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Steady Flow of Thermo-Viscous Fluid between Infinitely Stretched Porous Parallel Plates-A Perturbation Technique

Pothanna Nalimela[1,*], Srinivas Joshi[1], Aparna Podila[1], Padmaja Podila[2], Rajashekar Pemmaraju[1]

[1] Department of Mathematics, VNR Vignana Jyothi Institute of Engineering and Technology, Hyderabad, TS, India
[2] Department of Mathematics, Prasad V Potluri Siddhartha Institute of Technology, Vijayawada, AP, India

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ABSTRACT

The steady flow of a thermo-viscous incompressible fluid bounded between porous parallel plates is examined in this paper. The governing equations of the flow are coupled in the velocity and temperature field. The solutions of velocity and temperature are obtained using a powerful and most elegant method called perturbation technique. The closed form solutions of the velocity and temperature distributions are obtained when thermo-stress coefficient is far less compared to strain thermal conductivity coefficient and coefficient of cross viscosity. The variations of velocity and temperature distributions on the flow field have been discussed with the help of illustrations. The effect of various flow parameters on the flow field have been discussed with the help of graphs. The rate of variation of the velocity is observed as far less when compare to the temperature of the fluid. This effect is due to the porosity and suction/injection of the flow passes through the horizontal parallel plates. This study includes the applications in extraction of petrol and oils from porous rocks, chemical reactors, and human cardiovascular system and in several engineering devices such as heat and mass exchanges. The results of the present study will hopefully enable a better understanding of nuclear and clinical applications.

1. Introduction

The non-Newtonian nature of materials has been the subject of extensive study for over one and half centuries. It is only in last seven or eight decades that serious attempts have been made to extend this investigation in the realm of non-linearity. The failure of the linear theories in predicting to a reasonable extent the mechanical behaviour of materials such as liquid polymers, fluid plastic, the molten metals etc., subjected to stresses has been the motivating force behind study of the non-linear theories for material description. Some of the non-liner theories proposed so far (listed in references) have not taken into account the strong dependence of visco-elastic behaviour upon thermal conduction i.e. interaction/interrelation between mechanical and non-mechanical (such as thermal, chemical, electromagnetic etc.) effects even though the large amount data of experimental evidence indicate a strong dependence of visco-elastic nature of the fluid upon thermal behaviour.

* *Corresponding author.*
E-mail address: pothareddy81@gmail.com (Pothanna Nalimela)

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For example, materials such as solid propellants exhibit a mechanical behaviour at moderate temperature variations, whereas little or no correlation between them would be observed with that under the isothermal condition.

The concept of thermo-viscous fluids which reflect the interaction between thermal and mechanical responses in fluids in motion due to external influences was introduced by Koh and Eringin[1] in 1963. For such a class of fluids, the stress-tensor 't' and heat flux bivector 'h' are postulated as polynomial functions of the kinematic tensor, viz., and the rate of deformation tensor 'd':

$$d_{ij} = (u_{i,j} + u_{j,i})/2$$

and thermal gradient bivector 'b'

$$b_{ij} = \in_{ijk} \theta_k$$

Where u_i is the i^{th} component of velocity and θ is the temperature field.

A second order theory of thermo-viscous fluids is characterized by the pair of thermo-mechanical constitutive relations:

$$t = \alpha_1 I + \alpha_3 d + \alpha_5 d^2 + \alpha_6 b^2 + \alpha_8 (db - bd)$$

and

$$h = \beta_1 b + \beta_3 (bd + db)$$

With the constitutive parameters α_i , β_i being polynomials in the invariants of d and b in which the coefficients depend on density (ρ) and temperature (θ) only. The fluid is Stokesian when the stress tensor depends only on the rate of deformation tensor and Fourier-heat-conducting when the heat flux bivector depends only on the temperature gradient-vector, the constitutive coefficients α_1 and α_3 may be identified as the fluid pressure and coefficient of viscosity respectively and α_5 as that of cross-viscosity.

