



## Performance Analysis of Wet Porous Moving Fin under the Influence of Spherical Shaped $\text{TiO}_2$ – Ag Hybrid Nanoparticles in a Water Based Fluid

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### ABSTRACT

The study investigates the flow characteristics of spherical shaped  $\text{TiO}_2$  –Ag hybrid nanofluid with water as a base fluid passing through a wet porous rectangular moving fin with a focus on understanding the effects of nanoparticle concentration on the heat transfer rate. The fin under consideration are subjected to boundary conditions, insulated and convective tips. Hybrid nanofluid that combine nanoparticles with conventional base fluids have potentially enhanced thermal conductivity and heat transfer properties in engineering applications. The energy balance equation containing the parameters that effect the flow of heat transfer rate is non-dimensionalized and solved numerically using 3-stage Lobatto - IIIa formula with appropriate boundary conditions. The simulation result shows the impact of different parameters on the flow and heat transfer properties of the hybrid nanofluid obtained by mixing spherical shaped  $\text{TiO}_2$  – Ag hybrid nanoparticles with water as base fluid. It is observed that the fin shows significant heat transfer rate in a convective tip relative to an insulated tip. The findings contribute to the understanding of hybrid nanofluid flow and its potential application in the design and optimization of thermal management system. It also facilitates the ground work for research in the field of nano fluid based cooling system. The observation from the graphical illustration shows that the rise in the thermal conductivity of the base fluid by 23% increases the conduction heat transfer as well as the temperature distribution by 10%. The natural convection and radiation are the key parameters that determines the heat transfer rate from the surface to the surrounding. In our investigation, enhancing the  $Nc$ ,  $Nr$  parameters by 50% and 25%, the temperature distribution profile is reduced by about 13% and 6% respectively. The increase in the  $Pe$  number by 100% results in a rise in the temperature distribution by 8%.

## 1. Introduction

It has been a challenge for the researchers to come out with a novel method for the enhancement in heat transfer due to the ineffectiveness of conventional fluids such as water, ethylene glycol etc. These conventional fluids possess low thermal conductivity than the metals, CNT

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etc. Adding nanoparticles of dimension less than 100nm to conventional base fluids have shown promising results in enhancing the fluid flow and heat transfer rate. The nanoparticles generally extracted from metals, carbon nanotube possess relatively higher thermal conductivity than the conventional fluid that can intensify the heat transfer rate. The properties of nanofluids attracts the researchers to make use of it in various scientific areas such as biomedical engineering, electronic applications, tribology, environmental studies, Chemistry etc. The transfer of heat through fin structure with different geometries is a topic of significant interest in various fields, including engineering, physics and environmental sciences.

The effectiveness of a fully wet straight fin in various configurations, subjected to mass and heat transfer mechanism was investigated by Mostafa and Syed [1]. Wenhua Yu *et al.*, [2] have focussed their study in analysing the experimental outcomes pertaining to the enhancement in the heat transfer rate and thermal conductivity of nanofluids compared to conventional fluid. The parameters considered under the analysis are volume concentration, particle size, particle material, particle shape, temperature, acidity and base fluid material. The outcomes indicate that the heat transfer rate fall in the range 15% to 40%. Aziz and Bouaziz [3] obtained analytically the solutions for the fin efficiency and distribution of temperature over a longitudinal fin with a constant area. The non-linearity's due to temperature dependent internal heat generation and conductivity were taken care by using least square method. The analytical solutions represented graphically shows a deviation of less than 1% compare to the numerical solution, even with 60% rise in thermal conductivity and heat generation at base temperature relative to their values at sink temperature. Kamyar *et al.*, [4] have examined the coherence of various mathematical models such has traditional numerical methods with the experimental results and have proposed the adjustments in mathematical models. Adopting to two – phase models instead of single - phase models for simulation have shown improvement in accuracy while dealing with nanofluids. Abdul Aziza and Khani [5] have applied homotopy analysis method (HAM) to obtain analytic solution for heat transfer in a moving fin with variable thermal conductivity. The analytical solution derived shows a relative accuracy of three decimal places over direct numerical solution for a wide range of parametric values usually seen in thermal processing application. The solution obtained are graphically illustrated and interpreted.

Mohsen Torabi *et al.*, [6] have examined the impact of the various parameters on the distribution of temperature in a moving fin with a variable thermal conductivity. The analytical solution is obtained by using DTM technique that can be applied to wide range of differential equation. The results show the effectiveness of DTM technique in providing accurate solutions for highly nonlinear problems. Nuim Labib *et al.*, [7] have examined the heat transfer rate using two – phase mixture model involving two different base fluids with  $Al_2O_3$  as nanoparticles. The base fluids namely water and ethylene glycol are examined individually by incorporating  $Al_2O_3$  nanoparticles and the results are validated by comparing the outcomes with the experimental data found in the literature. It is found that the ethylene glycol exhibits relatively higher heat transfer rate than water. Further, a new concept of combined/ hybrid nanofluids that involves  $Al_2O_3$  nanoparticles and CNT mixture with water as base fluid is analysed and the computational results are validated with the experimental data found in the literature. The result shows a substantial rise in the heat transfer rate. The temperature distribution in  $Si_3N_4$  and Al Porous fin is simulated using DTM, CM and LS methods by Hatami *et al.*, [8]. All the three method outcomes were proven effective over numerical results. Various physical parameters such as convection heat transfer coefficient, internal heat generation, shape of the fin profile and thermal conductivity effects the heat transfer rate. Sobhan Mosayebidorcheh *et al.*, [9] analysed the temperature distribution in longitudinal fin of different shapes with variable cross section by solving governing equation using FDM and DTM techniques. Turkyilmazoglu [10] studied the effect of temperature that influence the efficiency of an

