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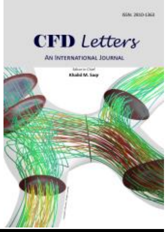


# Dual Solutions of Magnetohydrodynamics $Al_2O_3$+Cu Hybrid Nanofluid Over a Vertical Exponentially Shrinking Sheet by Presences of Joule Heating and Thermal Slip Condition

Muhammad Sajjad[1], Ali Mujtaba[2], Adnan Asghar[3,*], Teh Yuan Ying[3]

[1] Department of Mathematics & Physics, Riphah International University Faisalabad Campus, 44000 Faisalabad, Pakistan
[2] Department of Mathematics & Physics, University of Lahore, 53700 Lahore, Pakistan
[3] School of Quantitative Sciences, UUM College of Arts & Sciences, Universiti Utara Malaysia, 06010 UUM Sintok, Kedah Darul Aman, Malaysia

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**ABSTRACT**

Hybrid nanofluid has an extensive range of real-world applications. Hybrid nanofluid is a new and advanced nanofluid modification extensively used to increase thermal efficiency in fluid flow systems. The main objective of this research is to study magnetohydrodynamics hybrid nanofluid flow numerically in two dimensional over a vertical exponentially shrinking sheet, considering the effects of Joule heating and thermal slip condition. Furthermore, using the Tiwari-Das model, the influence of the suction parameter on variations of reduced skin friction and reduced heat transfer is also explored. The hybrid nanofluid in this research is an $Al_2O_3$+Cu/water hybrid nanofluid, in which water is the base fluid, and two types of solid nanoparticles, namely Alumina ($Al_2O_3$) and copper (Cu), are combined together. The governing partial differential (PDEs) equations are transformed into the ordinary differential equations (ODEs) using exponential similarity variables. The resulting ordinary differential equations (ODEs) are numerically solved using the three-stage Labatto III-A technique in the "MATLAB software's" bvp4c solver. Hybrid nanofluids have greater thermal efficiency than nanofluids and base fluid. Dual solutions are obtained in specified ranges of suitable parameters. The temperature profile $\theta(\eta)$ rises in both solutions as the Eckert value enhances. Besides, In the first and second solutions, the thermal boundary layer thickness decreased gradually as the thermal slip parameter increased. Finally, the conclusions presented that solution duality exists when the suction parameter $S \geq S_{ci}$, while no flow of fluid is possible when $S < S_{ci}$.

## 1. Introduction

Fluid dynamics study has attracted the interest of experts, scholars, and researchers from numerous fields in recent decades, owing to its several applications in engineering, science, and technology. Sakiadis [1] initially suggested the notion of a boundary layer steady flow on a stretching sheet for two dimensional. Later, Crane [2] modernized the thought of Sakiadis, and then he applied

* *Corresponding author.*
*E-mail address: asgharadnan675@gmail.com (Adnan Asghar)*

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it to steady flow two-dimensional linear stretching and exponential surfaces. He stated that the velocity with which a surface is stretched from a slit is proportional to its distance.

Furthermore, Choi [3] identified nanofluid as a mixture generated by dispersed nanoparticles in a normal fluid. He defined nanofluids as heat transfer dispersions with better thermal behaviour than base fluids or conventional liquids. Nanofluids are composed of tiny quantities of solid particles measuring 100 nm or less in size. Nanofluids often include nanoparticles with unique chemical and physical properties, for instance, carbon nanotubes, carbides, oxides, and metals (Nada *et al.,* [4]). Water, organic liquids, polymeric solutions, oils, and other common fluids are examples of base fluids (Wang *et al.,* [5]). It has been discovered that nanofluids have better thermal efficiency than their basic fluids. Consequently, nanofluids have abundant real-world applications, including heat transfer. Agriculture, drug delivery, solar energy, refrigerators, aerospace, building cooling and heating, microchips, and automobiles are just a few examples of applications (Lund *et al.,* [6]). Khan and Pop [7] examined the first time numerically on a stretching surface in a nanofluid with two-dimensional steady boundary layer flow. Miklavčič and Wang [8] are credited with being the first to explore the viscous three-dimensional flow across a shrinking surface using the suction effect. Bachok *et al.,* [9] examined a nanofluid with an unsteady boundary layer flow over a shrinking/stretching layer. Moreover, Dero *et al.,* [10] considered the two-dimensional model of a single phase of Tiwari-Das steady flow over-stretching and shrinking sheet over the analysis of Casson-based nanofluid.

Researchers have attempted to incorporate numerous solid nanoparticles through various types of base fluids in response to the increasing demand for heat transfer rates from various aspects of the industry, as previously mentioned, as a result, a hybrid nanofluid, the newest type of nanofluid, was developed. According to Devi and Devi [11], combining a metal nanoparticle in a small quantity/nanotube with an oxide/suspension of metal nanoparticles in a base fluid will dramatically boost thermal properties. Common fluids usually have more than one solid particle (glycol, paraffin oil, water, vegetable oil, engine oil, kerosene, and ethylene). According to Huminic and Huminic [12] can be noticed significant applications of hybrid nanofluid, such as these liquids, have been used in a lot of heat transfer applications, including heat exchangers, microchannels, warmth pipes, tube-shaped heat exchangers, air conditioning systems, little channel heat sinks, etc. Radiation's effects on a hybrid nanofluid of nonlinear stretching/shrinking surface two-dimensional steady flow were explored by Waini *et al.,* [13]. Moreover, Waini *et al.,* [14] studied the influence of transpiration steady flow through a stretching/shrinking surface by the uniform shear flow on a two-dimensional hybrid nanofluid. Numerous scholars investigated the stretching/shrinking sheet on nanofluids and hybrid nanofluids in different consideration models [15-38].

Magnetohydrodynamics (MHD) is a term that combines the words magneto (magnetic), hydro (fluids), and dynamics (motion). The study of the flow of electrically conducting fluids in the existence of a magnetic field is described as magnetohydrodynamics. Since the magnetic field occurs anywhere in the universe, MHD phenomena can also occur when conducting fluids are present in natural phenomena. Engineers use MHD concepts in the project and design of an extensive range of applications in industries (e.g., heat exchangers, generators, flow meters, nuclear waste disposal, nuclear reactor cooling, geothermal energy extractors, and MHD pumps, etc. (Yashkun *et al.,* [39]). So, it is vital. Devi and Devi [11] numerically examined steady magnetic flow over a stretching sheet with suction and Newtonian heating effect in a hybrid nanofluid. Furthermore, with Newtonian heating, Devi and Devi [40] continued their study to a three-dimensional steady flow. They reported that hybrid nanofluid heat transfers rate more quickly than normal nanofluid. After that, Devi and Devi [41] investigated the steady two-dimensional flow of a hybrid nanofluid past a stretching layer. Aly and Pop [42] studied the two-dimensional steady flow past a stretching and shrinking plate using suction, convective boundary conditions, and MHD effects in a hybrid nanofluid. Waini *et al.,* [43] explored the influence of hybrid nanofluid on steady fluid flow in an exponentially stretching/shrinking layer. They noticed that suction effects and shrinking effects had dual solutions. Lund *et al.,* [44] studied unsteady flow stretching/shrinking sheets with influence radiation and MHD in a hybrid nanofluid. Asghar and Ying [45] studied the MHD hybrid nanofluid flow through the influence of Joule heating in a three-dimensional rotating stretching/shrinking surface.

Joule heating, also known as ohmic heating, resistance, or resistive heating, is the process of producing heat by conducting an electric current over a conductor. Incandescent light bulbs, resistance ovens, food processing gear, electric warmers, electric fuses, electric stoves, soldering irons, and cartridge heaters are all examples of Joule heating (Yashkun *et al.,* [39]). Additionally, Khashi *et al.,* [46] examined the two-dimensional steady flow of a hybrid $Al_2O_3$-Cu/water nanofluid associated with a radially stretching/shrinking sheet by Joule heating, MHD, and suction influences. Moreover, Yan *et al.,* [47] explored an MHD hybrid nanofluid flow that passes across an exponential sheet, including the impact of Joule heating. More studies have been done on boundary layer flow over MHD can be found in these studies [48, 49].

Many applications of slip conditions contain the formation of interior cavities and heart valves and the cleaning of artificial heart valves (Jamil *et al.,* [50]). The notion of slip impact on boundary flow was initially proposed by Andersson [51]. Hayat *et al.,* [52] investigated rotating steady flow with partial slip and radiation impacts. They noticed that raising the slip parameter reduces temperature and velocity values. As mentioned above, Yan *et al*., [47] investigated magnetized hybrid nanofluid flow through the exponential sheet and slip conditions and Joule heating. More citations on the influence of slip parameters are presented in the literature [53, 54].

