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Unsteady Natural Convection in a Porous Square Cavity Saturated by Nanofluid Using Buongiorno Model: Variable Permeability Effect on Homogeneous Porous Medium

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ABSTRACT

This paper investigated numerically a natural convection in a porous cavity saturated by nanofluide. The left and right wall of the cavity are maintained at the hot-cold temperature respectively, the other walls are adiabatic. The two-phase Buongiorno model has been adopted to take account Brownian and thermophoretic diffusion in order to demonstrate the spatial distribution of the local nanoparticles concentration. After following the temporal evolution of the different structures ($0<\tau<5$), Numerical simulations are performed to explore the effect of density buoyancy ($10^4<Ra<10^6$), the permeability of the homogeneous porous medium ($10^{-5}<Da<10^{-2}$), on the hydrodynamic, thermal and mass behavior. An original motivation was also introduced in our work to examine the effect of linearly variable permeability along the opposite direction of the cavity by varying the initial Darcy number from ($10^{-5}<Da<10^{-2}$) and fixing the final Darcy number Daf $=10^{-5}$. The dimensionless partial differential equations are solved using the finite element method. The effects of the governing parameters on heat transfer are analyzed. Results indicate that the stationary regime is formed after the unsteady regime at dimensionless time $\tau = 0.3$. The movement of nanofluide is strongly influenced by thermal buoyancy forces and depends on the Darcy number, heat transfer is accentuated for the homogenous medium compared to this one with variable permeability. It is found that the convective flow in a homogeneous porous medium is considerably affected by the variation of permeability and consequently the heat transfer is reduced.

1. Introduction

The research activity in recent year on heat transfer is growing exponentially, in order to improve heat exchanger's energy efficiency in industry and develop efficient high- performance fluids composed of nanoparticles dispersed in the base fluid and which characterized by a high thermal conductivity and it is able to improve the heat transfer performance [1], the nanofluids are used in various applications for cooling systems, oil industry and in geothermal energy.

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Several authors have concentrated their efforts to quantify the parameters influencing heat transfer rate and physical phenomena induced by the nanofluids [2-5]. The incorporation of nanoparticles in pure water modifies the structure flow for low values of the Rayleigh number Boutra *et al*., [6], this nanoparticles are distributed due to inertia and buoyancy forces, Lahlou *et al*., [7 - 9]. Bhuiyana *et al*., [10] have observed that the heat transfer enhancement is strongly depends on the type of nanofluids, while the choice of hybrid nanofluids has been discussed by Goudarzi *at al.,* [11-17].

Recently, the modeling of the free convection by using nanofluids in triangular and inclined square cavities, Sheremet *et al*., [18, 19]. Natural convection flow suspended micropolar Casson fluid over a solid sphere has been studied by Alkasasbeh while [20], Bouras *et al.,* [21] have choose a horizontal annulus between an internal heated plane and an external cold half-elliptical for analyze the heat transfer rate. Alsabery *et al*., [22, 23] have revealed that the thermal conductivity and the size of the solid block in a partially heated and cooled cavity control perfectly the heat transfer.

In addition, Bouafia *et al*., [24] have added the effect of Rayleigh and Darcy numbers, concentration of nanoparticles except for the wall thickness of the solid portion in a square porous cavity having solid wall of finite thickness. Also, the fluid flow intensity increased. While, it decreased when the walls thickness increased Hussein *et al.,* [25].

To better describe the movement of nanoparticles; two parameters were introduced in the Buongiorno model, the Brownian motion and the thermophoresis parameter for nanofluids flow on a thin needle [26-28], this two-phase model was adopted by Hoghoughi *et al*., [29]. Mikhail *et al*., [30] added the dispersion effect and the mixed convection parameter was injected by Leony *et al*., [31]. The Brownian motion remains an imperative mechanism that contributes to the heat transfer enhancement Convectively, Zokri *et al*., [32]. The Buongiorno's nanofluid model has been used by Dero *et al.,* [33].

The importance of the application of nanofluide in porous medium received a great attention by many authors such heat exchangers [34], electronic cooling and solar system, according to the studies the use of both and nanofluide and porous media can enhance the efficiency of typical thermal systems, a mixed convection flow in a boundary layer saturated by a nanofluid and lodged in a porous medium have been studied by Abu Bakar *et al.,* [35]. , Douha *et al.,* [36] have proved that an increase in the heat transfer coefficients is observed with the raise of the porous layer permeability.

The behavior of saturated nanoliquids with different sizes in a porous medium has been studied by Lakshmi *et al*., [37], however Bourantas *et al*., [38] are interested to the effect of the porous medium in on the nanofluidic system and this effect has been taken account in the Buongiorno model, Sheremet and Pop [39]. We refer to the study of Sheikhzadeh, and Nazari [40].

Heat exchange by natural convection in porous medium remains a very attractive field for many researchers, generally this medium is an aquifer (geological formation) that govern the underground flow which is composed of multiple different layers with the permeability varies in the direction flow. The literature about the studies of heat and mass transfer induced by natural convection in an anisotropic porous media are limited compared to those of the isotropic medium [41 -44]. Porous anisotropic medium in natural convection with the Darcy-Brinkman formulation has been studied par by Bennacer *et al*., [45], they observed that a mass transfer maximum was obtained for a critical anisotropic permeability ratio, this ratio affects the convective flow along a vertical plate in a porous medium with isotropic permeability Degan [46]. Homogeneous porous media saturated by a binary fluid was chosen by Makhloufi *et al*., [47-51] to reveal the influence of anisotropic permeability rate on the decrease of heat transfers and the existence of a fully and the moderately convective flow. Ould-Amer *et al*., [52] have showed that convective flow is intensive in the first layer (of higher permeability) in a porous medium composed of three homogeneous layers saturated by a single fluid, on the other hand Chamkha, *et al*., [53] confirm that with the aid of a nanofluid enhances heat transfer even at a low permeable porous media, this enhancement can be also fulfilled by hybrid nanofluids for flow with anisotropic permeability at high Rayleigh number Bibi *et al*., [54, 55].

In particular, the theory of underground hydraulics is based on Darcy's law, this medium are highly heterogeneous, and uncertain, because it is impossible to measure their characteristics deterministically and they can be anisotropic [56].

Previous studies have focused on the application of Darcy's law for underground flow in any direction in space. The porous medium are considered homogeneous in the benchmark cavity. In the present investigation the porous medium is homogeneous with variable permeability. In fact, this law is quasi- linear, with permeability decreasing along the negative vertical direction.

2. Geometric Description

Laminar natural convection in square porous cavity saturated by nanofluids is numerically simulated using the finite element method, the height of the cavity is denoted by H. The problem is considered bidimensional, the left and right wall is maintained at constant temperature T_h and T_c ($T_h > T_c$) respectively, the other walls are adiabatic.

