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Effects of Throughflow and Gravity Modulation on Thermal Convection in a Couple Stress Fluid Saturated Porous Layer

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ABSTRACT

In this paper, we have investigated the effects of throughflow and gravity modulation on a couple stress fluid saturated porous layer using non-autonomous Ginzburg-Landau model. A small variation of disturbances has been considered to examine the nonlinear thermal instability in a couple stress fluid saturated porous media. At third order, the finite amplitude of convection has been calculated which determines heat transfer. The effect of throughflow i.e., inflow and outflow have dual nature of heat transfer. The couple-stress parameter has stabilizing nature on thermal instability. Further it is found that upward directed flow enhances and downward directed flow diminishes heat transfer.

1. Introduction

The fundamental interest in the field of the natural convection in a fluid saturated porous media is because of its real world applications, such as insulation of reactor vessels, use of geothermal energy, improved petroleum reservoirs recovery, storage for nuclear waste and ceramic processing, fabrication of polymers etc. A numerous study on gravity modulation have been documented by Bhadauria *et al.,* [1-3], Bhadauria and Kiran [4-9] and Govender [10, 11]. Chand *et al.,* [12] investigated the onset of thermal instability in the couple-stress nano fluid-saturated porous layer. They found that the couple-stress and porous parameters on the onset of convection, shows stabilizing effect, whereas nanoparticle variables have destabilizing effect. The first studies of the effects of gravitational modulation on the stability of thermal fluid layer were conducted by Gresho and Sani [13]. According to their findings, G-jitter improves the density of the layer that is being heated from below. The enormous amount of research that went into this area is extensively described in works of literature [14-16]. As a result, each of these applications, in addition to the case that porous material can be found in a variety of natural situations, various research has been conducted to investigate the consequences of various concepts associated with such media. We can also see from other research related to periodic stimulation of boundaries across time [17]. Kiran [18,

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19] and Kiran and Bhadauria [20, 21] studied the first nonlinear investigations on the throughflow over gravity modulation. They have been investigated several designs for double diffusional convection with modulated gravity or temperature fields of media. Kuqali *et al.,* [22] and Shivakumara *et al.,* [23] investigated throughflow and nonlinearity nature which is caused by the interaction of incompressible flow with temperature or the velocity and energy equations. Malashetty and Swamy [24], Malashetty and Padmavathi [25], Siddheshwar *et al.,* [26], Yang [27], and Nelson [28] have conducted numerous experimental and theoretical investigations dealing with materials processing and fluid physics in microgravity aboard a rotating spaceship. Nield and Bejan [15] provide us with an outstanding review of the majority of this research. Evaluation of coupled stress in comparison to classical Cauchy stress has led the current development of a number of fluid micro continuum concepts. Sharma and Sharama [29] investigated the instabilities of such a porous medium saturated with the couple-stress flow. They determined that its stability exchange equation is effective for the couple-stress fluids and that couple-stress control the onset of stationary which is extended by this parameter. During the last few years, the analysis of non-Newtonian rheological working fluid in the porous layer has attracted a lot of attention due to their practical importance in engineering filed and in industries, particularly in the extraction of crude oil through petroleum resources [29].

Additionally, non-Newtonian fluid flow through porous layer is appropriate for a variety of applications, including polymer formation, lubrication, and undesired removal drives. In the range of non-Newtonian liquids, couple stress liquid has different features, such polar properties in accumulation and the ability to maintain high viscosity. Srivastava *et al.,* [30] analysed the local effect of an internal heat source on double-diffusive thermal convection in a couple-stress fluid during an anisotropic porous medium. They noticed that when convection is arrived, the internal heat parameter increases. Sunil and Chandel [31] found that effect of an induced on the magnetic force is onset of convective motion using couple-stress fluids contained in a porous matrix. They provided a suitable condition for non-appearance of oscillatory convection. Stokes theory of couple stresses in fluids was introduced by Stokes [32]. Stokes, proposed the couple stress concepts for liquids and it represents the classical hypothesis.

Many scholars have investigated the thermal conduction in the porous media a couple-stress fluid has been saturated using Stokes [32] by considering the viscosity that is consistent into effect. Modulated gravity's effect on a convectively stable form can have a major impact on a system's stability by enhancing or diminishing its convective sensitivity. Umavathi and Malashetty [33] studied the Oberbeck convective flow of couple stress fluids in the presence of porous layer. They observed that the flow is governed by both the porosity and the coupling stress characteristics. Furthermore, with variable thermal conductivity in porous media for temperatures-dependent viscosity and couple-stress fluid is equally crucial in geophysical and with other applications [15, 30, 34]. When the system is vibrating vertically, a modified complex body force becomes significant. The corresponding buoyancy forces, which are formed when a density gradient combines with a force of gravity, have a complex spatial and temporal structure.

