tactic is utilized to estimate numerical outcomes of the obtained ordinary system of flow. The governing dimensionless equations are integrated subject to the aid of the bvp4c scheme in the built-in software of MATLAB to find out the solution.

Muhammad Jebran Khan *et al.,* [20] investigate the numerical analysis of an innovative model containing, bioconvection phenomena with a gyrotactic motile microorganism of magnetohydrodynamics Williamson nanofluids flow along with heat and mass transfer past a stretched surface. The effect of thickness variation and thermal conductivity feature is employed in the model. Bioconvection in nanofluid helps in bioscience such as in blood flow, drug delivery, micro-enzyme, biosensors, nanomedicine, content detection, etc. For the simulation procedure, the mathematical partial differential equations are converted into dimensionless systems owing to dimensionless variations such as magnetic field, power index velocity, Williamson parameter, wall thickness parameter, thermal conductivity variation, Prandtl number, thermal radiation, Brownian motion, Lewis number, Peclet number, and different concentration parameter, etc. For numerical simulation, the New Iterative Technique (NIM) numerical algorithm is adopted and employed for the linear regression planned for the proposed model.

Najiyah SafwaKhashiie *et al.,* [21] analyse the effect of MHD and viscous dissipation on radiative heat transfer of Reiner–Philippoff fluid flow over a nonlinearly shrinking sheet. By adopting appropriate similarity transformations, the partial derivatives of multivariable differential equations are transformed into the similarity equations of a particular form. The resulting mathematical model is elucidated in MATLAB software using the bvp4c technique. The suction effect has a noticeable influence on the Reiner–Philippoff fluid since increasing the suction parameter's value is seen to enhance the skin friction coefficient and the heat transfer performance. The dual solutions are established, leading to the stability analysis that supports the first solution's validity.

Iskandar Waini *et al.,* [22] study concerns the numerical investigation of the radiative non-Newtonian fluid flow past a shrinking sheet in the presence of an aligned magnetic field. By adopting proper similarity transformations, the governing partial derivatives of multivariable differential equations are converted to similarity equations of a particular form. The numerical results are obtained by using the bvp4c technique. According to the findings, increases in the suction parameter resulted in higher values of the skin friction and heat transfer rate. The same pattern emerges as the aligned angle and magnetic parameter are considered. On the other hand, the inclusion of the Bingham number, the Reiner–Philippoff fluid, and the thermal radiation parameters deteriorate the heat transfer performance. The dual solutions are established, which results in a stability analysis that upholds the validity of the first solution.

Yun-Xiang Li *et al.,* [23] investigation is to present an analysis of the bioconvection phenomenon for Darcy-Forchheimer flow of Reiner-Philippoff nanofluid induced by a stretched surface. The contribution of slip via higher relations is dissected for the flow. The radiative pattern is examined for the thermally developed flow. The heat and mass assessment has been examined with the help of modified CattaneoChristov expressions. The flow equations associated with momentum, volumetric friction and motile microorganism density are transformed into dimensionless form. Transmuted dimensionless non-linear equations are tracked with the shooting technique and results of prominent parameters are sketched via different graphs by using computational software MATLAB.

Sami UllahKhan *et al.,* [24] communicate the accessed dynamic feature of Reiner–Philippoff nanofluid with applications of the bioconvection phenomenon. The magnetic force impact and activation energy features are also intended to perform the radiative analysis of Reiner–Philippoff nanomaterial. The slip features higher-order relations are incorporated to analyze the flow. The modifications in the energy equation are suggested by using thermal radiation with the nonlinear relationship. The flow equations, which in turn to non-dimensionless form, are numerically tackled with a shooting scheme. A comprehensive thermal analysis for the endorsed parameters is presented. The numerical data is achieved to examine the fluctuation in heat, mass, and motile density functions.

Khan *et al.,* [25] suggested a two-dimensional flow of Eyring-Powell's nanofluid containing gyrotactic microorganisms has been developed by moving across a porous plate that is exposed to thermal radiation and surface suction. The Buongiorno nanofluid model was introduced to incorporate the energy and momentum equations, while the Roseland nonlinear approximation was introduced to incorporate solar radiation properties into the energy equations. The MATLAB 'bvp4c' scheme was implemented to find a numerical solution to the problem. The influence of various physical parameters on the velocity, temperature and concentration distribution is analyzed. Suction lowers the temperature but increases the heat transfer rate.

Olubode Kolade Koriko *et al.,* [26] analysis of the gravity-driven flow of a thixotropic fluid containing both nanoparticles and gyrotactic microorganisms along a vertical surface. To further describe the transport phenomenon, special cases of active and passive controls of nanoparticles are investigated. The governing partial differential equations of momentum, energy, nanoparticle concentration, and density of gyrotactic microorganism equations are converted and parameterized into a system of ordinary differential equations and the series solutions are obtained through the Optimal Homotopy Analysis Method (OHAM). The related important parameters are tested and shown on the velocity, temperature, concentration and density of motile microorganism profiles.

1.1 Gap and Significance of Study

The study of non-Newtonian fluids has attracted significant attention due to its wide-ranging applications in science and engineering. However, understanding the complex behaviour of these fluids, characterized by the intricate interaction between stresses and shear rate strain, remains a challenge. The Reiner-Philippoff fluid, which exhibits shear-thinning (Pseudo-plastic), shear-thickening (dilatant), and Newtonian behaviour, is a prime example. Additionally, other fluids like Power-law, Sisko, and Prandtl-Eyring share similar characteristics, further contributing to the complexity of non-Newtonian fluid dynamics. Researchers have explored the effects of various factors on these fluids, including thermal radiation, nanoparticles, and micropolar Ferrofluid. Still, there's a need for a comprehensive understanding of how these factors interact and impact the behaviour of non-Newtonian fluids. Moreover, the study introduces the concept of nanofluids, which are gaining importance in various technical, industrial, and scientific fields due to their enhanced thermal properties. The presence of nanoparticles in nanofluids significantly improves thermal conductivity, making them suitable for applications such as heating and cooling devices, nuclear systems, and more. However, there's a need to delve deeper into the thermophysical properties and behaviour of nanofluids to harness their full potential.