Waini *et al*., [15] investigated a two-dimensional hybrid nanofluid flow across a vertical exponentially shrinking surface utilizing mixed convection without considering MHD, thermal slip condition, radiation, and joule heating effects. Besides, the influences of MHD, mixed convection, and joule heating were considered by Yashkun *et al.,* [39] in a two-dimensional hybrid nanofluid flow containing an exponentially stretching/shrinking sheet, but these effects were not taken into consideration, nor were the effect of thermal slip condition, nor thermal radiation. However, employing the Tiwari-Das model (see Tiwari and Das [55]), the current research aims to fill in the gaps described in Waini *et al*., [15] and Yashkun *et al.,* [39] by focusing on the effect of MHD, Joule heating, and thermal slip condition. The current research objective is to explore the two-dimensional magnetohydrodynamics $Al_2O_3$+Cu hybrid nanofluid over a vertical exponentially shrinking sheet by the existence of Joule heating and thermal slip condition. Alumina ($Al_2O_3$) and copper (Cu) nanoparticles are utilized for a hybrid nanofluid in this research to achieve excellent convective heat transfer efficiency. In order to create a hybrid nanofluid ($Al_2O_3$+Cu/water), two nanoparticles of Alumina ($Al_2O_3$) and copper (Cu) are combined. The velocity and temperature profiles for various values of volume fraction copper, magnetic, Eckert number, thermal slip parameters and Prandtl number are examined in this study. Besides, the effect of volume fraction copper against suction on the variations of reduced skin friction and reduced heat transfer is also included in this study. The existing numerical findings are compared to the results of prior investigations for comparison purposes. According to our knowledge, this model is different and novel, and no related article has been found in the literature.

## 2. Methodology

### *2.1 Formulation of Mathematical Model*

The two-dimensional Magnetohydrodynamics incompressible and steady flow on a vertical exponentially shrinking sheet in a hybrid nanofluid is stated in Figure 1. The sheet movement is defined by the x-axis, while the y-axis is perpendicular to it. In the x-axis manner, the surface velocity is $u = u_w = U_w\, e^{x/L}$ $L$ is the characteristic length while $U_w$ is constant of the sheet. Furthermore, $T_w(x) = T_0\, e^{2x/L} + T_\infty$, $T_w(x)$ is the surface's temperature, $T_\infty$ is free stream temperature and $T_0$ characteristic temperature. Besides, along the y-axis, a varying magnetic field is $B(x) = B_0 e^{x/2L}$ in which $B_0$ represents the magnetic field's constant strength.

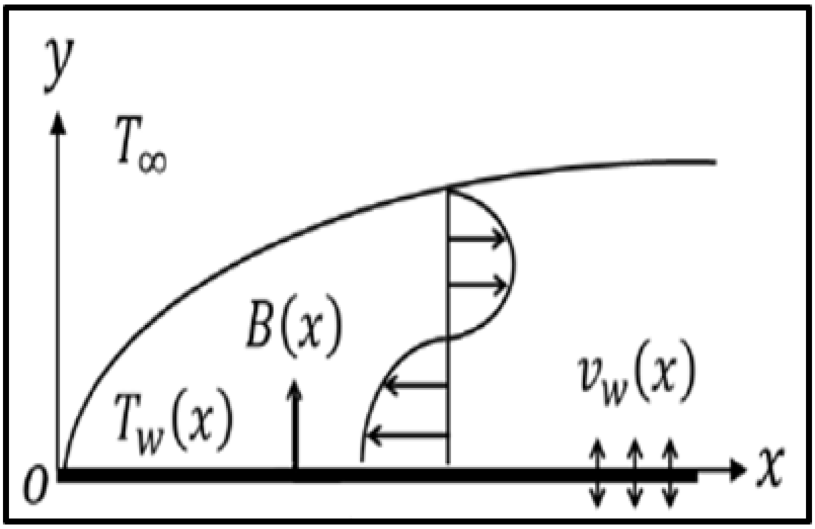


**Fig. 1.** The Physical model and coordinate system for shrinking sheet

The governing equation for a hybrid nanofluid is as follows (Waini *et al.,* [43]):

$$u_x + v_y = 0 \tag{1}$$

$$uu_x + vu_y = \frac{\mu_{hnf}}{\rho_{hnf}} u_{yy} - \frac{\sigma_{hnf}}{\rho_{hnf}} B^2 u, \tag{2}$$

$$uT_x + vT_y = \frac{k_{hnf}}{(\rho c_p)_{hnf}} T_{yy} + \frac{\sigma_{hnf}}{(\rho c_p)_{hnf}} B^2 u^2 \tag{3}$$

The boundary conditions are (Asghar *et al.,* [18]):

$$v = v_w(x),\ u = u_w,\ T = T_w + \varepsilon v_f (T_y) \ \text{ at } y = 0 \tag{4}$$
$$u \to 0,\ T \to T_\infty,\ \text{at}\ \ y \to \infty \tag{5}$$

The velocity components $u$, and $v$ along with the x-axis, and y-axis, respectively. However, $\varepsilon = \varepsilon_1\, e^{-x/2L}$ is the factor of thermal slip, and $\varepsilon_1$ signifies the initial values of the thermal factor. $T$ signify temperature. Hybrid nanofluid thermophysical characteristics were used to determine the above equation, as mentioned in Table 1 and Table 2. Solid nanoparticles volume fractions of Alumina ($Al_2O_3$) and copper (Cu) are expressed by $\varphi_1$ and $\varphi_2$ respectively. Moreover, $c_p,\ \mu,\ \rho,\ \sigma$ and $k$, which show the specific heat capacity, dynamic viscosity, density, electrical conductivity, and thermal conductivity, respectively. The fluid, nanofluid, hybrid nanofluid, solid nanoparticles 1 ( $\mathrm{Al_2O_3}$) and solid nanoparticles 2 (Cu) subscript denoted $f,\ nf,\ hnf,\ s1$ and $s2$, respectively.

**Table 1**
Hybrid nanofluid thermophysical characteristics [18,43].

| Names | characteristics |
|---|---|
| Dynamic Viscosity | $\mu_{hnf} = \dfrac{\mu_f}{(1-\varphi_1)^{2.5}(1-\varphi_2)^{2.5}}$ |
| Electrical conductivity | $\sigma_{hnf} = \dfrac{\sigma_2 + 2\sigma_{nf} - 2\varphi_2(\sigma_{nf} - \sigma_2)}{\sigma_2 + 2\sigma_{nf} + \varphi_2(\sigma_{nf} - \sigma_2)} \times (\sigma_{nf})$ where $(\sigma_{nf}) = \frac{\sigma_1 + 2\sigma_f - 2\varphi_1(\sigma_f - \sigma_1)}{\sigma_1 + 2\sigma_f + \varphi_1(\sigma_f - \sigma_1)} \times (\sigma_f)$ |
| Thermal conductivity | $k_{hnf} = \dfrac{k_{s2} + 2k_{nf} - 2\varphi_2(k_{nf} - k_{s2})}{k_{s2} + 2k_{nf} + \varphi_2(k_{nf} - k_{s2})} \times (k_{nf})$ where $k_{nf} = \frac{k_{s1} + 2k_f - 2\varphi_1(k_f - k_{s1})}{k_{s1} + 2k_f + \varphi_1(k_f - k_{s1})} \times (k_f)$ |
| Heat capacity | $(\rho c_p)_{hnf} = (1-\varphi_2)\left[(1-\varphi_1)(\rho c_p)_f + \varphi_1(\rho c_p)_{s1}\right] + \varphi_2(\rho c_p)_{s2}$ |
| Density | $\rho_{hnf} = (1-\varphi_2)\left[(1-\varphi_1)\rho_f + \varphi_1\rho_{s1}\right] + \varphi_2\rho_{s2}$ |

Table 2 shows the physical characteristics of solid nanoparticles and (water) base fluid [18,43].