Flow of incompressible homogeneous thermo-viscous fluids satisfies the usual conservation equations:

Equation of continuity

$$v_{i,i} = 0$$

Equation of momentum

$$\rho\left[\frac{\partial v_i}{\partial t} + v_k v_{i,k}\right] = \rho F_k + t_{ji,i}$$

and the energy equation

$$\rho c \dot{\theta} = t_{ij} d_{ij} - q_{i,i} + \rho\gamma$$

Where

$F_k = k^{th}$ Component of external force per unit mass,
$c =$ Specific heat,
γ =Thermal energy source per unit mass
$q_i = i^{th}$ Component of heat flux bivector = $\in_{ijk} h_{jk} / 2$
$t_{ij} =$ The components of stress tensor
$d_{ij} =$ The components of rate of deformation tensor

The development of non-linear theory reflecting the interaction/interrelation between thermal and viscous effects has been preliminarily studied by Koh and Eringen [1] and Coleman *et al.,* [7] studied Existence of caloric equations of state in thermodynamics. Langlois and William [2] examined steady flow of a slightly viscoelastic fluid between rotating spheres. Rivlin [3] examined the solution of problems in second order elasticity theory. Yamamoto and Yoshida [4] studied flow through a porous wall with convective acceleration. Beavers *et al.,* [5] studied boundary conditions at a naturally permeable wall. A systematic rational approach for such class of fluids has been developed by Green and Nagdhi [6]. Coleman *et al.,* [7] studied existence of caloric equations of state in thermodynamics. In 1965 Kelly [8] examined some simple shear flows of second order thermo-viscous fluids. Pothanna *et al.,* [9] studied flow of slightly thermo-viscous fluid in a porous slab bounded between two permeable parallel plates. Pothanna *et al.,* [10] examined effect of strain thermal conductivity on slightly thermo-viscous fluid in a porous slab bounded between two parallel plates. The problem of steady flow of a second order thermo-viscous fluid over an infinite plate was studied by Nageswara Rao and Pattabhi Ramacharyulu [11]. Aparna *et al.,* [12] studied uniform Flow of Viscous Fluid Past a Porous Sphere Saturated with Micro Polar Fluid. Aparna *et al.,* [13] examined Flow generated by slow steady rotation of a permeable sphere in a micro-polar fluid. Aparna *et al.,* [14] examined Viscous Fluid Flow Past a Permeable Cylinder. Aparna *et al.,* [15] studied Rotary Oscillations of a Permeable Sphere in an Incompressible Couple Stress Fluid. Padmaja *et al.,* [16] analyzed numerical solution of singularly perturbed two parameter problems using exponential splines. Pothanna *et al.,* [17] examined a numerical study of coupled non-linear equations of thermo-viscous fluid flow in cylindrical geometry. Pothanna *et al.,* [18] Analytical and Numerical Study of Steady Flow of Thermo-Viscous Fluid between Two Horizontal Parallel Plates in Relative Motion. Bakar *et al.,* [19] studied stability analysis on mixed convection nano fluid flow in a permeable porous medium with radiation and internal heat generation. Juwari *et al.,* [20] examined simulation of dispersion and explosion in petrol station using 3d computational fluid dynamics flacs software. Benkara-mostefa *et al.,* [21] examined heat transfer and entropy generation of turbulent flow in corrugated channel using nano fluid. Pparasa *et al.,* [22] investigated oscillatory flow of couple stress fluid flow over a contaminated fluid sphere with slip condition. Khan *et al.,* [23] studied heat and mass transfer of williamson nano fluid flow yield by an inclined lorentz force over a nonlinear stretching sheet. Khan *et al.,* [24] examined 3-D axisymmetric carreau nano fluid flow near the homann stagnation region along with chemical reaction: application fourier’s and fick’s laws. Khan *et al.,* [25] investigated change in internal energy of carreau fluid flow along with ohmic heating: a von karman application. Chu *et al.,* [26] examined thermal impact of hybrid nano fluid due to inclined oscillatory porous surface with thermo-diffusion features. Li shuguang *et al.,* [27] studied entropy optimized flow of sutterby nanomaterial subject to porous medium: buongiorno nano fluid model.

Perturbation technique is most powerful and elegant method which is used to solve the many complexes, highly nono-linear and coupled differential equations. The problem in the present investigation studied using the perturbation technique. The solutions of governing equations obtained also presented in form of graphs and effect of various material parameters of problem discussed and explained with different numerical values. The present work is very much useful to the researchers and scientist to solve their industry and research related problems. Porous parallel plates are of immense practical importance in industrial and engineering systems. The human cardiovascular system and in several engineering devices such as heat and mass exchanges, chemical reactors, chromatography columns and other processing equipment. Owing to the wide range of applications, the interest in the study of flow characteristics in these configurations has grown enormously during the last decades.

