CFD Letters

Journal homepage:
https://semarakilmu.com.my/journals/index.php/CFD_Letters/index
ISSN: 2180-1363

Slip Velocity Effect on Unsteady Free Convection Flow of Casson Fluid in a Vertical Cylinder

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ARTICLE INFO

Article history:
Received 28 June 2022
Received in revised form 20 July 2022
Accepted 19 August 2022
Available online 1 May 2023

Keywords:
Casson Fluid; Slip Velocity; Free Convection; Laplace Transform; Finite Hankel Transform

ABSTRACT

Many researchers study the Casson fluid flow in the cylinder since it imitates human blood flow in the small arteries. However, only a few researchers considered slip velocity at the boundary. The slip velocity is crucial in blood flow study due to naturally occurs during stretchable movement of the arteries. Hence, the study aims to obtain analytical solutions and understand the fluid flow behaviour with the slip velocity effect for the unsteady free convection flow of Casson fluid in a cylinder. The analytical solutions are obtained by using the joint methods of the finite Hankel transform and the Laplace transform. All initial and boundary conditions are satisfied by the analytical solutions that were obtained. The behaviour of velocity and temperature profiles are plotted and discussed graphically. It is evident from the results that increasing the slip velocity, Grashof number and time will enhance blood velocity while increasing the Casson parameter causes a decrement of blood velocity. Besides, the Prandtl number increases resulting in blood velocity and the blood temperature falling. Lastly, the obtained analytical solution is validated by comparing it with the previous study and found to be in good mutual agreement. The obtained analytical solution is significant to check the accuracy of the numerical solutions.

1. Introduction

Fluids are liquids and gases where flow behaviours are influenced by density and viscosity. In real life, inviscid fluid does not exist. Researchers have recently paid a lot of attention to research on viscous complex fluid, also known as non-Newtonian fluid. It is due to its extensive applications in the field of industries, biological sciences and engineering [1]. This type of fluid behaviour has a nonlinear relationship or comprises yield stress between shear stress and strain, which means it contradicts Newton's Law of Viscosity [2]. There are several models of non-Newtonian fluids that have been proposed for study. Among the famous viscosity models of fluid is the Casson fluid model which was introduced by Casson in 1959 to estimate the pigment-oil suspension flow behaviour [3]. He also investigated the model's accuracy in describing blood flow characteristics under low shear rates. Currently, it has become one of the most useful models to obtain accurate characterization and blood flow predictions in small arteries and capillaries under low shear rates. Other examples of

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https://doi.org/10.37934/cfdl.15.5.2941

the Casson fluid model applications in industries are paper pulp, paints, gypsum pastes, cosmetics, liquid detergents, lubricants, melted chocolate, concentrated fruit juices as well as being used to examine the characteristics of cement slurry in petroleum fields and others [4]. Casson fluid falls within the shear thinning with yield stress category, behaving as an elastic solid if yield stress is more dominant than applied external shear stress. Meanwhile, it starts to move or deform when yield stress is less significant as compared to shear stress [5, 6]. An example of an early study for Casson fluid flow in a cylinder was done by Dash *et al.,* [7] which yielded numerical results. Goud *et al.,* [8] obtained numerically the impact on free convection flow of Casson fluid past an oscillating plate. Then, Das *et al.,* [9], Hussanan *et al.,* [10] and Kataria *et al.,* [11, 12] obtained analytical solutions for Casson fluid flow through an oscillating plate while Khan *et al.,* [13] solved similar problems between two plates. They used the Laplace transform approach to address the problems analytically. Then, Ali *et al.,* [14] used the Laplace transform and Hankel transforms methods to investigate the analytical solution of the Casson fluid flow in a fixed cylinder with the pressure gradient and MHD effects.