exponential wet porous fin. The results show the efficiency and temperature at the fin tip is relatively better in exponential wet porous fin than the straight porous wet fins. The efficiency and enhancement in the heat transfer rate is analysed on a stretching and shrinking surface of rectangular fin by Turkyilmazoglu [11]. The result indicates that the shrinking provides relatively higher efficiency than stretching fins. Dogonchi and Ganji [12] have studied the effect of heat generation gradient, Peclet number and radiation conduction parameter on the fin tip temperature. The variation of convection and radiation parameters in a fully wet porous straight fin is analysed by Darvishi *et al.*, [13]. The analysis is carried out by generating heat equation using Darcy's model. The resultant differential equation is numerically solved using spectral collocation method. The variation in temperature influences the behaviour of hybrid nanofluid. Masoud Afrand *et al.*, [14] experimentally investigated the behaviour of  $\text{Fe}_3\text{O}_4\text{-Ag/EG}$  hybrid nanofluid at different temperatures and concluded that the samples possess shear thinning behaviour at all the subjected temperature. Thermophysical properties of nanofluids influence the heat transfer behaviour. Minea Alina Adriana [15] carried out the numerical investigation of oxide based hybrid nanofluid. The thermos-geometric parameters, heat transfer coefficient and the temperature dependent conductivity are some the important parameters that influence the temperature distribution and performance of longitudinal fin. The effect of these parameters was investigated, solved and validated by Sobamowo [16]. The result indicates the enhancement in the stability of thermo-geometric parameter with respect to temperature over a wide range. Gireesha *et al.*, [17] studied the performance of temperature on wet porous fin in radial profile with velocity constant by applying FEM method. The thermal conductivity is an important parameter in the study of temperature distribution in a moving fin. Parvinder Kaur & Surjan Singh [18] have analysed the distribution of temperature in a moving fin using MM, LWCM and LSM methods. The thermal conductivity under consideration is taken as constant, exponential, linear and quadratic. Hosseinzadeh *et al.*, [19] investigated the magnetic field impact on the thermal profile of wet porous moving fin in hybrid nanofluid by solving governing equations using Akbari-Ganji's method. Fallah Najafabadi *et al.*, [20] analysed the distribution of temperature in a moving fin by solving the governing equations by radial basis function method and validated the result using fourth order Runge –Kutta method. A study on distribution of heat in semi spherical shaped porous fin was presented by Atouei *et al.*, [21]. The governing equations were solved using Least Square Method (LSM), fourth order Runge-Kutta method (NUM) and Collocation Method (CM). Hosseinzadeh *et al.*, [22] numerically examined the solidification process characteristics of PCM within triplex tube LHTESS featuring innovative hollow fins in ternary hybrid nanoparticles (THNPs) by solving the governing equation using GFE method. Gireesha *et al.*, [23] analysed the thermal performance of moving longitudinal fin in hybrid nanofluid ( $\text{MgO-Ag/water}$ ) and illustrated the effect graphically. Nizamuddin Razali *et al.*, [24] have numerically investigated the thermal performance of natural wire and tube condensers in refrigerators by suspending  $\text{TiO}_2$  nanoparticles in HFC134a refrigerants. The improvement in the performance is seen around 1.2% for 5% addition  $\text{TiO}_2$  particles in base fluid. Asmahani Nayan *et al.*, [25] conducted numerical analysis of MHD flow of hybrid nanofluid over a vertical plate in porous medium. A comparative study from the results obtained shows that Nusselt number of  $\text{CuO - water}$  is less than  $\text{Ag - CuO / water}$ . The nonlinear partial differential equations were solved by using Keller Box method.

In our study, the focus is on  $\text{TiO}_2 - \text{Ag/water}$  hybrid nanofluid, as the addition of  $\text{TiO}_2$  nanoparticles to base fluid displays exceptional characteristics in contrast to conventional heat transfer fluids, notably heightened thermal conductivity.  $\text{TiO}_2$  is widely preferred in research due to its availability and the amount of information in literature. Its key characteristics and properties makes it easy to produce at a reduced cost as well as recyclable. An added advantage is its ability to

synthesize into nanostructure compared to most of the other materials employed in catalyst. Also, it is found stable in aqueous media.

Inspired by the above mentioned characteristics of  $TiO_2$  and the extensive applications of hybrid nanofluids and fins in heat transfer, a detail investigation of thermal performance of wet porous fin moving with steady velocity in hybrid nanofluid ( $TiO_2 - Ag / water$ ) is undertaken. The results shows an considerable improvement in heat transfer rate, as the temperature distribution has improved by an average of around 8% compared to other hybrid nanofluid such as  $MgO - Ag / water$  available in the literature [23], for various parameters  $Nr, Nc, Pe, Bi, m_2$  and  $n$ . Furthermore, a comparative analysis is performed between the convective and insulated fin tips. The formulated equations are numerically solved and the results are comprehensively discussed through graphical interpretations.