The theories proposed earlier have not been discussed the effects of thermo-viscous material parameters on the steady flow of a thermo-viscous incompressible fluid bounded between porous parallel plates. This paper attempts to study the effects of material parameters such as Suction/Injection parameter(S), thermo-mechanical interaction coefficient and thermal conductivity coefficient on the steady flow of a thermo-viscous incompressible fluid bounded between porous parallel plates in the absence of viscous dissipation.

2. Mathematical Formulation

Consider the steady flow of a second order thermo-viscous fluid characterized by constitute equations between two horizontal parallel porous plates (see Figure 1). The flow is generated by a constant pressure gradient in a direction parallel to the plates. Further, the plates are assumed to be porous allowing a constant injection at the lower plate and equal suction at the upper plate. Let v_0 be the injection/suction velocity.

With reference to a coordinate system OXYZ with origin on the plate, the X-axis in the direction of the fluid flow, Y-axis perpendicular to plates. The plates are represented by y=0 and y=h. The two plates are maintained at constant temperatures θ_0 and $\theta_1$respectively.

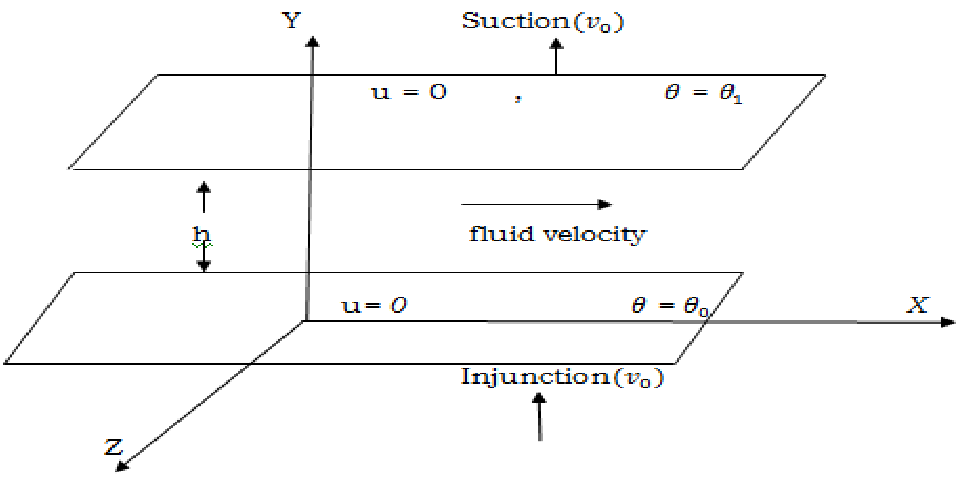

Fig. 1. Physical model and co-ordinate system

Let the steady flow between the two plates is characterized by the velocity field [u(y), v_0, 0] and temperature field θ(y). This choice of velocity satisfies the continuity equation. The equations of motion in the absence of external forces and internal energy sources reduces to

$$\rho v_0 \frac{\partial u}{\partial y} = - \frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - \alpha_6 \frac{\partial \theta}{\partial x} \frac{\partial^2 \theta}{\partial y^2} \tag{1}$$

$$\mu_c \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^2 + \rho F_y = 0 \tag{2}$$

$$\alpha_8 \frac{\partial}{\partial y}\left(\frac{\partial \theta}{\partial y} \frac{\partial u}{\partial y}\right) + \rho F_z = 0 \tag{3}$$

And the energy equation reduces to

$$\rho c \left(u \frac{\partial \theta}{\partial x} + v_0 \frac{\partial \theta}{\partial y}\right) = \mu \left(\frac{\partial u}{\partial y}\right)^2 - \alpha_6 \frac{\partial \theta}{\partial x} \frac{\partial u}{\partial y} \frac{\partial \theta}{\partial y} + k \frac{\partial^2 \theta}{\partial y^2} + \beta_3 \frac{\partial \theta}{\partial x} \frac{\partial^2 u}{\partial y^2} \tag{4}$$

Together with the boundary conditions:

$$u=0 \quad \theta = \theta_0 \quad \text{at} \quad y=0$$
$$u=0 \quad \theta = \theta_1 \quad \text{at} \quad y=h \tag{5}$$

