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Numerical Analysis of a Laminar Transient Flow of a Pseudo-Plastic Fluid

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ABSTRACT

This study is devoted to a numerical modeling of a transient pseudo plastic fluid flow in an elastic pipe. The set partial equations for both the fluid are derived from the law conservations of mass, momentum and energy for the fluid and Hooke's law for the wall pipe. The system governing this problem is presented and then solved numerically. The non-Newtonian character behavior of the fluid s modeled by the power law. The coupled method of characteristics, finite differences and Runge Kutta are used for spatial and temporal discretization respec-tively. Some results obtained are in good agreement with those found in the literature.

1. Introduction

Transient flows, non-Newtonian fluid flow are mainly found in natural and artificial systems, such as the hydraulic system, the human arterial network. This fact can cause great pressure leading to harmful forces [1]. The flow of viscoelastic fluids in pipes can manifest itself in a wide range of industrial process ap-plications and many natural systems. In this work, we consider non-Newtonian fluids [3], in which the viscosity can be described using the power law, or cross models, the capacity of these models has been studied by several researchers, in particular Pinho and Whitelaw [4], Toms [5] , Bird *et al.,* [2] with experimental and numerical studies. For this type of fluid, the strain rate must be evaluated, and it needs a two-dimensional analysis to have the unstable pressure and veloci-ty profile. Many studies in this context have been made by many researchers [6, 7], Pezzinga [8, 9] gave a quasi two-dimensional model of unsteady turbulent flow, of a network of pipes with better results compared to 1D models. Vardy and Brown [10] have also made remarkable studies for non-Newtonian un-steady flows in pipes, in particular the modeling of fluids whose viscosities de-pend on time. Recently, there is Wahba [11] who compared the fluids of thinning and shear using the power law model.

This study is focused on the unstable flow in conduct of a pseudo-plastic fluid. After performing these simulations for the power law and Cross models, there are alternative works [12] on the

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transients of these fluids that can be pro-posed for future research. ex. Ahmadi and Keramat [12, 13], Soares *et al.,* [14], Hadj-Taïeb [15]. Pezzinga *et al.,* [16] transients in polymer pipes under pressure using a two-dimensional (2D) KelvinVoigt viscoelastic model. Brunone *et al.,* [17], Kim [18] took pressure and energy dissipation, as well as unstable friction in laminar transient flows, comparing their numerical results with those from experiment. Meniconi *et al.,* [19, 20] studied the turbulent flow of rapidly decelerating pipes, so they proposed a new approach to estimate the energy dissipation as well as the pressure decay, and other very recent studies [21-24] have been carried out. In the, non-Newtonian transient laminar flow of pipes is simulated using the power law model. For this purpose, in this work the water hammer equations for a pseudo-plastic fluid are given, and solved by numerical methods based on Finite Difference, Characteristics and Runge-Kutta.

2. Governing Equations

2.1 Basic Equations

The phenomena can being described by the following system [25] as shown in Eq. (1) and Eq. (2),

$$\begin{cases} \dfrac{\partial \mathrm{H}}{\partial \mathrm{t}} + \dfrac{a^2}{g}\dfrac{\partial \mathrm{V}}{\partial \mathrm{z}} = 0 \\ \dfrac{\partial \mathrm{H}}{\partial \mathrm{z}} + \dfrac{\partial \mathrm{V}}{\partial \mathrm{z}} + \mathrm{J_q} = 0 \end{cases} \tag{1}$$

$$\begin{cases} \dfrac{1}{\rho a^2}\dfrac{\partial P}{\partial t} + \dfrac{\partial v}{\partial z} = 0 \\ \rho\dfrac{\partial v}{\partial t} = -\dfrac{\partial P}{\partial z} + \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial \tau}{\partial r}\right) \end{cases} \tag{2}$$

where V is average velocity, H is presser head averaged over the cross-sectional area of flow,a is wave speed, ρ is density,t is time and J_q is steady friction, and v is the axial velocity, P is presser head,a is wave speed, ρ is density, t is time and τ is shear stress.

2.2 Non-Newtonian Fluid Equations

The shear stress of non-Newtonian fluid is described as Eq. (3),

$$\tau = \mu\dot{\gamma} \tag{3}$$

where μ and $\dot{\gamma}$ are apparent viscosity and shear rate respectively.

The power law is described by the Eq. (4) [22, 23], with the fewest possible parameters.

$$\mu = \mathrm{m}(\dot{\gamma})^{\mathrm{n}-1} \tag{4}$$

where m and n are two empirical curve fitting parameters, also known the fluid consistency coefficients and the flow behavior index respectively. So, if n equals one and m is set to μ_0, the Newtonian fluid is obtained. In our study, m is fixed to μ_0, and quantity for n is selected.

