

# Contribution of Soret and Dufour aspects on Hybrid Nanofluid over 3D Magneto Radiative Stretching Surface with Chemical Reaction

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## **1. Introduction**

Many researchers and designers have researched distinct heat transfer applications, using nano liquids improvement, to reach cooling challenge needs such as electronics, transportation, and energy furnishing industries in the last few decades. The highest heat transfer rates are found by new Nano Fluids (NF) that are called HNFs. Towards hybrid nanofluids, Babar *et al.,* [1] analyzed the preparation of HNFs and their applications with challenges. Suresh *et al.*, [2] reported about Al<sub>2</sub>O<sub>3</sub>-CuO/H2O hybrid nanofluid used 2-step methods and its physical-thermos properties. Thoughtfully these HNFs are a new group of NFs that have scores of believable uses in every field, including heat transfer namely micro fluids, fabricating, conveyance, refutation, therapeutic, seafaring, acoustics etc. With the concept of HNFs, there are many experimental research articles are published. At now, scientists are interested in learning more about HNFs in a variety of flow geometries. A novel technology idea known as HNF has been the subject of several published experimental studies. Its already excellent results are further improved by the use of HNF. Momin *et al.,* [3] experimented

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with the effects of composite convection using an HNF Al<sub>2</sub>O<sub>3</sub>-CuO/H<sub>2</sub>O inclined Tube with the laminating flow.