Objective of the Study: The primary objective of this study is to investigate the complex behaviour of non-Newtonian fluids, particularly Reiner-Philippoff fluids, under various conditions. This investigation includes examining the impact of factors like thermal radiation, nanoparticles, and micropolar Ferrofluid on fluid dynamics. Additionally, the study aims to explore the behaviour of nanofluids, specifically those containing gyrotactic microorganisms, and how they respond to magnetic fields. Bioconvection, a phenomenon where smaller-scale physical rules lead to larger-scale occurrences, is a key aspect of this investigation. Furthermore, the study seeks to bridge gaps in the current understanding of these fluid systems and their interactions by employing mathematical modelling and numerical analysis. The ultimate goal is to provide insights that can contribute to advancements in science and engineering, particularly in the context of fluid dynamics and thermal applications. Figure 1 depicts the flow diagram of the proposed work.

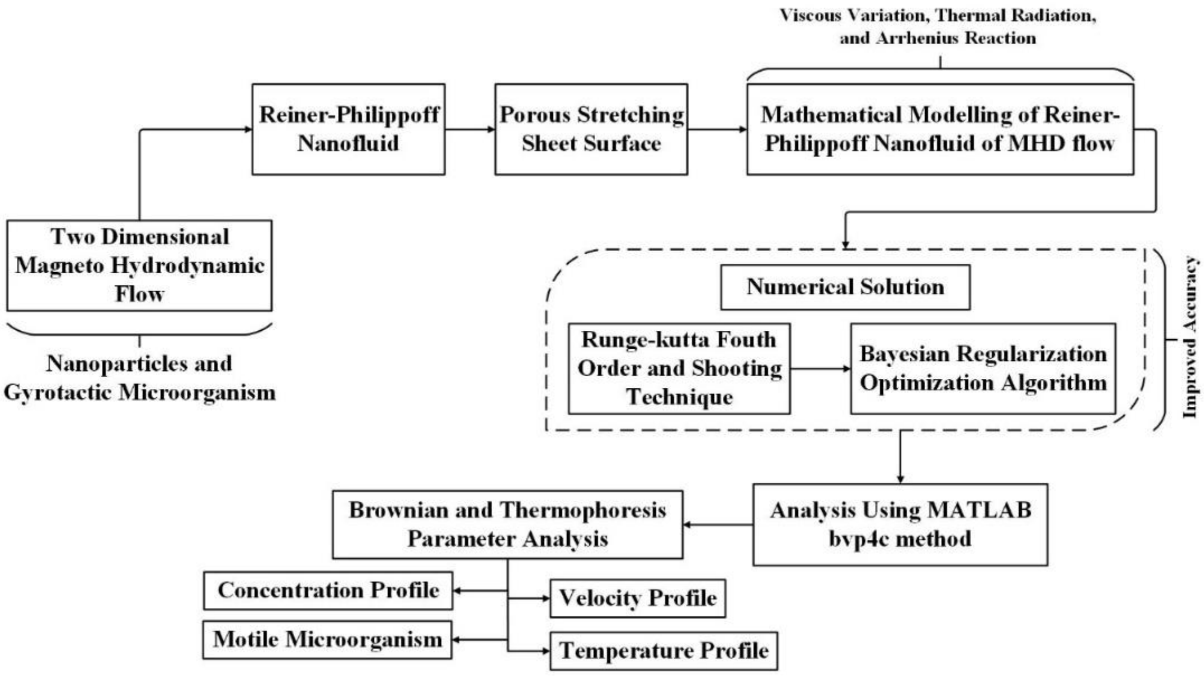

Fig. 1. Flow diagram of the proposed work

This article considers the steady two-dimensional magnetohydrodynamic flow of Reiner-Philippoff nanofluid enclosing both nanoparticles and gyrotactic microbes through a vertical porous medium of stretching sheet by taking the impact of thermal radiation. Both the horizontal *y*-axis and the vertical *x*-axis are subjected to a variable magnetic field. We assume that the surface is stretching in the *x*-direction with a stretching rate further the fluid is subjected to the applied magnetic field of strength. The Brownian, as well as thermophoresis distinctiveness, is observed via the Buongiorno model.

2. Mathematical Modelling

The surface is stretched in the direction of flow with velocity components u which is objected along with the sheet while v is taken towards the normal direction. The magnetic force is usually imposed on the flow direction. The nanofluid temperature (T), concentration (C) and motile density (N) are considered to be uniform in the flow domain. For thermally radiative flow analysis, the energy equation is modified by using specific relations.

Figure 2 illustrates the geometrical configuration of two-dimensional nanofluid flow over a stretching sheet. The Reiner-Philippoff fluid shows a non-Newtonian nature between these two values of viscosity μ. The shear stress and deformation rate relations associated with the Reiner–Philippoff fluid model are defined as

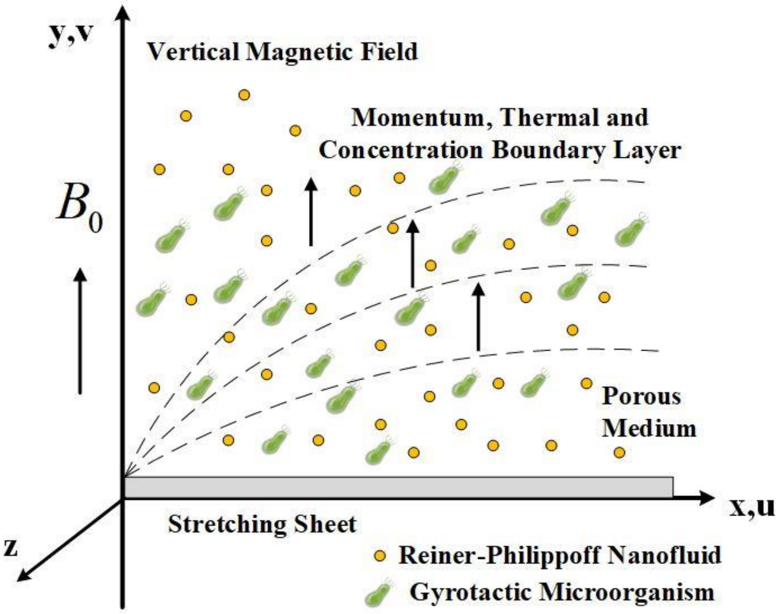

Fig. 2. Geometrical configurations

$$\frac{\partial u}{\partial y}=\frac{\tau}{\mu_\infty+\dfrac{\mu_0-\mu_\infty}{1+\left(\dfrac{\tau}{\tau_s}\right)}} \tag{1}$$