## 2. Methodology

A porous longitudinal moving fin with a uniform cross section area is considered with a flow of  $TiO_2 - Ag/water$  hybrid nanofluid over it. The configuration is illustrated in Figure 1. As the fin under consideration is porous, Darcy's model is employed in the analysis. It is assumed that the initial condition of the fin is at rest position with ambient and base temperatures  $T'_a$  and  $T'_b$  respectively. Subsequently, the fin loses heat by natural convection, radiation as the surface moves horizontally with steady velocity  $U'$ . Further, the analysis is simplified with following assumptions.

- i. The temperature and flow characteristics exhibit no variation with respect to time.
- ii. The temperature of the fin is constant.
- iii. The contact resistance between the fin base and the prime surface is neglected
- iv. The fin's base temperature is constant.
- v. The shape of the nanoparticles under consideration is spherical.
- vi.  $TiO_2 - Ag$  nanomaterials and liquid in the continuous phase are in thermal equilibrium state.

The governing energy balance equation at cross section  $dx$  of a fin under the flow of hybrid nanofluid is given by:

$$\frac{d^2T'}{dx'^2} - \frac{(\rho c_p)_{hnf} U'}{k'_{hnf}} \frac{dT'}{dx'} - \frac{2h_D i_{fg} (1 - \bar{\phi})(\omega' - \omega'_a)}{tk'_{hnf}} - \frac{2h'(1 - \bar{\phi})(T' - T'_a)}{tk'_{hnf}} - \frac{2gK(\rho c_p)_{hnf}(\rho\beta)_{hnf}(T' - T'_a)^2}{k'_{hnf}\mu_{hnf}t} - \frac{2\varepsilon\sigma F_{f-a}(T'^4 - T'^4_a)}{tk'_{hnf}} = 0 \quad (1)$$

Where,  $\beta_{hnf}$ ,  $\mu_{hnf}$ ,  $(C_p)_{hnf}$ ,  $k'_{hnf}$ ,  $\rho_{hnf}$ ,  $U'$  and  $T'$  denotes the volumetric coefficient of thermal expansion, effective dynamic viscosity, specific heat with constant pressure, thermal conductivity, effective density, Constant velocity of the fin and local fin temperature respectively. The subscript  $f$ ,  $nf$  and  $hnf$  denotes fluid, nanofluid and hybrid nanofluid. The axial distance is denoted by  $x'$  where as  $K$ ,  $F_{f-a}$ ,  $g$ ,  $h'$ ,  $t$ ,  $i_{fg}$ ,  $\bar{\phi}$ ,  $\omega'$ ,  $\omega'_a$ ,  $\varepsilon$ ,  $\sigma$ ,  $h_D$  are Permeability, Shape factor for radiation heat transfer, Acceleration due to gravity, Heat transfer coefficient, Fin Thickness, Latent heat of water evaporation, Porosity, Humidity ratio of the saturated air, Humidity ratio of the surrounding air, Surface emissivity of fin, Stefan –Boltzmann constant, Uniform mass transfer Coefficient respectively.

Mathematically, it is addressed as

$$k'_{hnf} = \frac{k'_{bf}(k'_{p_2} + (\alpha - 1)k'_{bf}) - (\alpha - 1)\varphi_2(k'_{bf} - k'_{p_2})}{(k'_{p_2} + (\alpha - 1)k'_{bf}) + \varphi_2(k'_{bf} - k'_{p_2})} \quad (2)$$

$$k'_{bf} = \frac{k'_f(k'_{p_1} + (\alpha - 1)k'_f) - (\alpha - 1)\varphi_1 k'_f(k'_f - k'_{p_1})}{(k'_{p_1} + k'_f) + \varphi_1(k'_f - k'_{p_1})} \quad (3)$$

Where ' $\alpha$ ' is the empirical constant. For spherical shaped nanoparticle,  $\alpha = 3$ .

$$\mu_{hnf} = \frac{\mu_f}{[(1 - \varphi_2)(1 - \varphi_1)]^{2.5}} \quad (4)$$

$$\rho_{hnf} = \varphi_2 \rho_{p_2} + [(1 - \varphi_1)(1 - \varphi_2)\rho_f + \varphi_1 \rho_{p_1}(1 - \varphi_2)] \quad (5)$$

$$(\rho c_p)_{hnf} = \varphi_2 (\rho c_p)_{p_2} + [(1 - \varphi_1)(1 - \varphi_2)(\rho c_p)_f + \varphi_1(1 - \varphi_2)(\rho c_p)_{p_1}] \quad (6)$$

$$(\rho\beta)_{hnf} = \varphi_2 (\rho\beta)_{p_2} + [(1 - \varphi_1)(1 - \varphi_2)(\rho\beta)_f + \varphi_1(1 - \varphi_2)(\rho\beta)_{p_1}] \quad (7)$$

Subscript  $p_1$  and  $p_2$  stand for solid nanoparticle 1 (Ag) and solid nanoparticle 2 ( $TiO_2$ ).  $\varphi_1, \varphi_2$  represents solid volume fraction of Ag and  $TiO_2$ .