Many researchers have been studying heat transfer analysis in Casson fluid due to its application in industrial, engineering, and medical purposes. Among those studies on heat transfer issues, free convection receives particular focus. Free convection flow, often referred to as natural convection flow, is produced by the buoyancy force and gravity force due to the density differences that come from temperature dispersion. Most scientists found solutions to these issues that occurred on the plates. For example, Hussanan *et al.,* [15], Khalid *et al.,* [16-18] and Ali *et al.,* [19] solved analytical problems of free convection flow of Casson fluid past through an oscillating vertical plate. Then, Khan *et al.,* [20] studied the free convection flow of Casson fluid with the effect of arbitrary shear stress at the plate. Next, Hari *et al.,* [21] did further research on the free convection flow of Casson fluid through an oscillating plate with additional effects of MHD, porous medium, and mass transfer. Besides that, the free convection flow of Casson fluid between two plates or in the channel had been solved by Mohammad *et al.,* [22]. All the problems are solved analytically by using the Laplace transform method. None of them managed to address the Casson fluid's free convection flow inside the cylinder.

Recently, researchers are paying more attention to studying free convection flow in a cylinder because it has more applicability in the actual world such as thermal storage systems, nuclear energy fields, heat exchangers, and others [23]. Khan *et al.,* [24] studied the natural convection flow of Newtonian fluid in an oscillating cylinder. The study had been extended by Ahmed *et al.,* [25] by using a time-fractional derivative model. Then, Shah *et al.,* [26] investigated free convection flow in a fixed cylinder by additional effects of pressure gradient and MHD. Besides that, Javaid *et al.,* [27] solved the problem of Second-grade fluid flowing through an oscillating cylinder naturally via convection. To obtain analytical solutions in cylindrical domain problems, all the researchers used the Laplace transform concerning the time variable and the finite Hankel transform concerning radial coordinate. However, the researchers have yet to solve the problems related to the Casson fluid model.

Numerous scholars are interested in studying the free convection flow of the Casson fluid model in a cylinder. The earliest study had been done by Ali *et al.,* [28-30] which resolved heat transfer issues resulting from free convection flow in a cylinder with various boundary conditions, including stationary, constant velocity, and oscillation. Then, Imtiaz *et al.,* [31] solved the free convection flow problem with the addition of nanoparticles through a fixed boundary cylinder. Next, Maiti *et al.,* [32–34] extended the heat transfer problem because of thermal radiation in a cylinder with fixed boundary conditions. All researchers obtained analytical solutions by solving the heat transfer problems with Laplace transform and finite Hankel transform method. All of them did not, however, analyse the heat transfer of a Casson fluid flow in a cylinder with a slip velocity effect at the boundary condition.

The slip velocity is the finite velocity of a fluid at or near a boundary that has been defined by Nubar [35]. In investigating the fluctuations of fluid velocity, slip velocity is essential [36-39]. Broad applications of slip conditions in industry and chemical engineering, such as when a thin film of oil is deposited on a moving plate, and in medical science such as slippage between blood and the arteries, have raised significant concerns [40-42]. For example, Ullah *et al.,* [43] studied the effects of the slip effect for Casson fluid flow over a nonlinearly stretching sheet and solved it numerically by using the Keller-box method. Besides, Imran *et al.,* [44] examined the free convection Casson fluid flow over an oscillating plate with the constant wall temperature and slip effect. Then, Saqib *et al.,* [45] extended the slip problem on an oscillating plate with the effect of radiation and mass transfer. Both of them obtained analytical solutions to slip velocity problems on the plate by using Laplace transform. Besides that, researchers also show interest in the further study of the relation with slip velocity problem in cylinders since it exists in real-life applications such as oil and gas drilling process and blood flow in arteries. Hayat *et al.,* [46] investigated the solution of the Casson fluid flow through a vertical cylinder with slip effects. Additionally, El-Aziz *et al.,* [47] used the shooting approach to solve a similar problem numerically with the addition of MHD and heat transfer effects. However, none of them managed to derive analytical answers for the Casson fluid flow inside the cylinder with the impact of slip velocity at the boundary. Some scholars used the Laplace transform and the Hankel transform to solve the slip velocity problem in a cylinder analytically. For example, Jiang *et al.,* [48] and Shah *et al.,* [49] provided an analytical solution to the slip velocity problem in a circular microchannel for Oldroyd-B fluid. The implications of slip velocity and absence of slip velocity for the Jeffrey fluid flow are then discussed by Padma *et al.,* [50, 51]. However, researchers have not attempted to solve problems for the Casson fluid model by using Laplace transform and Hankel transform yet.

