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Effects of Electrification and Transverse Force on Dusty Flow over a Linear Stretching Sheet

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ABSTRACT

The numerical investigation of the flow and heat transfer of steady dusty flow over a linear stretching sheet has been carried out. The effects of Transverse force and electrification has been incorporated in this problem. Modelling of the problem comprises of highly nonlinear partial differential equations that have been transferred to systems of ordinary differential equations by implementing suitable transformations. Since the equations are of boundary value problems in nature, have been transferred to initial value problem by using shooting method and then solved by RungeKutta 4th order technique. All the above numerical methods incorporated in the BVP4C tool of MATLAB and has been solved the systems of differential equations by using MATLAB software. The effects of different parameter like Prandtl number, Eckert number, transverse force, and electrification parameter on flow and heat transfer profile has been presented through graphs and tables. The results have been validated with previous authors. From the graph it is observed that the transverse force reduces the velocity of the particle phase in the flow where as electrification of particles raises the temperature of particle phase in the flow. The computations carried out with tolerance of order less than O (10^{-6}).

1. Introduction

The two-phased steady flow and heat transfer technology with transverse force over a stretchable sheet have a big impact on engineering processes and on variety of manufacturing industries. The principles of continual multi-phase flow over stretchable sheets have been applied in numerous industrial processes, such as paper manufacture and sheet coloring. The quality of the final product is influenced by the heat flux across each particular plane. Due to the huge advantages, several researchers have expressed their interest in researching the flow and heat transmission of fluid particle suspension across stretchy sheet.These research fields play a significant role in industrial applications. Sakiadis [1] conducted the initial investigation on boundary layer flow on a stretchy surface in 1961. Crane [2] was the first to explore that a linearly stretched sheet was the cause of

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the boundary layer flow. Then, several studies have been done on linearly stretching sheets by taking various physical aspects into account. Rubinow and Keller [3] have done a numerical and analytical investigation on the fluid flow around a sphere travelling through a viscous fluid with a low Reynolds number. Asmolov [4] has studied about the transverse force acting on a sphere-shaped particle travelling through a laminar boundary layer flow. Marble [6] used the series expansion method to solve the boundary layer equation, which is essentially correct for the downstream area of the plate when the slip velocity is low. Jain and Ghosh [5] took the particle momentum equation's effect on the flow field into account in the transverse direction. A numerical investigation about the two phase boundary layer flow and heat transfer with a non-uniform grid was conducted by Tripathy *et al.,* [7]. Das *et al.,* [8] have investigated the MHD free convection flow past in an infinite vertical plate with heat source in presence of radiation. The effects of electrifying particles on the flow geometry of horizontal plates, inclined stretched sheets, and jet flows have been explored by Samantara *et al.,* [9-12]. The study of transversal force has a bigger influence on both fluid and particle flow. Mei [13] has formulated an approximate equation for the shear lift force on a spherical particle at an infinite Reynolds number. Another investigation into the transverse force on a tiny sphere in a slow shear was conducted by Saffmann [14]. Swain *et al.,* [15] have numerical studied on the nano particles aggregation on radiative 3D flow of maxwell fluid over a permeable stretching surface with thermal radiation and heat source/sink. Turkyilmazoglu [16] have analyzed the flow and heat transfer of dusty fluid with Navier slip condition.

In the dusty flow, the particles colloids with each other as well as with the wall of the sheet. Thus electricity generates and effects flow and heat transfer phenomenon. Similarly as particles flows along with fluid, it rotates and thus transverse force also effects the effects flow and heat transfer phenomenon

Transverse force and electrification have some effect in velocity and heat transfer of particle phase in dusty flow. No literature available to investigate the effects of Transverse force and electrification simultaneously in the flow of dusty fluid over the stretching sheet.

2. Formulation of the Flow Problem

Assume that fluid is flowing in a boundary layer across a horizontal stretching sheet as shown in Figure 1.The dusty fluid is on the surface of sheet and the flow is created due to sudden movement of stretchable sheet. Two interacting reverse forces are applied to the wall, stretching it linearly. The Y-axis is normal to the flow, and the X-axis is reflected along it.