Introducing the non-dimensional quantities

$$y = hY, \quad u = (\mu/\rho h)\, U, \quad T = \frac{\theta-\theta_0}{\theta_1-\theta_0}, \quad \frac{\partial \theta}{\partial x} = \frac{\theta_1-\theta_0}{h} c_2, \quad -\frac{\partial p}{\partial x} = \frac{\mu^2}{\rho h^3} c_1$$
$$p_r = \frac{c\mu}{k}, \quad S = \frac{\rho h v_0}{\mu}, \quad B_3 = \frac{\beta_3}{\rho h^2 c}, \quad A_1 = \frac{\mu^2}{\rho h^2 c(\theta_1-\theta_0)}, \quad A_6 = \frac{\alpha_{6}(\theta_1-\theta_0)^2}{\mu^2} \tag{6}$$

Where c_1 and c_2 are non-dimensional pressure and temperature gradient respectively. In terms of these non-dimensional quantities the momentum and energy equations reduces to

$$S \frac{dU}{dY} = c_1 + \frac{d^2U}{dY^2} - A_6 c_2 \frac{d^2T}{dY^2} \tag{7}$$

$$U c_2 + S \frac{dT}{dY} = A_1 \left[\left(\frac{dU}{dY}\right)^2 - A_6 c_2 \frac{dU}{dY} \frac{dT}{dY}\right] + \frac{1}{p_r} \frac{d^2T}{dY^2} + B_3 C_2 \frac{d^2U}{dY^2} \tag{8}$$

The boundary conditions in Eq. (5) reduces to

$$U(0) = 0 \qquad T(0) = 0$$
$$U(1) = 0 \qquad T(1) = 1 \tag{9}$$

The fluid is assumed at slightly thermo-viscous in such a way that the interaction between the mechanical stress and thermal gradients characterized by coefficients α_6, β_3 of a lower order in magnitude compare to magnitude of viscous dispensation $\mu^2 \frac{d^2U}{dY^2}$ and non Fourier heat transfer B_3 i.e. terms containing A_6 in momentum and energy balance equation taken to be smaller than the other terms in the energy equation.

3. Perturbation Method

The velocity and temperature fields characterized by the perturbation method, by taking A_6 as the perturbation parameter.

$$U(Y) = U^{(0)}(Y) + A_6 U^{(1)}(Y) + A_6^2 U^{(2)}(Y) + A_6^2 U^{(2)}(Y) + \ldots\ldots\ldots\ldots\ldots \quad (10)$$

$$T(Y) = T^{(0)}(Y) + A_6 T^{(1)}(Y) + A_6^2 T^{(2)}(Y) + A_6^2 T^{(2)}(Y) + \ldots\ldots\ldots\ldots\ldots \quad (11)$$

Substituting Eq. (10) and Eq. (11) in Eq. (7) and Eq. (8) and collecting terms of like powers of A6, the successive approximations are as follows.

3.1 Basics or Zeroth Order Approximation (i.e. Terms Independent of A6)

The equations in this approximation are

$$S\frac{dU^{(0)}}{dY} = c_1 + \frac{d^2U^{(0)}}{dY^2} \quad (12)$$

$$U^{(0)}c_2 + S\frac{dT^{(0)}}{dY} = A_1\left(\frac{dU^{(0)}}{dY}\right)^2 + \frac{1}{p_r}\frac{d^2T^{(0)}}{dY^2} + B_3c_2\frac{d^2U^{(0)}}{dY^2} \quad (13)$$

With the boundary conditions:

$$U^{(0)}(0) = 0 \qquad U^{(0)}(1) = 0 \quad (14)$$

$$T^{(0)}(0) = 0 \qquad T^{(0)}(1) = 1 \quad (15)$$

From Eq. (12) and using the boundary conditions in Eq. (14), the velocity distribution is obtained as

$$U^{(0)}(Y) = \frac{C_1}{S}\left(Y - \frac{1-e^{SY}}{1-e^{S}}\right) \quad (16)$$