The thermophysical properties of water and nanofluids are at 25°c (see Table 1). The configuration of the cavity and the flow is shown in Figure.1

Table 1
Thermophysical properties of water

Physical property	Water
C_p (J/kg)	4179
ρ (kg /m^3)	997.1
K (W/m k)	0.613
B (k^{-1})	21e-5
μ (kg/m s)	8.55e-4

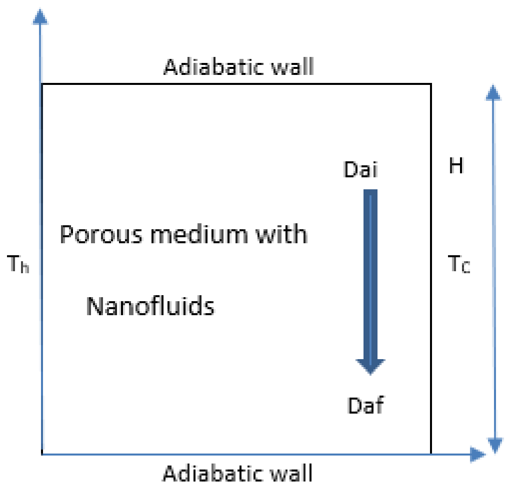


Fig. 1. Physical problem studied

3. Mathematical Model

The two -dimensional equations governing the stationary flow of nanofluids in natural convection inside a square cavity using Buongiorno mathematical model are described as follows.

$$\frac{\partial \rho}{\partial t}+\rho\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0 \tag{1}$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial^2 x}+\frac{\partial^2 u}{\partial^2 y}\right)-\frac{\mu}{K}u \tag{2}$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial^2 x}+\frac{\partial^2 v}{\partial^2 y}\right)-\frac{\mu}{K}v+\left[(1-C_c)\rho_f(T-T_c)\beta_f-(C-C_c)(\rho_s-\rho_f)\right]g \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\delta\left\{D_B\left(\frac{\partial T}{\partial x}\frac{\partial C}{\partial x}+\frac{\partial T}{\partial y}\frac{\partial C}{\partial y}\right)+\frac{D_T}{D_C}\left[\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2\right]\right\} \tag{4}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+\frac{D_B D_T}{D_C}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{5}$$

$$D_B=\frac{K_B T_0}{3\pi d_b \mu} \tag{6}$$

$$D_T=\frac{\beta\mu C_0}{\rho_f} \tag{7}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{8}$$

For an incompressible flow, the density of the fluid is constant as a function of the spatial position (ρ = constant), the mass equation simply becomes:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{9}$$

To generalize the phenomenon, a set of dimensionless variables has been introduced. It is defined as follows:

$$(X,Y)=\frac{(x,y)}{H},(U,Y)=\frac{H(u,v)}{\alpha_f},\theta=\frac{T-T_c}{T_h-T_c},\varphi=\frac{C-C_C}{C_h-C_c},P=\frac{pH^2}{\rho_f\alpha^2{}_f} \tag{10}$$

By introducing these dimensionless variables into the dimensional equations, we obtain:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{11}$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)-\frac{\Pr}{Da}U \tag{12}$$

$$\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)-\frac{\Pr}{Da}V+Ra\Pr(\theta-Nr\varphi) \tag{13}$$

$$\frac{\partial \theta}{\partial t}+U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}+Nb\left(\frac{\partial \theta}{\partial X}\frac{\partial \varphi}{\partial X}+\frac{\partial \theta}{\partial Y}\frac{\partial \varphi}{\partial Y}\right)+Nt\left[\left(\frac{\partial \theta}{\partial X}\right)^2+\left(\frac{\partial \theta}{\partial Y}\right)^2\right] \tag{14}$$

$$\frac{\partial \varphi}{\partial t}+U\frac{\partial \varphi}{\partial X}+V\frac{\partial \varphi}{\partial Y}=\frac{1}{Le}\left(\frac{\partial^2 \varphi}{\partial X^2}+\frac{\partial^2 \varphi}{\partial Y^2}\right)+\frac{Nt}{LeNb}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{15}$$

New parameters appear in the dimensionless equations Prandtl number, Darcy number, Rayleigh number, buoyancy ratio, Brownian motion, thermophoresis and Lewis number.

$$\Pr=\frac{\mu Cp}{k} \tag{16}$$

$$Da=\frac{K}{H^2} \tag{17}$$

$$Ra=\frac{(1-C_c)\beta_f g(T_h-T_c)H^3}{\nu_f\alpha_f} \tag{18}$$

$$Nr=\frac{(C_h-C_c)(\rho_s-\rho_f)}{(1-C_c)\beta_f\rho_f(T_h-T_c)} \tag{19}$$

$$NB=\frac{(C_h-C_c)D_B(\rho Cp)_s}{\alpha_f(\rho Cp)_f} \tag{20}$$

$$NT=\frac{D_T}{T_c}\frac{(\rho Cp)_s}{(\rho Cp)_f}\frac{(T_h-T_c)}{\alpha_f} \tag{21}$$

$$Le=\frac{\alpha_f}{D_B} \tag{22}$$

3.1 Boundary Condition

The proposed problem is provided with boundary condition in the dimensionless form of the cavity walls:

At τ = 0 we have in the whole domain $U = 0, V = 0, \theta = 1$ and $\varphi = 1$ for $\tau\rangle = 0$

-On the left wall: $U = 0, V = 0, \theta = 1$ and $N_B\, \partial\theta/\partial X + N_T \partial\varphi/\partial X = 0$

-On the right wall: $U = 0, V = 0, \theta = 0$ and $N_B\, \partial\theta/\partial X + N_T\, \partial\varphi/\partial X = 0$

- On the top wall: $\partial U/\partial Y = 0, \partial V/\partial Y = 0, \partial\theta/\partial Y = 0$ and $\partial\varphi/\partial Y = 0$

- On the bottom wall: $\partial U/\partial Y = 0, \partial V/\partial Y = 0, \partial\theta/\partial Y = 0$ and $\partial\varphi/\partial Y = 0$

3.2 The Average Nusselt Number

The heat transfer is characterizes by dimensionless parameter which is Nusselt number comparing heat transfer rate by convection and conduction. The local and average Nusselt number on the heated wall is given by:

$$Nu = \frac{hL_c}{k} \tag{23}$$

h, the heat transfer coefficient is given by:

$$h = \frac{q}{T_h - T_c} \tag{24}$$

$$Nu = -\left.\frac{\partial\theta}{\partial X}\right|_{X=0} \tag{25}$$

$$\overline{Nu} = \frac{1}{L_c}\int_0^{L_c} Nu\,dY \tag{26}$$

Where L_c the total chord length of the hot wall.

4. Numerical Method

The finite element method based on the Galerkin discretization scheme was used to solve the differential equations with boundary conditions. A triangular mesh appears more adequate (see Figure 2). The convergence criterion is of the order of 1e-6 for each variable. The analysis of the different properties values (stream function, temperature, concentration and average Nusselt number) are obtained for several meshes with a total number of elements of 51438 (see Table 2).