**Table 2**
Thermophysical characteristics of solid nanoparticles and (water) base fluids.

| Fluids | $\rho$ (kg/m$^3$) | $\sigma(S/m)$ | $c_p$(J/kg K) | k (W/m K) | $Pr$ |
|---|---|---|---|---|---|
| Water ($H_2O$) | 997.1 | 0.05 | 4179 | 0.613 | 6.2 |
| Alumina ($Al_2O_3$) | 3970 | $3.69\times10^7$ | 765 | 40 | - |
| Copper (Cu) | 8933 | $5.96\times10^7$ | 385 | 400 | - |

The similarity variables stated below are employed in this case [18,43].

$$u = U_w e^{x/L} f'(\eta);\ v = -\sqrt{\frac{U_w \nu_f}{2L}}\, e^{x/2L}\left(f(\eta) + \eta f'(\eta)\right);\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty};\ \eta = y\sqrt{\frac{U_w}{2\nu_f L}}\ e^{x/2L} \tag{6}$$

Here, prime shows the differentiation for $\eta$, $\nu_f$ kinematic viscosity of the fluid. $v_w = -(U_w \nu_f / 2L)^{1/2}\, e^{x/2L} S$, $S$ stands for the injection/suction parameter, when $S < 0$, flow is the injection, and when the $S > 0$ shows suction.
Eq. (1) identically satisfied, by Eq. (6) put in Eqs. (2-3), consequently, Eqs. (2-3) transformed into the following ODEs Equation.

$$\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f} f''' + f''f - 2(f')^2 - \frac{\sigma_{hnf}/\sigma_f}{\rho_{hnf}/\rho_f} M f' = 0 \tag{7}$$

$$\frac{1}{Pr(\rho c_p)_{hnf}/(\rho c_p)_f}\left[\frac{k_{hnf}}{k_f}\right]\theta'' + \theta' f - 4\theta f' + \frac{\sigma_{hnf}/\sigma_f}{(\rho c_p)_{hnf}/(\rho c_p)_f} MEc\left(f'^2\right) = 0 \tag{8}$$

Subject to boundary conditions:

$$f(0) = S, f'(0) = -1, \theta(0) = 1 + \delta_T \theta'(0) \tag{9}$$

$$f'(\eta) \to 0,\ \theta(\eta) \to 0 \text{ as } \eta \to \infty \tag{10}$$

The parameters are $M$ is the magnetic parameter, $\delta_T$ thermal slip parameter, $Ec$ is Eckert number and $Pr$ Prandtl number which is described by $M = \frac{2LB_0{}^2\sigma_f}{U_w\rho_f}$, $\delta_T = \varepsilon_1(\frac{\nu_f U_w}{2L})^{1/2}$, $Ec = \frac{u_w{}^2}{(T_w - T_\infty)(c_p)_f}$, and $Pr = \frac{(\mu c_p)_f}{k_f}$ respectively.

The skin friction coefficients $C_f$ and local Nusselt number $Nu_x$, which is specified as:

$$C_f = \frac{\mu_{hnf}}{\rho_f u_w^2}\left(u_y\right)_{y=0}, \; Nu_x = \frac{2L}{k_f(T_w - T_\infty)}\left[-k_{hnf}\left(T_y\right)_{y=0}\right]. \tag{11}$$

Become by using Eqs. (6) and (11).

$$\left(Re_x\right)^{1/2}C_f = \frac{\mu_{hnf}}{\mu_f} f''(0), \;\; \left(Re_x\right)^{-1/2}Nu_x = -\left[\frac{k_{hnf}}{k_f}\right]\theta'(0), \tag{12}$$

where $Re_x = \frac{2u_w L}{\nu_f}$ is the local Reynolds number.

*2.2 Numerical Method*

By using software MATLAB bv4pc solver numerically solved to nonlinear ODEs Eqs. (7)-(8) with the boundary Eqs. (9)-(10). Jacek Kierzenka and Lawrence F. Shampine of Southern Methodist University in Texas built the bvp4c solver (Hale [56]). The bvp4c solver is a finite difference process that employs the three-stage Lobatto IIIA implicit Runge–Kutta technique and returns numerical results of fourth-order precision. We included the bvp4c solver into our physical model by following the steps described below.

**STEP 1:** For the first system of nonlinear higher-order (ODEs) equations, generate the new variables of Eq. (13) utilizing Eqs. (9)-(10).

$$y(1) = f, y(2) = f', y(3) = f'', y(4) = \theta, y(5) = \theta'. \tag{13}$$

**STEP 2:** Applying the new variables from Eq. (13), reduce the system of higher-order nonlinear ODEs in Eqs. (7)-(8) to a system of first-order nonlinear ODEs:

$$f' = y(2),$$

$$f'' = y(3),$$

$$f''' = \left[2 * \left(y(2)\right)^2 - y(3) * y(1) + \frac{\sigma_{hnf}/\sigma_f}{\rho_{hnf}/\rho_f} M * y(2)\right]\frac{\rho_{hnf}/\rho_f}{\mu_{hnf}/\mu_f} \tag{14}$$

$$\theta' = y(5),$$

$$\theta'' = \frac{Pr(\rho c_p)_{hnf}/(\rho c_p)_f}{(k_{hnf}/k_f)}\left[4 * y(4) * y(2) - y(5) * y(1) - \frac{\sigma_{hnf}/\sigma_f}{(\rho c_p)_{hnf}/(\rho c_p)_f} M * Ec * \left(y(2)\right)^2\right]$$

**STEP 3:** Eqs. (9) and (10) are boundary conditions that are converted in terms of the new variables in Eq. (13):

$$y(1)_a = S, y(2)_a = -1, y(4)_a = 1 + \delta_T * y(5)_a,$$
$$y(2)_b = 0, y(4)_b = 0. \tag{15}$$

The subscript $'a'$ specifies the position of a sheet at $\eta = 0$ however the subscript $'b'$ denotes the location away from the sheet for a particular value of $\eta$. We employed a value of $\eta = 15$ for this research.

**STEP 4:** Obtain dual solutions by giving two distinct initial guesses to the bvp4c solver, one at a time. The first solution can be achieved with less restrictive initial guesses, but this is not every time accurate when getting the second solution. This procedure is repeated until the numerical solutions asymptotically satisfy the boundary conditions at infinity (i.e., Eq. (10)).

## 3. Results and Discussion

The solutions duality has been achieved in the figures by using distinct initial guesses for $f''(0)$ and $-\theta'(0)$, with the outcome that both velocity and temperature profiles satisfied the boundary condition $\eta \to \infty$ asymptotically. Before starting to discuss the results of the current study, we have compared the coding of a numerical method to make sure that our computer code is working properly, At first, to validate the coding of a numerical scheme in this study, the reduced skin friction $f''(0)$ for pure water when $\varphi_1 = \varphi_2 = 0 = M = Ec = \delta_T = 0$, and $Pr = 6.2$ is compared with varying values of $S$ issued by Waini *et al*., [13], Ferdows *et al*., [57], Cortell [58], and Rashidi *et al*., [59] as in Table 3 for stretching sheet $\lambda = 1$. Furthermore, Table 4 compares the values of $-\theta'(0)$ under various findings of Waini *et al*., [13], Ferdows *et al*., [57], Cortell [58], Rashidi *et al*., [59], and Raju *et al*., [60]. Tables (3) and (4) show remarkable consistency between those findings. The findings are consistent with those of preceding studies, and they are in a good deal of support.

**Table 3**
Comparison values of the reduced skin friction coefficient $f''(0)$ under $\varphi_1 = \varphi_2 = M = Ec = \delta_T = 0$, $Pr = 6.2$ and $\lambda = 1$.

| $S$ | Waini *et al*., [13] | Ferdows *et al*., [57] | Cortell [58] | Rashidi *et al*., [59] | Current study |
|---|---|---|---|---|---|
| 0.75 | -0.984439 | -0.984439 | -0.984417 | -0.9844401 | -0.9844392 |
| 0.5 | -0.873643 | -0.873643 | -0.873627 | -0.8736447 | -0.87364275 |
| 0 | -0.677648 | -0.677648 | -0.677647 | -0.6776563 | -0.67764784 |
| -0.5 | -0.518869 | -0.518869 | -0.518869 | -0.5188901 | -0.51886957 |
| -0.75 | -0.453523 | -0.453523 | -0.453521 | -0.4535499 | -0.45352331 |

**Table 4**
Comparison values of the reduced heat transfer $-\theta'(0)$ under $\varphi_1 = \varphi_2 = M = Ec = \delta_T = 0$, $Pr = 2$ and $\lambda = 1$.

| $S$ | Waini *et al.,* [13] | Ferdows *et al.,* [57] | Cortell [58] | Rashidi *et al*., [59] | Raju *et al*., [60] | Current study |
|---|---|---|---|---|---|---|
| 0.5 | 0.399100 | 0.398951 | 0.3989462 | 0.3990842 | 0.3990842 | 0.39909965 |
| 0 | 0.764357 | 0.764374 | 0.7643554 | 0.7643525 | 0.7643527 | 0.76435624 |
| 0.5 | 1.230792 | 1.230952 | 1.2307661 | 1.2307912 | 1.2307916 | 1.23079181 |