Employing this velocity distribution in Eq. (13) and using the boundary conditions in Eq. (15), the temperature distribution is obtained as

$$\begin{aligned} T^{(0)}(Y) = &\left[\begin{matrix} 1 - \frac{C_1C_2}{2S^2} - \frac{C_1C_2}{p_r S^3} + \frac{C_1C_2}{S^2(1-e^S)} - \frac{A_1C_1^2}{S^3} \\ -\frac{p_rC_1C_2(1-e^S)}{S^3(1-p_r)} - \frac{2p_rA_1C_1^2}{S^3(1-p_r)} - \frac{B_3C_1C_2p_r}{S(1-p_r)} - \frac{p_rA_1C_1^2}{2S^2(2-p_r)}\frac{1+e^S}{1-e^S} \end{matrix}\right]\left(\frac{1-e^{Sp_rY}}{1-e^{Sp_r}}\right) + \\ &\left[\frac{p_rC_1C_2}{S} + \frac{2A_1p_rC_1^2}{S(1-e^S)} + \frac{B_3C_1C_2p_rS}{1-e^S}\right]\frac{1-e^{SY}}{S^2(1-p_r)} + \frac{p_rA_1C_1^2}{2S^2(2-p_r)(1-e^S)^2}\left(1-e^{2SY}\right) + \frac{C_1C_2}{S^2}Y^2 + \\ &\frac{1}{Sp_r}\left[-\frac{p_rC_1C_2}{S}\left\{-\frac{1}{Sp_r} + \frac{1}{1-e^S}\right\} + \frac{p_rA_1C_1^2}{S^2}\right]Y \end{aligned} \quad (17)$$

3.2 First Order Approximation (i.e. Terms Containing A_6)

The equations are

$$S\frac{dU^{(1)}}{dY} = \frac{d^2U^{(1)}}{dY^2} - c_2\frac{d^2T^{(0)}}{dY^2} \tag{18}$$

$$U^{(1)}c_2 + S\frac{dT^{(1)}}{dY} = A_1\left[2\frac{dU^{(0)}}{dY}\frac{dU^{(1)}}{dY} - c_2\frac{dU^{(0)}}{dY}\frac{dT^{(0)}}{dY}\right] + \frac{1}{p_r}\frac{d^2T^{(1)}}{dY^2} + B_3c_2\frac{d^2U^{(1)}}{dY^2} \tag{19}$$

With the boundary conditions:

$$U^{(1)}(0) = 0,\ U^{(1)}(1) = 0 \tag{20}$$

$$T^{(1)}(0) = 0,\ T^{(1)}(1) = 0 \tag{21}$$

Which are homogeneous conditions

From the Eq. (16), Eq. (17), Eq. (18) and Eq. (19) together with boundary conditions in Eq. (20) and Eq. (21), the velocity field and temperature field are obtained as

$$\begin{aligned}
U^{(1)}(Y) = {} & \frac{p_rc_1c_2}{2S^3(1-e^S)}(c_1-c_2)[(S+2)(1-e^{SY}) - Y(SY+2)(1-e^S)] \\
& -- \frac{c_1c_2}{S^3(1-e^S)}\left[c_2 + 2A_1p_rc_1 + \frac{pS}{1-e^S}(c_1-c_2)\right][(1-e^{SY}) - Y(1-e^S)] \\
& + \frac{p_rc_1c_2}{S^2(1-p_r)(1-e^S)^2}\begin{bmatrix}(2p_r-1)B_3c_2S^2 + p_rc_2(1-e^S) \\ +c_1(1-p_r+2A_1)\end{bmatrix} \\
& + \frac{p_rA_1c_1^2c_2(p-1)}{S^2(2-p_r)(1-e^S)^3}[(e^S-e^{SY}) - e^{2S}(1-e^{SY}) + e^{2SY}(1-e^S)] \\
& + \frac{p_rc_2}{(1-p_r)(1-e^{Sp_r})(1-e^S)}\begin{bmatrix} 1 - \frac{C_1C_2}{2S^2} - \frac{C_1C_2}{p_rS^3} + \frac{C_1C_2}{S^2(1-e^S)} \\ -\frac{A_1C_1^2}{S^3} - \frac{C_1C_2p_r(1-e^S)}{S^3(1-p_r)} - \frac{2p_rA_1C_1^2}{S^3(1-p_r)} \\ -\frac{B_3C_1C_2p_r}{S(1-p_r)} - \frac{p_rA_1C_1^2}{2S^2(2-p_r)}\frac{1+e^S}{1-e^S}\end{bmatrix}\begin{bmatrix}(e^S-e^{Sp_r}) - e^{SY}(1-e^{Sp_r}) \\ +e^{Sp_rY}(1-e^S)\end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
T^{(1)}(Y) = {} & \frac{1}{sp_r}\begin{Bmatrix}\left[a_1 + \left(\frac{1}{2}+\frac{1}{Sp_r}\right)a_2 + \left(\frac{1}{3}+\frac{1}{Sp_r}+\frac{2}{S^2p_r^2}\right)a_3\right][1-e^{Sp_rY}] \\ -\left[a_1Y + \left(\frac{Y^2}{2}+\frac{Y}{Sp_r}\right)a_2 + \left(\frac{Y^3}{3}+\frac{Y^2}{Sp_r}+\frac{2Y}{S^2p_r^2}\right)a_3\right][1-e^{Sp_r}]\end{Bmatrix} \\
& + \frac{1}{S^2(1-p_r)}\left[a_4 - \frac{2-p_r}{S(1-p_r)}a_8\right][(1-e^S)(1-e^{Sp_rY}) - (1-e^{Sp_r})(1-e^{SY})] \\
& + \frac{1}{2S^2(2-p_r)}\left[a_5 + \frac{4-p_r}{2S(2-p_r)}a_9\right][(1-e^{2S})(1-e^{Sp_rY}) - (1-e^{Sp_r})(1-e^{2SY})] \\
& + a_6\frac{1}{3S^2(3-p_r)}[(1-e^{Sp_r})(1-e^{3SY}) - (1-e^{3S})(1-e^{Sp_rY})] \\
& + a_7\frac{1}{Sp_r}[Ye^{Sp_rY}(1-e^{Sp_r}) - e^{Sp_r}(1-e^{Sp_rY})] \\
& + a_{10}\frac{1}{S^2(1+p_r)}\left[e^{S(1+p_r)}(1-e^{Sp_rY}) + e^{Sp_rY}(1-e^{SY}) - e^{Sp_r}\left(1-e^{S(1+p_r)Y}\right)\right]
\end{aligned}$$