Synthesis, Characterization, and heat transfer impact on heat turbulent water-based  $(AI_2O_3-$ CuO/H2O) HNF aspects examined by the previous studies of Suresh *et al.,* and Suresh *et al.,* [4, 5]. Madhesh *et al.,* [6] proposed a heat transfer convective characteristic of CuO -TiO<sup>2</sup> HNFs. Arshad *et al.,* [7] presented hybrid conventional nanofluids heat transfer with Hall Effect. Madhesh *et al.,* [8] conducted an HNF coolant analysis. Heat and mass transport-water interpretation in a rotating media is of paramount relevance because of the wide range of fields in which it may be used, including astrophysics, liquid design, and geophysics. Alam *et al.,* [9] compared the three different types of NFs, NF Cu/H<sub>2</sub>O; HNF (Al<sub>2</sub>O<sub>3</sub>-Cu/H<sub>2</sub>O) and trihybrid nanofluid THNF (TiO<sub>2</sub>-Cu- Al<sub>2</sub>O<sub>3</sub>/H<sub>2</sub>O) and examined flow-rate. Hayat *et al.*, [10] studied, Al<sub>2</sub>O<sub>3</sub>-CuO/H<sub>2</sub>O HNFs heat transfer improvement. Sidik *et al.,* [11] reviewed the latest developments in HNFs for flow-rate. Arshad *et al.,* [12] addressed nanofluid flow with mass transmission over a permeable flat surface with a constant magnetic regime. Arshad *et al.,* [13] scrutinized single-phase flow over a permeable stretchable region with variable permeability, using Buongiorno model. Anuar *et al.,* [14] studied surfaces that has been altered by radiation by being either stretched or shrunk for the HNF steady rotating flow. Several academics have studied the rotating stretching/shrinking regime in HNF models with diverse concerns. Venkateswarlu *et al.*, [15] explained that CuO-Al<sub>2</sub>O<sub>3</sub>/H<sub>2</sub>O HNF flow past a permeable stretchable region with viscous dissipation. Lund *et al.,* [16] discussed porous-Forchheimer impact on rotating HNF on linearly shrinking/stretching sheets. Hayat *et al.,* [17] interpreted numeric analysis of rotating HNF Al<sub>2</sub>O<sub>3</sub>-CuO/H<sub>2</sub>O with heat generation /absorption. Also, there are several applications for research into MHD flow over a stretched sheet, including polymer extrusion, metallic surface cooling, and plastic regimes extrusion. Irfan *et al.,* [18] studied Maxwell nanofluid flows towards a stretched cylinder that has been convectively heated and MHD, stagnation point, heat sink/source, thermal radiation, and chemical reactions are also discussed. Asghar *et al.,* [19] demonstrated radiation influence of hybrid nanofluids on 3-dimensional magnetization. Hafeez *et al.,* [20] analyzed HNFs' thermal properties influenced by Soret and Dufour effects of Casson HNF based on ethylene glycol. Narayanaswamy *et al.,* [21] investigated Stefan Blowing influences upon Hybrid liquids on stretchable cylinders with thermal radiation, Dufour, and Soret effects. Venkateswarlu *et al.,* [22] reported temperature viscosity on HNF flow past a porous stretchable regime. Mohd Sohut *et al.,* [23] examined numerically unsteady 3-D flow for rotational HNFs. Khan *et al.,* [24] also researched unsteady 3-D flow with variable magnetic field for rotational HNFs,  $H_2O$ -CNTs-ferrous oxide with a variable magnetic regime. In the 3-dimensional flow of a sheet that is being stretched, a HNF is being used as a heat source and sink, investigated by Farooq *et al.,* [25]. Irfan *et al.,* [26] scrutinized the homogeneous-heterogeneous responses of Powell-Eyring ferromagnetic flow over flat plate with two equivalents magnetic dipoles. Sharma *et al.,* [27] studied HNF flow, on a curved stretching regime with Dufour-Soret effects. Khashi'ie *et al.*, [28] deliberated velocity slip and convective conditions for the above flow geometry. Nonlinear convection, the Soret-Dufour effect, and zero mass flow concepts were studied by Irfan *et al.,* [29]. Waini *et al.,* [30] explored transpiration persuasions on HNF flow rates across a stretchable regime with homogenous flow. Mahabaleshwar *et al.,* [31] utilized MHD Newtonian HNF flow owing super-linear permeable stretched regime. Butt *et al.,* [32] investigated viscous dissipation impact in 3-dimensional flow and heat transfer over a bidirectional stretched surface with a magnetic region. Irfan *et al.,* [33] explored Joule heating aspect with convective conditions in Oldroyd-B nanofluid. Hassan *et al.,* [34] examined the 3-dimensional rotating flow of magneto hybrid nanofluids under radiation over a bi-directional stretchable regime. Arshad *et al.,* [35] explored hybrid nanofluid over a permeable-stretchable plate with heat source and chemical reaction. Zin *et al.,* [36] scrutinized how heat radiation and absorption influence the flow of Jeffrey fluid across an infinite vertical plate in an unstable magnetohydrodynamic (MHD) free convection flow. Arshad *et al.,* [37] analyse the 3-dimensional magnetohydrodynamic Nanofluid flow with an inclined magnetic regime with heat radiation and chemical reaction. Knasara *et al.,* [38] moduled material-based thermal control is being explored to improve the thermal management of electronic components/detectors. Rafiq *et al.,* [39] elaborated on the radiation and activation energy phenomena in a chemically reactive Maxwell nanofluid near its stagnation point, the phenomena of activation energy and radiation in chemically reactive stagnation point Maxwell nanofluid have been elaborated. Kumar *et al.,* [40] deliberated on the influence of magnetic field, joule heating, rotation parameter, Hall current, and nonlinear thermal radiation on a spinning hybrid Fe3O4/Al2O3 nanofluid overstretched plate in the presence of a chemical reaction. Azmi *et al.,* [41] focused on the Maxwell hybrid nanofluid over mixed convective radiative across a stretching/shrinking inclined plate with nanoparticle form influence. Copper and aluminium oxide were added to sodium alginate as a base fluid to construct the problem, and the influence of shape factor is investigated using spherical, bricks, cylindrical, and platelet nanoparticles. Hashim *et al.,* [42] discussed a numerical investigation of natural convection in a trapezoidal chamber with left heated wall, right cold wall, and filled with Cu-Al2O3 and water HNF. Ishak *et al.*, [43] focused the thermal radiation and mixed convection flow of hybrid nanofluids across a permeable moving and stationary wedge. Maheswari *et al.,* [44] elaborated the effects of the Brownian motion, thermophoresis parameter, velocity ratio parameter, magnetic parameter, slim needle similarity radius, Prandtl number, and thermal radiation on a steady-state, laminar, MHD hybrid nanofluid composed of MgO-Ag/H2O fluid along a horizontal hot thin needle. From the previous studies it is observed that there is no numerical study on the 3 dimensional HNF (CuO-Ag/H<sub>2</sub>O) by the effects of the Dufour-Soret effect with magnetic region.

The goal of this article is to explore Du and Sr impacts on a linearly stretching sheet of a Hybrid Nanofluid (Cuo-Ag/Water). The boundary-layered PDEs of the physical flow are transformed into ODEs using similarity transformation, and an effective mathematical tool, the bvp5c code in MATLAB, was built to solve them. As a limiting condition, the code's correctness is confirmed by comparison with earlier studies. The effects of the relevant factors are graphically represented, and the computational results produced are contrasted with the available reports. The finding helps engineers and scientists alike gain insight into the boundary layer flow's behaviour and has consequences for several disciplines. The following primary research issues are addressed by this related study.