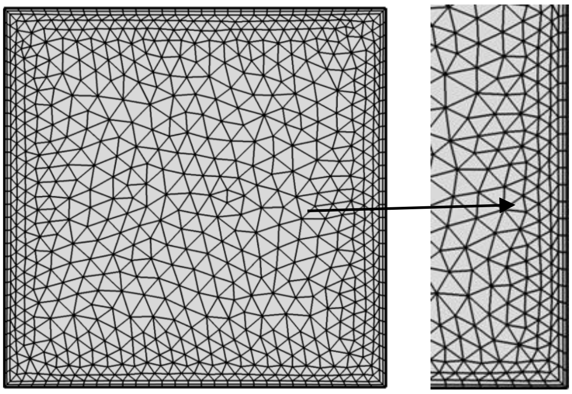


Fig. 2. Computation domain mesh

Table 2
Mesh grid analysis for the present configuration

Elements	Time	Temperature	Concentration	Stream function	Nusselt
7728	285 s	0.500246	1.001061	-4.076689	2.515511
15298	561 s	0.500247	0.998697	-4.077702	2.515782
25154	945 s	0.500246	1.000351	-4.078194	2.515860
51438	8188 s	0.500248	0.999748	-4.078669	2.516023
86560	11341 s	0.500248	0.999840	-4.078874	2.516102

5. Results and Discussion

An analysis of the different parameters was performed to understand the physical phenomenon, numerical calculations were obtained by fixing the height of the cavity (H = 2m), the Prandtl number of the base fluid (Pr = 5. 82), the Rayleigh number (Ra =10^5), the Darcy number (Da = 10^{-2}), the Lewis number (Le = 1), the ratio of buoyancy forces, the Brownian motion and the thermophoresis parameter (Nr = Nb = Nt= 0.1). On the other hand, we varied the$(10^4 \leq \mathrm{Ra} \leq 10^6)$, $(10^{-5} \leq \mathrm{Da} \leq 10^{-2})$, initial Darcy number $(10^{-5} \leq \mathrm{Dai} \leq 10^{-2})$ and final Darcy number Daf = 10^{-5}. The flow is supposed to be laminar and incompressible.

5.1 Temporal Evolution of the Mass and The Heat Structures

In order to better describe the variables of dynamic and thermal fields, the unsteady regime was chosen in our configuration by given the distribution of the streamlines, isotherms and iso-concentrations with dimensionless time $0 \leq \tau \leq 0.30$. We reveal the following results in Figure3.

At the initial time $\tau = 0$, a weak streamline is observed, isotherms and nanoparticles are distributed uniformly in the cavity, the buoyancy force does not stimulate the nanofluids flow. We result that the heat exchange is insignificant.

From the dimensionless time $\tau = 0.01$ to 0.05 a recirculation cell is formed near the left hot wall in the clockwise direction and continue to increase in intensity. In fact, a heat flux is created by the nanofluids flow moving away from the left hot wall towards the right cold wall.

At the moment $\tau = 0.01$, the isotherms begin to extend near the left wall, where the temperature gradient becomes important and the iso-concentrations deform from this moment.

From $\tau = 0.05$, the isotherms close the left vertical wall in addition an important accumulation of iso-concentrations is noticed at the bottom of the cavity near the left hot wall. We can interpret this alteration by a high concentration of nanoparticles due to the buoyancy forces which are submissive to an important temperature gradient.

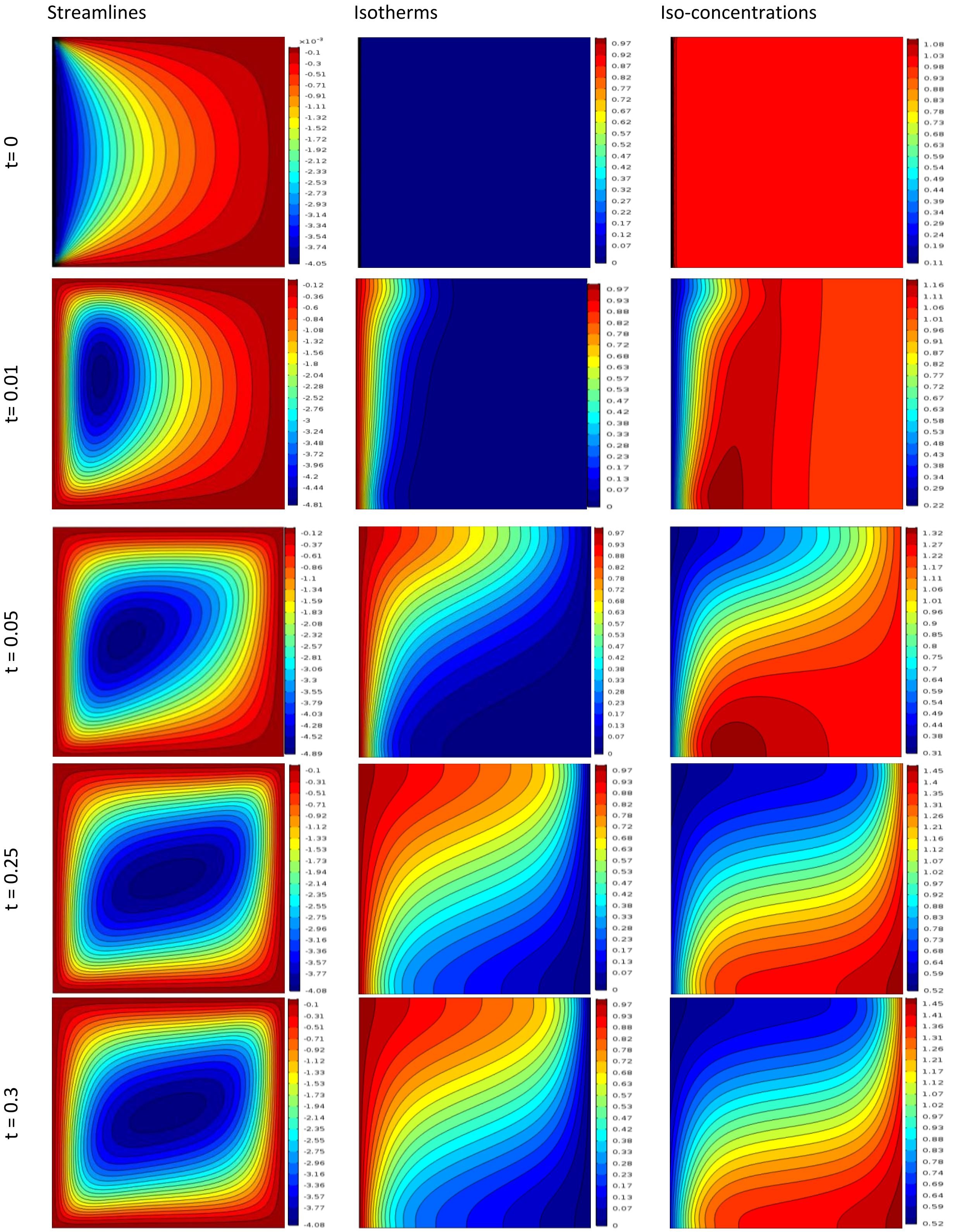
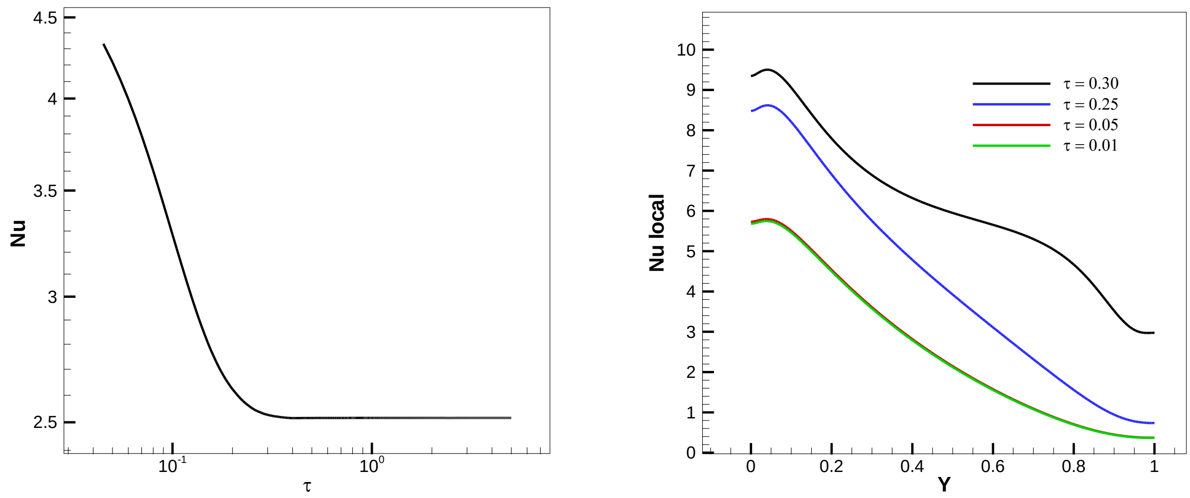