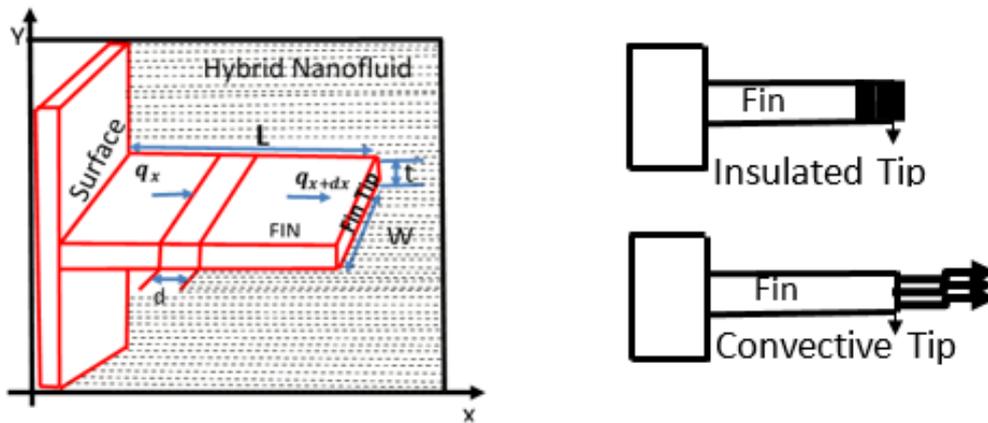


Fig. 1. Schematic of rectangular fin with Hybrid nanofluid with two different tips

In the investigation, we study the performance of  $TiO_2$ -Ag nanoparticles with water as base fluid. The properties of these material are shown in Table 1. The volume fraction of  $TiO_2$ -Ag are taken as  $\varphi_1$  and  $\varphi_2$ . The coefficient of heat transfer  $h'$  is defined as Where,  $h'_a$  convective heat transport coefficient at temperature  $T'_a$ .

$$h' = \frac{(T' - T'_a)^n h'_a}{(T'_b - T'_a)^n} \quad (8)$$

The problem is solved using two different types of boundary condition i.e the fin with insulated and convective tip.

Case1: Fin with an insulated tip

$$T'(0) = T'_b \text{ and } \frac{dT'(L)}{dx'} = 0 \text{ at } x' = 0 \text{ and } L \text{ respectively.} \quad (9)$$

Case 2: Convective fin tip

$$T'(0) = T'_b \text{ and } h'_a T'(L) + k'_{hnf} \frac{dT'(L)}{dx'} = 0 \text{ at } x' = 0 \text{ and } L \text{ respectively.} \quad (10)$$

Where L is the length of the fin.

**Table1**

Properties of each content of hybrid nanofluid

Physical properties	$\rho(\text{kg/m}^3)$	$c_p (\text{J/kg K})$	$k'(\text{W/mK})$	$\beta(1/\text{K})$
Ag	10,500	235	429	$1.89 \times 10^{-5}$
TiO <sub>2</sub>	4250	686	8.9538	$0.98 \times 10^{-5}$
Water	997.1	4179	0.613	$21 \times 10^{-5}$

The differential equation Eq. (1) is Non –dimensionalized using the following parameters

$$\begin{aligned} (\omega' - \omega'_a) &= b_2(T' - T'_a), & \theta' &= \frac{T'}{T'_b}, & \theta_a &= \frac{T'_a}{T'_b}, & X' &= \frac{x'}{L}, \\ Nc &= \frac{2(\rho c_p)_f(\rho\beta)_f g K T'_b L^2}{k'_f \mu_f t}, & Nr &= \frac{2\varepsilon\sigma L^2 T_b'^3}{t k'_f}, & pe &= \frac{(\rho c_p)_f U' L}{k'_f}, & Bi &= \frac{h'_a L}{k'_f}, \\ m_1 &= \frac{2b_2 L^2 h'_a i_{fg}(1-\bar{\phi})}{(Le)^{2/3} t C_{pf} k'_f}, & m_0 &= \frac{2L^2 h'_a(1-\bar{\phi})}{t k'_f}, & m_1 &= m_2 - m_0, \end{aligned} \quad (11)$$

On substituting the Eq. (8) and Eq. (11) in Eq. (1), we obtain the ODE in dimensionless form as

$$\begin{aligned} \frac{d}{dX'} \left( \frac{d\theta'}{dX'} \right) - pe \left( \frac{(\rho c_p)_{hnf}}{(\rho c_p)_f} \right) \left( \frac{k'_f}{k'_{hnf}} \right) \left( \frac{d\theta'}{dX'} \right) - Nr \left( \frac{k'_f}{k'_{hnf}} \right) (\theta'^4 - \theta_a^4) \\ - (m_1 + m_0) \frac{(\theta' - \theta_a)^{1+n}}{(1 - \theta_a)^n} \left( \frac{k'_f}{k'_{hnf}} \right) - Nc(\theta' - \theta_a)^2 \left( \frac{\mu_f}{\mu_{hnf}} \right) \left( \frac{k'_f}{k'_{hnf}} \right) \left( \frac{(\rho\beta)_{hnf}}{(\rho\beta)_f} \right) = 0 \end{aligned} \quad (12)$$

The non dimensionalised boundary conditions for convective and insulated tips are given by

$$\begin{aligned} \theta'(0) = 1 \text{ and } \theta'(1) = 0 \text{ at } X' = 0 \text{ and } X' = 1. \\ \theta'(0) = 1 \text{ and } \left( \frac{d\theta'}{dX'} \right) = -\theta'(1) \frac{k'_f}{k'_{hnf}} Bi \text{ at } X' = 0 \text{ and } X' = 1. \end{aligned} \quad (13)$$

In the equation,  $\theta_a, m_2, Bi, pe, n, Nr$  and  $Nc$  are non-dimensional ambient temperature, wet porous parameter, Biot number, Peclet number, power index, radiative parameter and convective parameter respectively. The dimensionless axial distance and temperature are  $X'$  and  $\theta'$  where as  $m_1, m_0$  are constants.