Here a_1 , a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9 and a_{10} and which are given.

The velocity and temperature distribution up to the first order approximation in A_6 are as follows.

The velocity distribution

$$U(Y) = U^{(0)}(0) + A_6U^{(1)}(0)$$

and the temperature distribution

T(Y) = T $^{(0)}$ (Y) + A_6 T $^{(1)}$ (Y)

4. Results and Discussion

Numerical estimates of the velocity and temperature fields was carried for different values of strain thermal conductivity coefficient $B_3 = (1,3,5)$ and also the Suction/Injection parameter S= (0.3, 0.4, 0.5) taking $C_1 = 1, \mathrm{A}_1 = 1, p_r = 1.5$ and $\mathrm{A}_6 = 0.001$ and these are illustrated graphically.

To get the physical insight in to the problem the velocity and temperature fields have been discussed by assigning numerical values to various material parameters such as the Suction/Injection parameter(S) , thermo-mechanical interaction coefficient(a_6), the strain thermal conductivity coefficient(b_3), cross viscosity coefficient(μ_c) and Prandtl number(p_r) which characterise the flow phenomena . The influences of these parameters on the velocity and temperature have been studied and are presented graphically.

Figure 2, Figure 3 and Figure 4 shows the curvature of the velocity profile reduces as suction parameter increases. This is independent of thermo-viscous nature of the fluid. As the values of 'y' increases the fluid velocity up to middle of the plate and then decreases to attain the velocity of the upper plate and all the velocity profiles coincides at the upper plate.