In space experimental studies, the time-dependent gravity field is interesting other uses include crystal formation & large-scale atmospheric convection. According to Wadih *et al.,* [35] and Wadih and Roux [36] vibrations are possible significantly to improve or retard heat transport in the layer. Mahajan and Tripathi [37] and Tripathi and Mahajan [38] studied the effects of different types of basic temperature and concentration gradients on a layer of reactive fluid under variable gravity field are analyzed using linear and non-linear analysis. They explore the double-diffusive instability in a reactive fluid layer with velocity slip by supervising linear and nonlinear stability analyses. The linear analysis is performed with the help of normal mode technique; however, the nonlinear stability analysis is regulated using the energy technique. An investigation is made to analyse the effect of slip length and non-constant gravity field on the stability of the system.

Therefore, in this paper, the non-autonomous Ginzburg-Landau model was used to investigate the effect of throughflow with gravity modulation on such a couple of stress fluid-saturated porous layers. Small variations of disturbances have been considered to find nonlinear heat transfer in a couple stress fluid saturated porous layers. For the third order, the finite magnitude of convection has been determined. The effect of throughflow has dual nature on heat transfer for low values of couple-stress flow. On convective instability, couple-stress parameter has a stabilizing effect. Throughflow and gravity modulation effects have been studied in the presence of couple-stress parameter.

2. Governing Equation

We consider a non-Newtonian fluid-saturated infinitely extended horizontally porous media bounded within two boundaries that are completely free- free at z = 0 and z = d as heated from the bottom. ΔT is the fixed variation in temperature all over the porous media. We have used the reference in Cartesian terms with the origin at the bottom as well as z - axis moving upwards in a vertical direction. Its schematic diagram is shown in Figure 1.

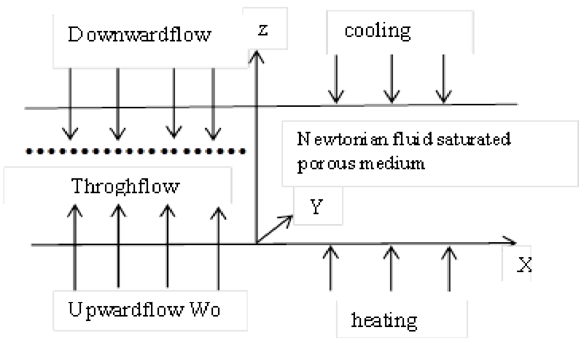

Fig. 1. A sketch of the Physical Problem

Here we consider throughflow in vertical directions along with z axis. Furthermore, we consider these assumptions are taken under Darcy Brinkman law and the Oberbeck Boussinesq approximations, the equations which represent the flow model are given by Bhadauria and Kiran [4] and Kiran and Bhadauria [20, 21].

$$\nabla.\vec{q} = 0, \tag{1}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial \vec{q}}{\partial t} = -\nabla P + \rho \vec{g} - \frac{\mu}{K}\vec{q} + \frac{\mu_c}{K}\nabla^2 \vec{q}, \tag{2}$$

$$\gamma \frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = k_T \nabla^2 T, \tag{3}$$

$$\rho = \rho_0\left[1 - \alpha_T (T - T_0)\right], \tag{4}$$

where $\vec{q}$ is velocity, K is permeability, μ is viscosity, P is pressure, μ_c is couple stress fluid, k_T is the coefficient of thermal expansion, ρ is density, T_0 is the temperature at which $\rho = \rho_0$ is the standard density and the heat capacity ratio, γ (here γ is taken unity for simplicity). The following are the externally imposed thermal and periodic gravity field:

$$T = T_0 + \Delta T \quad \text{at} \quad Z = 0, \tag{5}$$
$$T = T_0 \quad \text{at} \quad Z = d,$$

$$\vec{g} = g_0(1+\delta_2 \cos(\omega t))\hat{k}, \tag{6}$$

where δ_2 is magnitude of gravity modulation & ω is frequency of modulation.

Therefore, in this stage, the basic state is considered quiescent, with the following quantities:

$$\vec{q} = ((0,0,w_0(z)),\ \rho = \rho_b(z),\ P = P_b(z),\ T = T_b(z,t) \tag{7}$$

Substituting Eq. (7) into Eq. (1) to Eq. (4), obtain the following expressions, they help to define basic state of pressure and temperature:

$$\frac{dp_b}{dz} = \frac{\mu}{K} w_0 - \rho_b g, \tag{8}$$

$$w_0 \frac{dT_b}{dz} = k_T \frac{d^2 T_b}{dz^2}, \tag{9}$$

$$\rho_b = \rho_0[1-\alpha_T(T_b - T_0)], \tag{10}$$

The amplitude solution of the Eq. (9) when subjected to thermal boundary condition in Eq. (5) is provided by:

$$T_b = \frac{e^{Pez} - e^{Pe}}{1-e^{Pe}} \tag{11}$$

Disturbances of finite-amplitude are introduced to the solution of the basic state are:

$$\vec{q} = q_b + q',\ \rho = \rho_b + \rho',\ P = P_b + P',\ T = T_b + T'. \tag{12}$$