### 3. Numerical Procedure

The differential equation obtained in Eq. (12) is in non-linear form and is numerically integrated using Lobatto IIIA method. The excellent stability and good convergence properties of Lobatto IIIA method makes it a useful technique to solve boundary value problems. The method is found useful to capture the curvature of the solution with high accuracy. The BVP is solved using BVP4C code in MATLAB simulation tool.

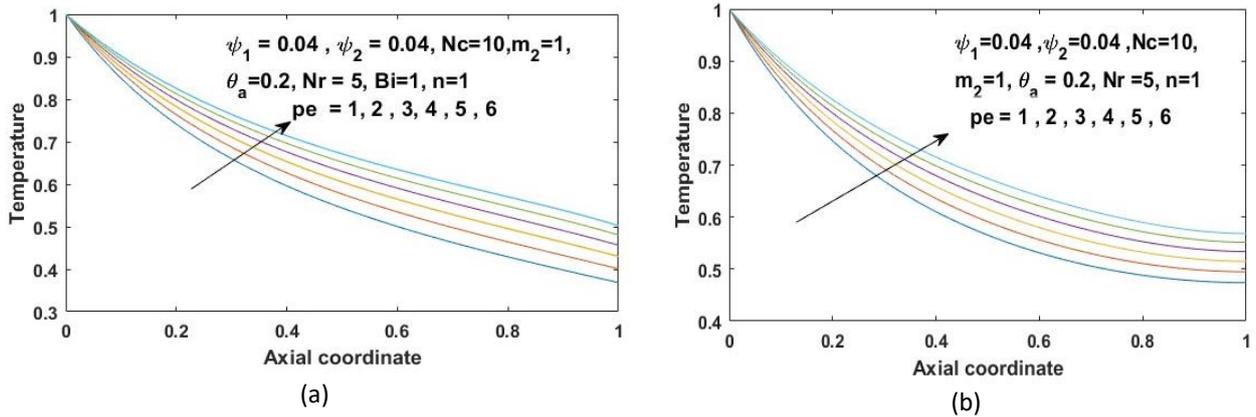
### 4. Results and Discussion

Initially, to confirm the accuracy of our proposed physical model under investigation, the outcome  $\theta'(X')$  for  $\varphi_1 = 0.04, \varphi_2 = 0.04$  and different values of  $Nc, Nr, Bi, n, m_2, pe$  are compared with the existing results obtained from Gireesha *et al.*, [23]. The outcome of the comparison is tabulated in Table 2. The % change in  $\theta'(X')$  clearly indicates the improvement in the efficiency of the model we have considered.

**Table 2**  
 $\theta'(X')$  values for  $\varphi_1 = 0.04, \varphi_2 = 0.04$  at  $x = 0.4$  in case of convective fin tip

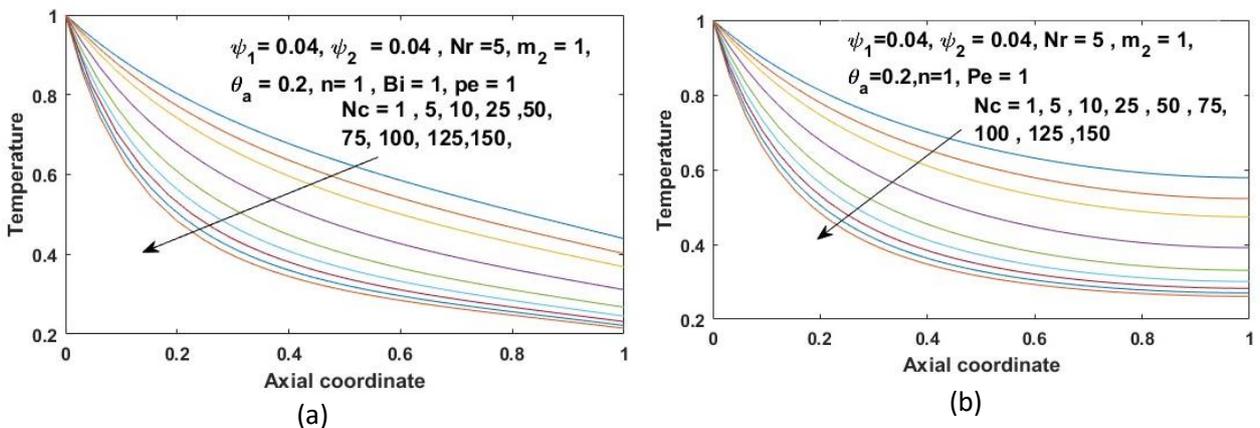
$pe$	$Nc$	$Nr$	$m_2$	$n$	$Bi$	Gireesha <i>et al.</i> , [23] $\theta'(X')$	Current outcome $\theta'(X')$	% change in $\theta'(X')$
1	10	5	1	1	1	0.6	0.59	1.45
2	10	5	1	1	1	0.64	0.62	3.13
3	10	5	1	1	1	0.66	0.65	1.68
4	10	5	1	1	1	0.69	0.67	1.90
5	10	5	1	1	1	0.71	0.69	2.81
1	1	5	1	1	1	0.69	0.67	2.46
1	5	5	1	1	1	0.63	0.63	0
1	10	5	1	1	1	0.6	0.59	2.17
1	50	5	1	1	1	0.45	0.44	2.22
1	10	1	1	1	1	0.65	0.63	3.23
1	10	5	1	1	1	0.61	0.59	3.11
1	10	10	1	1	1	0.55	0.54	1.82
1	10	5	0.1	1	1	0.62	0.6	2.9
1	10	5	1	1	1	0.6	0.59	1.45
1	10	5	10	1	1	0.53	0.49	8.11
1	10	5	1	0	1	0.59	0.58	1.69
1	10	5	1	1	1	0.6	0.59	1.5
1	10	5	1	2	1	0.61	0.6	1.31
1	10	5	1	1	1	0.59	0.58	1.69
1	10	5	1	1	2	0.6	0.59	1.50
1	10	5	1	1	3	0.61	0.6	1.31