Fig. 3. Temporal evolution of Streamlines, isotherms and iso-concentrations for different instants at Ra = 10^5, Da =10^{-3}, $N_B = N_T = N_r = Le = 0.1$.

Between a time interval τ = (0.25, 0.30), the recirculation cell occupies the middle of the cavity and keeps the same shape. It is observed that the isotherms do not change shape, even for iso-concentrations which become tight next to the right cold wall. Therefore, it is concluded that the flow becomes stable and the heat transfer reaches it's a maximum. The dimensionless time necessary to reach different variables dynamic and thermal field is about τ = 0.30.A steady state regime are formed after an unsteady flow. This is reflected in the flow stability and the dominance of convection heat transfer.

5.2 Calculation of the Average and Local Nusselt Number

The evolution of the average and local Nusselt number as a function of the dimensionless time τ (a) and along the hot wall with Y (b) is shown in Figure 4.



(a) (b)

Fig. 4. Variations of unsteady average Nusselt number with τ (a), local Nusselt number with Y (b)

We notice in Figure 4 (a) that at the initial dimensionless time τ = 0, There is a strong decrease in of temporal average Nusselt number due to the low gradient temperature, after it's does not change in value regardless of the increase time factor at τ =0.3 et becomes stable, which means that the dynamic and thermal field tend to reach the steady state therefore, we concluded that the steady state is established.

In the figure 4 (b) the evolution of the local Nusselt number along the hot wall for various dimensionless time is shown, at the dimensionless time 0.01≤ τ ≤ 0, 30. When the time goes from τ = 0, 05 to 0.30, we observe obvious that the local Nusselt number increases with the dimensionless time. At time τ = 0.30, we notice that high values of the local

Nusselt number are obtained at the hot wall a y = 0, we conclude that the heat transfer is more intense (Nu_{loc} = 9.5), after Nu_{loc} decreases rapidly at the top hot wall with a value of 2.8. We consider that the convective heat transfer is more significant due to the incorporation of nanoparticles in the base fluid. As a result, the convective heat transfer decreases until it reaches the values of the steady state.

5.3 Stationary Flow of Natural Convection in a Homogeneous Porous Cavity Saturated by a Nanofluid
5.3.1 Buoyancy effect

The impact of the Rayleigh number on the distribution of the streamlines, isotherms and iso-concentrations is shown in Figure 5. For the streamlines, we observed a single circulation cell which intensifies with increasing Rayleigh number (10^4≤ Ra ≤ 10^6), the convective flow induced by the

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thermal buoyancy forces is very intense. At value of Rayleigh number Ra=10^4, the isotherms and iso-concentrations are almost parallel to the vertical wall and its change shape when the Rayleigh number increases (Ra=10^5).