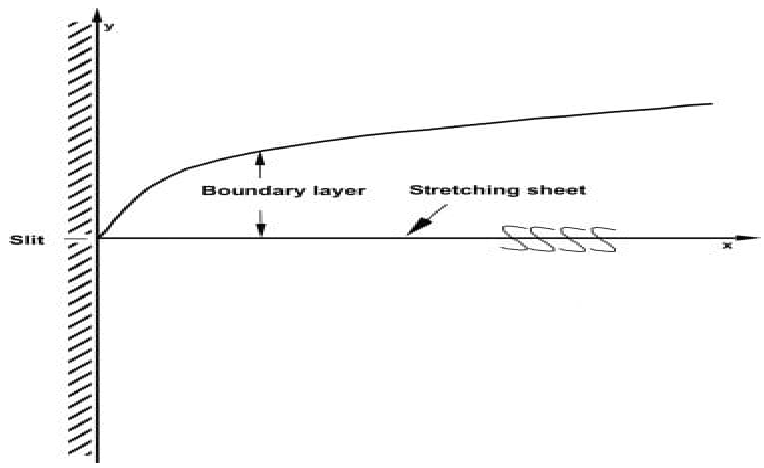

Fig. 1. Geometry of the flow problem

Despite taking into account this and the commonly used boundary layer approximation, the governing equations of continuity, momentum, and energy for both the fluid and dusty phases are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial}{\partial x}\left(\rho_p u_p\right)+\frac{\partial}{\partial y}\left(\rho_p v_p\right)=0 \tag{2}$$

$$(1-\varphi)\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=(1-\varphi)\mu\frac{\partial^2 u}{\partial y^2}-\frac{1}{\tau_p}\varphi\rho_s\left(u-u_p\right)+\varphi\rho_s\left(\frac{e}{m}\right)E \tag{3}$$

$$\varphi\rho_s\left(u_p\frac{\partial u_p}{\partial x}+v_p\frac{\partial u_p}{\partial y}\right)=\frac{\partial}{\partial y}\left(\varphi\mu_s\frac{\partial u_p}{\partial y}\right)+\frac{1}{\tau_p}\varphi\rho_s\left(u-u_p\right)+\varphi\frac{0.73(\rho/\rho_s)^{1/2}}{\tau_p^{1/2}}\left|\frac{\partial u}{\partial y}\right|^{1/2}\left(u-u_p\right)+\varphi\rho_s\left(\frac{e}{m}\right)E \tag{4}$$

$$\varphi\rho_s\left(u_p\frac{\partial v_p}{\partial x}+v_p\frac{\partial v_p}{\partial y}\right)=\frac{\partial}{\partial y}\left(\varphi\mu_s\frac{\partial v_p}{\partial y}\right)+\frac{1}{\tau_p}\varphi\rho_s\left(v-v_p\right) \tag{5}$$

$$(1-\varphi)\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=(1-\varphi)\,k\frac{\partial^2 T}{\partial y^2}+\frac{1}{\tau_T}\varphi\rho_s c_s\left(T_p-T\right)+\frac{1}{\tau_p}\varphi\rho_s\left(u-u_p\right)^2+(1-\varphi)\mu\left(\frac{\partial u}{\partial y}\right)^2+\varphi\rho_s\left(\frac{e}{m}\right)Eu_p \tag{6}$$

$$\varphi\,\rho_s c_s\left(u_p\frac{\partial T_p}{\partial x}+v_p\frac{\partial T_p}{\partial y}\right)=\frac{\partial}{\partial y}\left(\varphi k_s\frac{\partial T_p}{\partial y}\right)-\frac{1}{\tau_T}\varphi\rho_s c_s\left(T_p-T\right)-\frac{1}{\tau_p}\varphi\rho_s\left(u-u_p\right)^2+\varphi\mu_s\left[u_p\frac{\partial^2 u_p}{\partial y^2}+\left(\frac{\partial u_p}{\partial y}\right)^2\right]\varphi\rho_s\left(\frac{e}{m}\right)Eu_p \tag{7}$$

Where E is the electric field created by electric charge “e” of the particle of mass “m”. Other notations are of standard notations for fluid phase and particle phase.