The variation of $f''(0)$ and $-\theta'(0)$ with three values of volume fraction copper $\varphi_2 = 0.0, 0.01, \text{and } 0.02$ are presented in Figures 2-3 against suction parameter $S$ with the occurrence of different parameters values such as $\varphi_1 = 0.1, Pr = 6.2, M = 1.0$, and $\delta_T = Ec = 0.1$. The range of volume fraction copper $\varphi_2$ is 0 to 0.02. Here $i = 1, 2, 3. S_{ci}$ shows the critical point at the suction parameter where both solutions meet each other. It can be shown that when $S < S_{ci}$ no solution exists. Furthermore, the value of $S_{ci}$= 2.2320, 2.1219, and 2.0410 are relative critical points of $\varphi_2 = 0, 0.01$, and 0.02 respectively. It is worth revealing here that when $\varphi_2 = 0$, it is purely $Al_2O_3$/water-based nanofluid and $S_{c1} = 2.2320$, after that 1% of $\varphi_2$ is added and got $S_{c2} = 2.1219$. Moreover, the value of $S_{c3} = 2.0410$ appeared to rise as 2% of the volume fraction of copper $\varphi_2$ is added in the hybrid nanofluid. From Figure 2, in the first solution, when values of $\varphi_2$ enhance, reduced skin friction $f''(0)$ increase, and on the other side, the values of $f''(0)$ decrease in the second solution. Besides, as can be seen in Figure 3, when the value of $\varphi_2$ is a rise, the magnitude of $-\theta'(0)$ also declined in the first and second solutions. Physically, the boundary layer separation extends as the value of $\varphi_2$ increased. Similar outcomes can be seen in Asghar *et al.,* [18].

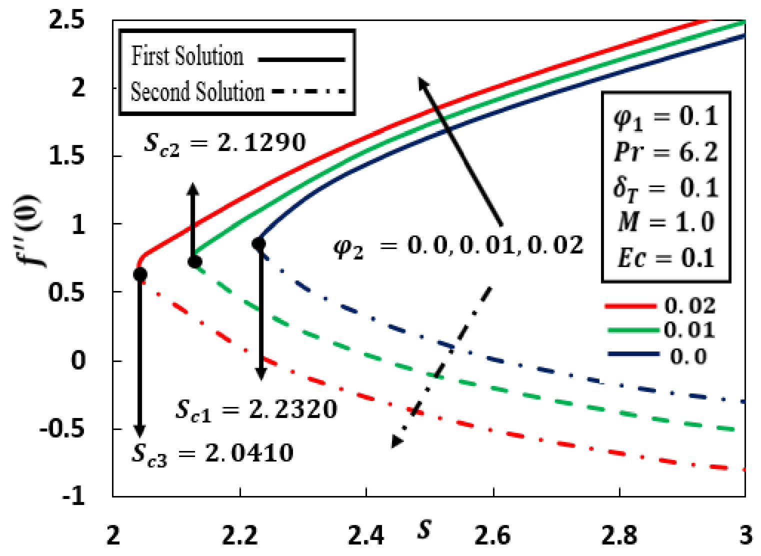


**Fig. 2.** Effect of $\varphi_2$ on $f''(0)$ against $S$

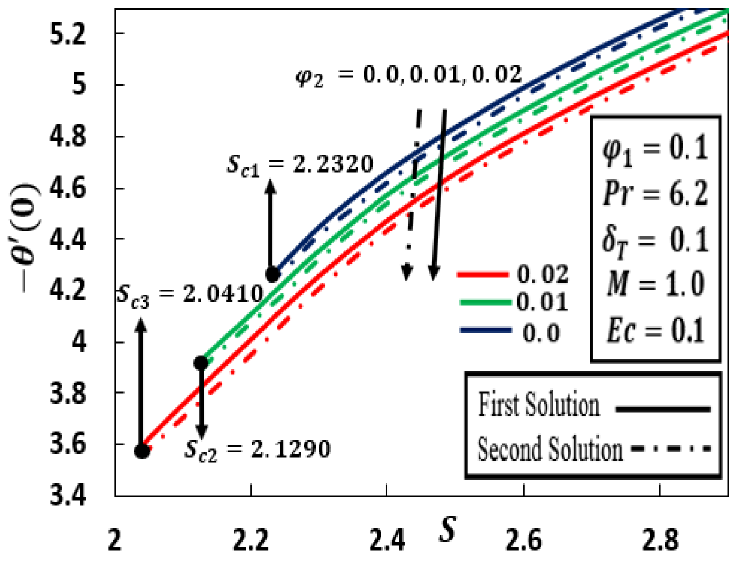


**Fig. 3.** Effect of $\varphi_2$ on $-\theta'(0)$ against $S$

Figures 4-5 depict the following on different values of $M$= 1.0, 2.0, and 3.0 for the velocity and temperature profiles $f'(\eta)$, and $\theta(\eta)$ respectively under various parameters such as $\varphi_1 = 0.1$, $\varphi_2 = 0.01$, $Pr = 6.2$, $S = 2.5$, $Ec = \delta_T = 0.1$. Figure 4 illustrates that velocity $f'(\eta)$ profile rise with an upsurge value of $M$ in the first solution and decrease in the second solution. As a consequence, the effect of the transfer rate is reduced as $M$ increases. Physically, the Lorentz force, which is caused by the magnetic field, makes the rate of transfer more resistant (Ishak [61]). Figure 5 displays that the temperature $\theta(\eta)$ profile grows as the value of $M$ increases in both solutions. Asghar *et al*., [19] found similar outcomes for $M$ parameter.

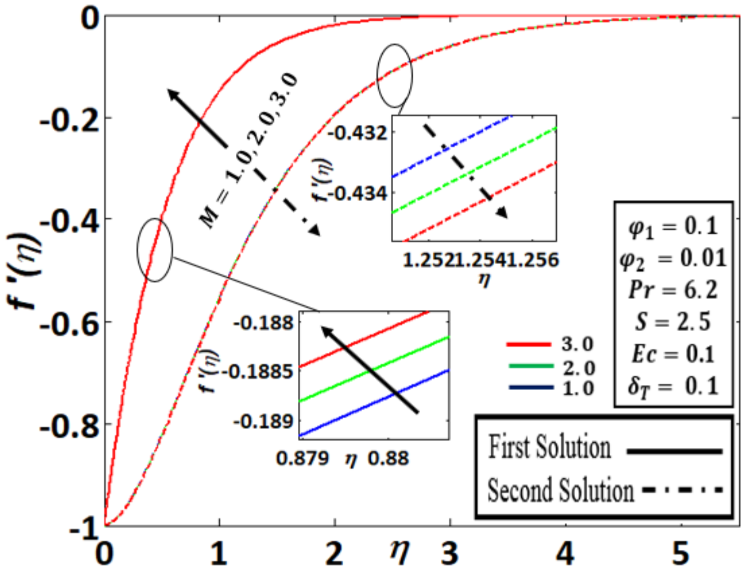


**Fig. 4**. Velocity profile $f'(\eta)$ for $M$ against $\eta$

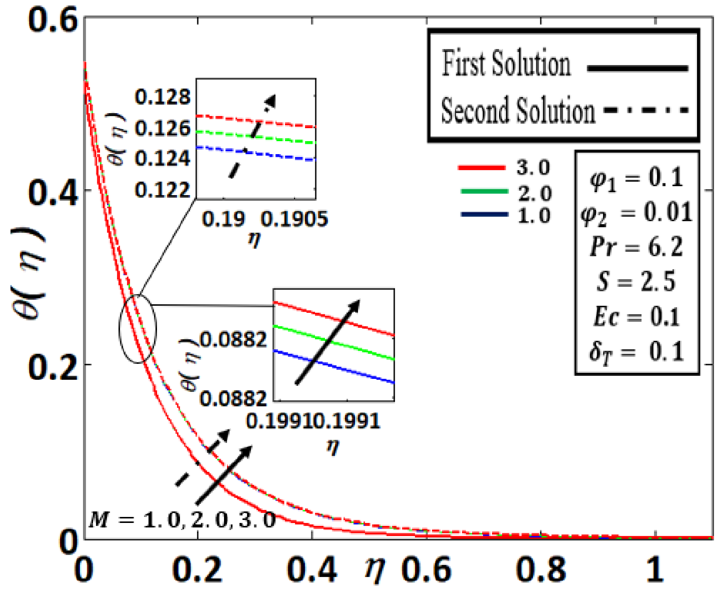


**Fig. 5**. Temperature profile $\theta(\eta)$ for $M$ against $\eta$

The plots of the velocity profile $f'(\eta)$ and temperature profile $\theta(\eta)$ against volume fraction copper $\varphi_2 = 0.0, 0.01,$ and $0.02$ are portrayed in Figures 6-7 with the existence of several parameters values $\varphi_1 = 0.1$, $Pr = 6.2$, $S = 2.5$, $M = 1.0,$ and $Ec = \delta_T = 0.1$ respectively. Figure 6 shows that the first solution rise and the second solution declined in the fluid velocity $f'(\eta)$ when enhanced, the value of $\varphi_2$. Besides, from Figure 7, the temperature profile $\theta(\eta)$ upsurges in both solutions when growing the value of $\varphi_2$. Physically heat transfer rate increased as the volume fraction of nanoparticles increased. Similar conclusions were found by Yan *et al.*, [47].

