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Control Effect on Rayleigh-Benard Convection in Rotating Nanofluids Layer with Double-Diffusive Coefficients

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ABSTRACT

Rayleigh-Benard convection in rotating nanofluids layer with feedback control and double-diffusive coefficients heated from below is examined. The model for nanofluid includes the mechanism of Brownian motion and thermophoresis. The system is considered for three types of lower-upper boundary conditions, free-free, rigid-free and rigid-rigid. Linear stability analysis based on normal mode technique is employed, the eigenvalue solution is obtained by Galerkin technique and solved by using Maple software. The effects of rotation, feedback control, double-diffusive cofficients and nanofluids parameters have significant impact to the system. Based on the observation, the effect of increasing the value of rotation, feedback control, Dufour parameter and solutal Rayleigh number are observed to stabilize the system. Meanwhile, the effect of increasing the value of Soret parameter, nanofluids Lewis number, nanoparticles concentration Rayleigh number and modified diffusivity ratio are found to destabilize the system. The effect of modified particles density increment in the system is very small and can be neglected. The effects of the parameters in the system are discussed and presented graphically.

1. Introduction

Nanofluids are the latest generation of engineered fluids that attracted many researchers interest due to their importance in science and technology. Nanofluids are colloidal mixture of nanosized particles (1-100nm) and base fluids [1]. Earliest investigation on nanofluids was explored by several authors [2-5]. In 2020, Menni *et al.,* [6] presented a global insight into the different applications of nanofluids in various heat exchangers, that is, heat pipe and plate-fin heat exchangers. Seven slip mechanisms that are, inertia, Brownian motion, thermophoresis, diffusiophoresis, Magnus effect, fluid drainage and gravity settling were discussed and it is found that Brownian motion and thermophoresis mechanism are crucial for convective transport in nanofluid model [7]. Farhana *et al.,* [8] experimented a study on thermo-physical properties of nanofluids specifically nanocellulose-aqueous ethylene glycol nanofluids. Tzou [9,10] employed Buongiorno's model to study the thermal

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instability problem in nanofluids layer. Nield and Kuznetsov [11] studied the onset of convection in nanofluids layer of finite depth. Yadav *et al.,* [12] performed the linear stability analysis of Rayleigh-Benard convection in nanofluids layer. Hadad *et al.,* [13] reported that thermophoresis and Brownian motion mechanism are significant in thermal enhancement of the natural convection in nanofluids layer. Douha *et al.,* [14] performed a naturel convection in partially porous cavity in the presence of nanofluids. Aliouane *et al.,* [15] investigated the development of laminar convection flow at different Rayleigh numbers in a rectangular Rayleigh-Benard cell. The enclosure is differently or partially heated, and it is filled with air or nanofluid. Mahammedi *et al.,* [16] numerically studied the turbulence flows and heat transfer of alumina water nanofluids in a circular tube. Abdelkader *et al.,* [17] investigated the onset of convection and friction factor of magnetic Ni nanofluids within cylindrical pipes. Phu *et al.,* [18] examined numerically the nanofluid-based solar collector duct equipped with baffles opposite to the heated wall. Heat transfer enhancement for mono and hybrid nanofluids flow in a straight pipe is studied numerically by Azman *et al.,* [19].

Dufour diffusion, known as thermodiffusion and Soret diffusion, known as thermal diffusion are important in both Newtonian and non-Newtonian convective heat and mass transfer. Hurle and Jakeman [20] demonstrated the Soret-driven thermosolutal convection both theoretically and experimentally by employing water-methanol mixture. Platten and Chavepeyer [21] employed water-ethanol mixture and water-isopropanol mixture for investigation on the period of oscillations for the Benard problem in two component systems with thermal diffusion. Caldwell [22] extended the investigation using water-salt solution. Knobloch and Moore [23] investigated linear stability of experimental Soret convective instability of water-ethanol mixture under different types of boundary conditions with an emphasis on the Biot number. Nield and Kuznetsov [24] extended their investigation executed in Nield and Kuznetsov [11] on linear stability analysis of double-diffusive convection in nanofluids layer. The effect of magnetic field on the onset of convection in nanofluids layer has been studied by Chand and Rana [25] with the aid of Soret effect. The derived eigenvalue solution is solved numerically by employing the Galerkin technique. In stationary convection, effects of magnetic field stabilized the system, while Soret effect destabilized the system. Meanwhile, Akbar *et al.,* [26] proposed an article of MHD double-diffusive natural convection in nanofluids layer based on Buongiorno's model and found that the heat transfer rate increased monotonically as the nanoparticles and salt are suspended in water. Abdulwahab *et al.,* [27] numerically studied on the investigation of fluid flow and heat transfer enhancement insight straight channel of magnetic nanofluids.

Coriolis force due to the rotation effect in fluids layer system has an important effect of the convective instability. Chandrasekhar [28] studied Rayleigh-Benard convection in fluids with linear temperature profile, with and without the effect of rotation. Namikawa *et al.,* [29] examined the effect of uniform rotation on thermal instability in horizontal layer of fluids by means of linear stability analysis and found that the Coriolis force inhibits the Marangoni-Benard convection. As for the micropolar ferrofluid, Sunil *et al.,* [30] used linear stability analysis and normal mode technique to investigate the stabilizing effect of rotation on convective instability for stationary and oscillatory mode of convection. There has been significant interest in thermal instability in rotating nanofluids layer. It is a known fact that rotation plays a significant effect in many practical applications of nanofluids in modern science and engineering with relatively rapid change in temperature and concentration [31-33]. Problem of rotating nanofluids layer on thermal instability associated to Brownian motion and thermophoresis mechanism is examined by Yadav *et al.,* [34, 35]. The Coriolis force effect on thermocapillary type of convection with other aspects of the problem has been studied by several authors [36-39]. Meanwhile, these authors have investigated the convective instability in rotating fluids layer induced by buoyancy and thermocapillary, which has received profound attention from engineering industry [40, 41]. Yadav *et al.,* [42, 43] studied the effect of rotation in nanofluids layer and reported that an increased in the value of nondimensional Taylor number parameter emerged from the dimensional equation increased the thermal critical Rayleigh number and stabilized the system.