- i. What effect do volume fractions and Magnetic fields have on profiles of concentration, temperature, and velocity?
- ii. Does an increase in the Dufour and Radiation parameters effect results in a higher rate of heat transmission and less skin friction?
- iii. How are the temperature and concentration profiles affected by chemical reactions and Soret numbers?
- iv. How do the skin frictions behave differently along both the axis, Nusselt number, and Sherwood number of distinct parameters?

In this study, these effects are considered and studied by the following problem formulation.

The goal of this work is to assess several parameters impacts such as the volume fraction of nanoparticles, including magnetic parameters, Soret and Dufour effects on hybrid Nanofluids overstretching sheets, and to explore their implications for various applications involving heat and mass transfer. Given its use in adjusting temperature distribution in a variety of industrial applications, generator cooling, nuclear system refrigeration, building ventilation, etc.

# **2. Problem Formulation**

The rotating, three-dimensional, linear HNF (CuO-Ag/Water) flow past stretching surface at Z = 0 is assumed. The problem flow arrangement is shown in Figure 1



Fig. 1. Flow geometry of the problem

The assumptions of the fluid flow problem are:

- i. The fluid occupies the half space at  $Z > 0$ .
- ii. With base fluid  $H_2O$ , the nano-sized particles CuO and Ag.
- iii. To evolve a targeted HNF (CuO-Ag/H2O) distinct volume fractions  $\varphi_1$ ,  $\varphi_2$  dispersed and taken along vertical axis, an angular velocity  $\Omega$  is persistent.

The governing equations Hayat *et al.,* [10] are:

$$
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0.
$$
 (1)

$$
u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - 2\Omega v = v_{\text{hurf}}\frac{\partial^2 u}{\partial z^2} - \frac{\sigma_{\text{hnf}}}{\rho_{\text{hnf}}}B_0^2 u.
$$
 (2)

$$
u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + 2\Omega u = v_{\text{inf}} \frac{\partial^2 v}{\partial z^2} - \frac{\sigma_{\text{inf}}}{\rho_{\text{inf}}} B_0^2 v.
$$
  

$$
u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \alpha_{\text{inf}} \frac{\partial^2 T}{\partial z^2} + \frac{Q}{(Q_0)} (T - T_{\infty}) - \frac{1}{(Q_0)} \frac{\partial q_r}{\partial z} + \frac{D_2}{(Q_0)} \frac{\partial^2 C}{\partial z^2}.
$$
(4)

$$
u\frac{\partial V}{\partial x} + v\frac{\partial V}{\partial y} + w\frac{\partial V}{\partial z} + 2\Omega u = V_{hnf}\frac{\partial V}{\partial z} - \frac{\partial h_{nf}}{\partial_{hnf}}B_0^2v.
$$
  

$$
u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \alpha_{hnf}\frac{\partial^2 T}{\partial z^2} + \frac{Q}{(\rho C_p)_{hnf}}(T - T_\infty) - \frac{1}{(\rho C_p)_{hnf}}\frac{\partial q_r}{\partial z} + \frac{D_2}{(\rho C_p)_{hnf}}\frac{\partial^2 C}{\partial z^2}.
$$
 (4)

$$
u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = \beta_{\text{lnif}}\frac{\partial^2 C}{\partial z^2} + D_1\frac{\partial^2 T}{\partial z^2} - \xi_1(C - C_\infty)^n.
$$
 (5)

Where,  ${}_{V_{mf}}$  ,  $\alpha_{_{hnf}}$  <sub>f</sub>and  $\beta_{_{hnf}}$  are three diffusions of HNF, correspondingly. In momentum equations the second term is called magnetic field effect. The heat generation/absorption coefficient is Q, last term in that equation is called Diffusion thermo effect (Dufour), and in concentration equation second term is called Thermo diffusion effect (Soret). Nonlinear chemical reaction is  $\zeta_{\text{\tiny{l}}}$  .

Radiation flux  $q_r$  is specified by using Rosenald approximation as

$$
q_r = \frac{-4\sigma}{3\chi} \frac{\partial T^4}{\partial z}.
$$
 (6)

Now expanding Taylor series about *T*

$$
T^4 \approx 4TT_{\infty}^3 - 3T_{\infty}^4 \tag{7}
$$

The boundary conditions of the above defined three-dimensional flow are by Hayat *et al.*, [10]  

$$
u = U_w = ax, v = V_w = by, w = 0, T = T_w, C = C_w, atz = 0.
$$
 (8)

$$
u \to 0, v \to 0, T \to T_{\infty}, C \to C_{\infty} a s z \to \infty.
$$
\n(9)

Using adequate similarity transformations described as may simplify the problem.