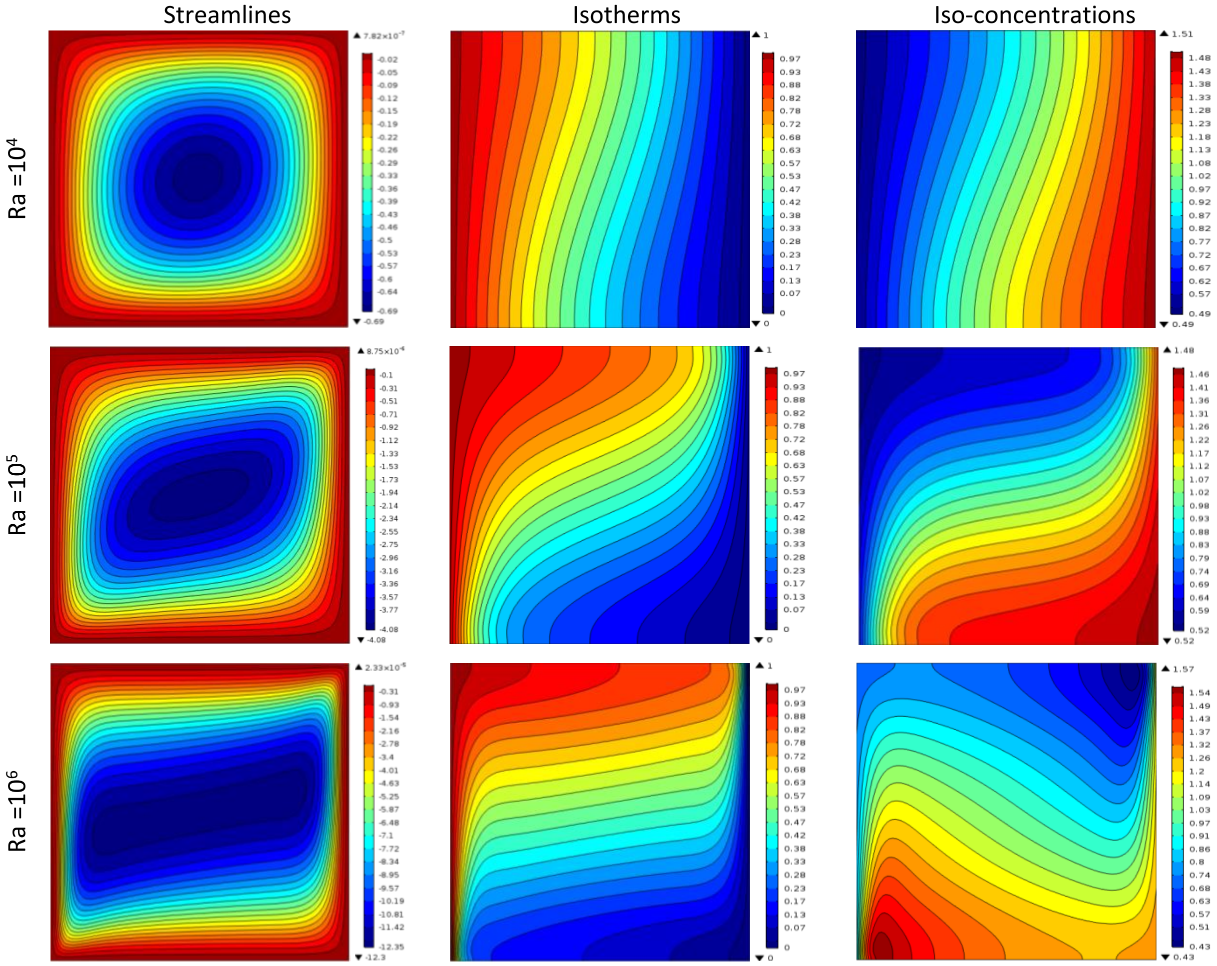


Fig. 5. Streamlines, isotherms and iso-concentrations for various Rayleigh number at Da =10^{-3} , N_B = N_T = N_r = Le = 0.1.

At Ra=10^5, the gradient of iso-concentrations becomes important near the cold right wall and at the bottom cavity. For a high value of the Rayleigh number Ra=10^6, the space between the isotherms is reduced, from this value we conclude that the heat transfer is significant. A stratification zone is detected at the bottom cavity near the left wall, which indicates that the nanoparticles are extremely concentrated in this region, we can observe that the iso-concentrations are reduced in value. The convective flow is strongly influenced by thermal buoyancy forces which contribute to enhance heat transfer.

5.3.2 Effect of the homogenous porous medium with isotropic permeability

The influence of Darcy number on the streamlines, isotherms and iso-concentrations is demonstrated in the Figure 6. We observe that the circulation cell decreases in size when the Darcy number decreases (Da = 10^{-5}). The flow is weakened for low value of the permeability. As a result, the flow is conductive. At high value of the Darcy number (Da=10^{-2}), the stream function increases from 0.07 to 8.62, which indicates that the flow circulation in the cavity is accentuated due to the increase in permeability which causes Nanofluids flow.

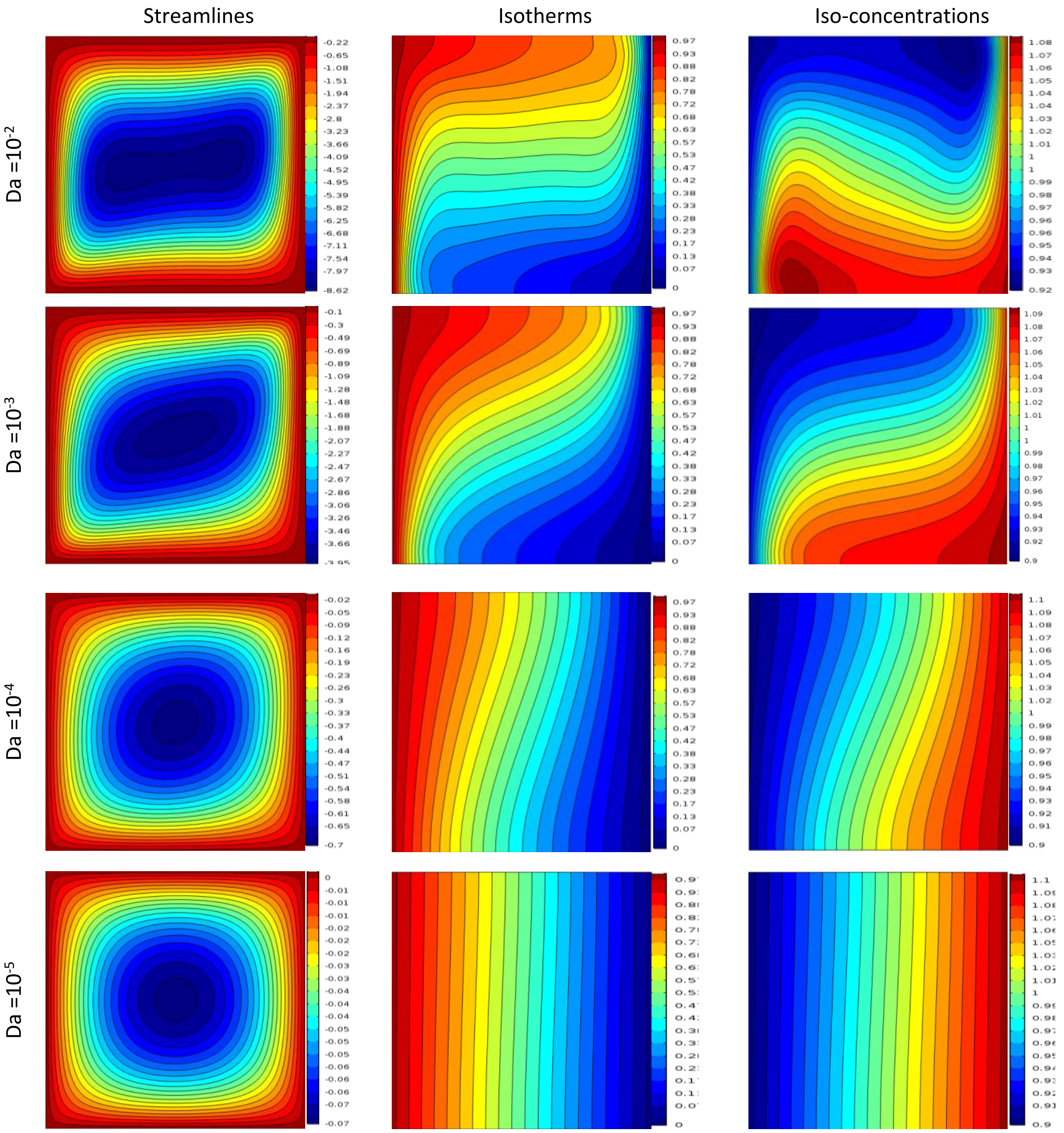


Fig.6. Streamlines, isotherms and iso-concentrations for various Darcy number at Ra $=10^5$, $N_B = N_T = N_r = Le = 0.1$

The isotherms and iso-concentrations are parallel to the vertical walls at value of Da $=10^{-5}$, which indicate that the heat transfer is dominated by pure conduction. The temperature and concentration fields are sensitive to the variation of the Darcy number between Da=10^{-4} and 10^{-2}. At a maximum value of Darcy number (Da $=10^{-2}$), the temperature and concentration gradients developed are important near the left hot wall and at the bottom cavity respectively. It is clear that the nanofluids through the pores without difficulty.