Boundary condition

$$u=U_w(x), v=0, T=T_w=T_\infty+A\left(\frac{x}{l}\right)^2;\ \text{as } y\to 0$$

$$u=0, u_p=0, v_p=v, T=T_p=T_\infty;\ \text{as } y\to\infty \tag{8}$$

To convert the governing Eq. (2) to (7) into a set of Ordinary differential equations, We used following transformation

$$u=cxf'(\eta),\ v=-\sqrt{c\nu}f(\eta),\ \eta=\sqrt{c/\nu}\,y,$$

$$u_p=cxF(\eta),\ v_p=\sqrt{c\nu}G(\eta),\ \rho_r=\rho_p/\rho=H(\eta) \tag{9}$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \theta_p(\eta)=\frac{T_p-T_\infty}{T_w-T_\infty}, \tag{10}$$

Where $T - T_\infty = A\left(\frac{x}{l}\right)^2 \theta(\eta)$, $T_p - T_\infty = A\left(\frac{x}{l}\right)^2 \theta_p(\eta)$

Non-Dimensional Equations

$$H' = -\frac{(HF + HG')}{G} \tag{11}$$

$$f''' = f'^2 - ff'' + \frac{1}{1-\varphi}\beta H(f' - F) + \frac{1}{1-\varphi}HE \tag{12}$$

$$F'' = \frac{1}{\epsilon}[F^2 + F'G - \beta(f' - F) - M] \tag{13}$$

$$G'' = \frac{1}{\epsilon}\left[GG' + \beta(f + G) - 0.73\sqrt{\beta}F_t\sqrt{f''}(f' - F)\right] \tag{14}$$

$$\theta'' = (2f'\theta - f\theta')Pr - \frac{2}{3}\frac{H\beta}{(1-\phi)}\left(\theta_p - \theta\right) - \frac{H\beta PrEc}{(1-\phi)}(f' - F)^2 - PrEcf''^2 - \frac{1}{1-\varphi}PrEcHMF \tag{15}$$

$$\theta_p'' = \frac{\epsilon}{Pr}\begin{bmatrix} 2F\theta_p + G\theta_p' + \beta\left(\theta_p - \theta\right) + \frac{3}{2}PrEc\beta(f' - F)^2 \\ -\frac{3}{2}\epsilon PrEc\left(FF'' + F'^2\right) - \frac{3}{2}PrEcMF \end{bmatrix} \tag{16}$$

With non-dimensional boundary condition

$$G'(\eta) = 0, f(\eta) = 0, f'(\eta) = 1, F'(\eta) = 0, \theta(\eta) = 1, \theta_p'(\eta) = 0; \text{as}\, \eta \to 0$$

$$f'(\eta) = 0, F(\eta) = 0, G(\eta) = -f(\eta), H(\eta) = \omega, \theta(\eta) \to 0, \theta_p(\eta) \to 0; \text{as}\, \eta \to \infty \tag{17}$$

3. Methodology

The coupled Eq. (11) to (16) with boundary condition Eq. (17) have been solved by using shooting technique followed by Runge-Kutta 4th order method. By using shooting technique the boundary value problems reduced to initial value problems and then the initial value problems have been solved by Runge-Kutta 4th order method whose accuracy, stability and convergence have been established earlier. Both the numerical methods are incorporated in BVP4C tool of MATLAB and hence the numerical solutions obtained as output of the MATLAB Programming. Using the assumption that the finite value of $\eta \to \infty$ at $\eta = 15$ with a specified tolerance order of less than $O(10^{-6})$.The results are obtained in both tabular as well as graphical form and has been validated with existing results as shown in Table 1.

Table1
Result validating table

Prandtl number, Pr	Ishak *et al.,* [15]	Subhas *et al.,* [16]	Giressha *et al.,* [17]	Chen [18]	Gurbka *et al.,* [19]	Mukhopadhya *et al.,* [20]	Current Study
0.72	----------	1.0885	1.0885	1.0885	1.0885	1.0885	1.0884
1.0	1.3333	1.3333	1.3333	1.3333	1.3333	1.3333	1.3333
3.0	2.5097	---------	2.5097	2.5097	----------	2.5097	2.5097
10.0	4.7969	4.7969	4.7969	4.7969	4.7969	----------	4.7969

4. Result Analysis

4.1 Effect of Prandtl Number (PR)

Figure 2 to 5 depicts the effects of Prandtl number on velocity and heat transfer profile of flow field of fluid phase as well as particle phase. From the figure it is concluded that velocity increases in particle phase with increasing of Pr whereas very negligible but opposite behaviour in case of fluid phase. In case of Temperature Profile, the impact is quite visible for both the phases. The heat transfer increases with lowering the value of Prandtl number.