Using adequate similarity transformations described as may simplify the problem:  
\n
$$
u = \alpha xf'(\eta), v = \alpha yg'(\eta), w = -\sqrt{av_f} (f(\eta) + g(\eta)), \eta = z \sqrt{\frac{a}{v_f}},
$$
\n
$$
\theta(\eta) = \frac{T - T_{\infty}}{T_w - T_{\infty}}, \phi(\eta) = \frac{C - C_{\infty}}{C_w - C_{\infty}}.
$$
\n(10)

differential equations.

$$
u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z} = \beta_{avg} \frac{\partial}{\partial z^2} + D_1 \frac{\partial}{\partial z^2} - \xi_1 (C - C_{\infty})^{\circ}.
$$
\n(5)  
\nWhere,  $v_{log}$ ,  $\alpha_{log}$  and  $\beta_{log}$  are three diffusions of HNF, correspondingly. In momentum equations the  
\nsecond term is called magnetic field effect. The heat generation/absorption coefficient is Q, last term  
\nin that equation is called Diffusion there (Soret). Nonlinear chemical reaction is  $\xi_1^{\circ}$ .  
\nRadiation flux  $q$ , is specified by using Rosenald approximation as  
\n
$$
q = \frac{-4\sigma}{3\chi} \frac{\partial T^4}{\partial z}.
$$
\n(6)  
\nNow expanding Taylor series about  $T_x$   
\n
$$
T^* \approx 4TT_x^{\circ} - 3T_x^{\circ}
$$
\n(7)  
\nNow expanding Taylor series about  $T_x$   
\n
$$
T^* \approx 4TT_x^{\circ} - 3T_x^{\circ}
$$
\n(8)  
\n
$$
u \rightarrow 0, v \rightarrow 0, T \rightarrow T_x, C \rightarrow C_x \text{ } \alpha z \rightarrow \infty.
$$
\n(9)  
\n
$$
u = \alpha x, v = V_w = by, w = 0, T = T_w, C = C_w, \text{ } \alpha z = 0.
$$
\n(9)  
\n
$$
u \rightarrow 0, v \rightarrow 0, T \rightarrow T_x, C \rightarrow C_x \text{ } \alpha z \rightarrow \infty.
$$
\n(9)  
\nUsing adequate similarity transformations described as may simplify the problem.  
\n
$$
u = \alpha x f'( \eta), v = \alpha y g'(\eta), w = -\sqrt{\alpha v_f} (f(\eta) + g(\eta)), \eta = z \sqrt{\frac{a}{v_f}},
$$
\n(10)  
\n
$$
u = \alpha x f'( \eta), v = \alpha y g'(\eta), w = -\sqrt{\alpha v_f} (f(\eta) + g(\eta)), \eta = z \sqrt{\frac{a}{v_f}},
$$
\n(10)  
\n
$$
u = \alpha x f'( \eta), v = \alpha y g'(\eta), w = -\sqrt{\alpha v_f} (f(\eta) + g(\eta)), \eta = z \sqrt{\frac{a}{v_f}},
$$
\n(10)  
\n
$$
f(\eta) = \frac{T
$$

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\n
$$
g'''(\eta) - (1 - \phi_1)^{2.5} (1 - \phi_2)^{2.5} \left( (1 - \phi_2) \left\{ (1 - \phi_1) + \phi_1 \left( \frac{\rho_{s1}}{\rho_f} \right) \right\} + \phi_2 \left( \frac{\rho_{s2}}{\rho_f} \right) \right)
$$
\n
$$
\left( g'^2(\eta) - g''(\eta) (f(\eta) + g(\eta)) - 2 \left( \frac{\varepsilon}{\gamma} \right) f'(\eta) \right) - \left( \frac{\sigma_{\text{lntf}}}{\sigma_f} \right) (1 - \phi_1)^{2.5} (1 - \phi_2)^{2.5} Mg'(\eta) = 0.
$$
\n(12)

$$
\left(g^{2}(\eta) - g''(\eta)(f(\eta) + g(\eta)) - 2\left(\frac{\varepsilon}{\gamma}\right)f'(\eta)\right) - \left(\frac{\sigma_{\text{huf}}}{\sigma_{\text{f}}}\right)(1 - \phi_{1})^{2.5}(1 - \phi_{2})^{2.5}Mg'(\eta) = 0.
$$