To the best of the authors' knowledge, most of the researchers studied analytically the Casson fluid flow in the vertical cylinder with the free convection heat transfer process. However, none of them consider the occurrence of the slip velocity at the boundary for the Casson fluid problems. Hence, the purpose of this study is to obtain an analytical solution and investigate the influence of the slip velocity on the Casson fluid flow in the vertical cylinder with the free convection heat transfer effect. It is significant in real-life applications since Casson fluid model imitates human blood flow in the small blood vessels with the occurrence of the slip. Slip velocity in a blood vessel exists when there is a velocity difference between fluid particle movement and stretching movement of the blood vessel's wall. Hence, in order to obtain analytical solutions for fluid velocity and temperature of the proposed problem, the combination methods of Laplace transform and finite Hankel transform are applied. Then, using Maple software, the analytical result is plotted and graphically analysed with the involved parameters.

2. Problem Formulation

Consider the movement of an infinitely large vertical cylinder with radius, r_0 filled with incompressible Casson fluid. The radial coordinate r is assumed to be normal to the z-axis, which is considered as flowing along the cylinder's axis in a vertically upward direction. Figure 1 illustrates the convective Casson fluid flow with boundary slip velocity and buoyancy force. Initially, at time $t^*=0$, the fluid and cylinder are both at rest and the temperature is ambient, T_∞. When u_s occurs at the cylinder's edge and the fluid begins to flow uniformly along the axis at time $t^*>0$. At the same time, the cylinder temperature was raised from ambient temperature T_∞ to the wall temperature T_w and thereafter it is maintained constant. Assume that the velocity and temperature are the functions of r and t only. All fluid parameters are also assumed to be constant, except for the density in the buoyancy term, which is determined by the standard Boussinesq's approximation. Under these assumptions, a well-defined problem of unsteady Casson fluid flow in an axisymmetric cylinder in terms of the corresponding partial differential equation for momentum and energy [33] is given as

$$\rho \frac{\partial u^*}{\partial t^*} = \mu\left(1+\frac{1}{\beta}\right)\left(\frac{\partial^2 u^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial u^*}{\partial r^*}\right) + g\rho\beta_T\left(T^* - T_\infty\right), \tag{1}$$

and

$$\rho c_p \frac{\partial T^*}{\partial t^*} = k\left(\frac{\partial^2 T^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial T^*}{\partial r^*}\right), \tag{2}$$

with the associated initial and boundary conditions [24, 50]

$$\begin{aligned}
&u^*(r^*,0)=0 \qquad && \mathrm{T}^*\left(r^*,0\right)=\mathrm{T}_\infty \qquad && ;\ r\in[0,r_0]\,, \\
&\frac{\partial u^*(0,t^*)}{\partial r^*}=0 \qquad && \frac{\partial T^*(0,t^*)}{\partial r^*}=0 \qquad && ;\ t^*>0, \\
&u^*(r_0^*,t^*)=u_s^* \qquad && \mathrm{T}^*\left(r_0^*,t^*\right)=\mathrm{T}_w \qquad && ;\ t^*>0.
\end{aligned} \tag{3}$$

where ρ is the density of the fluid, u^* is the velocity component along the z-axis, μ is the dynamic viscosity of the fluid, β is the non-Newtonian Casson parameter, g is the gravitational acceleration, β_T is the coefficient of thermal expansion, T^* is the temperature of the fluid, T_∞ is the ambient temperature, c_p is the specific heat capacity of fluid at a constant temperature, k is thermal conductivity, v is the kinematic viscosity of the fluid. Introducing the following dimensionless variables [24, 50] as

$$t=\frac{t^*\nu}{r_0^2},\ \ \mathrm{r}=\frac{r^*}{r_0},\ \ u=\frac{u^*}{u_0},\ \ u_s=\frac{u_s^*}{u_0},\ \ \theta=\frac{T-T_\infty}{T_w-T_\infty}. \tag{4}$$