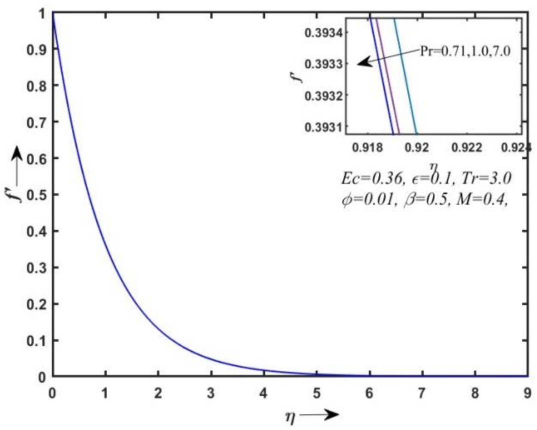

Fig. 2. Effect of Prandtl number on fluid velocity

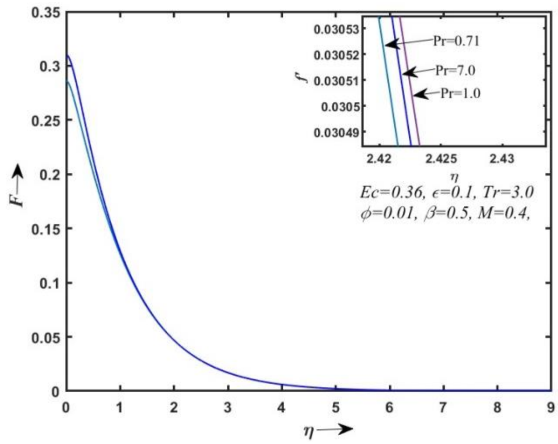

Fig. 3. Effect of Prandtl number on particle velocity

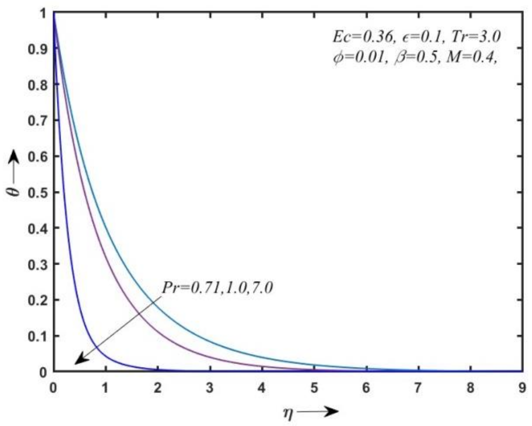

Fig. 4. Effect of Prandtl number on fluid temperature

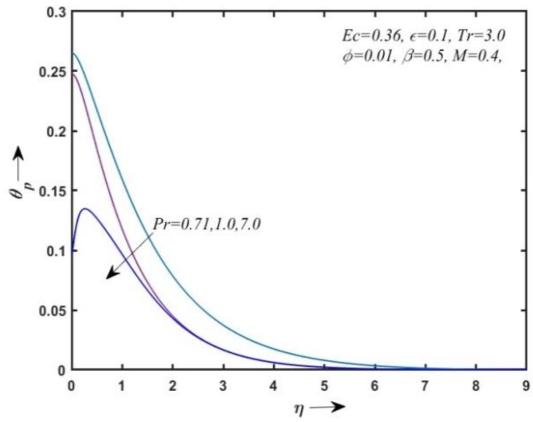

Fig. 5. Effect of Prandtl number on particle temperature

4.2 Effect of Electrification Parameter (M)

Figure 6 to 7 depicts the effects of Electrification Parameter (M) on heat transfer profile of flow field of fluid phase as well as particle phase .From the figure it is concluded that it has negligible effect in case of fluid phase but has significant effect in particle phase. Temperature of particle phase rises with increasing of electrification parameter. The increasing of M value causes the particle random motion rapid that makes more collision to generate more heat in particle phase.