$$
\begin{aligned}\n\left(\frac{R_{\text{Inf}}}{K_f} + \frac{4R}{3}\right) \theta \text{ }^n(\eta) + \beta \text{ }^n(\eta) - 2\left(\frac{1}{\gamma}\right)^n \text{ }^{n}(\eta)\n\end{aligned}\n\right) = \left(\frac{R_{\text{Inf}}}{K_f} + \frac{4R}{3}\right) \theta \text{ }^n(\eta) + \Pr\left((1 - \phi_2)\left\{(1 - \phi_1) + \phi_1\left(\frac{(\rho c_p)_{s1}}{(\rho c_p)_f}\right)\right\} + \phi_2\left(\frac{(\rho c_p)_{s2}}{(\rho c_p)_f}\right)\right) \\
\left(f(\eta) + g(\eta)\right) \theta \text{ }^n(\eta) + \delta \Pr(\theta(\eta) + Du \Pr(\phi \text{ }^n(\eta)) = 0.\n\end{aligned}
$$
\n(13)

 $(f(\eta) + g(\eta))$ 

$$
(f(\eta) + g(\eta))\theta'(\eta) + \delta \Pr \theta(\eta) + Du \Pr \phi''(\eta) = 0.
$$
  

$$
\phi''(\eta) + \left(\frac{Sc}{(1-\phi_1)(1-\phi_2)}\right) (Sr\theta''(\eta) + (f(\eta) + g(\eta))\phi'(\eta) - Rc\phi(\eta)) = 0.
$$
 (14)

The corresponding boundary conditions are:

$$
f = 0, f' = 1, g = 0, g' = \lambda, \theta = 1, \phi = 1, at\eta = 0
$$
\n(15)

$$
f' \to 0, g' \to 0, \theta \to 0, \phi \to 0 \text{ as } \eta \to \infty
$$
\n(16)

The provided factors are

The provided factors are  
\n
$$
\varepsilon = \frac{\Omega}{a}, \lambda = \frac{b}{a}, \Pr = \frac{v_f (\rho C p)_f}{k_f}, R = \frac{4\sigma T^4}{\chi k_f}, \delta = \frac{Q}{a(\rho C p)_f}, Sc = \frac{v_f}{\beta_f}, Rc = \frac{\xi_1 (C - C_{\infty})^{n-1}}{a},
$$
\n
$$
Du = \frac{D_2 (C_w - C_{\infty})}{(\rho C p)_f v_f (T_w - T_{\infty})}, Sr = \frac{D_1 (T_w - T_{\infty})}{v_f (C_w - C_{\infty})}, M = \frac{\sigma_f B_0^2}{\rho_f a}.
$$
\n(17)

The physical quantities, the "Skin- friction"  $\,C_{_{fx}}$  ,(along x-axis),  $\,C_{_{fy}}$  (along y-axis), the "local Nusselt number"  $Nu_x$  and "Sherwood number"  $Sh_x$  provided as

$$
C_{fx} = \frac{\mu_{hnf} \left(\frac{\partial u}{\partial z}\right)_{z=0}}{\rho_f (ax)^2}, \quad C_{fy} = \frac{\mu_{hnf} \left(\frac{\partial v}{\partial z}\right)_{z=0}}{\rho_f (ax)^2},
$$
\n(18)

$$
Nu_x = -\frac{\chi k_{hnf}}{k_f (T - T_{\infty})} \left(\frac{\partial T}{\partial z}\right)_{z=0}, Sh_x = -\frac{\chi k_{hnf} \left(\frac{\partial C}{\partial z}\right)_{z=0}}{k_f (C - C_{\infty})}.
$$
\n(19)

## **3. Numerical Procedure**

An aimed at numerically solve the associated non-linear ODEs Eq. (11) - Eq. (14) with their boundary conditions provided in Eq. (15), the shooting process is used. The technique begins by simplifying the boundary-constrained system of Eq.  $(11)$  – Eq.  $(14)$  to first-order equations. The "bvp5c" MATLAB program was built to resolve the Eq. (11) through Eq. (14) along the boundary conditions using a shooting strategy Eq. (15). The boundary criteria are then used to choose starting predictions that are suitable. The shooting procedure is restated to ensure success in meeting the 10- 6 convergence requirement.