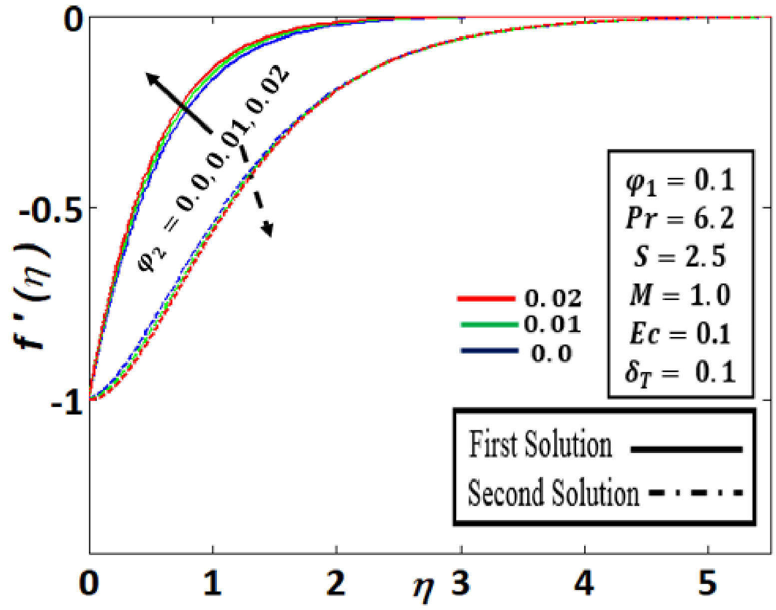


**Fig. 6**. Velocity profile $f'(\eta)$ for $\varphi_2$ against $\eta$

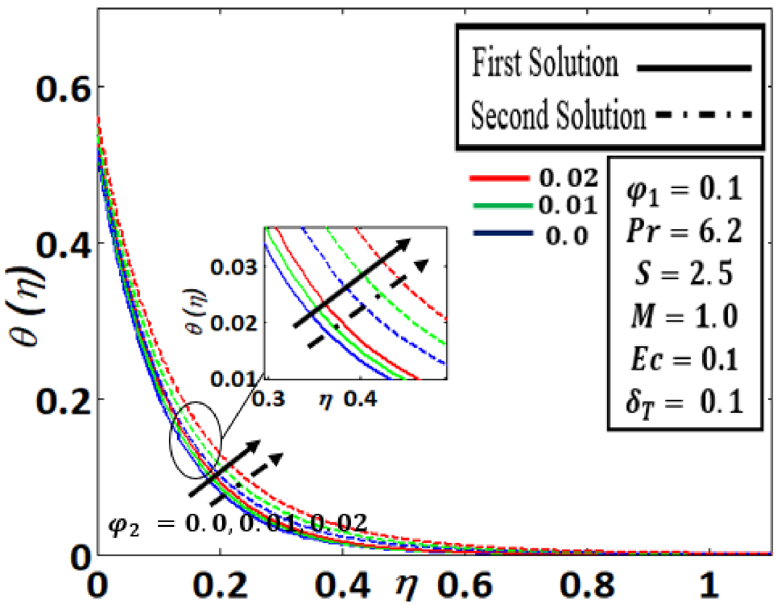


**Fig. 7.** Temperature profile $\theta(\eta)$ for $\varphi_2$ against $\eta$

Figure 8 illustrates the value of $Ec = 0.1, 0.2$, and $0.3$ at a distinct parameter $\varphi_1 = 0.1$, $\varphi_2 = 0.01$, $Pr = 6.2$, $S = 2.5$, $M = 3.0$, and $\delta_T = 0.1$ for the temperature profile $\theta\ (\eta)$. In the first and second solutions, the values of $Ec$ grow, causing $\theta\ (\eta)$ to improve as well. Physically, the heat transfer intensity rises as the value of $Ec$ increases due to the increasing heat created by Joule heating. Asghar and Ying [45] achieved similar trends.

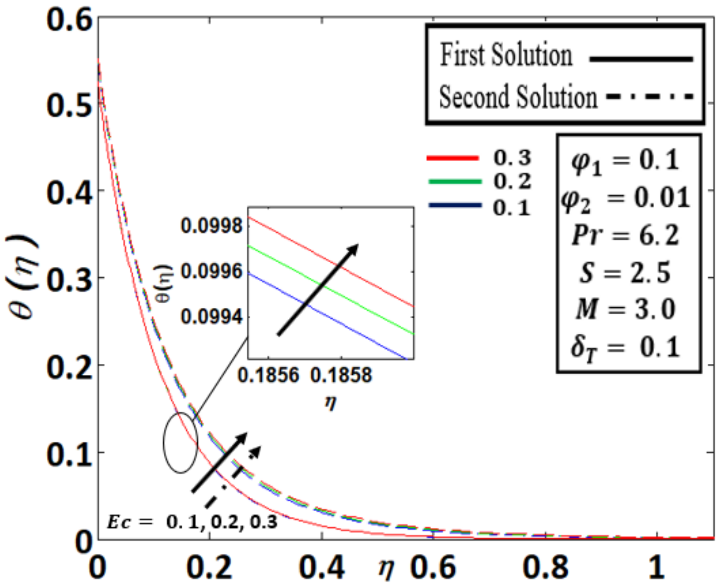


**Fig. 8.** Temperature profile $\theta(\eta)$ for $Ec$ against $\eta$

Figure 9 demonstrates the various values of the thermal slip parameters $\delta_T = 0.1, 0.2,$ and $0.3$ with several parameters $\varphi_1 = 0.1, \varphi_2 = 0.01, Pr = 6.2, S = 2.5, M = 1.0,$ and $Ec = 0.1$ for the temperature profile $\theta(\eta)$. The thermal boundary layer thickness constantly declined in the first and second solutions with a rising value of the thermal slip parameter $\delta_T$. Physically, when the thermal slip parameter $\delta_T$ upsurges, less heat is shifted from the surface to the fluid, lowering the temperature. A similar result regarding temperature profile $\theta(\eta)$ was reported by Asghar *et al.*, [19].

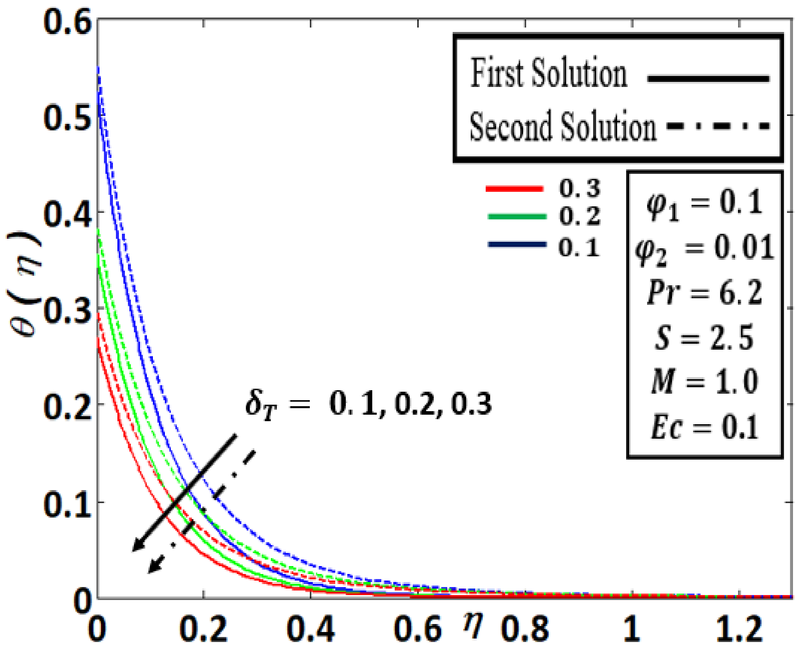


**Fig. 9.** Temperature Profile $\theta(\eta)$ for $\delta_T$ against $\eta$

Figure 10 investigated the effect of Prandtl number $Pr = 6.0, 6.1,$ and $6.2$ with different parameters $\varphi_1 = 0.1,\ \varphi_2 = 0.01,\ S = 2.5,\ M = 3.0,$ and $Ec = \delta_T = 0.1$. Physically conclusion reveals that an increment in $Pr$ would lower the temperature of the hybrid nanofluid for the first and second solutions. In other circumstances, thermal boundary layer thickness declines as $Pr$ enhances. Asghar and Ying [45] had similar results.