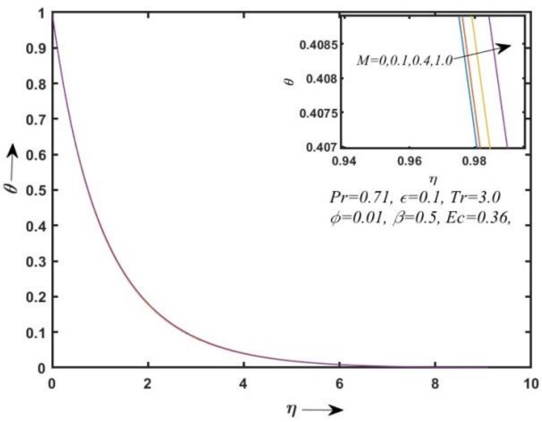

Fig. 6. Effect of electrification Parameter on fluid temperature

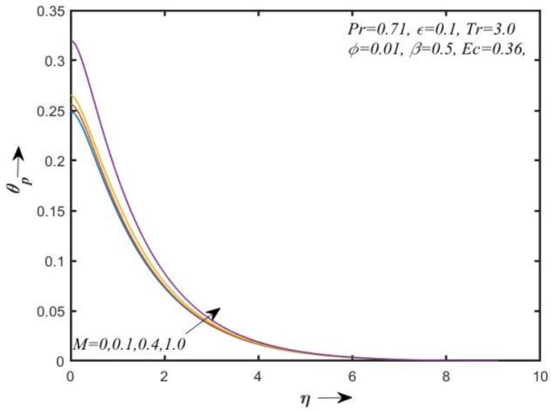

Fig. 7. Effect of electrification Parameter on particle temperature

4.3 Effect of Transverse Force (Tr)

Figures 8 to 11 depict the effects of transverse on the flow and heat transfer profile of the flow field for both the fluid phase and the particle phase. From the figure, it can be inferred that the fluid phase's velocity profile has far less of an impact than the particle phase does close to the stretching sheet's surface. The particle phase's velocity profile reduces close to the sheet's surface as the transverse force rises and coincides farther from the sheet. Both phases exhibit minimal effects from the transverse force on the temperature profile, with the fluid phase having far less impact than the particle phase.