Since, we introduce two-dimensional convection stream function ψ as $(u',0,w') = \left(\frac{\partial \psi}{\partial z}, 0, -\frac{\partial \psi}{\partial x}\right)$ which satisfy Eq. (1) and following non-dimensional physical variables are rescaled by:

$$x^* = \frac{x}{d}, y^* = \frac{y}{d}, z^* = \frac{z}{d},\ p' = \frac{\mu k_T}{K} p^*,\ t' = \frac{d^2}{k_T} t^*,$$

$q' = \frac{k_T}{d} q^*$, $T = \Delta T T^*$, $\psi = k_T \psi^*$, and $\Omega^* = \frac{k_T}{d^2}\Omega.$

Substituting the Eq. (12) into Eq. (1) to Eq. (4), the resulting dimensionless governing system (dropping its asterisk *) by using the dimensionless variables stated above and eliminating the pressure gradient term:

$$\nabla^2\psi + \frac{1}{\Pr_D}\frac{\partial}{\partial t}(\nabla^2\psi) = C\nabla^4\psi - Rag_m\frac{\partial T}{\partial x}, \tag{13}$$

$$\frac{\partial T}{\partial t} - \frac{\partial \psi}{\partial x}\frac{\partial T_b}{\partial z} - (\nabla^2 - Pe\frac{\partial}{\partial z})T = \frac{\partial(\psi, T)}{\partial(x,z)}. \tag{14}$$

where $Pe = \frac{w_0 d^2}{k_T}$ is Peclet number, $\Pr_D = \frac{\phi v d^2}{K k_T}$ is Darcy Prandtl number, $Ra = \frac{\alpha_T g \Delta T d K}{v k_T}$ is thermal Rayleigh number, $g_m = (1+\delta_2\cos(\omega t))\vec{k}$, and $C = \frac{\mu_c}{\mu d^2}$, is the couple stress parameter. The Eq. (14) shows that the factor $\frac{\partial T_b}{\partial z}$, has been given in Eq. (11), the basic state solutions have an effect on the stability problem. Considering small change of time and re-scaling it as $\tau = \varepsilon^2 t$, the convection in a stationary mode is to be discussed. The linear and non-linear system of Eq. (13) and Eq. (14) may be represented in the matrix form as follows:

$$\begin{bmatrix} \nabla^2 - C\nabla^4 & Rag_m\frac{\partial}{\partial x} \\ -\frac{\partial T_b}{\partial z}\frac{\partial}{\partial x} & -\nabla^2 + Pe\frac{\partial}{\partial z} \end{bmatrix}\begin{bmatrix} \psi \\ T \end{bmatrix} = \begin{bmatrix} -\frac{\varepsilon^2}{\Pr_D}\frac{\partial}{\partial \tau}\nabla^2 \\ -\frac{\varepsilon^2}{\Pr_D}\frac{\partial T}{\partial \tau} + \frac{\partial(\psi, V)}{\partial(x,z)} \end{bmatrix} \tag{15}$$

To evaluate the solution of this Eq. (15), the impermeable stress-free heat transfer boundary condition is used by Bhadauria and Kiran [4] and Kiran [18, 19].

$$\psi = T = 0 \qquad \text{on} \quad Z = 0 \text{ and } Z = 1 \tag{16}$$

3. Heat Transport and Stationary Instability

In order to derive the solution and to resolve nonlinearity the following asymptotic solutions are given in the above Eq. (15) [5, 6, 18, 19]:

$$\left.\begin{aligned} Ra &= R_0 + \varepsilon^2 R_2 + ... \\ \psi &= \varepsilon\psi_1 + \varepsilon^2\psi_2 + \varepsilon^3\psi_3 + ... \\ T &= \varepsilon T_1 + \varepsilon^2 T_2 + \varepsilon^3 T_3 + ... \\ \delta &= \delta_0 + \varepsilon\delta_1 + \varepsilon^2\delta_2 + \varepsilon^3\delta_3 + ... \end{aligned}\right\} \tag{17}$$

In the absence of gravitational modulations, R_0 would be the critical Rayleigh number where convection starts. The statement δ is suitable with a basic state solution such that if δ_0 disappears at the lower order [4, 10, 11]. Further, in addition, δ_1 vanishes, the equations that were derived in order $\varepsilon\, and\, \varepsilon^2$ shows that the solution has a singularity. These findings show that gravity modulation effect must be provided at $\delta = \varepsilon^2\delta_2$ enabling consistency. Now, system will be studied for different orders of ε [4].