The impact of Peclet number ( $Pe$ ) on the thermal profile of convective, insulated fin tips are demonstrated in Figure 2a and Figure 2b. Peclet number is a dimensionless quantity that reduces the interaction between the fin and surrounding as its value increases. This in turn, enhances the temperature of the fin in both insulated and convective tips. Consequently, the rate at which the heat is transferred decreases with the increase in  $Pe$  value. Enhancing the  $Pe$  value by 100%, the temperature distribution profile is reduced by 8% in the case of convective tip. Also, the investigation results show that the temperature distribution of the proposed model has improved by around 2% compared to the existing model discussed in Gireesha *et al.*, [23].



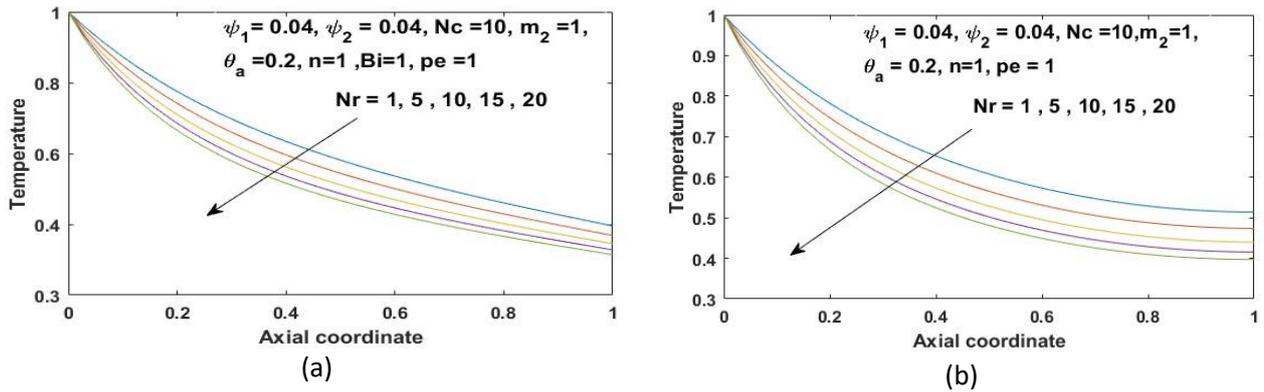
**Fig. 2.** (a) Temperature profile of convective fin tip for Peclet number range 1 to 6 (b) Temperature profile of insulated fin tip for Peclet number range 1 to 6

The impact of convective parameter on the temperature distribution for two different conditions of the fin tip is indicated in Figure 3a and Figure 3b. It is observed that the thermal profile reduces with the increase in convective variable. The change in the convective parameter varies Darcy’s or Rayleigh’s number, as it’s a combination of both the numbers. Scaling up the value of Convective parameter leads to increase in the value of either Darcy’s or Rayleigh’s number. This in turn, elevates the permeability or Buoyancy effect leading to enhancement in the heat transfer rate. Enhancing the  $Nc$  value by 50%, the temperature distribution profile is reduced by 13% in the case of convective tip. Also, the investigation results show that the temperature distribution of the physical model under consideration has improved by around 2% over the existing model proposed in Giresha *et al.*, [23].



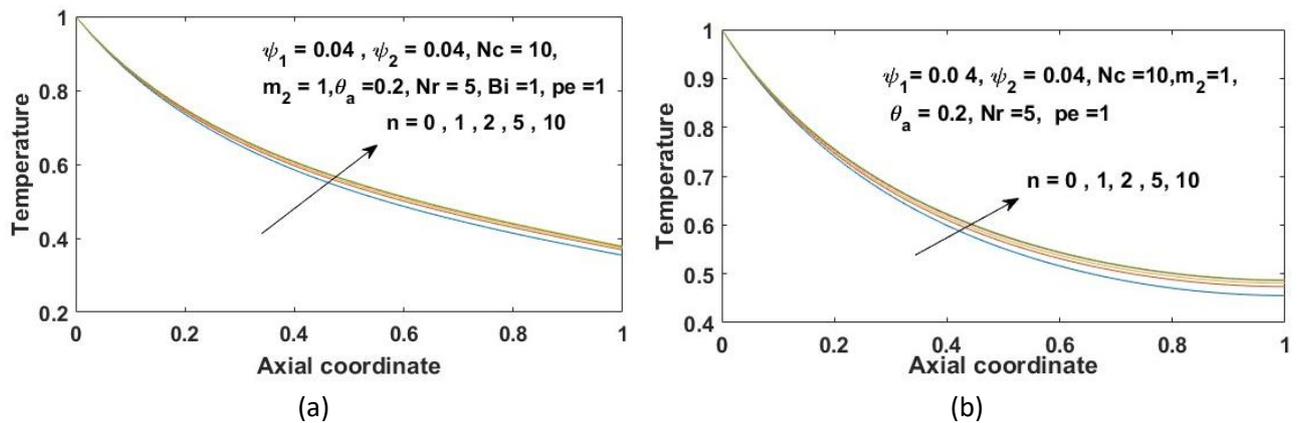
**Fig. 3.** (a) Temperature profile of convective fin tip for multiple values of convective parameters (b) Temperature profile of insulated fin tip for multiple values of convective parameters