5.4 Natural Convection in a Porous Cavity: Variable Permeability Effect

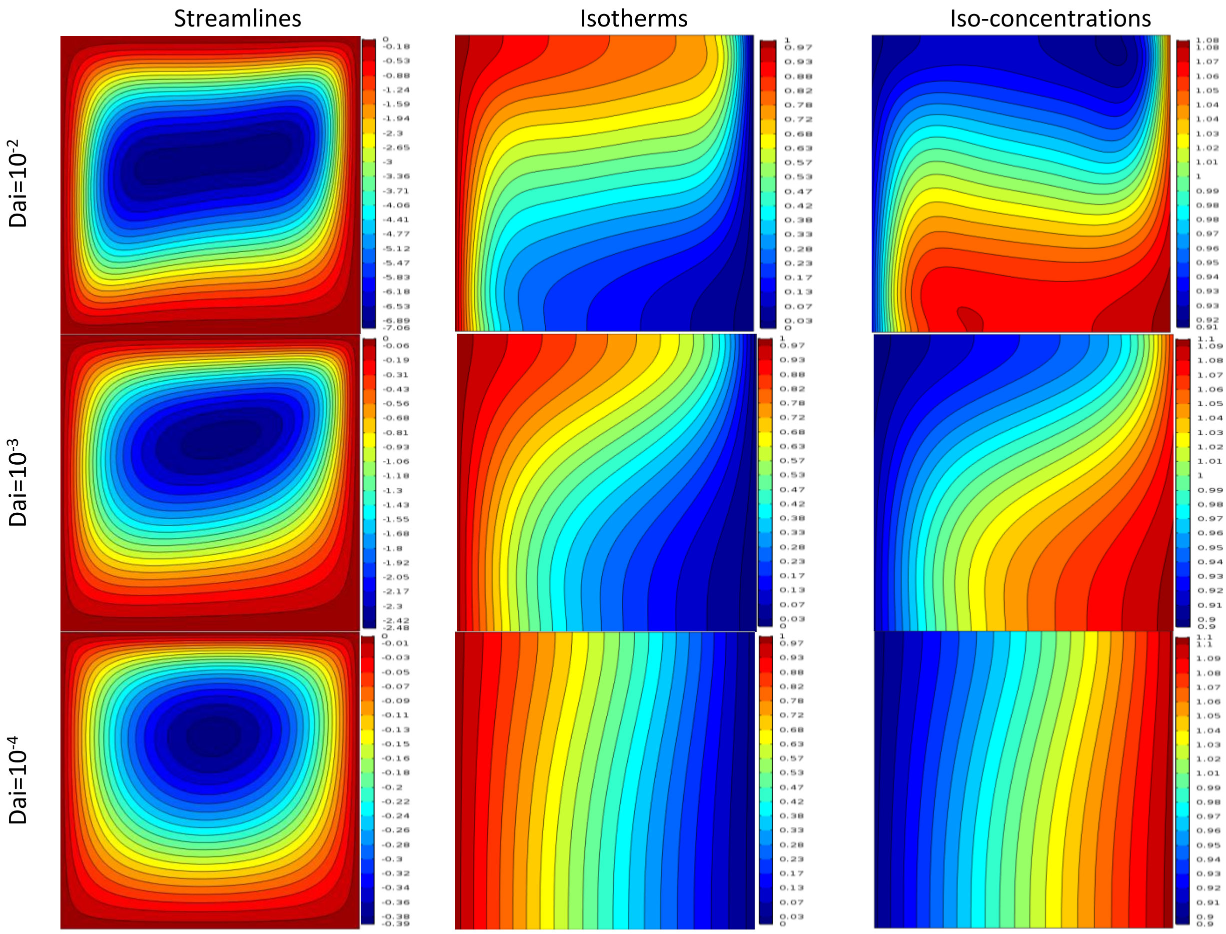
Aquifers present the variation in permeability along underground flow direction. In our configuration we represent a linear mathematical relationship which relates the Darcy number as a function of the depth of the cavity, leading to the deep layers which are characterized by a low permeability. Darcy's law is valid for Kx = Ky where Kx, Ky is the permeability along the horizontal x and vertical y axis:

$$Da = (Dai - Daf) * y + Daf \quad \text{(b)}$$

Our domain is limited by [0, H]. Initial Darcy number Dai (10^{-5}, 10^{-2}) and final Darcy number Daf $=10^{-5}$.

5.4.1 Homogenous porous medium with variable permeability

The same circulation cell was observed in Figure 7 for a low value of initial Darcy number (Dai $=10^{-5}$), If we increase the ($10^{-5}\leq$ Dai $\leq 10^{-4}$), the cell grows in intensity. The isotherms are parallel to the walls for a value of the Dai = 10^{-5}, from the variation of ($10^{-4}\leq$ Dai $\leq 10^{-2}$), its change shape which means that the heat the heat transfer is dominated by convection when permeability is higher. The distribution of iso-concentrations is identical to the isotherms for each variation of the Dai ($10^{-5}\leq$ Dai $\leq 10^{-3}$), except for the value of (Dai = 10^{-2}) we observe a difference in the distribution of nanoparticles.