With deformation rate $\frac{\partial u}{\partial y}$ and shear stress τ. Given boundary layer assumptions, the governing equations for the considered problem are

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{2}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho_f}\frac{\partial \tau}{\partial y}-\frac{\sigma_c B_0^2}{\rho_f}+\frac{g_1}{\rho_f}\begin{bmatrix}(1-C_f)\rho_f\beta^*(T-T_\infty)-\\(\rho_p-\rho_f)(C-C_\infty)-(N-N_\infty)\gamma^*(\rho_m-\rho_f)\end{bmatrix} \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\Lambda_f\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma^*}{3(\rho c)_f\kappa^*}\frac{\partial}{\partial y}\left(T^3\frac{\partial T}{\partial y}\right)+\frac{(\rho c_p)_p}{(\rho c_p)_f}\left[D_B\left(\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{4}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial y^2}\frac{\partial T}{\partial y}\right)+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}-Kr^2(C-C_\infty)\left(\frac{T}{T_\infty}\right)^n\exp\left(\frac{-E_a}{k_1 T}\right) \tag{5}$$

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}+\frac{\hat{b}\hat{w}}{(C_w-C_\infty)}\left[\frac{\partial}{\partial y}\left(N\frac{\partial C}{\partial y}\right)\right]=D_m\frac{\partial^2 N}{\partial y^2} \tag{6}$$

For the radiative phenomenon, the nonlinear thermal relations are incorporated into the energy equation by using the Roseland approximations Eq. (1)-(6). Moreover, the last term in Eq. (5) represents the activation energy which is derived by using the Arrhenius relations.
The boundary conditions are

$$\left.\begin{aligned} &u = u_w(x) = ax^{1/3} + u_{sl},\ u_{sl} = A\frac{\partial u}{\partial y} + B\frac{\partial^2 u}{\partial y^2},\ at\ \ v = 0 \\ &-k\frac{\partial T}{\partial y} = h_f\left(T_f - T\right),\ \ D_B\frac{\partial C}{\partial y} + \frac{DT}{T_\infty}\frac{\partial T}{\partial y}, N = N_w,\ at\ \ y = 0 \end{aligned}\right\} \tag{7}$$

$$u \to 0, T \to T_\infty, C \to C_\infty, N \to N_\infty \text{ as } y \to \infty \tag{8}$$

with v (kinematic viscosity), σ_c (electrical conductivity), B_0 (magnetic field strength), physical quantities ρ_f (fluid density), σ^* (Stefan Boltzmann constant), $\left(\rho c_p\right)_p$ (nanoparticles heat capacity), κ^* (applied Boltzmann constant), Λ_m (thermal diffusivity), $\hat{w}$ (swimming cells speed), $\Omega*$ (volume suspension coefficient), D_B (diffusion constant), $\hat{b}$ (chemotaxis constant), k^* (mean absorption coefficient), D_m (microorganisms diffusion constant), g (gravity), $\left(\rho c_p\right)_f$ (heat capacity), ρ_m (motile microorganism density) and ρ_p (nanofluid density). The following non-dimensional set of expressions is anticipated for the current flow model in Eq. (9)-(14):

$$\left.\begin{aligned} &\eta = \sqrt{\frac{a}{v}}\frac{y}{x^{1/3}},\ \psi = \sqrt{avx^{\frac{2}{3}}},\ \ \tau = \sqrt{a^3 vg(\eta)},\ \ \theta(\eta) = \frac{T - T_w}{T_w - T_\infty} \\ &\phi(\eta) = \frac{C - C_w}{C_\infty}, \chi(\eta) = \frac{N - N_w}{N_w - N_\infty} \end{aligned}\right\} \tag{9}$$

$$g = f''\frac{g^2 + A\gamma^2}{g^2 + \gamma^2} \tag{10}$$

$$g' = \frac{1}{3}f'^2 - \frac{2}{3}f\ f'' - M\ f' + Gr\left(\theta - Nr\phi - Rb\chi\right) = 0 \tag{11}$$

$$\begin{aligned} &\left[1 + \frac{4}{3}Rd\left\{1 + \left(\theta_w - 1\right)\theta\right\}^3\right]\theta'' + 4Rd\left(\theta_w - 1\right)\left[1 + \left(\theta_w - 1\right)\theta\right]^2\theta'^2 + \\ &\Pr\left[f\theta' + Nb\theta'\phi' + N_t\left(\theta'\right)^2\right] = 0 \end{aligned} \tag{12}$$

$$\phi'' + Le\Pr f\phi' + \frac{N_t}{N_b}\theta'' - \Pr Le\sigma\left(1 + \delta\theta\right)^n \exp\left(\frac{-E}{1 + \delta\theta}\right)\phi = 0 \tag{13}$$

$$\chi'' + Lbf\chi' - Pe\left[\phi''\left(\chi + \delta_1\right) + \chi'\phi'\right] = 0$$

$$\left.\begin{array}{l} f(0)=0,\ f'(0)=1+\Omega_1 f''(0)+\Omega_2 f'''(0), \theta'(0)=-Bi(1-\theta(0)) \\ \phi'(0)+\dfrac{N_t}{N_b}\theta'(0)=0, \chi(0)=1 \\ f'(\infty)\to 0, \theta(\infty)\to 0, \phi(\infty)\to 0, \chi(\infty)\to 0 \end{array}\right\} \tag{14}$$