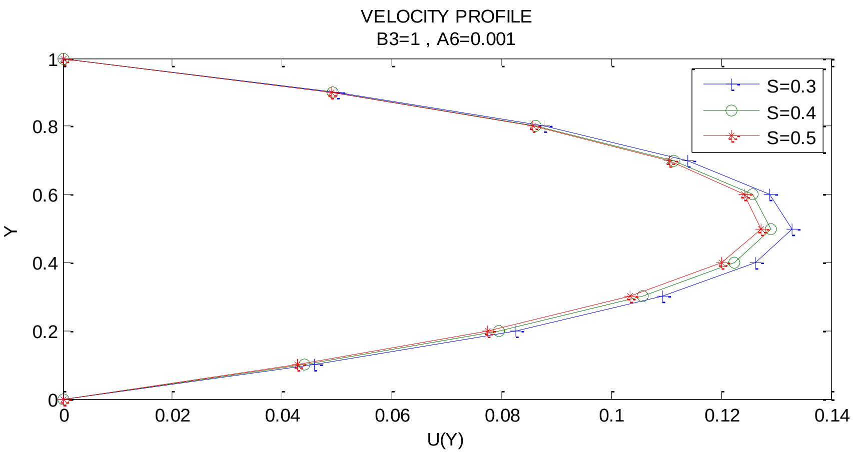

Fig. 2. Variations of the velocity profiles $U(Y)$ with S, A_6 and $B_3 = 1$

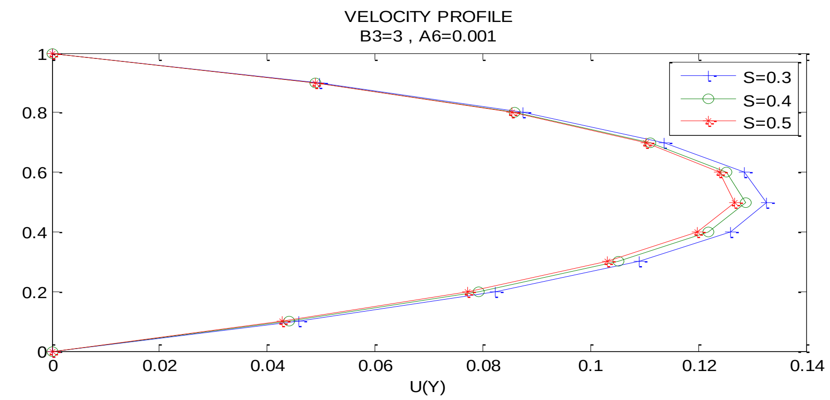

Fig. 3. Variations of the velocity profiles $U(Y)$ with S, A_6 and $B_3 = 3$

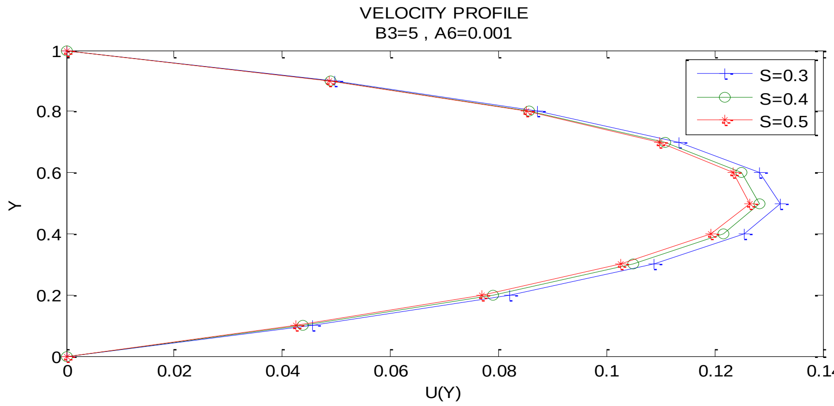

Fig. 4. Variations of the velocity profiles $U(Y)$ with S, A_6 and $B_3 = 5$

As suction parameter increases the temperature decreases and it is maximum at the middle of the plate. As B_3 increases the fluid gets heated slowly at the beginning of channel and rises up to middle of the plate and then decreases to attain the temperature of the upper plate. This is illustrated in Figure 5, Figure 6 and Figure 7. As the values of 'y' increases the fluid temperature up to middle of the plate and then decreases to attain the temperature of the upper plate and all the velocity profiles coincides at the upper plate.

The rate of increase of temperature as the values of Suction/Injection parameter (S) is at faster rate when compare to the rate of increase of velocity of the fluid.