The governing Eqs. (1)-(3) are transformed to the following dimensionless form by using Eq. (4), obtain as

$$\frac{\partial u}{\partial t} = \beta_1\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) + Gr\theta \tag{5}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\mathrm{Pr}}\left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r}\right) \tag{6}$$

together with the corresponding initial and boundary conditions

$$\begin{aligned} &u(r,0)=0, && \theta(r,0)=0 && ;\ r\in[0,1]\,, \\ &\frac{\partial u(0,t)}{\partial r}=0, && \frac{\partial \theta(0,t)}{\partial r}=0 && ;\ t>0, \\ &u(1,t)=u_s, && \theta(1,t)=1 && ;\ t>0 \end{aligned} \tag{7}$$

where $\Pr=\frac{\mu c_p}{k}$ is the Prandtl number, $Gr=\frac{g\beta_T(T_w-T_\infty)r_0^2}{\nu u_0}$ is the Grashof number, $\beta_1=\frac{1}{\beta_0}$ and $\beta_0=1+\frac{1}{\beta}$ are the constant parameters.

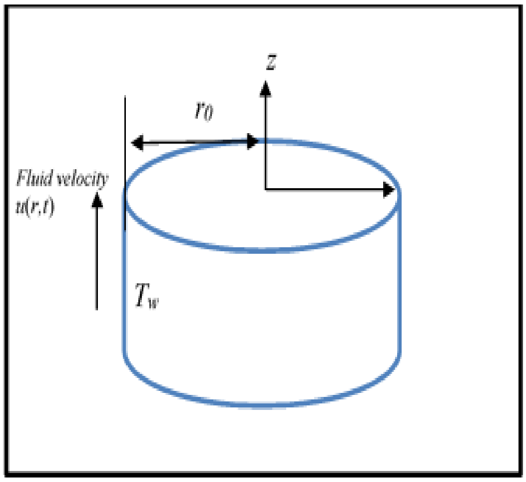

Fig. 1. Fluid flow's physical geometry

3. Results

Initial and boundary value problems are solved using the joint Laplace transform and Hankel transform.

3.1 Calculation of Temperature

Applying Laplace transform into Eqs. (6) and (7) subjected to the initial condition Eq. (7), yields

$$s\bar{\theta}(r,s)=\frac{1}{\Pr}\left[\frac{\partial^2\bar{\theta}(r,s)}{\partial r^2}+\frac{1}{r}\frac{\partial\bar{\theta}(r,s)}{\partial r}\right], \tag{8}$$

$$\bar{\theta}(1,s)=\frac{1}{s}, \tag{9}$$

where $\bar{\theta}(r,s)$ is the Laplace transform of the function $\theta(r,t)$ and s is the transformation variable. Then, applying finite Hankel transform of zero order to Eq. (8) and by using condition Eq. (9), give

$$\overline{\theta_H}(r_n,s)=\frac{J_1(r_n)}{r_n}\left[\frac{1}{s}-\frac{1}{s+\frac{r_n^2}{\Pr}}\right], \tag{10}$$

where $\bar{\theta}_H(r_n,s)=\int_0^1 r\bar{\theta}(r,s)J_0(rr_n)dr$ is the finite Hankel transform of the function $\bar{\theta}(r,s)$ and r_n with *n=0,1,...* are the positive roots of the equation $J_0(x)=0$, where J_0 is being the Bessel function of the first kind and zero order. Next, applying the inverse Laplace transform of Eq. (10), obtain as

$$\theta_H(r_n,t)=\frac{J_1(r_n)}{r_n}-\frac{J_1(r_n)}{r_n}\exp\left(-\frac{r_n^2}{\Pr}t\right). \tag{11}$$