With Philippoff fluid parameter $A=\dfrac{\mu_0}{\mu_\infty}$, buoyancy ratio parameter $Nr=\dfrac{(C_w-C_\infty)(\rho_P-\rho_f)}{\beta^*(1-C_f)T_\infty}$, Bingham number $\gamma=\dfrac{\tau_s}{\sqrt{a^3 v}}$, Brownian constant $N_b=(\rho c)_p D_B\left(\dfrac{(C_w-C_\infty)}{(\rho c)_f}\right)v$, Prandtl number $\Pr=\dfrac{v}{\Lambda_f}$, bioconvection Rayleigh number $Rb=\dfrac{\gamma^*(\rho_m-\rho_f)(N_w-N_\infty)}{\beta^*(1-C_f)T_\infty}$, thermophoresis constant $N_t=\dfrac{((\rho c)_p D_T(T_w-T_\infty))}{(\rho c)_f T_\infty v}$, activation energy $E=\dfrac{E_a}{k_1 T}$, chemical reaction parameter $\sigma=\dfrac{kr^2}{a}$, temperature difference parameter $\delta=\dfrac{T_w-T_\infty}{T_\infty}$, bioconvection Lewis number $Le=v/D_m$, microorganism difference parameter $\delta_1=\dfrac{N_\infty}{N_w-N_\infty}$, Peclet number $Pe=\dfrac{\hat{b}\hat{w}}{D_m}$.

The reduced Nusselt number, Sherwood number and motile density number in the dimensionless form are presented in Eq. (15).

$$\left.\begin{array}{l} Nu(\mathrm{Re}_x)^{-0.5}=-\left(1+\dfrac{4}{3}Rd(1+(\theta_w-1)\theta(0))^3\right)\theta'(0) \\ Nn(\mathrm{Re}_x)^{-0.5}=-\chi'(0) \\ Sh(Re_x)^{-0.5}=-\phi'(0) \end{array}\right\} \tag{15}$$

3. Numerical Solution of Governing Equation

The nonlinear coupled differential Eq. (7) along with the boundary conditions Eq. (8) form a two-point boundary value problem and is solved using the shooting technique together with the fourth-order Runge-Kutta integration scheme by converting it into an initial value problem. The governing equations are initially transformed into the set of ordinary differential equations (ODEs) by using suitable similarity transformations. Thereafter, this system of ODEs is numerically solved by employing the fourth-order Runge-Kutta-Fehlberg method. For numerical solution, the unbounded domain $[0,\infty)$ has been replaced by $[0,\eta_{\max}]$ where $\eta_{\max}$ is a real number chosen in such a way that the solution does not show any significant variations $\eta>\eta_{\max}$. It is noteworthy that $\eta_{\max}=7$ assures the expected level of convergence for all the numerical outcomes delineated in this article. The momentum Eq. (11) and Eq. (12) will be tackled collectively by the shooting method and then the temperature and concentration equations will be tackled by using f as a known function. Denoting f by y_1, f' by y_2, g by y_3, and the missing initial condition by s, the momentum Eq. (11) and Eq. (12) are converted into the following system of first-order ODEs.

$$\left.\begin{aligned} y_1' &= y_2 & y_1(0) &= 0 \\ y_2' &= \frac{y_3\left(y_3^2+\gamma^2\right)}{\left(y_3^2\right)}, & y_2(0) &= 1 \\ y_3' &= \frac{1}{3}y_2^2 - \frac{2}{3}y_1 y_2', & y_3(0) &= s \end{aligned}\right\} \tag{16}$$

The above system Eq. (16) has been handled numerically with the assistance of the Runge Kutta fourth-order (RK4). Furthermore, the missing initial conditions are updated with the help of Newton's scheme until the criteria stated below are met

$$max\{|y_2(\eta_{max}) - 0|\} < \epsilon \tag{17}$$

Where the symbol ϵ is a positive number having value $\epsilon = 10^{-6}$ and $\eta_{max} = 7$.

To solve the temperature Eq. (13), it is converted into the following system comprising of the first order differential expressions Eq. (16) signifying θ by u_1 and θ' by u_2 and using f as a known function. The following system of ODEs together with the initial conditions is achieved.

$$\left.\begin{aligned} u_1' &= u_2 \quad , u_1(0) = 1 \\ u_2' &= \frac{\left[1+4Rd(\theta_w-1)(1+(\theta_w-1)u_1)^2u_2^2+(2/3)Prfu_2+(A^*exp^{-\eta}+B^*u_1)\right]}{((1+u_1)+(4/3)Rd(1+(\theta_w-1)u_1)^3)} \\ &\quad , u_2(0) = 1 \end{aligned}\right\} \tag{18}$$

The system of Eq. (18) is treated the same way as Eq. (16) to obtain θ and θ'. The concentration Eq. (14) is transformed into the first-order ODEs by denoting ϕ by z_1 and ϕ' by z_2 and taking f as a known function. The following resulting system of equations is;

$$\left. z_1' = z_2 z_1(0) = 1, \quad z_2' = \frac{\left(z_2^2+(2/3)fz_2\right)}{1+z_1}, \quad z_2(0) = u \right\} \tag{19}$$

Further, the accuracy of numerical solutions is kept up with the optimization algorithm. The necessary description of the solution methodology adopted based on ANNs back propagated with Bayesian regularization.

3.1 Bayesian Regularization Optimization Algorithm

The necessary description of methodology based on neural networks, including the layer structure, hidden neurons, topology of the networks, and arbitrary selection of an input and target data set for training, testing, and validation samples, is determined to solve the problem. The neural networks environment in MATLAB software is exploited via 'nftool' for the execution of the developed scheme based on ANN back propagated with Bayesian regularization, i.e. ANN-BR. The solution procedure comprises a significant dataset description and an execution process for executing the proposed ANN-BR. The overall workflow of the solution methodology is presented in Figure 3.