The radiative parameter ( $Nr$ ) is one of the key parameters that effects the thermal profile of the fin. The impact of the parameter on the fin with convective and insulated tips is illustrated in Figure 4a and Figure 4b. The increase in the  $Nr$  value that indicates strong radiation effect decreases the temperature of the fin along its axial length, as it results in enhancement of the heat transfer from the surface of the fin to the hybrid nanofluid. Increasing the  $Nr$  value by 25%, the temperature distribution profile is reduced by 6% in the case of convective tip. Also, the investigation results show that the temperature distribution of the physical model under consideration has improved by around 2.5% compared to the existing model proposed in Giresha *et al.*, [23].



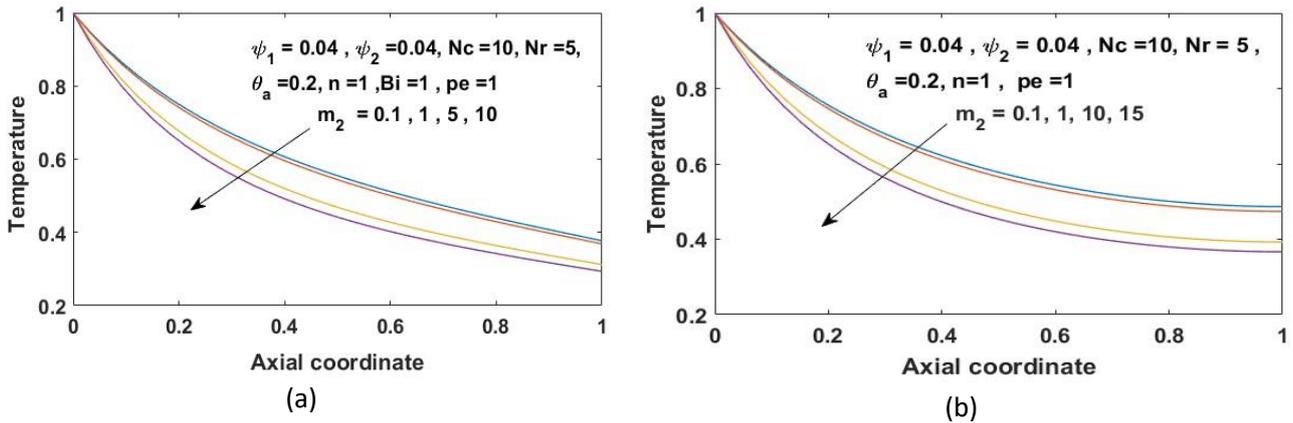
**Fig. 4.** (a) Temperature profile of convective fin tip for multiple values of radiative parameters (b) Temperature profile of insulated fin tip for multiple values of radiative parameters

Figure 5a and Figure 5b demonstrates the impact of power index ‘ $n$ ’ that represents different flow regime on the distribution of temperature on the fin. The temperature field is relatively lower for  $n = 0$  (linear case) than  $n > 0$  (non-linear case). This indicates that the heat transfer rate decreases with increase in the power index value. For the convective fin tip, the investigation results show that the temperature distribution of wet porous rectangular moving fin in  $\text{TiO}_2 - \text{Ag} / \text{water}$  hybrid nanofluid for different values of ‘ $n$ ’ has improved by around 1.5% over the existing model proposed in Gireesha *et al.*, [23].



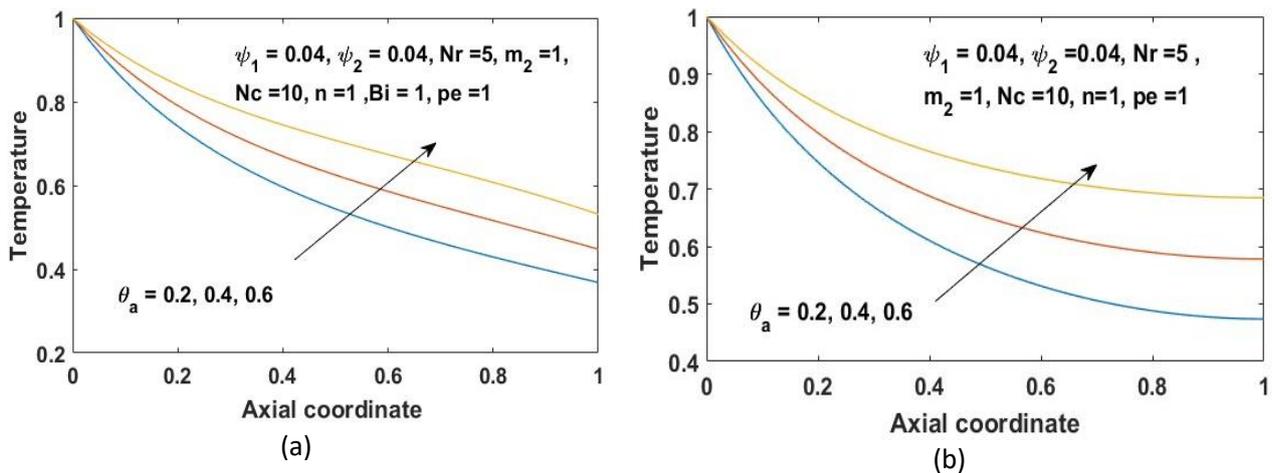
**Fig. 5.** (a) Temperature profile of convective fin tip for various power index (b) Temperature profile of insulated fin tip for various power index