Now the following procedure have been taken.

i. In the system of higher order nonlinear ODEs, new variables are introduced in Eq. (11) - Eq. (14) *Fg.* (14)<br> *ii.*  $y(1) = f, y(2) = f', y(3) = f'', y(4) = g, y(5) = g', y(6) = \theta, y(7) = \theta'$  (20)

$$
\sf{ii}.
$$

$$
y(1) = f, y(2) = f', y(3) = f'', y(4) = g, y(5) = g', y(6) = \theta, y(7) = \theta'
$$
\n(20)

iii. The system of higher order nonlinear ODEs in Eq. (9) - Eq. (10) are reduced as

$$
iv.\\
$$

$$
f'=y(2),
$$

 $f'' = y(3)$ ,

$$
f'' = y(3),
$$
  

$$
f''' = \left( (y(2))^2 - (y(1) + y(4))y(3) - 2\varepsilon y y(5) + \frac{\sigma_{\text{lnff}}/\sigma_f}{\rho_{\text{lnff}}/\rho_f} M.y(2) \right) \frac{\rho_{\text{lnff}}/\rho_f}{\mu_{\text{lnff}}/\mu_f},
$$
(21)

$$
g'=y(5),
$$

$$
g' = y(5),
$$
  
\n
$$
g'' = \left( y(5)y(5) - (y(1) + y(4))y(6) + 2\left(\frac{\varepsilon}{\gamma}\right)y(2) + \frac{\sigma_{\text{lnff}}/\sigma_f}{\rho_{\text{lnff}}/\rho_f} M.y(5) \right) \frac{\rho_{\text{lnff}}/\rho_f}{\mu_{\text{lnff}}/\mu_f},
$$
\n(22)

$$
\theta'=y(7),
$$

$$
\theta' = y(7),
$$
\n
$$
\theta'' = \left( \frac{(\rho_{cp})_{_{hnf}}}{(\rho_{cp})_{_f}} (y(1) + y(4)) y(10) - \delta y(7) + \left( \frac{D u.Sc.}{(1 - \phi_1)(1 - \phi_2)} (y(1) + y(4)). y(10) - Rcy(9)) \right) \right)
$$
\n
$$
\left( \Pr \left( \frac{k_{_{hnf}}}{k_f} + \frac{4R}{3} - \frac{D u.Pr.Sc.Sr}{(1 - \phi_1)(1 - \phi_2)} \right) \right),
$$
\n(23)

 $\phi' = y(9)$ .

$$
\phi'' = \frac{Sc}{(1 - \phi_1)(1 - \phi_2)} (Sr\theta'' + (y(1) + y(4), y(10) - Rcy(9))).
$$
\n(24)

## v. The boundary conditions are

The boundary conditions are  
\n
$$
y_a(1) = 0
$$
,  $y_a(2) = 1$ ,  $y_a(4) = 0$ ,  $y_a(5) = \lambda$ ,  $y_a(7) = 1$ ,  $y_a(9) = 1$ .  
\n $y_b(2) = 0$ ,  $y_b(4) = 0$ ,  $y_b(6) = 0$ ,  $y_b(8) = 0$ . (25)

Where a, b denotes the position of the sheet at  $\eta = 0$ .

vi. The Cfx, Nux, Shx transformed into the following form:

$$
\text{Re}^{\frac{1}{2}}C_{fx} = \frac{1}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}} f''(0), \quad \gamma^{-1} \text{Re}^{\frac{1}{2}}C_{fy} = \frac{1}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}} g''(0),
$$
\n
$$
\text{Re}^{\frac{-1}{2}}Nu_x = -\frac{k_{\text{inf}}}{k_f} \theta'(0), \qquad \text{Re}^{\frac{-1}{2}}Sh_x = -\frac{k_{\text{inf}}}{k_f} \phi'(0).
$$
\n(26)

\nHere Re is given as, Re =  $\frac{U_w x}{V_f}$ .

vii. For nonlinear ODEs in Eq. (20) to Eq. (24) with boundary conditions in Eq. (25), use the MATLAB technique in bvp5c solver.

The advantage of this approach converts the boundary value problem to get the initial conditions by the boundary conditions as a function of multivariate of the beginning condition at some points. In order to achieve the far field boundary condition asymptotically, the computed boundary values should achieve the real boundary values.

# **4. Results and Discussions**

In this study, we apply MHD to the 3-D flow of a rotating HNF across a stretched surface with Soret and Dufour repercussions. CuO nanoparticles with volume fraction  $\phi_{\!\scriptscriptstyle 1}$  are scattered on the base fluid H<sub>2</sub>O, resulting in CuO-H<sub>2</sub>O nanofluid. To create the CuO-Ag/H<sub>2</sub>O hybrid nanofluid, silver with volume fraction  $\phi_2$  is mixed with CuO-H<sub>2</sub>O nanofluid. To get insight into the issue, the non-linear differential equations have been numerically evaluated. Figure  $2$  – Figure 16 visually depict the impact of relevant physical factors on velocity, temperature, and concentration sketches.