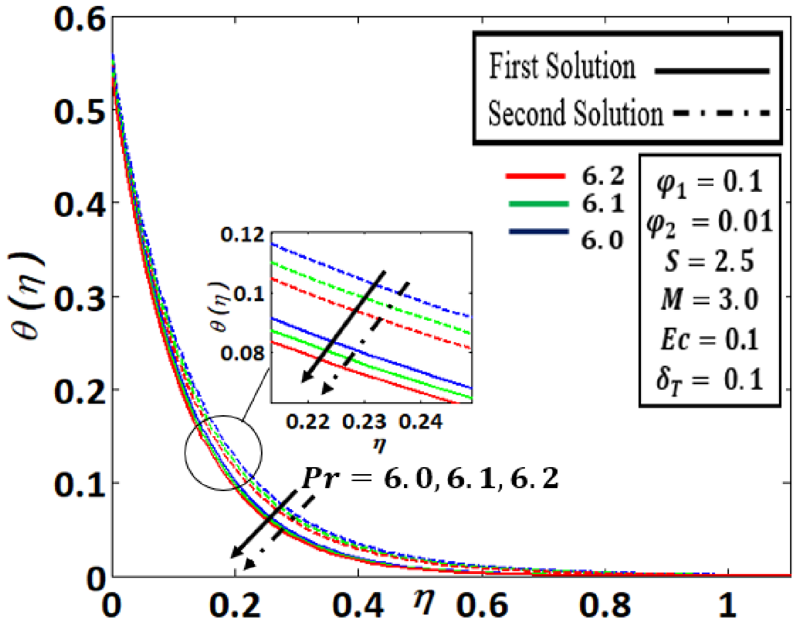


**Fig. 10**. Temperature Profile $\theta(\eta)$ for $Pr$ against $\eta$

## 4. Conclusions

Hybrid nanofluid possesses an extensive assortment of industrial applications, which motivates this study. In this study, two-dimensional Magnetohydrodynamics $Al_2O_3$+Cu/water hybrid nanofluid with heat transfer over an exponentially shrinking sheet by Joule heating and thermal slip has been investigated through a bvp4c solver on the MATLAB computing platform. This research concentrates on the behavior of the reduced skin friction $f''(0)$, reduced heat transfer $-\theta'(0)$, velocity and temperature profiles under the effect of suction/injection, MHD, thermal slip, and joule heating on the hybrid nanofluid flow. The key conclusions of the existing investigation are as follows:

i. Given an acceptable set of defined parameters, dual solutions have been proved to be possible.
ii. The hybrid nanofluid flow continues until it approaches a critical point $S < S_{ci}$, beyond which no fluid flow is possible.
iii. The insertion of volume fraction copper $\varphi_2$ delayed the separation of the boundary layer.
iv. Both solutions decrease when the value of the thermal slip parameter and Prandtl number is enhanced. Besides, both solutions enhanced when the value of the Eckert number increased.

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## References

[1] Sakiadis, Byron C. "Boundary-layer behavior on continuous solid surfaces: I. Boundary-layer equations for two-dimensional and axisymmetric flow." *AIChE Journal* 7, no. 1 (1961): 26-28. https://doi.org/10.1002/aic.690070108

[2] Crane, Lawrence J. "Flow past a stretching plate." *Zeitschrift für angewandte Mathematik und Physik ZAMP* 21, no. 4 (1970): 645-647. https://doi.org/10.1007/BF01587695

[3] Choi, S. US, and Jeffrey A. Eastman. *Enhancing thermal conductivity of fluids with nanoparticles*. No. ANL/MSD/CP-84938; CONF-951135-29. Argonne National Lab.(ANL), Argonne, IL (United States), 1995. https://www.osti.gov/servlets/purl/196525
[4] Abu-Nada, Eiyad, and Hakan F. Oztop. "Effects of inclination angle on natural convection in enclosures filled with Cu–water nanofluid." *International Journal of Heat and Fluid Flow* 30, no. 4 (2009): 669-678. https://doi.org/10.1016/j.ijheatfluidflow.2009.02.001
[5] Wang, Xiang-Qi, and Arun S. Mujumdar. "Heat transfer characteristics of nanofluids: a review." *International journal of thermal sciences* 46, no. 1 (2007): 1-19. https://doi.org/10.1016/j.ijthermalsci.2006.06.010
[6] Lund, Liaquat Ali, Zurni Omar, Sumera Dero, Dumitru Baleanu, and Ilyas Khan. "Rotating 3D flow of hybrid nanofluid on exponentially shrinking sheet: Symmetrical solution and duality." *Symmetry* 12, no. 10 (2020): 1637. https://doi.org/10.3390/sym12101637
[7] Khan, W. A., and I. Pop. "Boundary-layer flow of a nanofluid past a stretching sheet." *International journal of heat and mass transfer* 53, no. 11-12 (2010): 2477-2483. https://doi.org/10.1016/j.ijheatmasstransfer.2010.01.032
[8] Miklavčič, M., and C. Wang. "Viscous flow due to a shrinking sheet." *Quarterly of Applied Mathematics* 64, no. 2 (2006): 283-290. https://doi.org/10.1090/S0033-569X-06-01002-5
[9] Bachok, Norfifah, Anuar Ishak, and Ioan Pop. "Unsteady boundary-layer flow and heat transfer of a nanofluid over a permeable stretching/shrinking sheet." *International Journal of Heat and Mass Transfer* 55, no. 7-8 (2012): 2102-2109. https://doi.org/10.1016/j.ijheatmasstransfer.2011.12.013
[10] Dero, Sumera, Azizah Mohd Rohni, and Azizan Saaban. "Stability analysis of Cu− C6H9NaO7 and Ag− C6H9NaO7 nanofluids with effect of viscous dissipation over stretching and shrinking surfaces using a single phase model." *Heliyon* 6, no. 3 (2020): e03510.
[11] Devi, SP Anjali, and S. Suriya Uma Devi. "Numerical investigation of hydromagnetic hybrid Cu–Al2O3/water nanofluid flow over a permeable stretching sheet with suction." *International Journal of Nonlinear Sciences and Numerical Simulation* 17, no. 5 (2016): 249-257. https://doi.org/10.1515/ijnsns-2016-0037
[12] Huminic, Gabriela, and Angel Huminic. "Hybrid nanofluids for heat transfer applications–a state-of-the-art review." *International Journal of Heat and Mass Transfer* 125 (2018): 82-103. https://doi.org/10.1016/j.ijheatmasstransfer.2018.04.059
[13] Waini, Iskandar, Anuar Ishak, and Ioan Pop. "Hybrid nanofluid flow and heat transfer over a nonlinear permeable stretching/shrinking surface." *International Journal of Numerical Methods for Heat & Fluid Flow* (2019). https://doi.org/10.1088/1402-4896/ab0fd5
[14] Waini, Iskandar, Anuar Ishak, and Ioan Pop. "Transpiration effects on hybrid nanofluid flow and heat transfer over a stretching/shrinking sheet with uniform shear flow." *Alexandria Engineering Journal* 59, no. 1 (2020): 91-99. https://doi.org/10.1016/j.aej.2019.12.010
[15] Waini, Iskandar, Anuar Ishak, and Ioan Pop. "Mixed convection flow over an exponentially stretching/shrinking vertical surface in a hybrid nanofluid." *Alexandria Engineering Journal* 59, no. 3 (2020): 1881-1891. https://doi.org/10.1016/j.aej.2020.05.030
[16] Lund, Liaquat Ali, Zurni Omar, Ilyas Khan, and El-Sayed M. Sherif. "Dual branches of MHD three-dimensional rotating flow of hybrid nanofluid on nonlinear shrinking sheet." *Computers, Materials and Continua* 66, no. 1 (2020): 127-139. https://doi.org/10.32604/cmc.2020.013120
[17] Kolsi, Lioua, Sumera Dero, Liaquat Ali Lund, Umar F. Alqsair, Mohamed Omri, and Sami Ullah Khan. "Thermal stability and performances of hybrid nanoparticles for convective heat transfer phenomenon with multiple solutions." *Case Studies in Thermal Engineering* 28 (2021): 101684. https://doi.org/10.1016/j.csite.2021.101684
[18] Asghar, Adnan, Teh Yuan Ying, and Wan Mohd Khairy Adly Wan Zaimi. "Two-Dimensional Mixed Convection and Radiative Al2O3-Cu/H2O Hybrid Nanofluid Flow over a Vertical Exponentially Shrinking Sheet with Partial Slip Conditions." *CFD Letters* 14, no. 3 (2022): 22-38. https://doi.org/10.37934/cfdl.14.3.2238
[19] Asghar, Adnan, Teh Yuan Ying, and Khairy Zaimi. "Two-Dimensional Magnetized Mixed Convection Hybrid Nanofluid Over a Vertical Exponentially Shrinking Sheet by Thermal Radiation, Joule Heating, Velocity and Thermal Slip Conditions". *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 95, no. 2 (2022): 159-79. https://doi.org/10.37934/arfmts.95.2.159179
[20] Khan, Ansab Azam, Khairy Zaimi, Suliadi Firdaus Sufahani, and Mohammad Ferdows. "MHD flow and heat transfer of double stratified micropolar fluid over a vertical permeable shrinking/stretching sheet with chemical reaction and heat source." *Journal of Advanced Research in Applied Sciences and Engineering Technology* 21, no. 1 (2020): 1-14. https://doi.org/10.37934/araset.21.1.114
[21] Rohni, Azizah Mohd, Sumera Dero, and Azizan Saaban. "Triple solutions and stability analysis of mixed convection boundary flow of Casson nanofluid over an exponentially vertical stretching/shrinking sheet." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 72, no. 1 (2020): 94-110. https://doi.org/10.37934/arfmts.72.1.94110