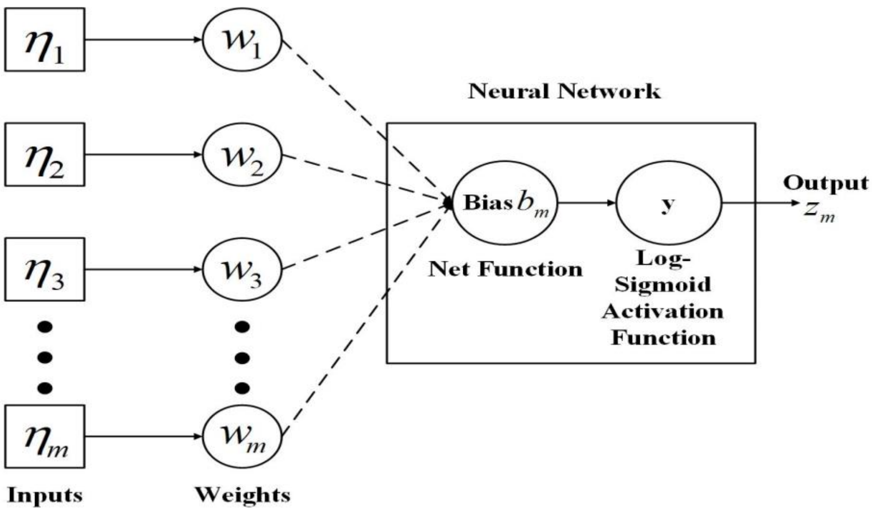

Fig. 3. Mathematical representation of Bayesian regularization network

The total amount of data for ANN-BR is 1001 found between 0 and 1 by setting 0.001 as the stepsize and using the Runge-Kutta technique through "NDSolve" in Mathematica. These data are distributed randomly for f, f', θ, and ϕ into sets: training, testing, and validation to achieve the best convergence. The reference datasets for the velocity profile, i.e., $f(\eta)$, $f'(\eta)$ the temperature profile, i.e. $\theta(\eta)$, $\theta'(\eta)$. Further, the concentration profile $\phi(\eta)$and $\phi'(\eta)$, and the motile density profile $\chi(\eta)$and $\chi'(\eta)$are generated for 61 inputs. In this problem, we chose the sets as 85% of the data are utilized for training, 10% for testing, and 5% for validation of the proposed ANN-BR using a neural network. The number of neurons is also arbitrary; selecting 20 gives a good accuracy of the computational results.

4. Analysis of Results and Discussion

This segment delivers a detailed discussion of the output attained from the numerical computations on Eq. (11)-(14) using the bvp4c package in the MATLAB software. The analysis covers the discussions on the consequence of numerous physical parameters that arise in the proposed model where the outputs of the computations are presented in graphical and tabular form. Analysis of the effects of viscous variation, thermal radiation, and Arrhenius reaction on the flow under consideration are presented as tables and figures using the default values of the dimensionless parameter. On velocity profiles $f'(\eta)$, temperature profiles $\theta(\eta)$, nanoparticle concentration profiles $\phi(\eta)$, and density of motile microorganisms profiles $w(\eta)$, the effects of certain selected parameters have been evaluated and presented. The theoretical values of the governing parameters are $M = Nr = Rb = 0.5$, $N_b = N_t = \delta_t = \delta_n = K_1$ and $K_2 = 0.1$, $Sh = Pe = 1.0$, $\Pr = Le = 2.0$. In Table 1 it is revealed that the local Nusselt number is a decreasing function of $\Pr, Nb$and $Nt$for the nanoparticles.

Table 1
Sh_x , Nu_x and Nn_x for various M and P_0

M	P_0	Nu_x	Sh_x	Nn_x
2	0	2.5231	-0.2700	2.8226
3	1	3.4493	-0.1697	3.5544
4	10	4.2423	-0.0506	4.2734
5	100	5.0333	0.0800	4.9863

The present results also designate that if the porosity of the medium increases then better convergence can be achieved for bioconvection parameters. No evident difference is noticed in the concentration Nn_x of motile microbes meanwhile the large size of pores (0 < P_0 < 100) do not ingest microorganisms (Table 1). As the porosity parameter does not involve the heat as well as concentration equation, therefore, it does not affect the rates of mass and heat transport significantly. Moreover, the only significant change is noticed in shear stress it is because of the presence of P_0 in the momentum equation. It is also noticed here that bioconvection through porous media is not affected in the presence of thermal radiation, and motile microbes' density Nn_x is enhanced with the thermal radiation effect. The outcomes reveal that Nn_x (the local motile microbe's density) is stable only when the porosity is large.

An increasing shift in heat transfer is noted due to $\Pr$, Nb and Nt while lower numerical data is achieved for Nr and Rd . The numerical results prepared in Table 2 visualized variation in $-\phi'(0)$ which claim that its rate of the mass transfer becomes slower with Nb, Nr and Nc .

Table 2
Numerical treatment of $-\phi'(0)$ against flow parameter

$\Pr$	λ	Nr	Nc	Rd	Nb	Nt	$-\phi'(0)$
0.1	0.4	0.4	0.2	0.6	0.4	0.4	0.63547
0.7							0.67878
1.1							0.71657
0.2	0.3						0.59786
	0.7						0.53653
	1.1						0.49775
	1.0	0.3					0.58855
		0.5					0.57435
		0.8					0.55534
		0.2	0.3				0.55764
			0.5				0.52764
			0.9				0.50134
				0.3			0.60744
				0.9			0.63754
				1.5			0.64743
2.0	1.0				0.1		0.68743
					0.5		0.71743
					0.9		0.73267
						0.1	0.62953
						0.7	0.59664
						1.3	0.53743

4.1 Brownian Motion and Thermophoresis Parameter Analysis

The overall effect is a reduction in the velocity of the fluid. Increasing the Brownian motion parameter was shown to lead to a decrease in the solute boundary layer. An increase in the Brownian motion parameter boosts the movement of particles. This causes the warming of the boundary layer which effectively causes nanoparticle to move away from the surfaces inside the inactive fluid. Figure 4 – Figure 8 shows the impact of Brownian motion and thermophoresis effect on the velocity, temperature, concentration profiles and motile microorganisms. Brownian motion is the random 'indecisive' movement of particles suspended in a fluid resulting from the collision with the fast-moving molecules of the fluid.