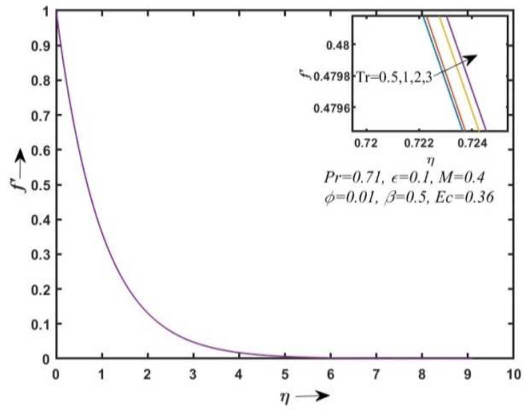

Fig. 8. Effect of transverse force on fluid velocity

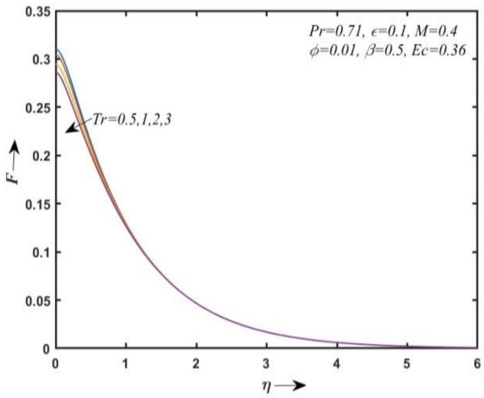

Fig. 9. Effect of transverse force on particle velocity

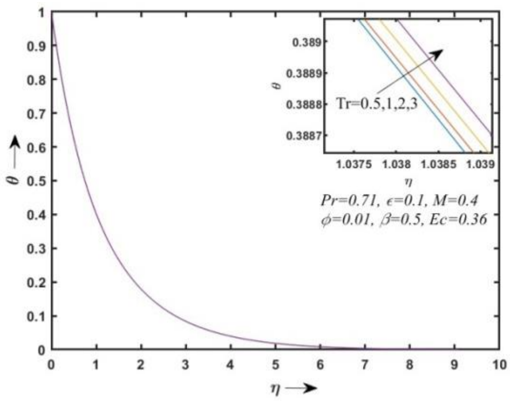

Fig. 10. Effect of transverse force on fluid temperature

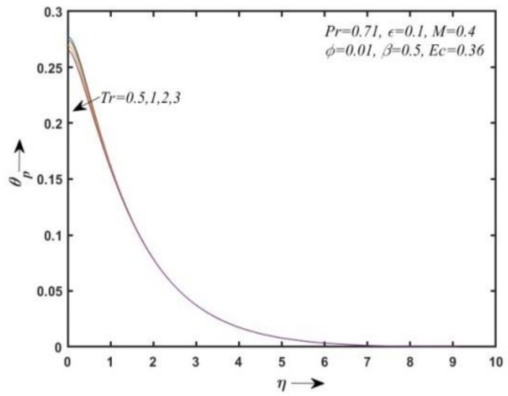

Fig. 11. Effect of transverse force on particle temperature

*4.4 Effect of Fluid Particle Interaction Parameter (*β)

The effects of the fluid particle interaction parameter (β) on the flow and heat transfer profile of the flow field of the fluid phase as well as the particle phase are shown in the figures from 12 to 15. The interaction parameter (β) plays a crucial influence in particle phase of velocity as well as Temperature profile, as can be seen in the image. The velocity and temperature profile of particle phase increases with larger value of interaction parameter (β) which transfers the energy more rapidly from particle to particle.

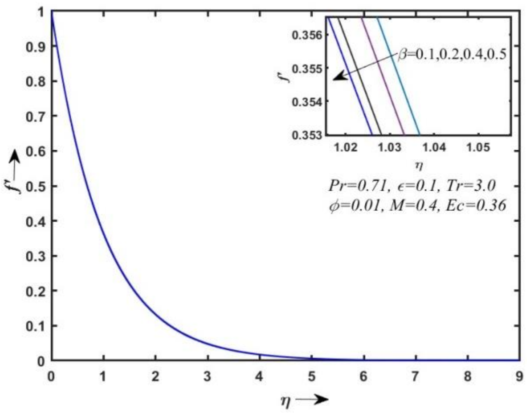

Fig. 12. Effect of fluid particle interaction parameter ($\boldsymbol{\beta}$) fluid velocity

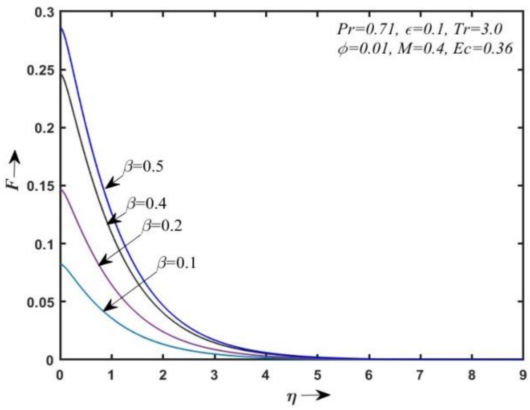

Fig. 13. Effect of fluid particle interaction parameter ($\boldsymbol{\beta}$) particle velocity

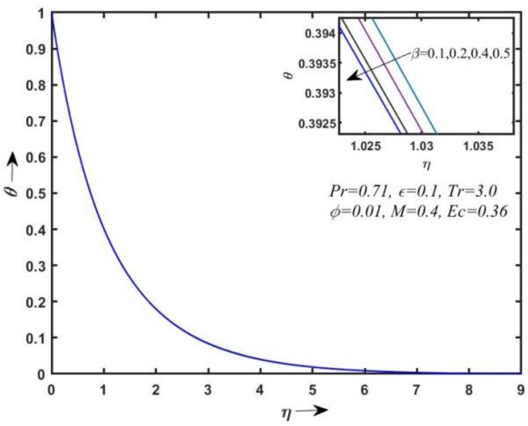

Fig. 14. Effect of fluid particle interaction parameter ($\boldsymbol{\beta}$) fluid temperature