3.1 First Order System

The system uses the following format at the lowest level:

$$\begin{bmatrix} \nabla^2 - C\nabla^4 & Rag_m \dfrac{\partial}{\partial x} \\ -\dfrac{\partial T_b}{\partial z}\dfrac{\partial}{\partial x} & -\nabla^2 + Pe\dfrac{\partial}{\partial z} \end{bmatrix} \begin{bmatrix} \psi_1 \\ T_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{18}$$

Lowest-order solution accoriding to initial conditions, Eq. (16) evaluated as follows:

$$\psi_1 = A\sin(k_c x)\sin(\pi z), \tag{19}$$

$$T_1 = -\frac{4k_c\pi^2 A}{c(4\pi^2 + Pe^2)}\cos(k_c x)\sin(\pi z), \tag{20}$$

where $c = k_c^{\,2} + \pi^2$ is square of horizontal wave number. Onset of stationary convection is quantitatively determined by using value of critical Rayleigh number with the related wave number and expressions are given by:

$$R_0 = \frac{(c - c^2 C)(4\pi^2 + Pe^2)}{4\pi^2 k_c^{\,2}}, \tag{21}$$

$$k_c = \pi \tag{22}$$

3.2 System of Second Order

Now, the system adopts the following form:

$$\begin{bmatrix} \nabla^2 - C\nabla^4 & Rag_m \dfrac{\partial}{\partial x} \\ -\dfrac{\partial T_b}{\partial z}\dfrac{\partial}{\partial x} & -\nabla^2 + Pe\dfrac{\partial}{\partial z} \end{bmatrix} \begin{bmatrix} \psi_2 \\ T_2 \end{bmatrix} = \begin{bmatrix} R_{21} \\ R_{22} \end{bmatrix} \tag{23}$$

The following terms of RHS in present system defined as:

$$R_{21} = 0, \qquad \text{and} \tag{24}$$

$$R_{22} = \frac{\partial(\psi, T)}{\partial(x, z)}, \tag{25}$$

The solutions of second-order subjected to initial conditions as in Eq. (16) (assuming first-order solutions) of system are given by:

$$\psi_2 = 0, \tag{26}$$

$$T_2 = \frac{-2k_c^{\,2}\pi^3}{c(4\pi^2 + Pe^2)^2} A^2 \sin(2\pi z) + \frac{-Pek_c^{\,2}\pi^2}{c(4\pi^2 + Pe^2)^2} A^2 \cos(2\pi z). \tag{27}$$

For convection in a stationary mode, the horizontally averaged Nusselt number Nu is calculated as follows:

$$\begin{aligned} Nu &= 1 + \left[\frac{k_c}{2\pi} \int_0^{\frac{2\pi}{k_c}} \frac{\partial T_2}{\partial z} \partial x \right] \div \left[\frac{k_c}{2\pi} \int_0^{\frac{2\pi}{k_c}} \frac{\partial T_b}{\partial z} \partial x \right] \\ &= 1 + \frac{4\pi^4 k_c^{\,2}(e^{Pe} - 1)}{cPe(4\pi^2 + Pe^2)^2} A^2. \end{aligned} \tag{28}$$

In the situation of a porous media which is isotropic in the absence of fluid flow, the following conclusions are found in Eq. (21), Eq. (22), and Eq. (28) are presented by Bhadauria *et al.,* [2], Siddheshwar *et al.,* [26], and Lapwood [39].

3.3 System of Third Order

Now for this point system takes the form as:

$$\begin{bmatrix} \nabla^2 - C\nabla^4 & Rag_m \dfrac{\partial}{\partial x} \\ -\dfrac{\partial T_b}{\partial z}\dfrac{\partial}{\partial x} & -\nabla^2 + Pe\dfrac{\partial}{\partial z} \end{bmatrix} \begin{bmatrix} \psi_3 \\ T_3 \end{bmatrix} = \begin{bmatrix} R_{31} \\ R_{32} \end{bmatrix} \tag{29}$$

Here, terms of RHS are given by:

$$R_{31} = -\frac{1}{\Pr_D} \frac{\partial \nabla^2 \psi_1}{\partial \tau} - R_0 \delta_2 \cos(\omega\tau) \frac{\partial T_1}{\partial x} - R_2 \frac{\partial T_1}{\partial x}, \tag{30}$$

$$R_{32} = -\frac{\partial T_1}{\partial \tau} + \frac{\partial T_2}{\partial z}\frac{\partial \psi_1}{\partial x}, \tag{31}$$

Now, putting first-order and second-order solutions into the following Eq. (30) and Eq. (31) and easily we get the expressions for R_{31} and R_{32}. Under solvability condition, we get Ginzburg-Landau equation for existence of third order system. The Ginzburg-Landau expression is given by:

$$Q_1 \frac{dA(\tau)}{d\tau} - Q_2(\tau)A(\tau) + Q_3 A(\tau)^3 = 0, \tag{32}$$

where Q_1, Q_2, Q_3 are coefficients:

$$Q_1(\tau) = \left(\frac{c - c^2 C}{\Pr_D} + \frac{4R_0 \pi^2 k_c^{\ 2}}{c^2(4\pi^2 + Pe^2)} \right),\ Q_3 = \frac{2R_0 \pi^4 k_c^{\ 4}}{c^2(4\pi^2 + Pe^2)^2},$$

$$Q_2(\tau) = \left(\frac{4\pi^2 k_c^{\ 2}}{c^2(4\pi^2 + Pe^2)} \left[R_0 + R_2 \delta_2 \cos(\Omega \tau) \right] \right).$$

Eq. (32) is known as Bernoulli equation, because of its non-autonomous structure, finding an analytical solution is very difficult in the presence of modulation. As a result, it was numerically solved by using Mathematica 12.0 built-in function ND Solution, when necessary initial condition at $A_0 = a_0$ where a_0 is defined as present initial convection magnitude. Its analytic solution of Eq. (32) for such an un-modulated case is as follows:

$$A = \frac{1}{\sqrt{\left(\frac{Q_3}{Q_2} + C_1 e^{\left[-\frac{2Q_2}{Q_1}\tau \right]} \right)}}, \tag{33}$$

where Q_1, Q_3 as same in Eq. (32), $Q_2(\tau) = \left(\frac{4R_2 \pi^2 k_c^{\ 2}}{c^2(4\pi^2 + Pe^2)} \right)$ and C_1 is an integration constant which appears in Eq. (33), which can be obtained by adopting appropriate boundary conditions.