Finally, the inverse finite Hankel transform is applied to Eq. (11) and written as

$$\theta(r,t)=1-2\sum_{n=1}^{\infty}\frac{J_0(rr_n)}{r_nJ_1(r_n)}\exp\left(-\frac{r_n^2}{\Pr}t\right). \tag{12}$$

3.2 Calculation of Velocity

Applying the Laplace transform to the Eqs. (5) and (7) subjected to the initial condition Eq. (7), yields

$$s\bar{u}(r,s)=\beta_1\left[\frac{\partial^2\bar{u}(r,s)}{\partial r^2}+\frac{1}{r}\frac{\partial\bar{u}(r,s)}{\partial r}\right]+Gr\bar{\theta}(r,s), \tag{13}$$

$$\bar{u}(1,s)=\frac{u_s}{s}, \tag{14}$$

where $\bar{u}(r,s)$ is the Laplace transform of the function $u(r,t)$. Then, applying finite Hankel transform of zero order to Eq. (13) and by using boundary condition Eq. (14), give

$$\bar{u}_H(r_n,s)=\frac{r_nJ_1(r_n)\beta_1}{s+\beta_1r_n^2}\frac{u_s}{s}+\frac{GrJ_1(r_n)}{r_n}\left[\frac{1}{s\left(s+\beta_1r_n^2\right)}-\frac{1}{\left(s+\frac{r_n^2}{\Pr}\right)\left(s+\beta_1r_n^2\right)}\right], \tag{15}$$

where $\bar{u}_H(r_n,s)=\int_0^1 r\bar{u}(r,s)J_0(rr_n)dr$ is the finite Hankel transform of the function $\bar{u}(r,s)$. Eq. (15) can be written in a more suitable form as

$$\bar{u}_H(r_n,s)=F_1(s)+F_2(s), \tag{16}$$

where

$$F_1(s)=\frac{r_nJ_1(r_n)\beta_1}{s+\beta_1r_n^2}\frac{u_s}{s}=\frac{J_1(r_n)}{r_n}\left[\frac{u_s}{s}-\frac{u_s}{s+\beta_1r_n^2}\right], \tag{17}$$

$$F_2(s)=\frac{GrJ_1(r_n)}{r_n}\left[\frac{1}{\beta_1r_n^2}\left(\frac{1}{s}-\frac{1}{s+\beta_1r_n^2}\right)-\frac{\Pr}{r_n^2(1-\beta_1\Pr)}\left(\frac{1}{s+\beta_1r_n^2}-\frac{1}{s+\frac{r_n^2}{\Pr}}\right)\right]. \tag{18}$$

Applying the inverse Laplace transform to Eq. (16), together with Eqs. (17) and (18) can be written as

$$u_H(r_n,t)=f_1(t)+f_2(t), \tag{19}$$

where

$$f_1(t)=\frac{J_1(r_n)}{r_n}\left(u_s-u_s\exp\left(-r_n^2\beta_1t\right)\right), \tag{20}$$

and

$$f_2(t)=\frac{GrJ_1(r_n)}{r_n}\left[\frac{1}{\beta_1r_n^2}\left(1-\exp\left(-\beta_1r_n^2t\right)\right)-\frac{\Pr}{r_n^2(1-\beta_1\Pr)}\left(\exp\left(-\beta_1r_n^2t\right)-\exp\left(\frac{-r_n^2}{\Pr}t\right)\right)\right]. \tag{21}$$