The capability to control complex heat transfer flow is essential in technology and fundamental sciences. In many technological processes, the naturally occurring dynamic flow patterns may not be the optimal ones. By controlling the heat flow, one may be able to optimize the process. The aid of using feedback control in thermal instability was studied and it is showed that the use of feedback control can significantly increase the thermal critical Rayleigh number, thus stabilized the system [44, 45]. Tang and Bau [46-48] and Howle [49-51] demonstrated experimentally that the feedback control stabilized the system. Bau [52] employed a linear controller to delay the onset of Marangoni-Benard instability and concluded that the similar control strategies are also effective in delaying the onset of Rayleigh-Benard convection. Further, many researchers have included other effects with feedback control [53-55] on thermal instability induced by buoyancy and surface tension driven.

In the paper, we intend to study the effect of double-diffusive coefficients with feedback control on the onset of Rayleigh-Benard convection in rotating nanofluids layer, since there is no research reported on this investigation from the previous researchers. This study is extended from the study done by Nield and Kuznetsov [24] by adding the combination effects of feedback control and rotation for the lower-upper boundary conditions of free-free, rigid-free and rigid-rigid. A linear stability analysis based on normal mode technique is performed, and the eigenvalue solution is obtained by employing the Galerkin technique. Numerical computations of the various relevant parameters are computed by Maple software and presented graphically. The study finds relevance in many applications particularly in manufacturing processes in industry.

2. Methodology

Cartesian coordinates (*x*,*y*,*z*) are used, where the *z*-axis points vertically upward. Consider a horizontal layer of a rotating incompressible nanofluids of thickness *L* confined between the planes $z^* \in [0, L]$ is heated from below subjected to feedback control, *K* and double-diffusive coefficients as shown in Figure 1. The nanofluids layer rotates about the vertical axis at a constant angular velocity, $\mathbf{\Omega}^* = (0, 0, \Omega)$. The stability of a horizontal rotating nanofluids layer in the presence of feedback control, *K* and double-diffusive coefficients are examined. The temperature, solute concentration and nanoparticles volumetric fraction of nanoparticles as the lower and upper walls are denoted by T_l^*, C_l^* and ϕ_l^* at *z* = 0 and; T_u^*, C_u^* and ϕ_u^* at *z* = *L*, respectively. The governing equations under the Boussinesq approximation under this model with the presence effects of rotation, feedback control and double-diffusive coefficients are written below

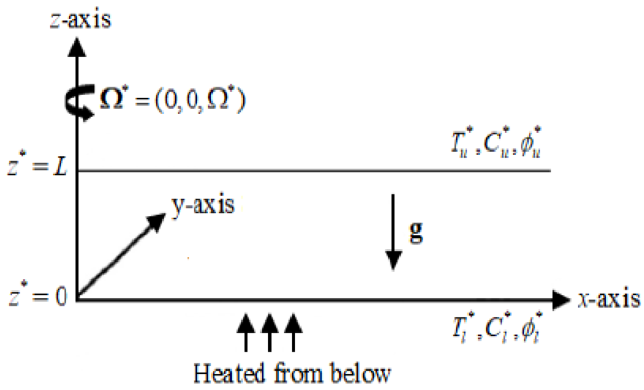

Fig. 1. Physical configuration and coordinate system

$$\nabla^* \cdot \boldsymbol{u}^* = 0, \tag{1}$$

$$\rho_f \left[\frac{\partial \boldsymbol{u}^*}{\partial t^*} + \left(\boldsymbol{u}^* \cdot \nabla^* \right) \boldsymbol{u}^* \right] = -\nabla^* p^* + \mu \nabla^{*2} \boldsymbol{u}^* + \boldsymbol{g} \left\{ \phi^* \rho_p + \left(1 - \phi^*\right) \rho_f \left[1 - \alpha_T \left(T^* - T_u^* \right) - \alpha_C \left(C^* - C_u^* \right) \right] \right\} -2\rho_f \left(\boldsymbol{\Omega}^* \times \boldsymbol{u}^* \right), \tag{2}$$

$$(\rho c)_f \left[\frac{\partial T^*}{\partial t^*} + \left(\boldsymbol{u}^* \cdot \nabla^* \right) T^* \right] = \kappa \nabla^{*2} T^* + (\rho c)_p \left[D_B \nabla^* \phi^* \cdot \nabla^* T^* + \left(\frac{D_T}{T_u^*} \right) \nabla^* T^* \cdot \nabla^* T^* \right] + (\rho c)_f \left(D_{TC} \nabla^{*2} C^* \right), \tag{3}$$

$$\left[\frac{\partial C^*}{\partial t^*} + \left(\boldsymbol{u}^* \cdot \nabla^* \right) C^* \right] = D_S \nabla^{*2} C^* + D_{CT} \nabla^{*2} T^*, \tag{4}$$