4. Results and Discussions

In this paper, we have investigated the effects of throughflow and gravity modification on convective instability in a porous media saturated with a couple stress fluid. To study the effects of gravity modulation and vertical throughflow on coupled stress fluid, the nonlinear study was made. The transport analysis of nonlinear phenomena has been evaluated with the help of Landau equation. There is only minimal amount of amplitude gravity modulation has been considered for Benard - Darcy convection where effects of gravity modulation had been assumed for third order O (ε^2). The objective for weakly nonlinear theory is to investigate transmission of heat in ways that a linear study could not. In order to study transfer of heat in porous media external regulations are important. The purpose of this study is to consider such a gravity modulation and vertical throughflow. Because, here we examine small amplitude modulation on heat transport, then the value of $\boldsymbol{\delta}_2$ may be very small, around 0.1. Furthermore, because low - frequency has the maximum effect on convection and heat transfer is more. The amplitude of gravitational modulation is considered to be minimal.

The numerical values for Nu derived from the expression in Eq. (28) were calculated on solving the amplitude Eq. (32) and that are in Figure 2 to Figure 7 results have been presented. It is seen that expression in Eq. (28) is in conjunction as for Eq. (32), in this case Nu is function of the system's variable. Figures 2 to Figure 7 indicates the effect on each variable of heat transfer, with plots of Nusselt number Nu vs. τ . The figures show that value of Nu starts at 1 and stays steady for a long period, indicating that the conduction is present in the system. Then number Nu increases over time, showing convections, and then increases again, representing the steady state.

Figure 2 shows that, as increase of Darcy-Prandtl number, the value of Nu increases, for lower values of time and values of Pr_D, effects are clear. As result of heat transfer, the effect diminishes and heat - transfer remains stagnant with increment of time. Further, the analysis of Pr_D may be compared with the following references; Bhadauria *et al.,* [1, 2], Kiran and Bhadauria [20], Bhadauria and Kiran [4].

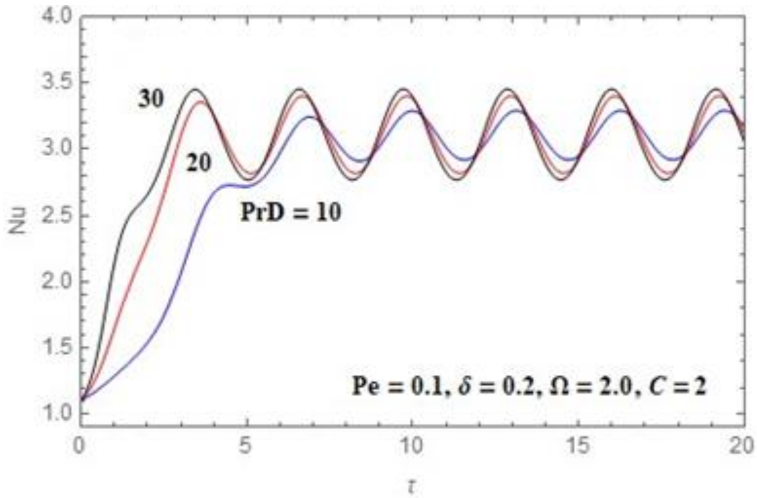

Fig. 2. Effect of Nusselt number for various values of Darcy-Prandtl number

In Figure 3, we investigate the effects of Pe on heat exchange for downwards and upwards throughflows, wherein upwards throughflow (Pe > 0) is destabilizing and downwards throughflow (Pe < 0) is stabilizing. Similar results are found by a Nield and Bejan [15] for a small amount of throughflows in a fluid layer. These observations are computable by Shivakumara *et al.,* [23] and Suma *et al.,* [40]. According to them, the destabilising effect could be caused by the throughflow changing the linear to the nonlinear transformation of the stable basic temperature variation.