The effects of volume fractions on velocities and temperature constituents  $f'(eta)$ ,  $g'(eta)$ ,  $\theta'(\eta)$  are represented in the following Figure 2 to Figure 7 respectively. The velocity profiles  $f'(\eta)$ ,  $g'(\eta)$  are evident from the graphs that it elevates when volume fraction parameters  $\phi_1$ ,  $\phi_2$  increase for hybrid nanofluid (CuO-Ag/water) from Figure 2 - Figure 5.



**Fig. 4.** Performance of  $\phi_2$  on  $f'(\eta)$ 



**Fig. 5.** Performance of  $\phi_2$  on  $g'(\eta)$ 

The temperature profile  $\theta(\eta)$  in Figure 6 and Figure 7 declines when the values of  $\phi_1$ ,  $\phi_2$  are increased due to the fluid transferring heat more slowly as a result of its larger viscosity, which also makes it more resistant to heat. This results in the removal of heat from the fluid more efficiently, thus reducing the fluid temperature. Finally, nanoparticles in the fluid cause turbulence, which enhances the heat transfer by stirring the fluid and increasing heat transfer. All these factors combined lead to the decreased HNF (CuO-Ag/water) temperature profile with a rising volume fraction.



By Figure 8, for M = 0.7,0.8,0.9 the velocity is decreased because when a transversal magnetic regime is applied, a drag force is created that is analogous to a Lorentz force, this force seems to oppose the flow of fluid, slowing it down.  $g'(\eta)$  is also increasing with M = 0.7,0.8,0.9, observed by Figure 9. We illustrate that an increment in R (radiation) causes the increment in temperature in

Figure 10. Physically, the thermal radiation transfer with conduction heat transfer is measured by the *f k* χ





Schmidt number impact on the concentration profile is demonstrated in Figure 11. Since the mass diffusivity of momentum is quantitatively related to the Schmidt number. Because of the Schmidt number, the diffusivity drops, which lowers the fluid's concentration.



From Figure 12, the values 0.1, 0.2, 0.3 of chemical reactions (Rc) lead to a decline in the concentration of the fluid and concentration boundary layer thickness increased.



From Figure 13 and Figure 14, it is shown that a rise in the Dufour number (0.1, 0.4 0.6) results in a cooling of the temperature distribution. As a consequence, the fluid gets less heat and its viscosity rises, the concentration sketch slightly improves owing to reduced friction, turn, boosts the concentration.





The typical thermophysical characteristics of a HNF are listed in Table 1, whereas these characteristics are shown in Table 2, at 25<sup>0</sup> C. While calculating  $-f''(0)$ ,  $-g''(0)$ ,  $-\theta'(0)$  and  $-\phi'(0)$ in Table 3 and Table 4, we have taken  $\phi_1 = 0.1, \phi_2 = 0.1, Sr = 0.5, \varepsilon = 0.1, \gamma = 0.09, M = 0.5, R = 0.5,$ the typical thermophysical characteristics of a finite are listed in Table 1, whereas these<br>characteristics are shown in Table 2, at 25<sup>0</sup>C. While calculating  $-f''(0)$ ,  $-g''(0)$ ,  $-\theta'(0)$  and  $-\phi'(0)$ <br>in Table 3 and Table 4, are same.

#### **Table 1**





Table 3 represents that the skin friction coefficients of HNF both,  $x$ ,  $y$ -directions is increasing with np volume fractions. The  $C_{f_x}$  is amplifying and  $C_{f_y}$  is in opposite behaviour for increasing Du, M. The skin friction is reducing in  $x$ -direction and intensifies in y-direction by the increment of rotation parameter (ε). There is no effect on  $C_{f_x}$  by increment of Sr, R,  $C_{f_y}$  is decreasing for Sr, and is increasing for Rc.

**Table 3**

Effect of  $-f''(0)$ ,  $-g''(0)$  on hybrid nanofluid (CuO-Ag/Water)

$\phi_{\scriptscriptstyle 1}$	$\phi_{\scriptscriptstyle 2}^{\phantom{\dag}}$	Du	Sr	M	Rc	$\lambda$	$\boldsymbol{\mathcal{E}}$	$-1$	$^{-1}$
								$\frac{(1-\overline{\phi_1})^{2.5}(1-\phi_2)^{2.5}}{1-\overline{\phi_1}} f''(0)$	$\frac{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}}{(1-\phi_2)^{2.5}} g''(0)$
0.02								2.168377	3.706013
0.03								2.213696	3.808651
0.04								2.239256	4.110922
	0.01							1.971499	3.343463
	0.02							2.032835	3.482947
	0.03							2.094840	3.623493
		0.1						2.558169	4.604153
		0.2						2.558170	4.604145
		0.3						2.558170	4.604143
			1.5					2.558169	4.604156
			2.0					2.558169	4.604154
			3.0					2.558169	4.604154
				0.7				2.694248	4.430973
				0.8				2.755653	4.426886
				0.9				2.814308	4.405777
					0.02			2.558169	4.604156
					0.03			2.558169	4.604155
					0.04			2.558169	4.604154
						0.2		2.306125	2.886370
						0.5		2.418121	3.260842
						0.9		2.526529	4.345198
							0.03	2.652571	3.031807
							0.04	2.642538	3.217904
							0.06	2.620893	3.609773