$$\left[\frac{\partial \phi^*}{\partial t^*} + \left(\boldsymbol{u}^* \cdot \nabla^* \right) \phi^* \right] = D_B \nabla^{*2} \phi^* + \frac{D_T}{T_u^*} \nabla^{*2} T^*, \tag{5}$$

where $\boldsymbol{u}^* = (u, v, w)$ is the Darcy velocity, ρ is the density of the base fluid, ρ_f is the nanofluids density at the reference temperature T_l^*, ρ_p is the nanoparticles mass density, t^* is the time, p^* is the pressure, μ is the viscosity, $\boldsymbol{g}$ is the gravitational force, ϕ^* is the volumetric fraction of nanoparticles, α_T is the thermal volumetric coefficient, α_C is the solutal volumetric coefficient, T^* is the temperature, C^* is the solute concentration, c is the specific heat, c_p is the specific heat of the nanoparticles, κ is the nanofluids thermal conductivity, D_{TC} is the Dufour diffusivity, D_B is the Brownian diffusion coefficient and D_T is the thermophoretic diffusion coefficient, D_S is the solutal diffusivity and D_{CT} is the Soret diffusivity. Eq. (1)-(5) are nondimensionalized using the following definition

$$\left(x^*,y^*,z^*\right)=L\left(x,y,z\right),p^*=\frac{p\mu\alpha_f}{L^2},t^*=\frac{tL^2}{\alpha_f},\phi=\frac{\phi^*-\phi_l^*}{\phi_u^*-\phi_l^*},C=\frac{C^*-C_u^*}{\Delta C^*},\psi_z^*=\frac{\psi_z\alpha_f}{L},$$
$$\left(u^*,v^*,w^*\right)=\frac{\alpha_f}{L}\left(u,v,w\right),T=\frac{T^*-T_u^*}{\Delta T^*}, \tag{6}$$

where $\alpha_f=\dfrac{\kappa}{\left(\rho c\right)_f}$ is the effective thermal diffusivity. Then, the nondimensionalized (1)-(5) takes the form

$$\nabla\cdot\boldsymbol{u}=0, \tag{7}$$

$$\frac{1}{\Pr}\left[\frac{\partial\boldsymbol{u}}{\partial t}+\boldsymbol{u}\cdot\nabla\boldsymbol{u}\right]=-\nabla p-\nabla^2\boldsymbol{u}-Rm\hat{\boldsymbol{e}}_z+\frac{Rs}{Le}C\hat{\boldsymbol{e}}_z+RaT\hat{\boldsymbol{e}}_z-Rn\phi\hat{\boldsymbol{e}}_z-\sqrt{Ta}\left(\boldsymbol{u}\times\hat{\boldsymbol{e}}_z\right), \tag{8}$$

$$\left[\frac{\partial T}{\partial t}+\boldsymbol{u}\cdot\nabla T\right]=\nabla^2T+Df\nabla^2C+\frac{N_B}{Ln}\nabla\phi\cdot\nabla T+\frac{N_AN_B}{Ln}\nabla T\cdot\nabla T, \tag{9}$$

$$\left[\frac{\partial C}{\partial t}+\boldsymbol{u}\cdot\nabla C\right]=\frac{1}{Le}\nabla^2C+Sr\nabla^2T, \tag{10}$$

$$\left[\frac{\partial\phi}{\partial t}+\boldsymbol{u}\cdot\nabla\phi\right]=\frac{1}{Ln}\nabla^2\phi+\frac{N_A}{Ln}\nabla^2T, \tag{11}$$

where $\hat{\boldsymbol{e}}_z=\left(0,0,1\right)$ is the unit vector in the *z*-direction, $\Pr=\dfrac{\mu}{\alpha_f\rho_f}$ is the Prandtl number, $Rm=\dfrac{\left[\rho_p\phi_l^*+\rho_f\left(1-\phi_l^*\right)\right]gL^3}{\mu\alpha_f}$ is the basic density Rayleigh number, $Rs=\dfrac{\rho_f g\alpha_C L^3\Delta C^*}{\mu D_S}$ is the solutal Rayleigh number, $Le=\dfrac{\alpha_f}{D_S}$ is the Lewis number, $Ra=\dfrac{\rho_f g\alpha_T L^3\Delta T^*}{\mu\alpha_f}$ is the thermal Rayleigh number, $Rn=\dfrac{\left(\rho_p-\rho_f\right)\left(\phi_u^*-\phi_l^*\right)gL^3}{\mu\alpha_f}$ is the nanoparticles concentration Rayleigh number, $Ta=\left(\dfrac{2\Omega^*\rho_f L^2}{\mu}\right)^2$ is the Taylor number, $N_B=\dfrac{\left(\rho c\right)_p}{\left(\rho c\right)_f}\left(\phi_u^*-\phi_l^*\right)$ is the modified particle density increment, $N_A=\dfrac{D_T\Delta T^*}{D_BT_u^*\left(\phi_u^*-\phi_l^*\right)}$ is the modified deffusivity ratio, $Ln=\dfrac{\alpha_f}{D_B}$ is the nanofluids Lewis number, $Df=\dfrac{D_{TC}\Delta C^*}{\alpha_f\Delta T^*}$ is the Dufour parameter, and $Sr=\dfrac{D_{CT}\Delta T^*}{\alpha_f\Delta C^*}$ is the Soret parameter.