[22] Dzulkifli, Nor Fadhilah, Norfifah Bachok, Nor Azizah Yacob, Ioan Pop, Norihan Arifin, and Haliza Rosali. "Stability Solution of Unsteady Stagnation-Point Flow and Heat Transfer over a Stretching/Shrinking Sheet in Nanofluid with Slip Velocity Effect." *CFD Letters* 14, no. 1 (2022): 66-86. https://doi.org/10.37934/cfdl.14.1.6686

[23] Mahat, Rahimah, Muhammad Saqib, Imran Ulah, Sharidan Shafie, and Sharena Mohamad Isa. "MHD Mixed Convection of Viscoelastic Nanofluid Flow due to Constant Heat Flux." *Journal of Advanced Research in Numerical Heat Transfer* 9, no. 1 (2022): 19-25.

[24] Ali, Kashif, Sohail Ahmad, Ozaira Baluch, Wasim Jamshed, Mohamed R. Eid, and Amjad Ali Pasha. "Numerical study of magnetic field interaction with fully developed flow in a vertical duct." *Alexandria Engineering Journal* 61, no. 12 (2022): 11351-11363. https://doi.org/10.1016/j.aej.2022.05.009

[25] Shahzad, Faisal, Wasim Jamshed, Rabia Safdar, Syed M. Hussain, Nor Ain Azeany Mohd Nasir, Mallinath Dhange, Kottakkaran Sooppy Nisar et al. "Thermal analysis characterization of solar-powered ship using Oldroyd hybrid nanofluids in parabolic trough solar collector: An optimal thermal application." *Nanotechnology Reviews* 11, no. 1 (2022): 2015-2037. https://doi.org/10.1515/ntrev-2022-0108

[26] Hafeez, Muhammad Bilal, Marek Krawczuk, Kottakkaran Sooppy Nisar, Wasim Jamshed, and Amjad Ali Pasha. "A finite element analysis of thermal energy inclination based on ternary hybrid nanoparticles influenced by induced magnetic field." *International Communications in Heat and Mass Transfer* 135 (2022): 106074. https://doi.org/10.1016/j.icheatmasstransfer.2022.106074

[27] Ali, Kashif, Aftab Ahmed Faridi, Sohail Ahmad, Wasim Jamshed, Nargis Khan, and Mohammad Mahtab Alam. "Quasi-linearization analysis for heat and mass transfer of magnetically driven 3rd-grade (Cu-TiO2/engine oil) nanofluid via a convectively heated surface." *International Communications in Heat and Mass Transfer* 135 (2022): 106060. https://doi.org/10.1016/j.icheatmasstransfer.2022.106060

[28] Jamshed, Wasim, Mohamed R. Eid, Syed M. Hussain, Aissa Abderrahmane, Rabia Safdar, Obai Younis, and Amjad Ali Pasha. "Physical specifications of MHD mixed convective of Ostwald-de Waele nanofluids in a vented-cavity with inner elliptic cylinder." *International Communications in Heat and Mass Transfer* 134 (2022): 106038. https://doi.org/10.1016/j.icheatmasstransfer.2022.106038

[29] Jamshed, Wasim, Nor Ain Azeany Mohd Nasir, Ameni Brahmia, Kottakkaran Sooppy Nisar, and Mohamed R. Eid. "Entropy analysis of radiative [MgZn6Zr-Cu/EO] Casson hybrid nanoliquid with variant thermal conductivity along a stretching surface: Implementing Keller box method." *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science* (2022): 09544062211065696. https://doi.org/10.1177/09544062211065696

[30] Bouslimi, Jamel, Ali A. Alkathiri, Abdulaziz N. Alharbi, Wasim Jamshed, Mohamed R. Eid, and Mohamed Lamjed Bouazizi. "Dynamics of convective slippery constraints on hybrid radiative Sutterby nanofluid flow by Galerkin finite element simulation." *Nanotechnology Reviews* 11, no. 1 (2022): 1219-1236. https://doi.org/10.1515/ntrev-2022-0070

[31] Jamshed, Wasim, Abdullah K. Alanazi, Siti Suzilliana Putri Mohamed Isa, Ramashis Banerjee, Mohamed R. Eid, Kottakkaran Sooppy Nisar, Hashem Alshahrei, and Marjan Goodarzi. "Thermal efficiency enhancement of solar aircraft by utilizing unsteady hybrid nanofluid: a single-phase optimized entropy analysis." *Sustainable Energy Technologies and Assessments* 52 (2022): 101898. https://doi.org/10.1016/j.seta.2021.101898

[32] Alkathiri, Ali A., Wasim Jamshed, Mohamed R. Eid, and Mohamed Lamjed Bouazizi. "Galerkin finite element inspection of thermal distribution of renewable solar energy in presence of binary nanofluid in parabolic trough solar collector." *Alexandria Engineering Journal* 61, no. 12 (2022): 11063-11076. https://doi.org/10.1016/j.aej.2022.04.036

[33] Jamshed, Wasim. "Finite element method in thermal characterization and streamline flow analysis of electromagnetic silver-magnesium oxide nanofluid inside grooved enclosure." *International Communications in Heat and Mass Transfer* 130 (2022): 105795. https://doi.org/10.1016/j.icheatmasstransfer.2021.105795

[34] Shahzad, Faisal, Jamel Bouslimi, Soumaya Gouadria, Wasim Jamshed, Mohamed R. Eid, Rabia Safdar, M. D. Shamshuddin, and Kottakkaran Sooppy Nisar. "Hydrogen energy storage optimization in solar-HVAC using Sutterby nanofluid via Koo-Kleinstreuer and Li (KKL) correlations model: A solar thermal application." *International Journal of Hydrogen Energy* 47, no. 43 (2022): 18877-18891. https://doi.org/10.1016/j.ijhydene.2022.04.039

[35] Jamshed, Wasim, M. Prakash, Syed M. Hussain, Mohamed R. Eid, Kottakkaran Sooppy Nisar, and Taseer Muhammad. "Entropy amplified solitary phase relative probe on engine oil based hybrid nanofluid." *Chinese Journal of Physics* 77 (2022): 1654-1681. https://doi.org/10.1016/j.cjph.2021.11.009