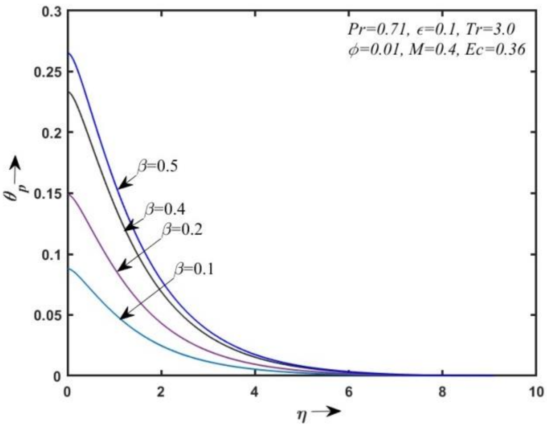

Fig. 15. Effect of fluid particle interaction parameter ($\boldsymbol{\beta}$) particle temperature

4.5 Effect of Diffusion Parameter (ϵ)

The figures from 16 to 19, respectively, indicate the effects of the diffusion parameter (ϵ) on the flow and heat transfer profiles of the flow field of the fluid phase as well as the particle phase. The diffusion parameter (ϵ) only little affects the fluid phase, but it significantly affects the particle phase. The graphic suggests that faster heat and flow transmission in the boundary layer occurs when the diffusion parameter (ϵ) is increased.

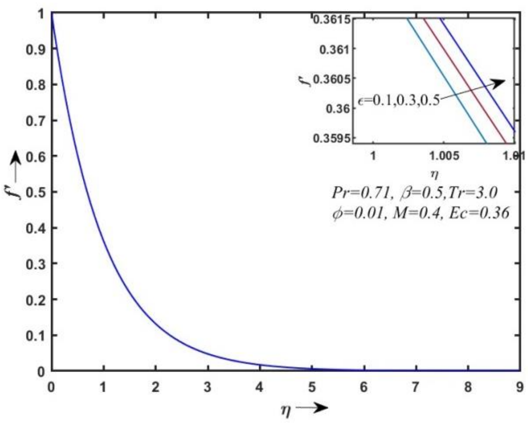

Fig. 16. Effect of diffusion parameter (ϵ) fluid velocity

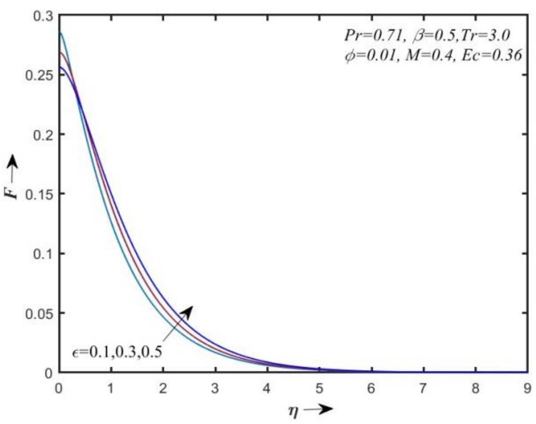

Fig. 17. Effect of diffusion parameter (ϵ) particle velocity

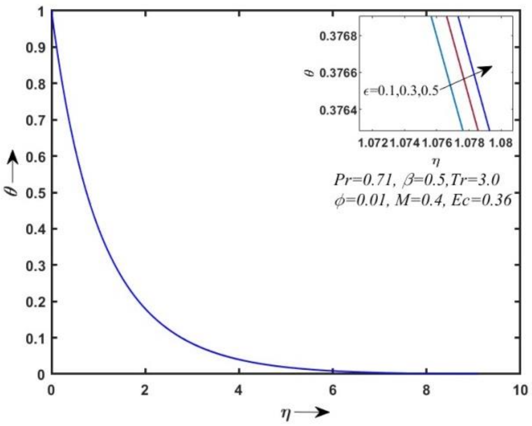

Fig. 18. Effect of diffusion parameter (ϵ) fluid temperature

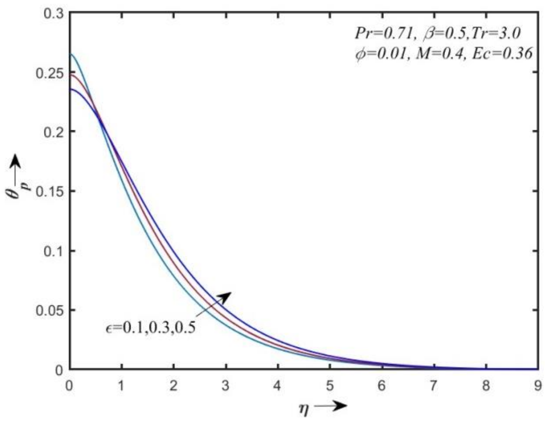

Fig. 19. Effect of diffusion parameter (ϵ) particle temperature

4.6 Effect of Eckert Number (Ec)

Figures 20 and 21 show how Eckert Number affects the flow's heat transfer profile. The graphic suggests that the temperature profile of both the fluid phase and the particle phase rises with increasing Eckert Number.