2.3 Initial Conditions

The initial conditions corresponds to the steady state flow. These conditions can be written as Eq. (5) and Eq. (6):

$$\frac{\partial \mathrm{v}}{\partial \mathrm{z}} = 0 \tag{5}$$

$$\frac{\partial \mathrm{P}}{\partial \mathrm{z}} = -\frac{2\tau_0}{\mathrm{R}} \tag{6}$$

2.4 Boundary Conditions

The velocity is set to zero after the valve closure, at the valve boundary. At the reservoir, a constant pressure head is associated. In contact, the flow boundaries with the pipe wall have zero velocity. So, these boundary equations can be reduced as follows in Eq. (7), (8), (9):

$$\mathrm{v}_{(\mathrm{r=R})} = 0 \tag{7}$$

$$\mathrm{v}_{(\mathrm{valve})} = 0 \tag{8}$$

$$\mathrm{H}_{\mathrm{reservoir}} = \mathrm{cte} \tag{9}$$

3. Numerical Method

The equations of the proposed unsteady flow system is solved using the Finite Difference and Characteristics methods. The system of equations over time, is solved using a fourth-order Runge-Kutta scheme. The systems in Eq. (1) and Eq. (2) are given in matrix form in Eq. (10)

$$\frac{\partial \{\mathrm{W}\}}{\partial \mathrm{t}} = -[\mathrm{B}]\frac{\partial \{\mathrm{W}\}}{\partial \mathrm{z}} + \{\mathrm{c}\} \tag{10}$$

with the unknown vector describe in Eq. (11)

$$\{\mathrm{W}\}=\{\mathrm{H}\ \mathrm{V}\} \tag{11}$$

4. Application and Numerical Results

In the present study the test is initiated by a sudden closing of the valve, which causes excision of the fluid flow to the valve, creating oscillations of pressure and velocity, which propagate along the pipe, a representation of the head pressure over time in midpoint and valve is given in Figure. 1 and Figure 2. The data base is gived in Table 1. Figure 1. and Figure 2. corresponds to the pressure heads.it shows significantly, pressure decay, as well as energy dissipation.

Table 1
Installation data

Parameters	Values
Tank height H_0 (m)	1
Internal pipe diameter (m)	0.25
Length of pipe (m)	36.09
High viscosity oil at 20 ° (Pa. s)	0.03484
Poisson's ratio	0.34
Young's modulus of copper (GPa)	120

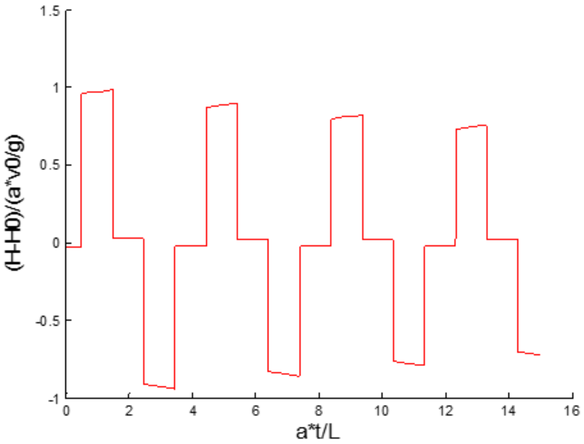

Fig. 1. Pressure time-history at midpoint of pipe

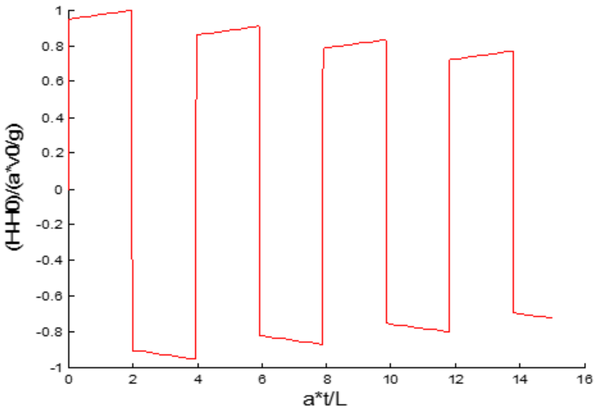

Fig. 2. Pressure time-history at valve of pipe

4.1 Pipe Length Effect

To investigate the effect of length pipe, keeping the same numerical data, we vary just the value of L. The results are shown in Figure 3, Figure 4 and Figure 5. Long transmission pipelines, usually large in diameter and flow rate as shown in Figure 3, Figure 4 and Figure 5. The negative pressure causes threatening problems like column separation and cavitation. The results indicated that using equipment when the pumps fail alone does not have the appropriate efficiency in eliminating the hazards in the transmission pipelines.