The upper boundary is assumed to be nondeformable and the basic state of the nanofluids is quiescent (at rest) with temperature, volumetric fraction of nanoparticles and solute concentration varying in the *z*-direction only that is a solution of the form described by

$$\left(u,v,w,p,T,C,\phi,\psi_z\right)=\left[0,0,0,p_b\left(z\right),T_b\left(z\right),C_b\left(z\right),\phi_b\left(z\right),\psi_{b,z}\right]+\left(u',v',w',p',T',C',\phi',\psi_z'\right). \tag{12}$$

We substitute Eq. (12) into Eqs. (7)-(11) and linearize them by neglecting the products of primed quantities and obtain

$$\nabla\cdot\boldsymbol{u}'=0, \tag{13}$$

$$\frac{1}{\Pr}\frac{\partial\boldsymbol{u}'}{\partial t}=-\nabla p'-\nabla^2\boldsymbol{u}'+\frac{Rs}{Le}C'\hat{\boldsymbol{e}}_z+RaT'\hat{\boldsymbol{e}}_z-Rn_d\phi'\hat{\boldsymbol{e}}_z-\sqrt{Ta}\left(\boldsymbol{u}'\times\hat{\boldsymbol{e}}_z\right), \tag{14}$$

$$\frac{\partial T'}{\partial t}-w'=\nabla^2T'+Df\nabla^2C'+\frac{N_B}{Ln}\left[\frac{\partial T'}{\partial z}-\frac{\partial\phi'}{\partial z}\right]-\frac{2N_AN_B}{Ln}\frac{\partial T'}{\partial z}, \tag{15}$$

$$\frac{\partial C'}{\partial t}-w'=\frac{1}{Le}\nabla^2C'+Sr\nabla^2T', \tag{16}$$

$$\frac{\partial\phi'}{\partial t}+w'=\frac{1}{Ln}\nabla^2\phi'+\frac{N_A}{Ln}\nabla^2T'. \tag{17}$$

Taking curl $\hat{\boldsymbol{e}}_z\cdot$ curl $\left(\nabla\times\right)$ twice of Eq. (14) using the curl identity together with Eq. (13), and retaining the *z*-component, we obtain

$$\left[\frac{1}{\Pr}\frac{\partial}{\partial t}\nabla^2-\nabla^4\right]w'=-\frac{Rs}{Le}\nabla_H^2C'+Ra\nabla_H^2T'-Rn\nabla_H^2\phi'-\sqrt{Ta}\frac{\partial\psi_z'}{\partial z}, \tag{18}$$

$$\left[\frac{1}{\Pr}\frac{\partial}{\partial t}-\nabla^2\right]=\psi_z'+\sqrt{Ta}\frac{\partial w'}{\partial z}, \tag{19}$$

where $\psi_z'=\nabla\times\boldsymbol{u}'$ is the vorticity. The normal mode expansion of the dependent variables is assumed in the form

$$\left(w',T',C',\phi',\psi_z'\right)=\left[W\left(z\right),\Theta\left(z\right),\eta\left(z\right),\Phi\left(z\right),\Psi\left(z\right)\right]e^{\left[i\left(a_xx+a_yy\right)+st\right]}, \tag{20}$$

then, substituting Eq. (20) into Eqs. (15)-(19) and by letting *s* = 0, we obtain

$$\left(D^2-a^2\right)^2W-a^2Ra\Theta-a^2\frac{Rs}{Le}\eta+a^2Rn\Phi-\sqrt{Ta}D\Psi=0, \tag{21}$$

$$W+\left[D^2-a^2-\frac{2N_AN_B}{Ln}D+\frac{N_B}{Ln}D\right]\Theta-\frac{N_B}{Ln}D\Phi+Df\left(D^2-a^2\right)\eta=0, \tag{22}$$

$$W+Sr\left(D^2-a^2\right)\Theta+\frac{1}{Le}\left(D^2-a^2\right)\eta=0, \tag{23}$$

$$W-\frac{N_A}{Ln}\left(D^2-a^2\right)\Theta-\frac{1}{Ln}\left(D^2-a^2\right)\Phi=0, \tag{24}$$

$$\left(D^2-a^2\right)\Psi+\sqrt{Ta}DW=0, \tag{25}$$

where $a=\sqrt{a_x^2+a_y^2}$ is the wavenumber and $D=\frac{d}{dz}$. Following the proportional feedback control [52], the continuously distributed actuators and sensors are arranged in a way that for every sensor, there is an actuator positioned directly beneath it. The determination of a control, $q(t)$ can be accomplished using the proportional-integral-differential (PID) controller of the form

$$q(t)=r+K\left[e(t)\right],e(t)=\hat{m}(t)+m(t), \tag{26}$$

where *r* is the calibration of the control, $e(t)=\hat{m}(t)+m(t)$ is an error or deviation from the measurement, $\hat{m}(t)$, from some desired reference value, $\hat{m}(t)$, while K is the scalar controller gain where $K=K_P+K_D\frac{d}{dt}+K_L\int_0^t dt$, and K_P is the proportional gain, K_D is the differential gain, and K_L is the integral gain. Based on (26), for one sensor plane and proportional feedback control, the actuator modifies the heated surface temperature using a proportional relation between the upper, *z* = 1 and the lower, *z* = 0, thermal boundaries for the perturbation field

$$T'(x,y,0,t)=-KT'(x,y,1,t), \tag{27}$$

where T' denotes the deviation of the temperature of fluid from its conductive state. Eq. (21)-(25) are solved subject to the appropriate boundary conditions. Considering the proportional controller, *K* positioned at the lower boundary of nanofluid layer, we will have