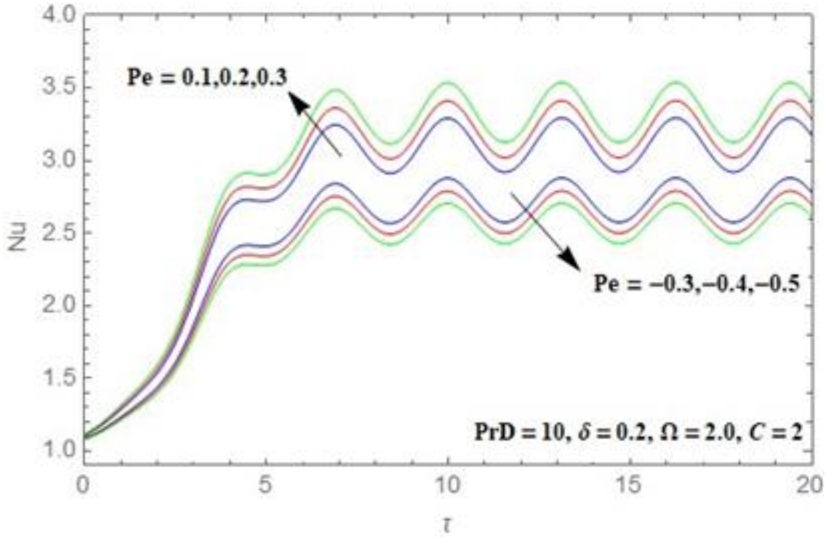

Fig. 3. Effect of Nusselt number for various values of Peclet number

The basic state temperature is a measure of this, and it can be read as the rate at which energy is transferred into the disturbances caused by the interactions of perturbed thermal convection with the basic convective motion for temperature difference.

It is important to note that Eq. (21)'s critical Rayleigh-Darcy number will be even function of Pe. Upon increasing Peclet number, as R_0 increases the beginning of convection is postponed due to through flows.

In Figure 4, we observe that the magnitude of Nu increases as that the modulation amplitude increases, resulting in increased transport of heat. The effect of amplitude on heat transport is in respect of following.

$$Nu_{\delta_2=0.2} < Nu_{\delta_2=0.3} < Nu_{\delta_2=0.4}$$

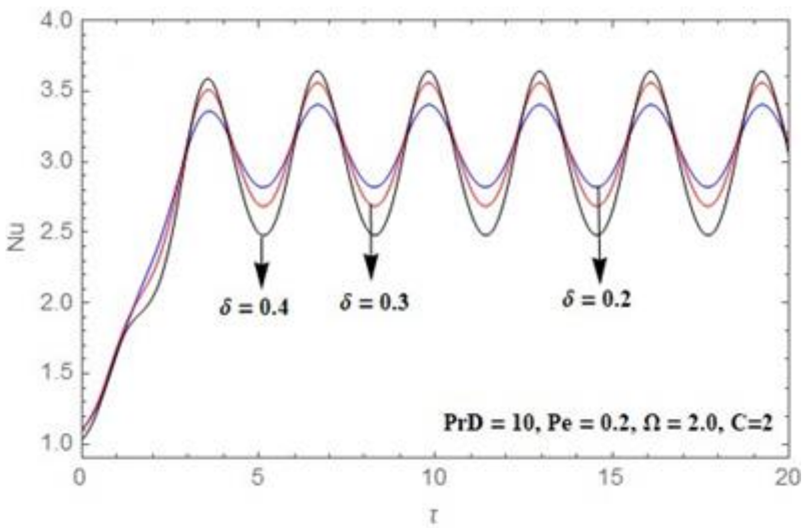

Fig. 4. Effect of Nusselt number for various values of amplitude modulation parameter

In Figure 5, we found that the magnitude of Nu diminishes as that the modulated frequency is increase.

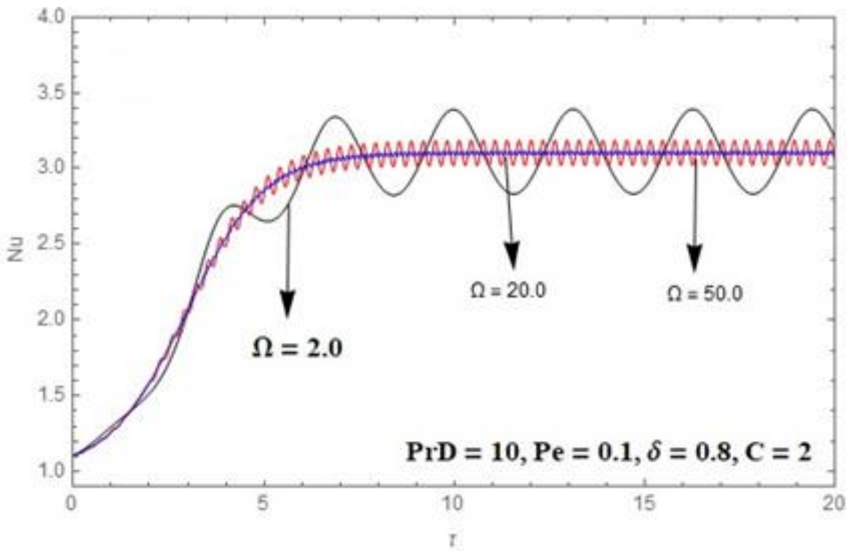

Fig. 5. Effect of Nusselt number for various values of Darcy-Prandtl number

Here, gravity modulation effects on thermal convection which vanishes completely at higher frequency.