The impact of parameter Brownian N_b and thermophoresis N_t on Reiner Philippoff fluid velocity function $f'(\eta)$ is presented in Figure 4. The curve of flow speed is reducing with a rise in the values of M, it is due to the interaction of opposing force namely Lorentz force increasing. The effects of Brownian motion parameter and thermophoresis parameter on the dimensionless velocity profiles for $\mathrm{Pr}=1$, $Le=1$, $M=N=0.5$. It is noticed that the velocity profile $f'(\eta)$ is enhanced for the bioconvection parameter η, whereas it is reduced for the Buoyancy ratio parameter N_r.

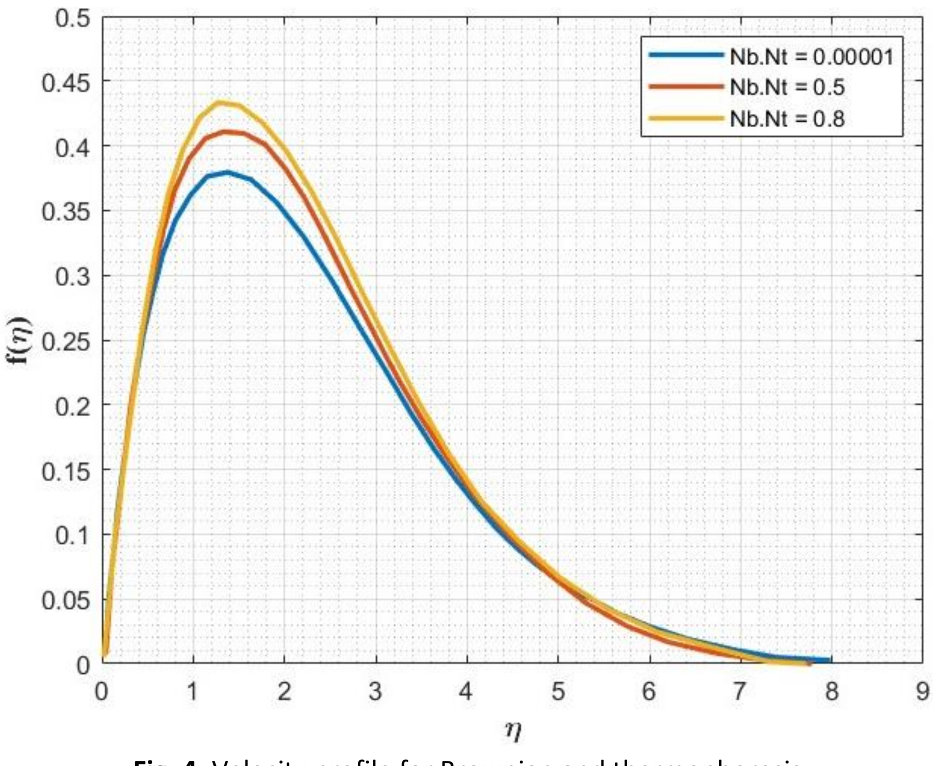

Fig. 4. Velocity profile for Brownian and thermophoresis

Figure 5 illustrates the temperature distribution under the Brownian motion parameter N_b and thermophoresis parameter N_t. In general, due to the irregular motion of the particles, causes a collision between these particles. Effects of Brownian motion parameter and thermophoresis parameter on the dimensionless temperature profiles for $\mathrm{Pr}=1$, $Le=1$, $M=N=0.5$. The influence of the Prandtl number is to decrease the temperature. A dimensionless quantity that puts the thermal conductivity in correlation with the viscosity of a fluid refers to the Prandtl number. This phenomenon tends to decrease the boundary layer thickness as well as the temperature. As a result, larger values N_t cause an increase in temperature, due to which the surface temperature also increases. Similarly, the increases in the values of λ in stretching case rise in the temperature profile $\theta(\eta)$.