For lower free and upper free boundaries

$$W=D^2W=\Theta(0)+K\Theta(1)=\eta=\Phi=\Psi=D\Psi=0 \text{ at } z=0,$$

$$W=D^2W=D\Theta=\eta=\Phi=D\Psi=0 \text{ at } z=0. \tag{28}$$

For lower rigid and upper free boundaries

$$W=DW=\Theta(0)+K\Theta(1)=\eta=\Phi=\Psi=D\Psi=0 \text{ at } z=0.$$

$$W=D^2W=D\Theta=\eta=\Phi=D\Psi=0 \text{ at } z=0. \tag{29}$$

For lower rigid and upper rigid boundaries

$$W=DW=\Theta(0)+K\Theta(1)=\eta=\Phi=\Psi=D\Psi=0 \text{ at } z=0.$$

$$W = DW = D\Theta = \eta = \Phi = D\Psi = 0 \text{ at } z = 0. \tag{30}$$

The Galerkin-type weighted residuals method is applied to find the approximate solution to the system. The variables are written in a series of basis functions

$$W = \sum_{i=1}^{n} A_i W_i, \Theta = \sum_{i=1}^{n} B_i \Theta_i, \eta = \sum_{i=1}^{n} C_i \eta_i, \Phi = \sum_{i=1}^{n} D_i \Phi_i, \Psi = \sum_{i=1}^{n} E_i \Psi_i, \tag{31}$$

where A_i, B_i, C_i, D_i and E_i are constants and the basis functions $W_i, \Theta_i, \eta_i, \Phi_i$ and Ψ_i where $i = 1, 2, 3, \ldots,$ will be chosen corresponding to the free-free, rigid-free and rigid-rigid lower-upper boundary conditions

$$\begin{aligned} &W_i = \sin(z\pi), \Theta_i = z(2-z), \eta_i = \Phi_i = \sin(z\pi), \Psi_i = z\left(3z - 2z^2\right), \\ &W_i = z^2(1-z)(3-2z), \Theta_i = z(2-z), \eta_i = \Phi_i = z(z-1), \Psi_i = z\left(3z - 2z^2\right), \\ &W_i = z^2(1-z)^2, \Theta_i = z(2-z), \eta_i = \Phi_i = z(z-1), \Psi_i = z\left(3z - 2z^2\right). \end{aligned} \tag{32}$$

Substitute (31) into Eqs. (21)-(25) and make the expressions on the left-hand sides of those equations (the residuals) orthogonal to the trial functions, thereby obtaining a system of *5N* linear algebraic equations in the *5N* unknowns. The vanishing of the determinant of the coefficients produces the eigenvalue equation for the system. One can regard *Ra* as the eigenvalue and thus *Ra* is found in terms of the other parameters.

3. Results and Discussion

The onset of Rayleigh-Benard convection in rotating horizontal layer of nanofluids with feedback control subjected to double-diffusive coefficients is developed. The model used for nanofluids includes the combination effects of Brownian motion and thermophoresis. Linear stability analysis is employed and the resulting eigenvalue, Rayleigh number, *Ra* parameter used to show the strength of buoyancy force, is extracted using the Galerkin technique. Three types of lower-upper boundary conditions are considered for free-free, rigid-free and rigid-rigid. The values of the chosen parameters are following the range values proposed by Chand and Rana [25] that are: *Rs* = 200, *Rn* = 2, *Sr* = 0.2, *Df* = 0.2, N_A = 5, N_B = 0.0001, *Le* = 0.2 and *Ln* = 200. As for Taylor number and feedback control parameters are fixed at *Ta* = 1000 and *K* = 3. The obtained results are presented graphically to illustrate the impact of various parameters on Rayleigh number, *Ra* versus wavenumber, *a* in Figures 2-8. Further, the critical Rayleigh number, Ra_c is plotted in Figures 9 and 10.

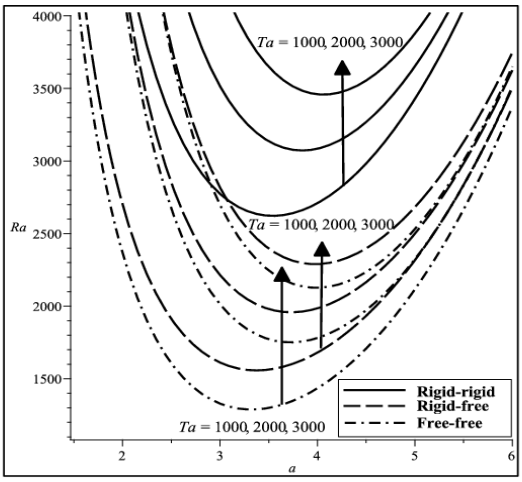

Fig. 2. *Ra* versus *a* for various values of *Ta*

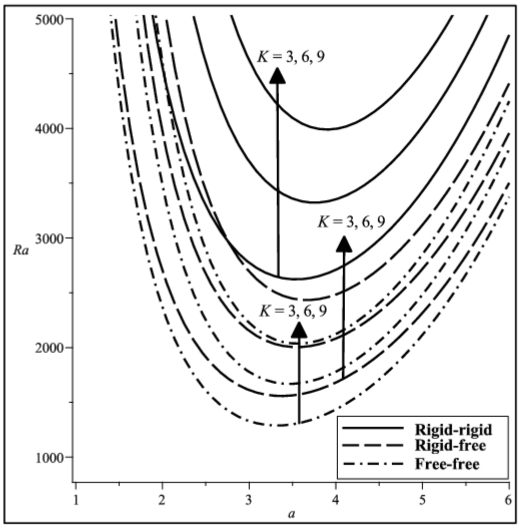

Fig. 3. *Ra* versus *a* for various values of *K*

Test computations have been performed and we compared our results with Chandrasekhar [28] and Yadav *et al.,* [43] for regular fluids in the absence of feedback control, *K* and double-diffusive coefficients in Table 1. As can be seen in the table, our results are in good agreement with those reported by Chandrasekhar [28] and Yadav *et al.,* [43], hence verify the accuracy of our analysis.