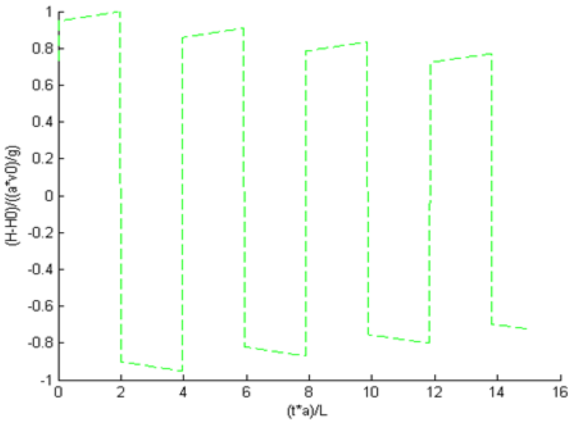

Fig. 3. Pressure time-history at valve of pipe L=36 m

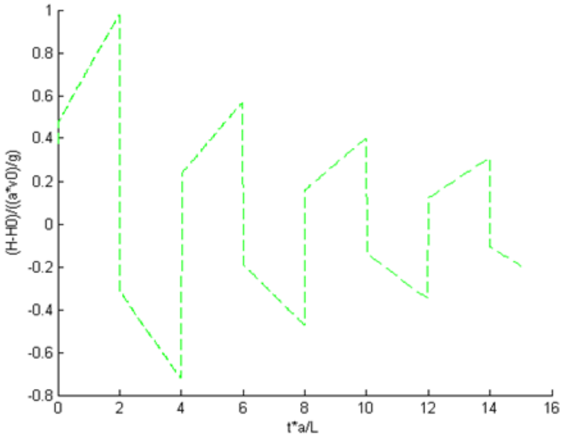

Fig. 4. Pressure time-history at valve of pipe L=360m

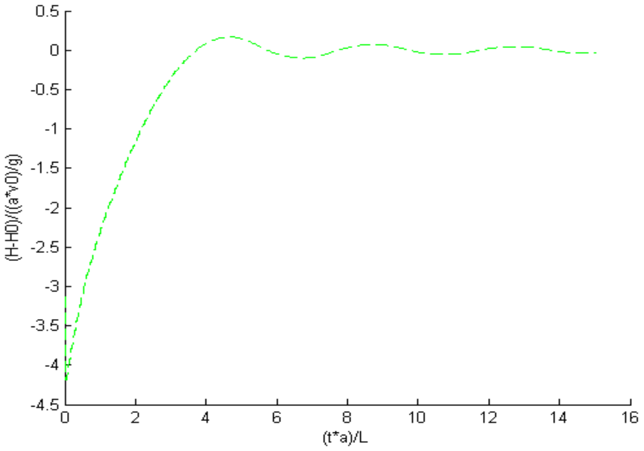

Fig. 5. Pressure time-history at valve of pipe L=3600 m

When adopting the classical boundary expression, the maximum pressures do not vary much along the pipe Figure 6. These observations illustrate that the pro-posed boundary expression not only improves the ability of the water hammer model to replicate the pressure damping pattern at the valve but also improves the prediction of the pressure distribution along the pipe over time.

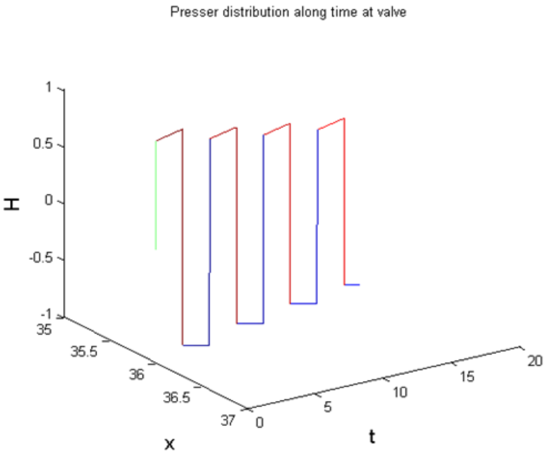

Fig. 6. Presser distribution

5. Conclusion

This study made it possible to model a transient laminar viscoplastic fluid flow in a pipe with a coupled finite difference, Runge Kutta and characteristic numerical schema. More so, it has clearly shown how the flow of fluids is affect-ed by some of the flow parameters such as fluid viscosity, pipe length.

This numerical code can easily be used alongside the existing codes to predict water-hammer in the laminar transient viscoelastic flow in pipes.

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