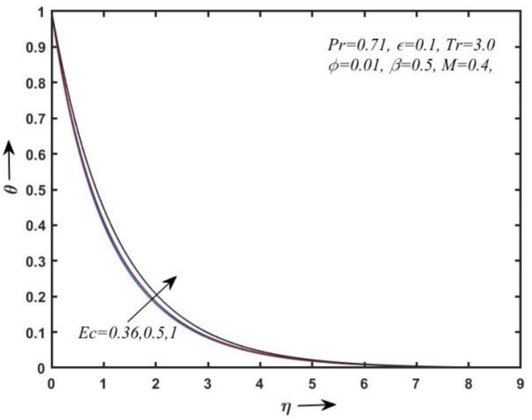

Fig. 20. Effect of diffusion parameter (ϵ) fluid temperature

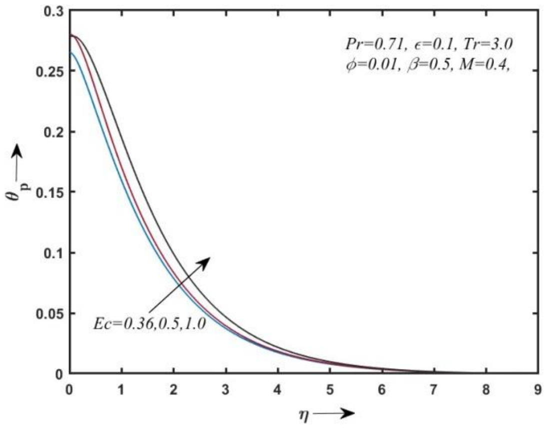

Fig. 21. Effect of diffusion parameter (ϵ) particle temperature

4.7 Skin Friction and Nusselt Number

The effect of different parameters on skin friction and and Nusselt number is represented in following Table 2. From the table it is observed that increasing the transverse force causes the lowering of Skin friction and Nusselt number.

Table 2
Effect of "Prandtl Number", "Eckert number", "Electrification parameter", "Transvers force", "Diffusion parameter" and "Fluid particle interaction parameter" on skin friction and Nusselt number

Pr	Ec	M	Tr	β	ϵ	$f''(0)$	$-\theta'(0)$
0.71	0.36	0.4	3.0	0.5	0.1	1.34618	1.25198
1.0						1.34889	1.34730
7.0						1.34889	2.79771
0.71	0.36	0.4	3.0	0.5	0.1	1.34618	1.25198
	0.5					1.34618	1.19403
	0.8					1.34618	0.98707
0.71	0.36	0	3.0	0.5	0.1	1.34889	1.25399
		0.1				1.34889	1.25376
		0.4				1.34889	1.25198
		1.0				1.34889	1.25173
0.71	0.36	0.4	0.5	0.5	0.1	1.34889	1.25308
			1.0			1.34797	1.25270
			2.0			1.34686	1.25224
			3.0			1.34618	1.25198
0.71	0.36	0.4	0.5	0.1	0.1	1.34221	1.25039
				0.2		1.34339	1.25082
				0.4		1.34534	1.25161
				0.5		1.34618	1.25198
0.71	0.36	0.4	3.0	0.5	0.1	1.34618	1.25198
					0.3	1.34644	1.25221
					0.5	1.34696	1.25248

5. Conclusion

The effects of different parameters on flow over Linear Stretching Sheet have been studied and conclusions are as follows.

(i) The transverse force which is applied to flow field is perpendicular to the flow and hence it hampers the velocity and heat transfer of the flow following the Skin friction and Nusselt number.
(ii) The electrification has no effect on Skin friction but has effect on Nusselt number. The nusselt number decreases with rise of electrification parameter.
(iii) In the future scope of this problem, different type of Stretching Sheet can be considered and the effects can be analyzed.

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