[36] Hussain, Syed M., R. Sharma, and Ali J. Chamkha. "Numerical and statistical explorations on the dynamics of water conveying Cu-Al2O3 hybrid nanofluid flow over an exponentially stretchable sheet with Navier's partial slip and thermal jump conditions." *Chinese Journal of Physics* 75 (2022): 120-138. https://doi.org/10.1016/j.cjph.2021.11.007

[37] Jamshed, Wasim, Rabia Safdar, Ameni Brahmia, Abdullah K. Alanazi, Hala M. Abo-Dief, and Mohamed Rabea Eid. "Numerical simulations of environmental energy features in solar pump application by using hybrid nanofluid flow: Prandtl-Eyring case."*Energy & Environment* (2022): 0958305X211073806. https://doi.org/10.1177/0958305X211073806

[38] Mabood, Fazle, Suriya Uma Devi Sathyanarayanan, Madheshwaran Prakash, and Iskander Tlili. "Influencing Al2O3–Cu in 20% water+ 80% EG hybrid nano coolant inflow on penetrable tensile surface." *Energy Sources, Part A: Recovery, Utilization, and Environmental Effects* (2020): 1-17. https://doi.org/10.1080/15567036.2020.1801905

[39] Yashkun, Ubaidullah, Khairy Zaimi, Anuar Ishak, Ioan Pop, and Rabeb Sidaoui. "Hybrid nanofluid flow through an exponentially stretching/shrinking sheet with mixed convection and Joule heating." *International Journal of Numerical Methods for Heat & Fluid Flow* (2020). https://doi.org/10.1108/HFF-07-2020-0423

[40] Devi, S. Suriya Uma, and SP Anjali Devi. "Numerical investigation of three-dimensional hybrid Cu–Al2O3/water nanofluid flow over a stretching sheet with effecting Lorentz force subject to Newtonian heating." *Canadian Journal of Physics* 94, no. 5 (2016): 490-496. https://doi.org/10.1139/cjp-2015-0799

[41] Devi, Suriya Uma, and SP Anjali Devi. "Heat transfer enhancement of cu− $ al_ {2} o_ {3} $/water hybrid nanofluid flow over a stretching sheet." *Journal of the Nigerian Mathematical Society* 36, no. 2 (2017): 419-433.

[42] Aly, Emad H., and Ioan Pop. "MHD flow and heat transfer over a permeable stretching/shrinking sheet in a hybrid nanofluid with a convective boundary condition." *International Journal of Numerical Methods for Heat & Fluid Flow* (2019). https://doi.org/10.1108/HFF-12-2018-0794

[43] Waini, Iskandar, Anuar Ishak, and Ioan Pop. "Hybrid nanofluid flow induced by an exponentially shrinking sheet." *Chinese Journal of Physics* 68 (2020): 468-482. https://doi.org/10.1016/j.cjph.2019.12.015

[44] Lund, Liaquat Ali, Zurni Omar, Ilyas Khan, and El-Sayed M. Sherif. "Dual solutions and stability analysis of a hybrid nanofluid over a stretching/shrinking sheet executing MHD flow." *Symmetry* 12, no. 2 (2020): 276. https://doi.org/10.3390/sym12020276

[45] Teh, Yuan Ying, and Adnan Asghar. "Three Dimensional MHD Hybrid Nanofluid Flow with Rotating Stretching/Shrinking Sheet and Joule Heating." *CFD Letters* 13, no. 8 (2021): 1-19. https://doi.org/10.37934/cfdl.13.8.119

[46] Khashi'ie, Najiyah Safwa, Norihan Md Arifin, Roslinda Nazar, Ezad Hafidz Hafidzuddin, Nadihah Wahi, and Ioan Pop. "Magnetohydrodynamics (MHD) axisymmetric flow and heat transfer of a hybrid nanofluid past a radially permeable stretching/shrinking sheet with Joule heating." *Chinese Journal of Physics* 64 (2020): 251-263. https://doi.org/10.1016/j.cjph.2019.11.008

[47] Yan, Liang, Sumera Dero, Ilyas Khan, Irshad Ali Mari, Dumitru Baleanu, Kottakkaran Sooppy Nisar, El-Sayed M. Sherif, and Hany S. Abdo. "Dual solutions and stability analysis of magnetized hybrid nanofluid with joule heating and multiple slip conditions." *Processes* 8, no. 3 (2020): 332. https://doi.org/10.3390/pr8030332

[48] Asghar, Adnan, Liaquat Ali Lund, Zahir Shah, Narcisa Vrinceanu, Wejdan Deebani, and Meshal Shutaywi. "Effect of Thermal Radiation on Three-Dimensional Magnetized Rotating Flow of a Hybrid Nanofluid." *Nanomaterials* 12, no. 9 (2022): 1566. https://doi.org/10.3390/nano12091566

[49] Shoaib, Muhammad, Muhammad Asif Zahoor Raja, Muhammad Touseef Sabir, Saeed Islam, Zahir Shah, Poom Kumam, and Hussam Alrabaiah. "Numerical investigation for rotating flow of MHD hybrid nanofluid with thermal radiation over a stretching sheet." *Scientific Reports* 10, no. 1 (2020): 1-15. https://doi.org/10.1038/s41598-020-75254-8

[50] Jamil, Muhammad, and Najeeb Alam Khan. "Slip effects on fractional viscoelastic fluids." *International Journal of Differential Equations* 2011 (2011). https://doi.org/10.1155/2011/193813

[51] Andersson, Helge I. "Slip flow past a stretching surface." *Acta Mechanica* 158, no. 1 (2002): 121-125. https://doi.org/10.1007/BF01463174

[52] Hayat, Tanzila, S. Nadeem, and A. U. Khan. "Rotating flow of Ag-CuO/H2O hybrid nanofluid with radiation and partial slip boundary effects." *The European Physical Journal E* 41, no. 6 (2018): 1-9. https://doi.org/10.1140/epje/i2018-11682-y

[53] Aly, Emad H., and I. Pop. "MHD flow and heat transfer near stagnation point over a stretching/shrinking surface with partial slip and viscous dissipation: Hybrid nanofluid versus nanofluid." *Powder Technology* 367 (2020): 192-205. https://doi.org/10.1016/j.powtec.2020.03.030

[54] Zainal, Nurul Amira, Roslinda Nazar, Kohilavani Naganthran, and Ioan Pop. "Unsteady stagnation point flow of hybrid nanofluid past a convectively heated stretching/shrinking sheet with velocity slip." *Mathematics* 8, no. 10 (2020): 1649. https://doi.org/10.3390/math8101649

[55] Tiwari, Raj Kamal, and Manab Kumar Das. "Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids." *International Journal of heat and Mass transfer* 50, no. 9-10 (2007): 2002-2018. https://doi.org/10.1016/j.ijheatmasstransfer.2006.09.034

[56] Hale, Nicholas, and Daniel R. Moore. "A sixth-order extension to the MATLAB package bvp4c of J. Kierzenka and L. Shampine." (2008).
[57] Ferdows, M., Md Jashim Uddin, and A. A. Afify. "Scaling group transformation for MHD boundary layer free convective heat and mass transfer flow past a convectively heated nonlinear radiating stretching sheet." *International Journal of Heat and Mass Transfer* 56, no. 1-2 (2013): 181-187. https://doi.org/10.1016/j.ijheatmasstransfer.2012.09.020
[58] Cortell, Rafael. "Heat and fluid flow due to non-linearly stretching surfaces." *Applied Mathematics and Computation* 217, no. 19 (2011): 7564-7572. https://doi.org/10.1016/j.amc.2011.02.029
[59] Rashidi, Mohammad Mehdi, Behnam Rostami, Navid Freidoonimehr, and Saeid Abbasbandy. "Free convective heat and mass transfer for MHD fluid flow over a permeable vertical stretching sheet in the presence of the radiation and buoyancy effects." *Ain Shams Engineering Journal* 5, no. 3 (2014): 901-912. https://doi.org/10.1016/j.asej.2014.02.007
[60] Raju, C. S. K., N. Sandeep, C. Sulochana, V. Sugunamma, and M. Jayachandra Babu. "Radiation, inclined magnetic field and cross-diffusion effects on flow over a stretching surface." *Journal of the Nigerian Mathematical Society* 34, no. 2 (2015): 169-180. https://doi.org/10.1016/j.jnnms.2015.02.003
[61] Ishak, Anuar. "MHD boundary layer flow due to an exponentially stretching sheet with radiation effect." *Sains Malaysiana* 40, no. 4 (2011): 391-395.