The inverse finite Hankel transform of order zero is used to determine fluid flow velocity for Eq. (19), which is given as

$$\begin{aligned}u(r,t)=u_s-2u_s\sum_{n=1}^{\infty}\left[\frac{J_0(rr_n)\exp\left(-r_n^2\beta_1t\right)}{r_nJ_1(r_n)}\right]\\+\frac{2Gr}{\beta_1\Pr-1}\sum_{n=1}^{\infty}\left[\frac{J_0(rr_n)}{r_n^3J_1(r_n)}\left(\frac{\beta_1\Pr-1}{\beta_1}+\frac{\exp\left(-r_n^2\beta_1t\right)}{\beta_1}-\Pr\exp\left(\frac{-r_n^2}{\Pr}t\right)\right)\right].\end{aligned} \tag{22}$$

4. Discussion

The fluid flow behavior of the obtained analytical solutions for the fluid velocity and temperature are plotted and analyzed by using Maple code. A limiting case for the present result Eq. (22) also has been observed in this problem to ensure that the current outcome is accurate by comparing it with the published results done by Khan *et al.,* [24]. This comparison is shown in Figure 2. The present result Eq. (22) is identical to the solution obtained by Khan *et al.,* [24]. Hence, the accuracy of the solution to this problem is confirmed.

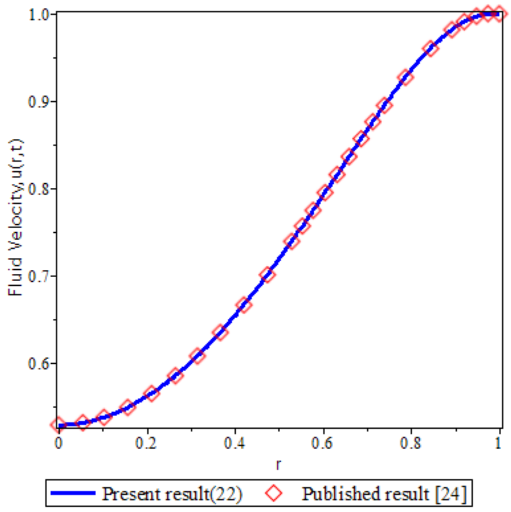

Fig. 2. Comparison of velocity profile $u(y,t)$ from equation (22) when $\beta \rightarrow \infty$, u_s=1.0 with Khan *et al.,* [24] when ω= 0

The numerical results of the velocity and temperature profiles are produced and graphically shown in Figures 3-7 with the related parameters such as Casson parameter β, Grashof number Gr, Prandtl number Pr, slip velocity parameter u_s and time parameter t. For the numerical calculations, the following variables are fixed: u_s=0 for no-slip condition, u_s=0.1 for slip condition, Pr=21.0 Prandtl number for blood, β=0.8, t=2.0, Gr=1.0 and the range of relevant parameter values have been approximated as β=0.4,0.8,1.2, Gr=1.0,2.0,3.0 and Pr=5.0,7.2,21 based on the physical data provided in the previous studies [22, 24, 50]. Besides, a broad spectrum of parameter values is being considered in the current study's findings which can be described as u_s=0,1.0,2.0 and t=1.0,2.0.

The influence of the Casson parameter, β on the fluid velocity distributions with the presence of slip and no-slip conditions is displayed in Figure 3. From the observation, the fluid velocity decreases as the Casson parameter increases for both slip and no-slip conditions. It is caused by the fluid's shear thickening factor and increasing internal friction. Therefore, the fluid thickens and becomes more viscous, which causes a reduction in the fluid's velocity.

Figure 4 illustrates velocity profiles with the various thermal Grashof number, Gr as slip velocity and no-slip velocity present at the boundary. It shows that increasing the Grashof number with the existence of the slip or no-slip effects will cause the fluid velocity increases. This is because the thermal buoyancy force is dominant during the free convection process. It causes differences in fluid density and temperature. A fluid with a high temperature has a low density, which causes it to rise as a result of buoyancy, whereas a fluid with a low temperature has a high density, which causes it to fall as a result of gravity. Therefore, it has less impact on the viscous force in the momentum equation, which increases the fluid velocity.