The above conclusions are very similar to the temperature modulation linear theory as results, where change in critical Rayleigh number at higher frequencies is due to the thermal modulation reduces the frequency almost zero, as a result it holds [41-43].

In research of Malashety and Padmavathi [25], Bhadauria and Kiran [5, 6], Kiran [19], Kiran and Bhadauria [43], and Kiran and Narasimhulu [44], gravity modulation is described for Newtonian or non-Newtonian fluids with a stationary or oscillatory convective mode. For a given wave number, the following can be observed:

$$Nu_{\Omega=50} < Nu_{\Omega=20} < Nu_{\Omega=2}.$$

Figure 6 shows that the effects of Pe > 0 (=1) is as an example of upward throughflow. The system is now more stable in this condition than when there is no horizontal throughflow, the opposite conclusions are obtained in a situation of downwards throughflows Pe < 0 (Pe =-1). The fact that the horizontal Rayleigh Number Pe does have an even function whereas the average Nusselt number is the result of the odd Pe function provided in Eq. (28). As a consequence, the amplitude of the Nusselt number has an effect on the convection problem, these results are found in a study by Kiran [42]. The author may note that heat transfer results are comparable to Rebhi *et al.,* [45], Khan *et al.,* [46], Najib *et al.,* [47], and Mahadi *et al.,* [48] for different heat transfer modes.

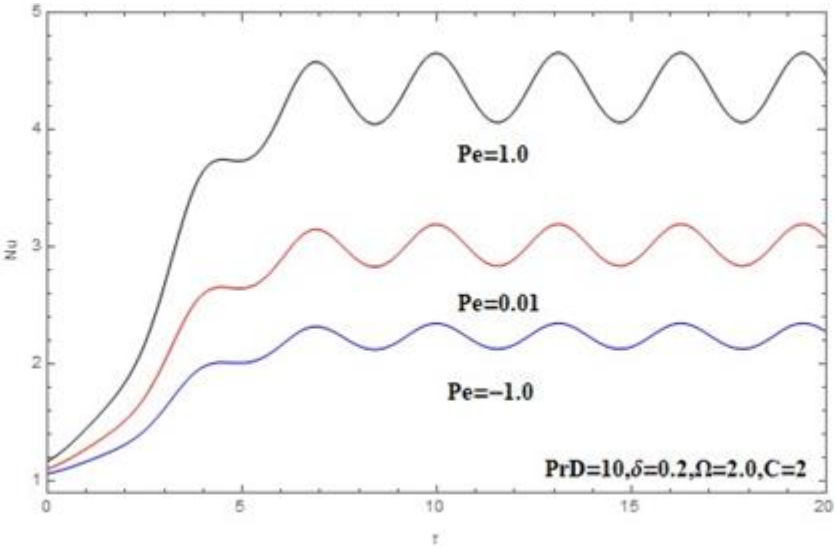

Fig. 6. Effect of Nusselt number for various values of Peclet number

An evaluation of the numerical solution to the current problem may also be found in Figure 7. We see that the amplitudes of Nusselt numbers are smaller in un-modulated case than those in case with a modulation.

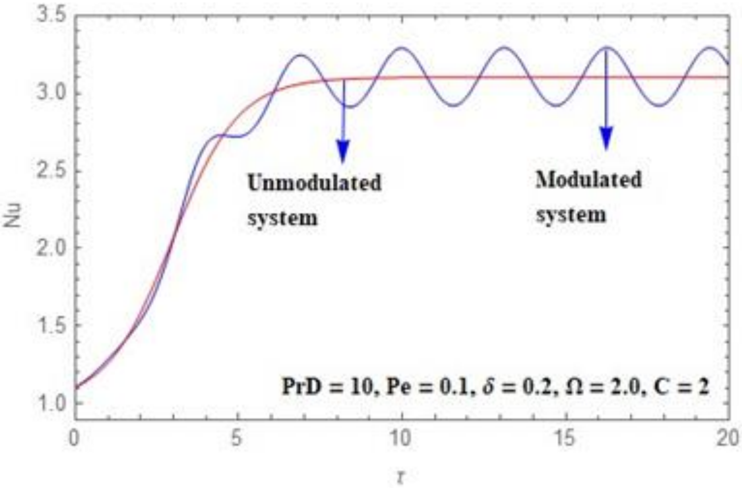

Fig. 7. Effect of Nusselt number for modulated and unmoulded systems

Figure 8 can be observed that, Nu increases with increase of couple stress parameter, for small values of C and shorter time period, the effect is obvious.