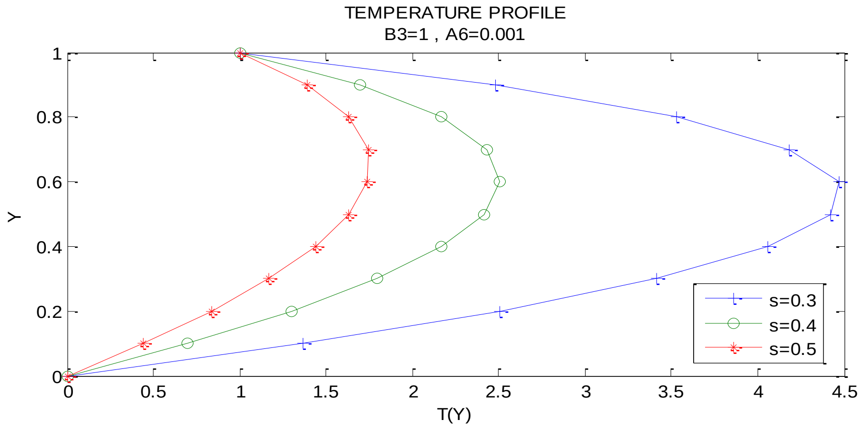

Fig. 5. Variations of the temperature profiles $T(Y)$ with S, A_6 and $B_3 = 1$

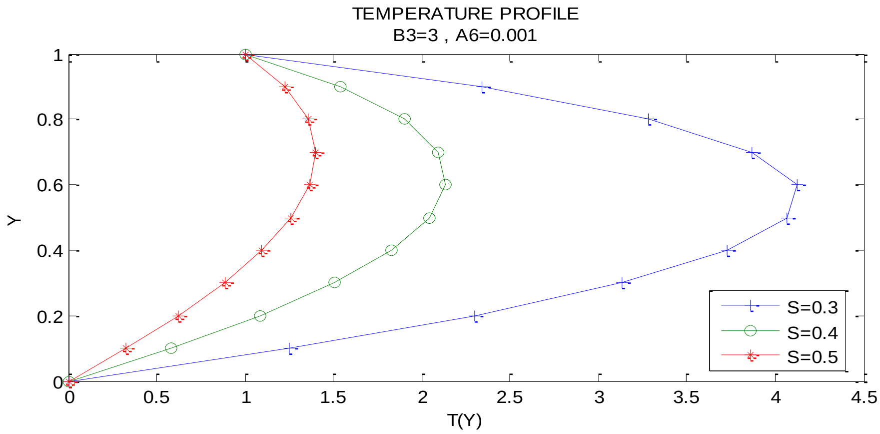

Fig. 6. Variations of the temperature profiles $T(Y)$ with S, A_6 and $B_3 = 3$

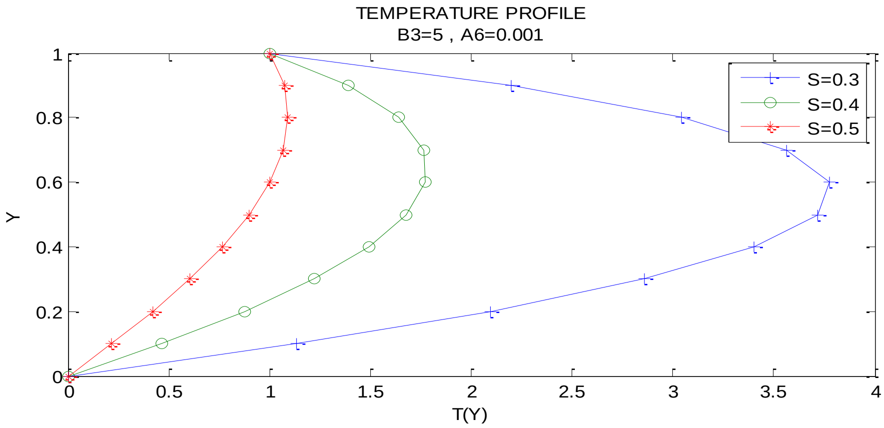

Fig. 7. Variations of the temperature profiles $T(Y)$ with S, A_6 and $B_3 = 5$

5. Conclusions

In this paper, the steady flow of a thermo-viscous incompressible fluid bounded between infinitely stretched porous parallel plates is examined in this paper the resulting governing steady, non-linear and coupled equations have been solved by using Perturbation technique. The computations are carried out for different values of thermo-mechanical interaction coefficient, strain thermal conductivity coefficient, Suction/Injection parameter and for the fixed values of remaining physical parameters.

i. The fluid velocity increases with the increase of Suction/Injection parameter up to the center of the channel and then decreases.
ii. The fluid temperature increases with the increase of strain thermal conductivity coefficient up to the center of the channel and then decreases.
iii. Both the fluid velocity and temperature increases, as the values of 'y' increases up to middle of the plate and then decreases.
iv. The rate of increase of velocity is far less when compared to the temperature of the fluid.
v. The present work can be extended to thermos-viscous flows in porous medium.
vi. The flows in porous medium as the future work can also be obtain the numerical solutions using MATLAB and MATHEMATICA solvers.
vii. The present study can also be extended to solve the governing equations in cylindrical and spherical geometry.

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