The effects of Prandtl number, Pr concerning slip velocity and non-slip velocity conditions on fluid velocity profiles are discussed in Figure 5. It is found that an increment of the Prandtl number will lead to the decrement of fluid velocity with the occurrence of the slip or no-slip effect. As the Prandtl number increases, fluid viscosity increases which indicates that the viscous force dominates over the thermal force. As a result, the fluid thickens and increases frictional force, which causes the fluid to move slowly.

Figure 6 is plotted to discuss the behavior of velocity profiles for no-slip velocity and slip velocity conditions at the walls of the cylinder (r=1) with the changes in time. It shows that the fluid velocity flow increases with an increase of the slip velocity, u_s effect. It is due to the velocity gradient that exists between two distinct mediums that interact with the fluid flow and the cylinder wall. The slip velocity impact is important since it can be found in practical applications like blood flow in the arteries. Besides that, as the time parameter t increases, so does the fluid velocity. It is due to the fluid system more stable with the larger time interval.

Lastly, the influence of the Prandtl number on the temperature profile, θ (r, t) versus radial coordinate r has been depicted in Figure 7 for different values of the time t. Based on the graph observation, increases in Prandtl number lead to fluid temperature decrease. It is due to the rapid thermal diffusion of the heat that takes place for a larger Prandtl number. It causes fluid to cool down faster and decreases the fluid temperature.

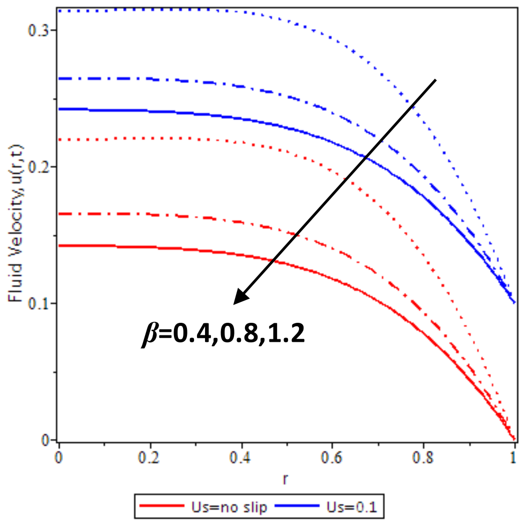

Fig. 3. Velocity profiles for different Casson Parameter and slip velocity with Pr=21.0, Gr=1.0 and t=2.0.

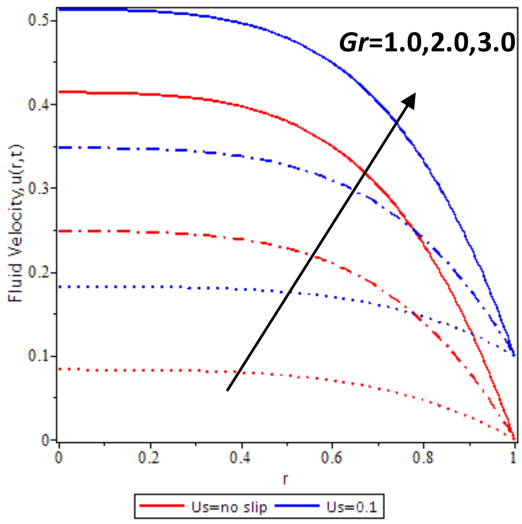

Fig. 4. Velocity profiles for different Grashof number and slip velocity with Pr=21.0, β =0.8 and t=2.0.

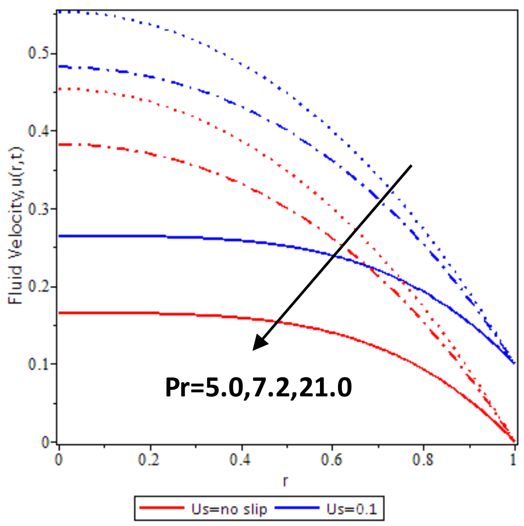

Fig. 5. Velocity profiles for different Prandtl number and slip velocity with β =0.8, Gr=1.0 and t=2.0.