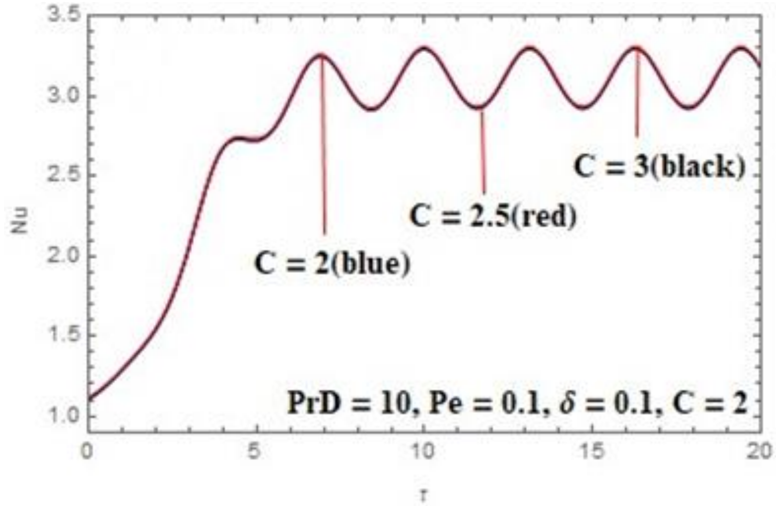

Fig. 8. Effect of Nusselt number for various values of couple stress parameter

Hence, thermal conduction further, as time passes, the effect diminishes. These results are quite interest that couple stress parameter reduces the velocity profile of the flow for linear models. But, in the present nonlinear study due to interaction with flow velocity there is a reverse trend in transport phenomenon. We have drawn stability curves (Figure 9 and Figure 10) to check onset thermal instability in the layer. We see that effect of couple stress parameter C and Peclet number Pe on stability curves. Both figures show that for linear case, C and Pe stabilizes the system upon enhancement. The trend is reverse in heat transfer case is due to nonlinearity flow model. Rayleigh number gives assumption that stability region and onset instability. The reader may note that both throughflow and couple stress fluid stabilizes for linear and destabilize for nonlinear case.

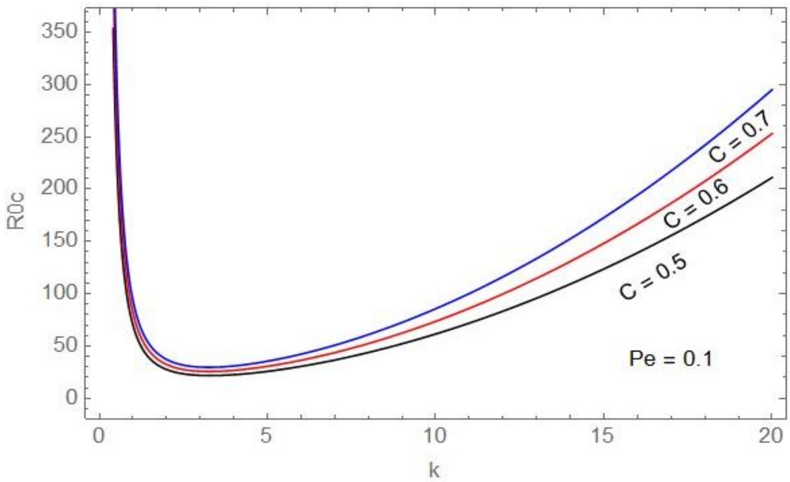

Fig. 9. Effect of C on Stability curves for Rayleigh number versus wavenumber

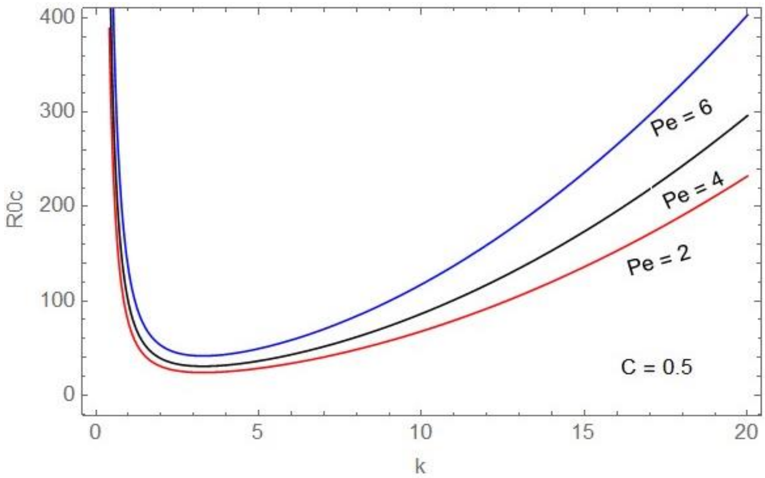

Fig. 10. Effect of Pe on Stability curves for Rayleigh number versus wavenumber

5. Conclusions

We have analysed the effect of throughflow and gravitational modulation on couple stress fluids saturated porous layer. Ginzburg-Landau model has been used to find finite amplitude. The conclusions of the above results are as follows:

i. The effects of Pr_D are that they increase heat transfer at lesser values of time and disappear at larger values of time.
ii. Throughflow enhances heat transfer in the upwards direction (Pe > 0) and the converse in the downwards direction (Pe < 0).
iii. Heat transfer in porous media decreases as the modulation frequency increases
iv. At large values of Ω, the effect of g-jitter gets negligible.
v. When value of the couple stress parameter (C) increases heat transfer (Nu).
vi. The variation of C is not visible for smaller values of time t and for larger values of time heat transport enhances.

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