Effect of $-\theta$ (0), $-\phi$ (0) on (CuO-Ag/Water) hybrid hano fluid									
$\phi_1$	$\phi_2$	Du	Sr	M	Rc	λ	ε	$\frac{k_{\textit{hnf}}}{\theta} \theta'(0)$ $k_f$	$k_{\ensuremath{\textit{inf}}\xspace}$ $\phi'(0)$ $k_f$
0.02								3.116592	0.147002
0.03								3.106912	0.145502
0.04								2.890108	0.144002
	0.01							3.194824	0.148502
	0.02							3.174026	0.147002
	0.03							3.153209	0.145502
		0.1						3.047379	0.135008
		0.2						3.047432	0.135004
		0.3						3.047449	0.135003
			1.5					1.39586	0.135001
			2.0					0.901102	0.135000
			3.0					0.662618	0.135000
				0.7				3.14062	0.135002
				0.8				3.162121	0.135002
				0.9				3.174970	0.135002
					0.02			0.908802	0.135001
					0.03			0.953357	0.135001
					0.04			0.997913	0.135001
						0.2		2.759282	0.135001
						0.5		2.958639	0.135001
						0.9		2.997022	0.135001
							0.03	3.429220	0.135000
							0.04	3.399714	0.135002
							0.06	3.329759	0.135002

**Table 4** Effect of  $-\theta'(0), -\phi'(0)$  on (CuO-Ag/Water) hybrid nano fluid

 From Table 4 We notice that by enhancing nanoparticle volume fractions, and soret, diminishes Nusselt number, Sherwood number. The Nusselt number is elevates with Du, M, Rc,  $\lambda$  increment. There is no influence of M, Rc on Sherwood number. Local Nusselt number is increasing with Du, M, Rc, λ. Finally, Nusselt number is decreasing and Sherwood number is increasing with rotation parameter. Table 5 depicts validation of results of skin friction with earlier published work references Hayat *et al.,* [10], Butt *et al.,* [32] and observed that splendid agreement done.



The investigation shows that there is excellent agreement between the current computations and the literature that has been published.

# **5. Conclusions**

 In this research, we do a computational study of the magnetohydrodynamics of a threedimensional flow of a rotating HNF (CuO-Ag/Water) across a stretchable surface subject to Soret and Dufour impacts on the magnetic regime. Positively monumental findings are as follows:

- i. The escalation in the volume fractions of CuO ( $\phi_{\!\scriptscriptstyle (}1)$  and Ag ( $\phi_{\!\scriptscriptstyle 2}$ ) nanoparticles leads to the enhancement of the velocities along the x-axis and y-axis.
- ii. For the increasing values (0.5, 0.7, 0.9) of Schmidt number, (0.1, 0.2, 0.3) Chemical reaction the concentration profiles drop.
- iii. The temperature boundary layers are enlarged with the rise of Thermal radiation.
- iv. The concentration and temperature profiles decline with the values of Dufour and for Soret the temperature declines as well as the concentration rises.
- v. The present research work is 99.9% consistent with earlier work Butt *et al.,* [32] for the variable  $\lambda$  = 0, 0.1, 0.2, 0.3, 0.4, 0.5 values.

#### **6. Future Scope**

In the future years, researchers are prepared to explore a wide range of properties related to the magnetohydrodynamics of a three-dimensional flow of a rotating HNF (CuO-Ag/Water) across a stretchable surface impact on the magnetic regime. This complete study will include aspects such as the chemical reaction, subject to Soret and Dufour effects. This work will be carried out to exploring the characteristics of hybrid nanofluids in regulated temperature conditions might give useful insights for a number of energy-related applications.

Studying the characteristics of hybrid nanofluids in regulated temperature conditions might give useful insights for a variety of energy-related applications includes concentrating solar power systems, optimizing heat transfer efficiency, reducing energy usage, and more.

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## **Conflict of interest**

The authors declare that they have no conflict of interest.

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