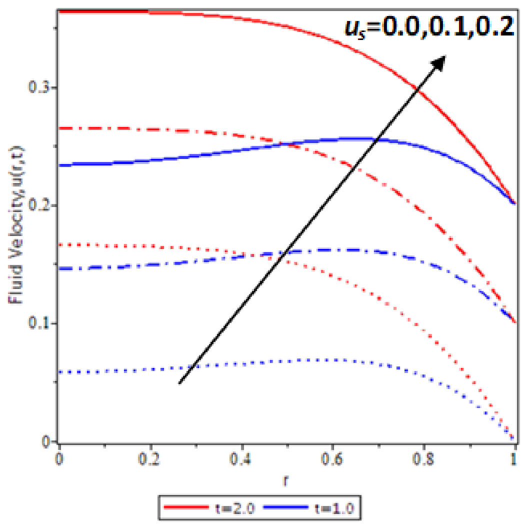

Fig. 6. Velocity profiles for different slip velocity and time parameter with Pr=21.0, β =0.8 and Gr=1.0.

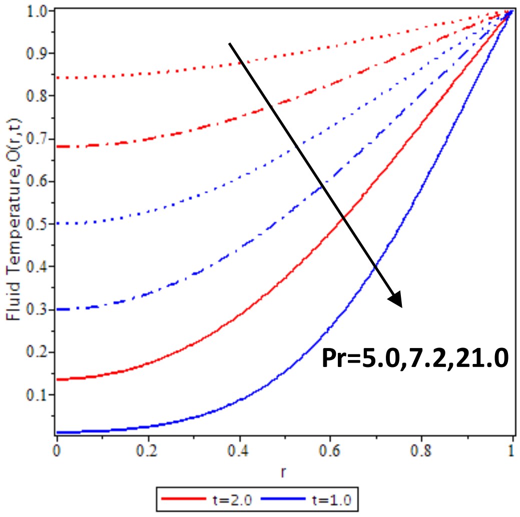

Fig. 7. Temperature profiles for different Prandtl number and time parameter.

5. Conclusions

The investigation is conducted into the problem of heat transfer resulting from free convection of Casson fluid flow through a vertical cylinder with the slip velocity effect at the boundary. By utilizing a combination of the Laplace transform and finite Hankel transform techniques, the analytical solutions of fluid temperature and velocity are achieved. The corresponding initial and boundary conditions are satisfied by the analytical results. The obtained analytical solution is highly beneficial for verifying the accuracy of the numerical results. Besides, the obtained result is significant to study human blood flow behaviour in the small blood vessels. Future research on this topic can incorporate boundary conditions for other biofluid models, and additional effects such as MHD, porous medium, radiation and chemical reaction. Moreover, the main findings from the discussion above are as follows:

i. The limiting case of the obtained result Eq. (22) is found in an excellent mutual agreement with the previous study by Khan *et al.,* [24]. Thus, the accuracy of the obtained result is validated.
ii. Enhancement of the fluid velocity when Gr, u_s and t are increased.
iii. Fluid velocity decreases as β and Pr increase.
iv. Temperature profiles decrease when Pr increases and t decreases.
v. Slip velocity enhances fluid flow, particularly along the cylinder's wall when r=1.

Acknowledgement
The authors would like to acknowledge the Ministry of Higher Education Malaysia and Research Management Centre-UTM, University Technology Malaysia (UTM) for financial support through vote numbers 4B748 and 03M77.

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