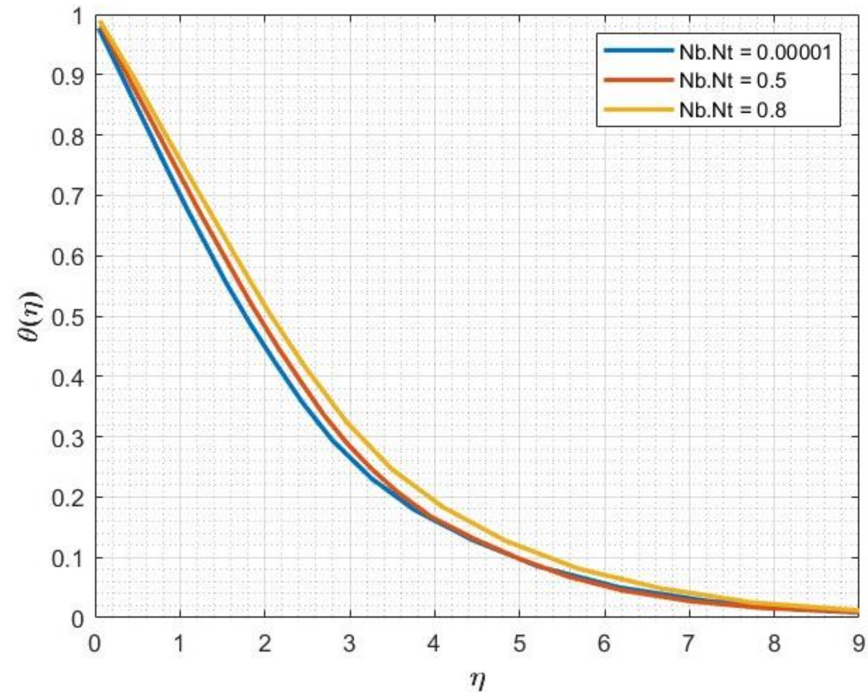

Fig. 5. Temperature profile for Brownian and thermophoresis

Figure 6 revealed the significance of the chemical reaction parameter, Kr on the fluids' concentration N_b, N_t. A decrease in concentration profile is attributed to the enhancement in the chemical reaction parameter. Meanwhile, the decrease is higher in the case of viscous fluid ($M = 1$). It is evident from Figure 6 that the rising value N_t increases the concentration distribution in the absence of Kr. Table 3 shows that enhancing the viscous variation parameter increases the skin friction coefficients at the rate of 0.2786 and the rate of heat transfer at 0.02703. But lowers the rate of mass transfer at −0.0569 and the density of motile microorganisms at −0.0321.

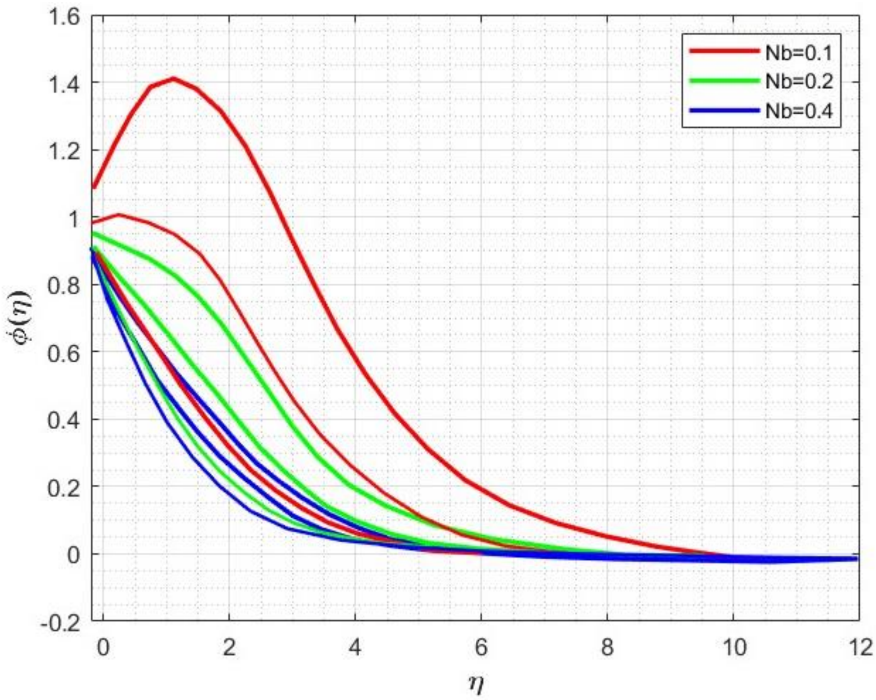

Fig. 6. Concentration profile for Brownian motion and thermophoresis

Table 3
Variation in $f''(0), -\theta'(0), -\phi'(0)$ and $-\chi'(0)$ with M when $\eta_{\max} = 7$ and $Kr = 1$

M	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$	$-\chi'(0)$
1	0.6254	0.0025	1.2345	1.1423
2	0.7893	−0.0489	1.6485	1.3645
3	1.1256	−0.0421	1.5875	1.2678

4.2 Motile Microorganism Profile

The unstable density stratification generates the bioconvection. In the recent work, the movement of the microbes is associated with viscous drag and gravitational torques in the flow (gyrotaxis). The research utilized the gyrotactic microorganism with the nanoparticles to estimate the density of the bioconvective fluid. Figure 7 and Figure 8 illustrate the motile microorganism density with the Brownian motion and thermophoresis parameter.

Figure 7 is portrayed against the motile microbes' distribution for several estimations of the bioconvection Lewis and buoyancy number. The elevating values Le and Br tend to depress the concentration profiles. A declaration in the motile microbe's distribution N_b is observed with an increase in the values of bioconvection Peclet number and motile microbes' parameter χ .

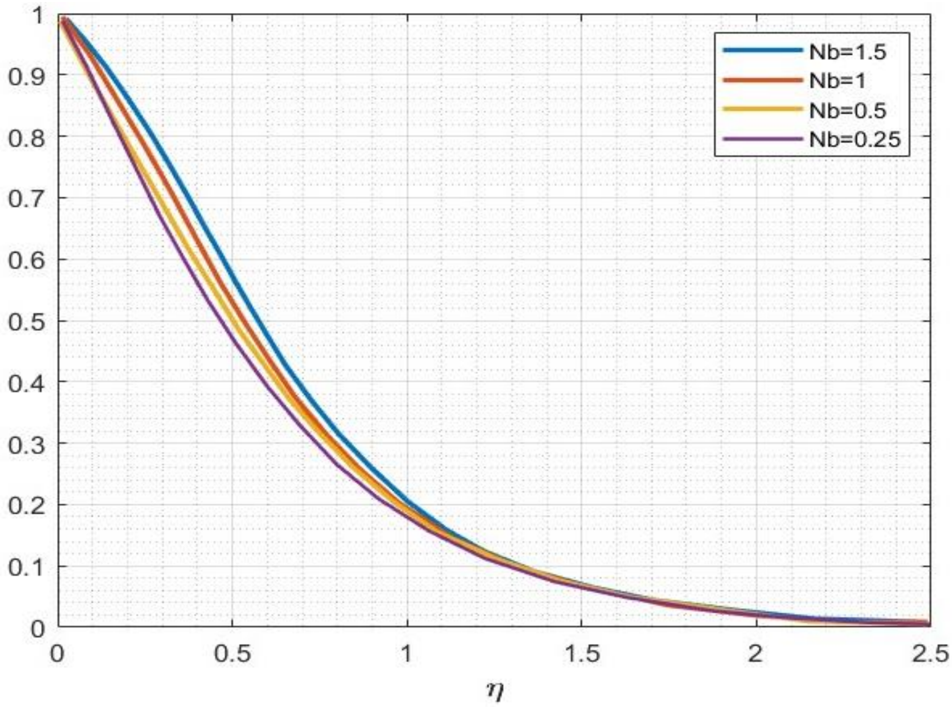

Fig. 7. Density of the motile microorganism Brownian motion parameter analysis

Figure 8 illustrates the motile microorganism density with thermophoresis parameter analysis. The Peclet number and cell swimming speed are inversely proportional to Dn (the microorganism's diffusivity) and directly proportional to each other. Consequently, expanding values of the Peclet number depreciate the motile microbe's density profile and escalates the wall motile microorganisms flux. For the diverse variation of Pe and N_t the motile microorganism density number increased.