Table 1
Comparison of critical Rayleigh number, Ra_c for different values of *Ta* with Chandrasekhar [28] and Yadav *et al.,* [43] in rigid-rigid boundary conditions for regular fluids

Taylor number, *Ta*	Chandrasekhar [28]	Yadav *et al.,* [43]	Present study
0	1707.80	1707.83	1707.76
10	1713.00	1713.00	1713.00
100	1756.60	1756.41	1756.33
500	1940.30	1940.26	1940.08
1000	2151.70	2151.39	2151.32
2000	2530.50	2530.18	2530.10
5000	3469.20	3468.58	3468.46
10000	4713.10	4712.13	4712.05

Figures 2 and 3 shows increasing values of Taylor number, *Ta* = 1000, 2000, 3000 and feedback control, *K* = 3, 6, 9 to the plot of *Ra* versus *a*. Both figures shift upwards as their values are increased for various types of boundary conditions. In Figure 2, the plot of increasing values of *Ta* indicates that the Coriolis force due to the rotation inhibits the convection where the nanofluids move to the horizontal plane with higher velocity due to the vorticity introduced by the rotation mechanism. Meanwhile, the velocity of the nanofluids in the vertical plane is reduced, thereby minimizing the amount of thermal convection [28]. As for Figure 3, the plot of increasing values of *K* indicates that an active controller is capable to delay the convection where the sensors detect the departure of the nanofluids from its conductive state and direct the actuator to suppress any disturbances [53].

In Figures 4 and 5 show the plot of the two important effects of interdiffusion, namely Dufour and Soret effects that arise due to the combination of concentration gradient and temperature. The plot of *Ra* represents the obtained results for both effects under free-free, rigid-free and rigid-rigid boundary conditions. Figure 4 indicates the plot of increasing values of Dufour effect *Df* = 0.2, 0.4, 0.6 on the system is found to stabilize the system. The reason is because the energy flux becomes more significant and increases the solute concentration by driving mass gradient within the system, thus delay the thermal convection. Meanwhile, Figure 5 represent the plot of increasing values of Soret, *Sr* = 0.2, 0.4, 0.6 on the system and destabilized the system. The increase in *Sr* increases the temperature flux, which contributes to the acceleration of Rayleigh-Benard convection.