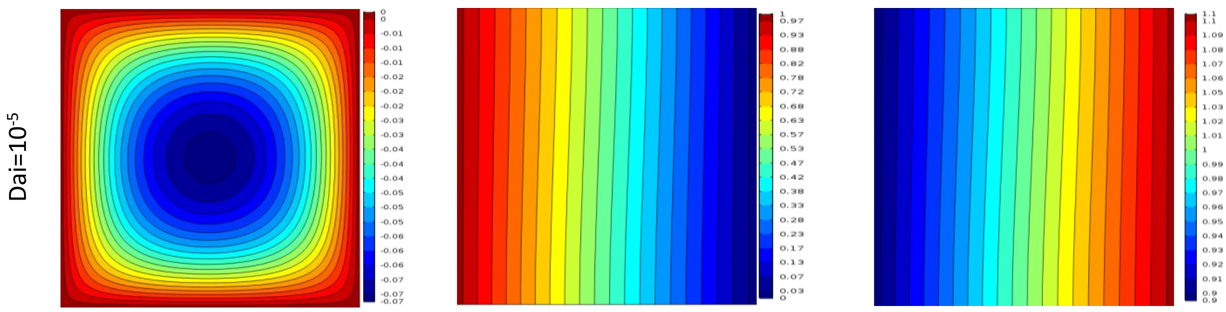
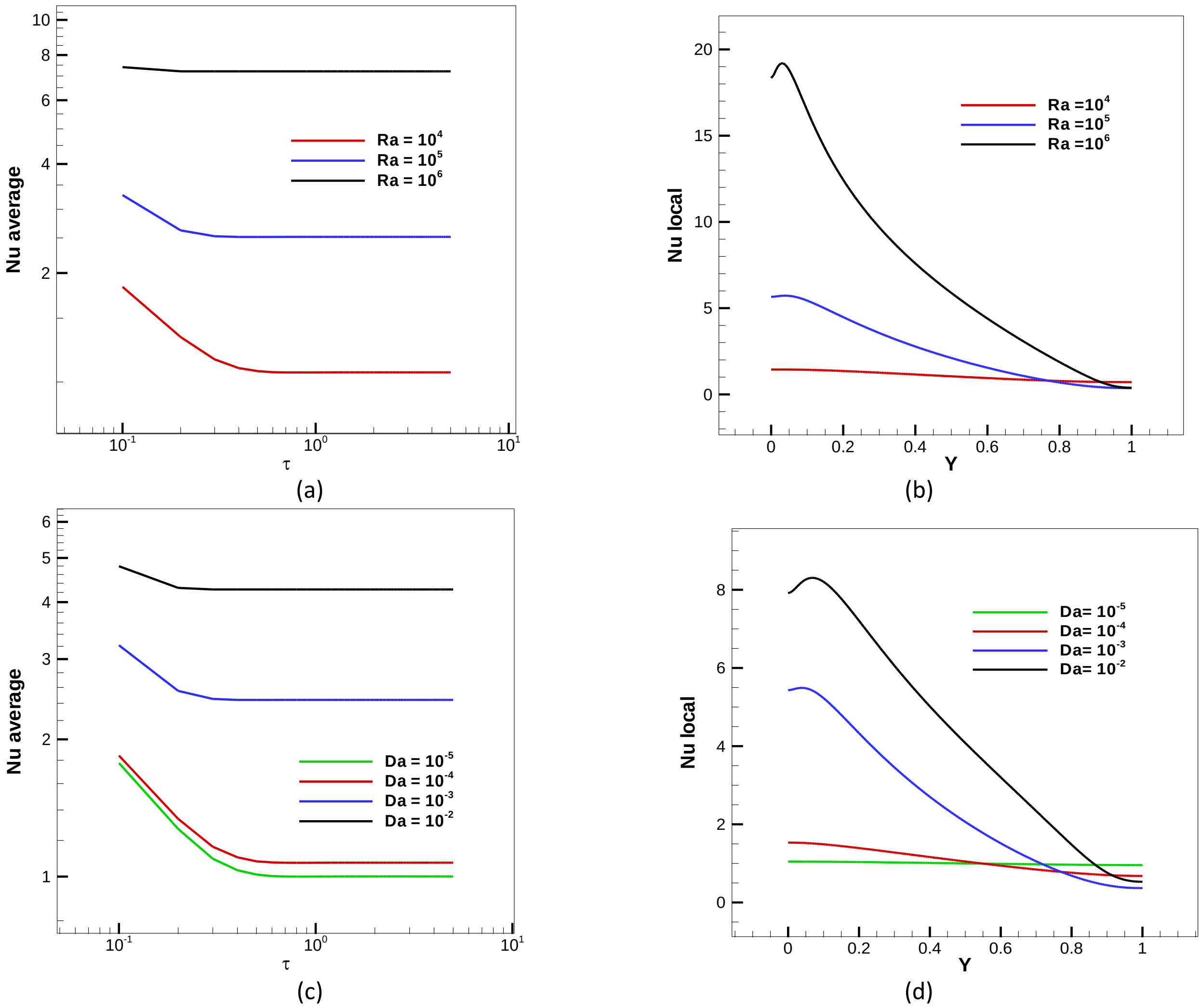
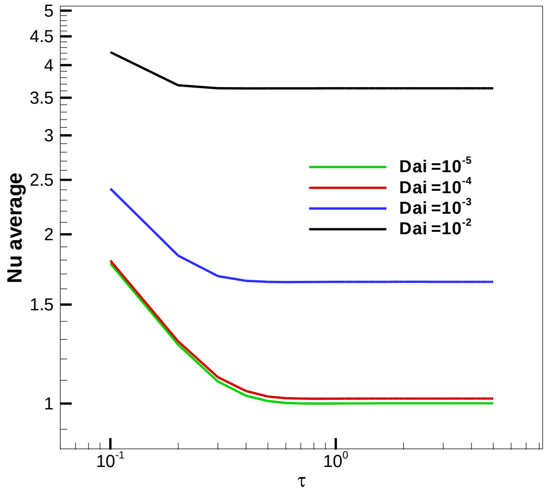


Fig. 7. Streamlines, isotherms and iso-concentrations for various initial Darcy number in porous medium with variable permeability at Ra $=10^5$, $N_B = N_T = N_r =$ Le = 0.1 and $Da_f = 10^{-5}$

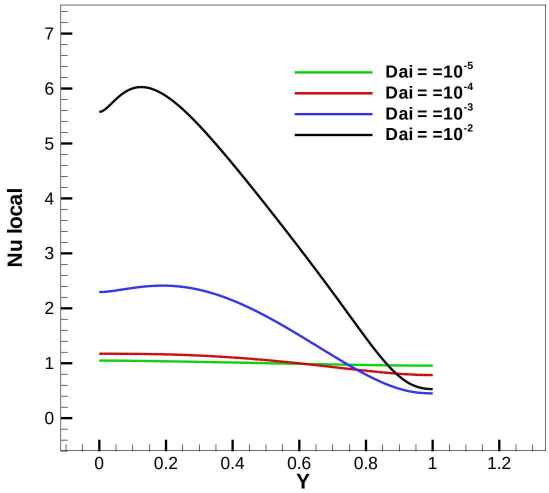
5.5 Calculation of the Average and Local Nusselt Number

The variation of the average and local Nusselt number is showen in Figure 8. with Rayleigh number ($10^4 \leq$ Ra $\leq 10^6$), Darcy number ($10^{-5} \leq$ Da $\leq 10^{-2}$), initial Darcy number ($10^{-5} \leq$ Dai $\leq 10^{-2}$) and fixing Daf $=10^{-5}$.





(e)



(f)

Fig. 8. Variations of steady average Nusselt number with τ (a), local Nusselt number with Y (b) for different Ra, Da and Dai.

We notice in Figure 8 (a) that the average Nusselt number is an increasing function of the Rayleigh number Ra. This increase is due to the buoyancy forces that favors a maximum heat transfer for Ra which reaches a value of 10^6.

In Figure 8 (b), we observe at Ra = 10^4 that the local Nusselt number decreases slightly along the left hot wall due to the existence of the nanoparticles in the base fluid. The local Nusselt number increases considerably along the hot wall with the increase of the Rayleigh number from 10^5 to 10^6, in this interval the maximal value of Nu_{loc} is due to the important temperature gradient which motivates the convective heat transfer.

The existence of nanoparticles in the base fluid increases the local rate of heat transfer [57]. In the figure 8 (c) the average Nusselt number (Nu_{avg}) increases with the Darcy number (Da=10^{-2}), due to the high permeability which augments the convective heat transfer. The low values of the Darcy number (Da=10^{-5}) is reflected by the conductive flow.

We observe in Figure 8 (d), the variation of the local Nusselt number along the hot wall as a function of the Darcy number (Da), we notice that for the low value of Da =10^{-5} the local Nusselt is equal to unit, the conduction dominates the flow. The high values of Nu_{loc} from Da = 10^{-4} to 10^{-3} indicate that the heat exchange carried out by convection for high values of Darcy number. This increase is significant at Da = 10^{-2}. We explained these phenomena due to the high permeability.