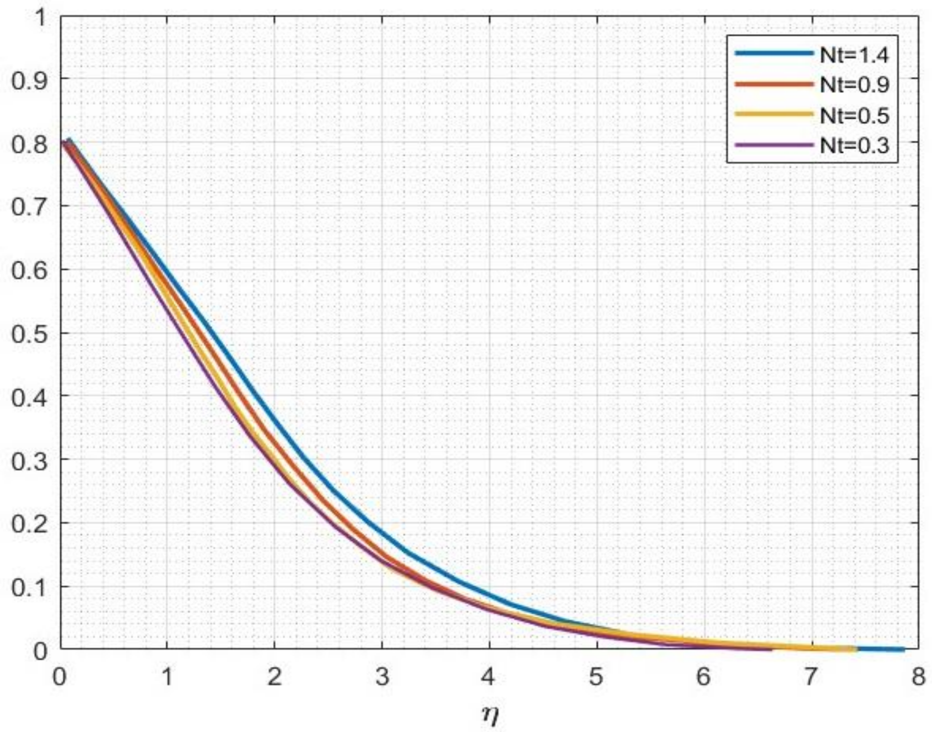

Fig. 8. Density of the Motile Microorganism Thermophoresis Parameter Analysis

Table 4 presents the values of local Nu_x for various values of existing parameters in the flow problem. It is recognized that Nu_x scale back for Reiner Philippoff fluid, Newtonian fluid and dilatant fluid. Further, Nu_x enhances pseudoplastic fluid, Newtonian fluid and dilatant fluid for rising values of R, P r and θ_w. Furthermore, Nu_x is more enhanced in Reiner Philippoff fluid when compared to Newtonian fluid and dilatant fluid.

Table 4
Comparison of various emerging parameters values on Nusselt number

Pr	Rb	θ_w	Nu_x		
			Newtonian Fluid $(\lambda = 1)$	**Dilant Fluid** $(\lambda < 1)$	**Reiner Philippoff Fluid** $(\lambda > 1)$
0.5			1.170370	0.849563	1.271570
1.0			0.985130	0.718474	1.271570
1.5			0.727497	0.411036	1.117839
2.0	1.0		1.167123	0.605998	0.986096
	1.5		1.337873	0.813848	1.648060
	2.0		1.467557	0.909926	1.272527
		0.5	0.865201	1.001301	1.506275
		1.2	1.167123	0.635582	1.465852
		1.5	1.368815	0.925486	1.235462

5. Research Conclusion

Nanofluids have already proven great potential in the thermal amplification of several manufacturing industries and have been widely employed in energy technologies in recent times. Therefore, the current framework investigates the characteristics of the Reiner Philippoff nanofluid MHD flow with the influence of viscous variation, thermal radiation, and Arrhenius reaction over a porous stretching sheet. The remarkable properties of nanofluid are demonstrated by thermophoresis and Brownian motion characteristics. Thermophoresis has relevance in mass transport processes in much higher temperature gradient operating systems. An appropriate similarities transformation is utilized to make it convenient to partial differential equations into ordinary differential equations. The procedure is assumed to employ MATLAB. The effects of velocity, temperature, concentration, and motile density profiles on various physical parameters such as the magnetic number, the Brownian motion parameter, the Prandtl number, the Lewis number, the thermophoresis parameter, and the Peclet number are estimated with velocity, temperature, concentration and motile microorganism.

i. To determine the solution, the governing dimensionless equations are integrated with the help of the bvp4c scheme in the MATLAB built-in program.
ii. The numerical computations for the collection of equations described in coupled and nonlinear forms are carried out using the shooting method of Runge Kutta's fourth order.
iii. The R-P fluid parameter increases fluid velocity, but the Bingham number and permeability of porous space show a decreasing change in velocity.
iv. For the diverse variation of Pe and N_t the motile microorganism density number increased.
v. The rising value N_t increases the concentration distribution in the absence of Kr

The concentration of the nanofluid is improved by the activation energy and thermophoretic constants, whereas the Prandtl number, Lewis number, and concentration relaxation parameter show fading results.

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