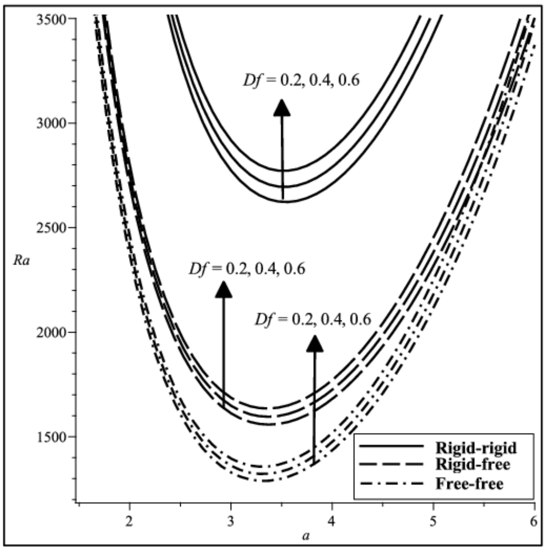

Fig. 4. *Ra* versus *a* for various values of *Df*

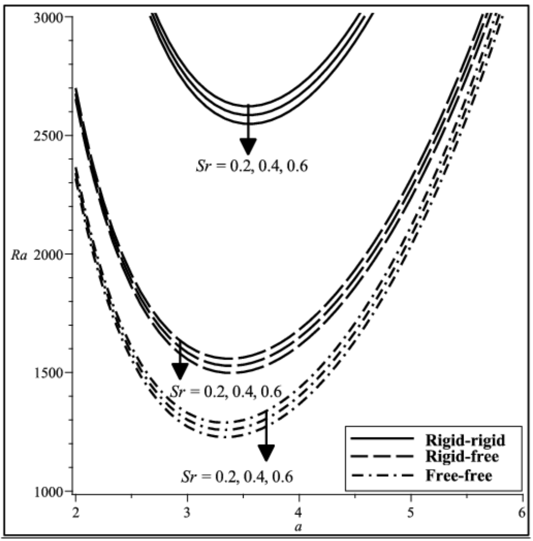

Fig. 5. *Ra* versus *a* for various values of *Sr*

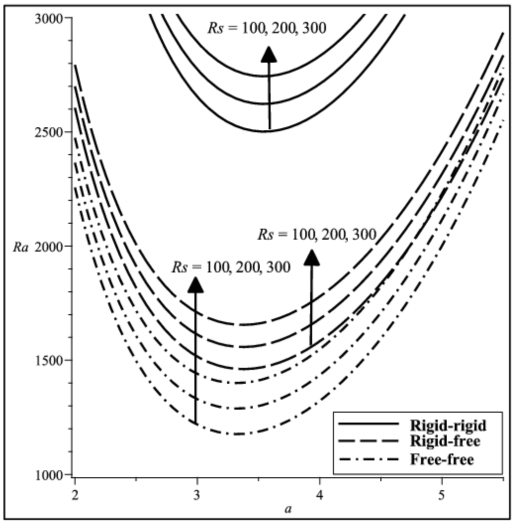

Fig. 6. *Ra* versus *a* for various values of *Rs*

The influence of solutal Ryleigh number, *Rs* and nanoparticles concentration Rayleigh number, *Rn* on the onset of Rayleigh-Benard convection in nanofluids layer are illustrated in Figures 6 and 7. The increase values of *Rs* = 100, 200, 300 plotted in Figure 6 increase the Rayleigh number, *Ra* spontaneously, thus stabilized the system. Therefore, the onset of convection can be postponed due to the greater amount of solute concentration than that of the solvent, leading to a decrease in the overall temperature within the system. As for the Figure 7, shows the *Ra* decreases rapidly as the values of *Rn* = 1, 2, 3 increased and destabilized the system. The increased in *Rn* increase the volumetric fraction of nanoparticles that results in a combination of Brownian motion and thermophoresis diffusion within the system where it is noted that Brownian motion and thermophoresis are known as the primary mechanism to enhance the thermal instability.

The linear instability thresholds on the Rayleigh number Ra of the modified diffusivity ratio N_A are plotted in Figure 8 for N_A = 2, 6 and 10. The values of Rayleigh number Ra decreases only slightly with increasing of the modified diffusivity ratio N_A under three lower-upper boundary conditions. This phenomenon occurs because the parameter N_A is directly proportional to the thermophoresis diffusion coefficient D_T, where and increasing D_T indicates an instability in the temperature difference within the nanofluids layer. Therefore, as N_A increases, the thermal instability also increase and destabilizes the system.

Figures 9 and 10 show the variation of critical Rayleigh number, Ra_c in the function of Taylor number, *Ta* for different values of Soret parameter *Sr* = 0.2 and 0.8 and nanoparticles concentration Rayleigh number, *Rn* = 1, 3 respectively. From these figures, it is observed that the critical Rayleigh number Ra_c increases with the increase in Taylor number *Ta* representing that its effect delays the onset of convection in the existence of destabilizing effects of Soret parameter *Sr* and nanoparticles concentration Rayleigh number *Rn*.

Finally, the influence of Dufour effect, *Df* = 0.4, 0.8 is the reciprocal phenomenon to the Soret effect, *Sr* as shown in Figure 11. The critical Rayleigh number Ra_c for the various values of Soret parameter *Sr* increases slightly with the increase in the values of Dufour parameter *Df*. Therefore, the effect of increasing Dufour parameter *Df* effect has been observed to delay the convection and stabilized the system.