Figure 6a and Figure 6b shows the influence of variable wet porous ( $m_2$ ) on the temperature characteristics of the fin. The variation clearly shows that the fin temperature decreases with the increase in the value of  $m_2$ . This is due to the wet porous nature of the fin. For the convective fin tip, the investigation results show that the temperature distribution of wet porous rectangular moving fin in  $\text{TiO}_2 - \text{Ag} / \text{water}$  hybrid nanofluid for various values of  $m_2$  has improved by an average of around 5% over the existing model proposed in Gireesha *et al.*, [23].



**Fig. 6.** (a) Temperature profile of convective fin tip for multiple  $m_2$  values (b) Temperature profile of insulated fin tip for multiple  $m_2$  values

The effect of ambient temperature on the distribution of temperature along the fin’s surface is depicted in Figure 7a and Figure 7b. As the value of  $\theta'$  (ratio of ambient temperature to the base temperature) increases, the temperature of the surrounding liquid gets enhanced which in turn hampers the heat transfer from the surface of the fin. As a result, enhancement in the temperature profile is observed with the rise in  $\theta'$  value.



**Fig. 7.** (a) Temperature profile of convective fin tip for various dimensionless ambient temperatures (b) Temperature profile of insulated fin tip for various dimensionless ambient temperatures

The influence of Biot number ( $Bi$ ) on the fin temperature is depicted in Figure 8. The heat transport rate is enhanced with the increase in the value of  $Bi$ , as it is observed that the temperature of the fin reduces with the rise in  $Bi$  value. For the convective fin tip, the investigation results show that the temperature distribution of wet porous rectangular moving fin in  $TiO_2 - Ag$  /water hybrid nanofluid for various values of  $m_2$  has improved by an average of around 2.5% over the existing model proposed in Gireesha *et al.*, [23].

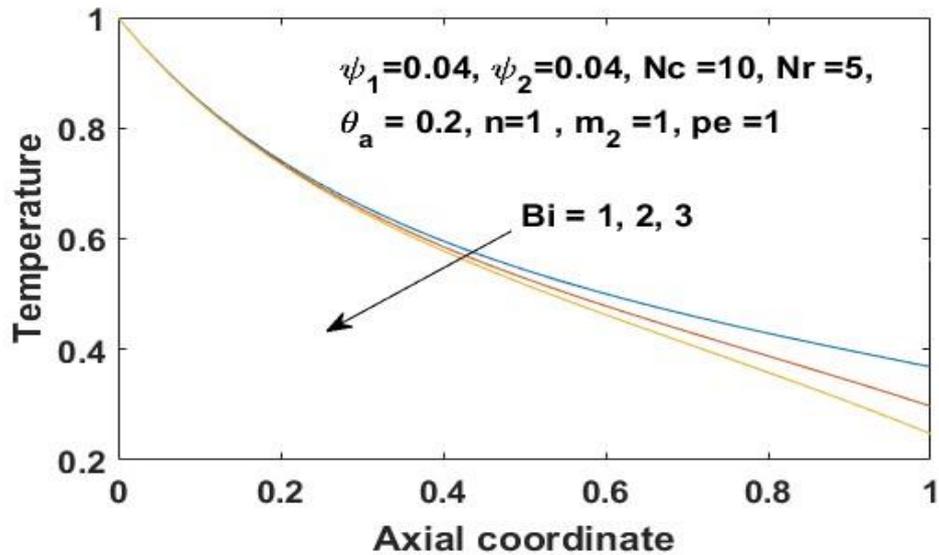


Fig. 8. Temperature profile of convective fin tip for various Biot numbers

## 5. Conclusions

The analysis investigates the thermal behaviour of a wet porous rectangular moving fin in spherical shaped  $\text{TiO}_2 - \text{Ag}$  / water hybrid nanofluid subjected to two boundary conditions. The rectangular fins are simple and effective structures used to increase the heat transfer rate from surface to the fluid. The thermophysical properties of the hybrid nanofluid such as specific heat, thermal conductivity, viscosity and convective heat transfer plays an important role in determining the heat transfer performance of the moving fin in hybrid nanofluid. The addition of high thermal conductive material  $\text{TiO}_2 - \text{Ag}$  to the base fluid relatively improves the thermal performance i.e the heat transfer rate from fin to the fluid. Also, a noticeable improvement in the heat transfer rate is observed relative to other hybrid nanofluids found in the literature with the rise in  $Nc$ ,  $Nr$ ,  $Bi$ ,  $n$ ,  $m_2$  and  $pe$  values;

- i. The fin cooling is significantly getting impacted by thermal radiation and natural convection.
- ii. The Enhancement in the wet porous parameter values increases the heat transport rate by increasing the wetness around the fin.
- iii. The interaction with the surrounding enhances with the decrease in Peclet number, in turn enhances heat transport rate.
- iv. The convective and radiative parameters improve the heat transfer rate. The heat transport efficiency improves with the  $\text{TiO}_2 - \text{Ag}$  nanoparticles added to the base fluid.

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