We notice **in** figure 8 (e) that the average Nusselt number at Dai =10^{-5}, equal to unit, when Dai = 10^{-4}, there is an almost negligible increase, the average Nusselt number increase proportionally with Da from 10^{-4} to 10^{-2}. At Dai =10^{-2}, a high permeability favors movement of nanofluids through the porous medium.

Figure 8 (f) presents the variation of Nu_{loc} with y for various values of Dai, it is observed that Nu_{loc} is equal to unit at low value of initial Darcy number Dai = 10^{-5}., we constate that Nu_{loc} is very high (Nu_{loc} = 6) for high value of Dai varies from 10^{-4} to 10^{-2}. We notice that the value of Nu_{avg} (4.8) at Da =10^{-2} is higher compared to Dai = 10^{-2} (Nu_{avg} = 4.3), and for the local Nusselt number respectively (Nu_{loc} =2.95, 6).

5.6 Comparison between Isotropic Medium and Homogeneous Medium with Variable Permeability

The numerical results are obtained for the case of an isotropic homogeneous porous medium with a constant permeability are compared with those of the homogeneous porous medium with variable permeability, with the variation of Da and Dai=10^{-5} à 10^{-2}. According to the values obtained from the different variables for the Darcy number Da and initial Darcy number Dai, we sign that the dynamic and thermal field variables don't change except for Da, Dai = 10^{-2}, the absolute maximum value of the stream functions decreases from 8.62 to 7.08 for the Da and Dai respectively.

The heat and masse transfer is more improved for the Darcy number Da compared to the intial Darcy number Dai. The flow intensity is weakened due to the low values of variable permeability and therefore the heat transfer is reduced. According to Figure 9, we notice that the average Nusselt number equal to unit from Da = Dai = 10^{-5} to 10^{-4} for the two mediums, which explains that the heat transfer is dominate by pure conduction, for the high values of the average Nu the heat transfer is convective. From Da = 10^{-4} to $10^{-3}$the two curves move away in order to approach again at 10^{-2} we observe that the average Nusselt number of homogeneous porous medium with isotropic permeability is higher than this one of the homogeneous porous medium with variable permeability. The difference in the decrease of the average Nusselt number is about 13 .95%.

The effects of the variable permeability in the homogenous porous medium on the convection heat transfer are progressively inhibited. When Da is large Da >10^{-2} (absence of inertia forces), we approach the pure fluid medium [46].

When the Darcy number is very low (low porosity) Da ≤ 10^{-5}, the permeability variation has a strong influence on the heat transfer which is reduced. When Da increases, the variable permeability effect is insignificant on heat transfer which is maximum due to the high permeability. It can be concluded that permeability have an impact on heat transfer.

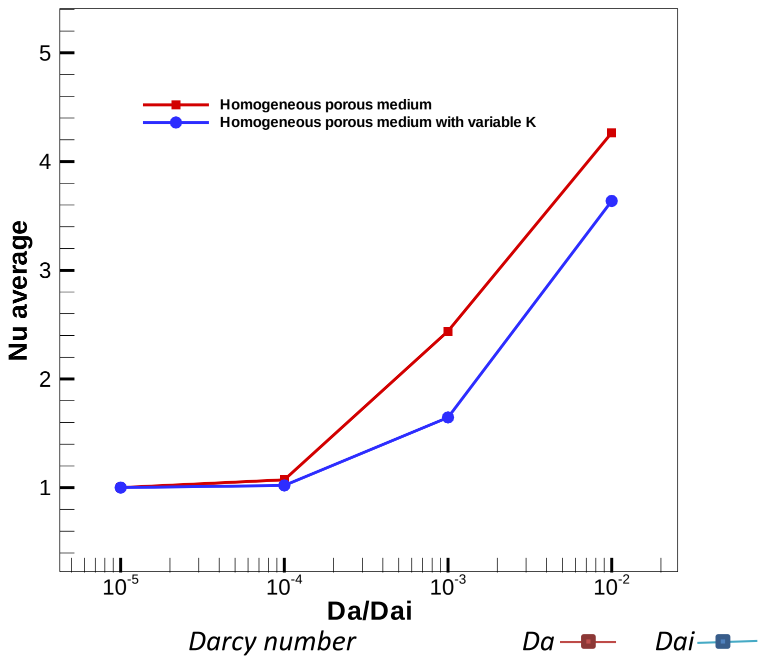


Fig. 9. Evolution of the average Nusselt number according to the Darcy and intial Darcy number

6. Numerical validation

The validation of the reference geometry is used by de Sheikhzadeh, S. Nazari was done under the same conditions, the equations used were similar and we can conclude that the same results are obtained for the iso-concentrations cases (see Figure 10).

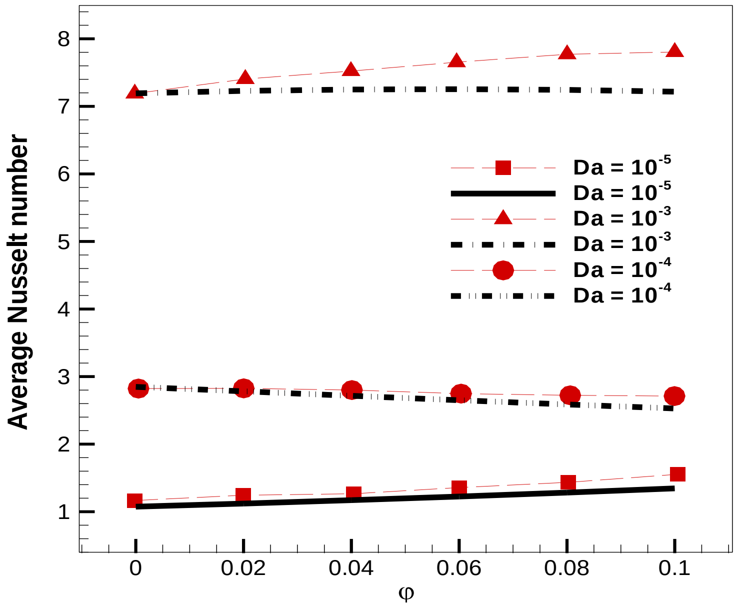


Fig. 10. Numerical comparison results of average Nusselt number with Sheikhzadeh, S. Nazari ------ and a present work------

7. Conclusions

Laminar natural convection in porous square cavity filled with nanofluids was investigated. The Buongiorno's model are used to simulate nanofluids flow and porous medium, the dimensionless equations were solved numerically by using the Galerkin finite element method. The results obtained are found in agreement with those of Nazari's publication. From the present study, we reveal that:

i. The dimensionless time necessary to reach different variables dynamic and thermal field is about τ = 0.30, which is reflected by the flow stability and the dominance of convection heat transfer, it is strongly influenced by thermal buoyancy forces which contribute to enhance heat transfer.
ii. The flow intensity is weakened due to the low values of permeability and consequently the heat transfer is reduced while the high value permeability facilitates the nanofluids movement.
iii. The convective heat transfer is more significant due to the incorporation of nanoparticles in the base fluid. The decreases of average Nusselt number value is about 13 .95%. From this, we conclude that the heat and masse transfer is more improved for the homogeneous porous medium compared to the homogeneous porous medium with variable permeability.

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