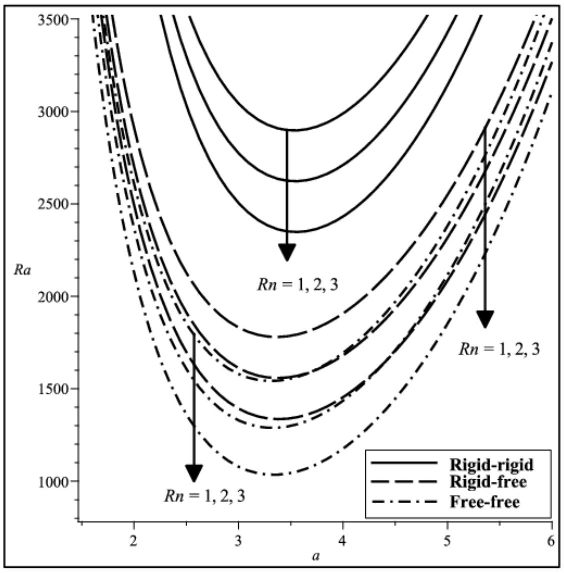

Fig. 7. *Ra* versus *a* for various values of *Rn*

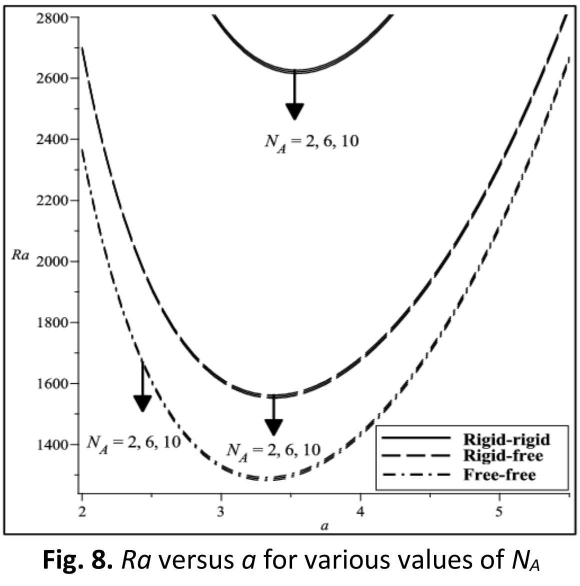

Fig. 8. *Ra* versus *a* for various values of N_A

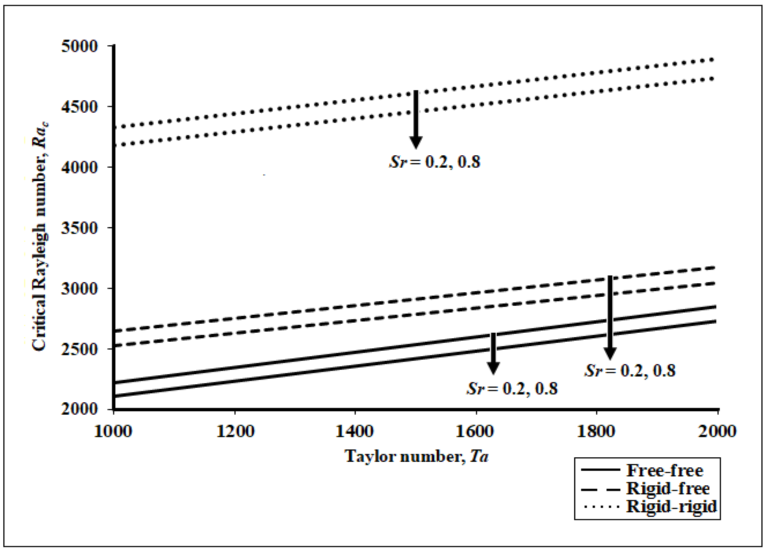

Fig. 9. Plot of Ra_c in the function of Ta for selected values of Sr

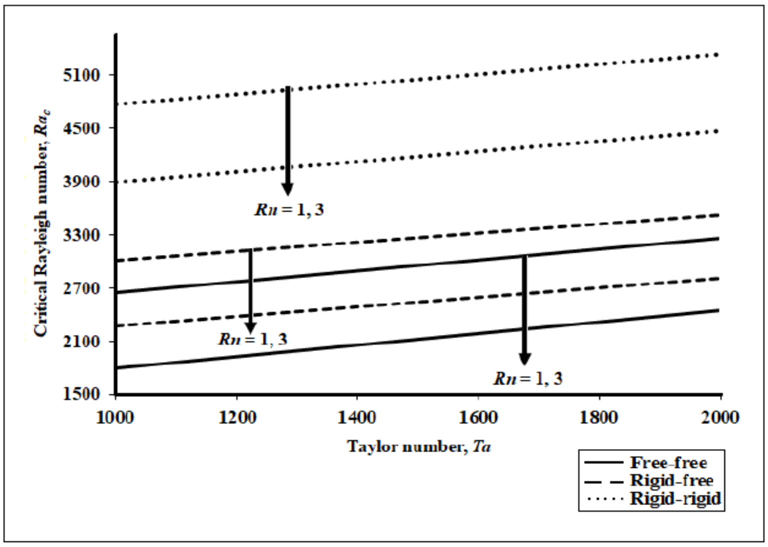

Fig. 10. Plot of Ra_c in the function of Ta for selected values of Rn

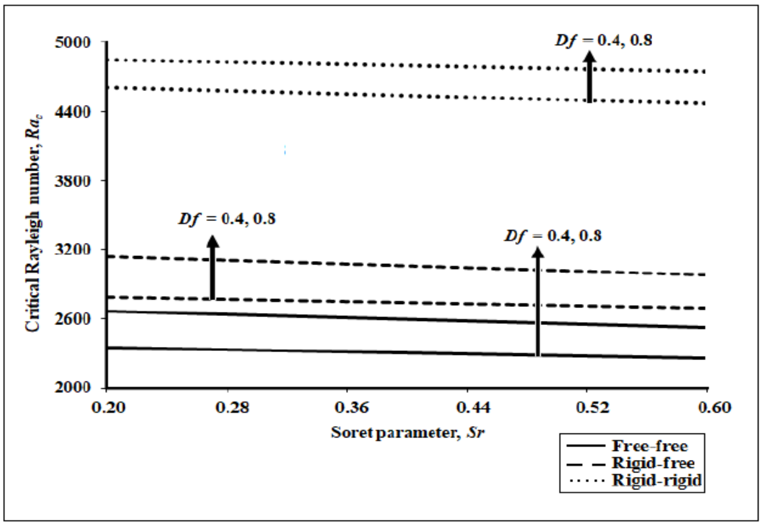

Fig. 11. Plot of Ra_c in the function of Sr for selected values of Df

4. Conclusions

The onset of Rayleigh-Benard convection in rotating nanofluids layer associated with feedback control subjected to double-diffusive coefficients has been studied theoretically for free-free, rigid-free and rigid-rigid. The employed model of nanofluids combined the effect of Brownian motion and thermophoresis. Linear stability analysis is used where the eigenvalue solution is obtained from normal mode analysis. Then, the eigenvalue solution is solved by applying Galerkin method and computed numerically through Maple software.

The implementation of the effects of feedback control K, Taylor number Ta, Dufour parameter Df, and solutal Rayleigh number Rs are found to produce a lag on Rayleigh-Benard convection in nanofluids layer system when the values of these parameters are increased thus stabilized the system. The opposite outcome can be seen in the implementation effects of Soret parameter Sr, nanoparticles concentration Rayleigh number Rn and modified diffusivity ratio N_A advance the onset of Rayleigh-Benard convection in nanofluids layer system when the values of these parameters are increased thus destabilized the system. The implication of modified particle density increment N_B on the onset of Rayleigh-Benard convective instability in nanofluids layer is too small and can be neglected [43].

Based on the obtained results, the system of lower-upper free-free, rigid-free and rigid-rigid boundary conditions are found to be the most effectively stable with rigid-rigid boundaries whereas free-free is found to be the most unstable boundaries in the system.

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Conflict of interest
The authors would like to declare that there is no conflict of interest.

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