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Numerical Comparison of Three Rotors for Gravitational Water Vortex Turbine

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ABSTRACT

Renewable energy sources have gained significant attention due to the increasing demand for clean energy production. The gravitational vortex turbine (GVT) is one of the emerging technologies in the field of renewable energy that has gained attention for its simple and low-cost manufacturing process. The turbine operates by utilizing the energy of wastewater or other liquid flows to generate power on-site, making it a potentially viable solution for small-scale power generation. However, the optimization of the turbine's design is necessary to improve its efficiency and to make it a more competitive source of renewable energy. Previous research on GVT has mainly focused on the chamber's design to improve the formation of the vortex. However, little attention has been paid to the rotor design, which is also a critical parameter affecting the turbine's performance. The current study aimed to investigate the performance of three different rotors for the turbine, including the Savonius, H-Darrieus, and a standard rotor with straight blades, using numerical simulations. The numerical simulations were performed using ANSYS software, with ICEM modules for discretization and CFX for simulation. The results showed that the straight-bladed rotor outperformed the other two rotors, with an increase in efficiency of 40% and 79% compared to the Savonius and H-Darrieus geometry blades, respectively. The study highlights the importance of considering the rotor design in the optimization of the gravitational vortex turbine. The results provide valuable insights into the design parameters that can be used to enhance the turbine's performance. These findings can contribute to the development of more efficient and cost-effective gravitational vortex turbines for on-site power generation and consumption.

1. Introduction

The turbine is a crucial component in any hydroelectric power generation system as it converts the energy contained in water into rotational mechanical energy to generate electricity [1]. Selecting the most suitable turbine for a particular installation site depends on the characteristics of the site,

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with flow and head being the most significant ones. The commonly used turbines in hydroelectric installations worldwide are Francis, Kaplan, and Pelton turbines, and to a lesser extent, crossflow, Turgo, and gravitational turbines. Figure 1 illustrates the Gravitational Vortex Turbine's operating range compared to other conventional hydraulic generation technologies like the Pelton, Francis, and Michell Banki turbines. The latter requires deeper tributaries and more complex mounting devices than the GVT due to its configuration.

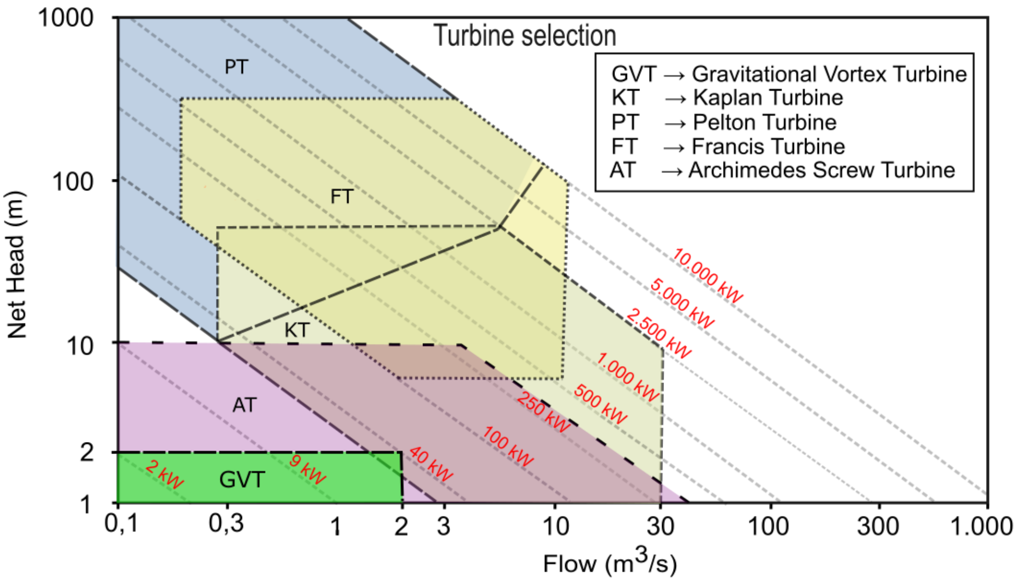

Fig. 1. Selection chart for different hydraulics turbines according to the net head and the volumetric flow [2]

The GVT represents a renewable energy alternative that relies on a small-scale turbine to convert the kinetic energy of fluid into electrical energy using a generator [3]. As illustrated in Figure 2, the turbine, also referred to as the standard tank, operates by allowing fluid to enter horizontally and tangentially. The circular geometry of the tank, combined with the difference in elevation between the inlet and outlet of the chamber, generates a gravitational vortex, a rotation of the fluid with respect to the outlet hole. This vortex results from the gravitational force and the Coriolis force acting together [4]. As water flows through the chamber tank, it follows a spiral path that rotates around the axis of the air core formed by the conservation of momentum between two fluids, water, and air, in this case.

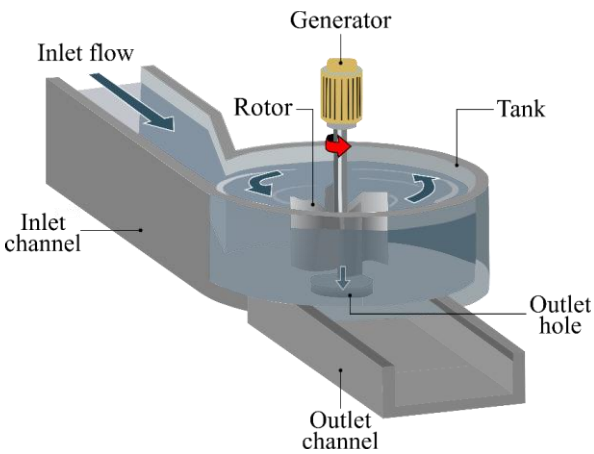

Fig. 2. GVT parts

According to the literature review on GVTs, especially at the runner and chamber geometry, the Table 1 summarize. The mains studies founded in the literature related to the GVTs.

Table 1
Literature review summary

Parameter	Study	Findings	Configuration	Reference
Water inlet height	CFD and experimental	Result differences between 0% and 7%	No runner Standard tank	[5–7]
Turbulence model	Analytical, CFD and experimental	Baseline Reynolds stress model	No runner Standard tank	[8]
Tank geometry	CFD and experimental	Cylindrical	No runner	[9]
		Conical	No runner	[10]
		Concave and convex	No runner	[11]
Tank diameter Angle of the reduction zone Width of the inlet channel Angle of the cone Channel height	CFD	0.8 m 70° 0.125 D 23° 0.2 D	No runner Standard tank	[12]
Outlet diameter for a conical tank	CFD	0.3 D	No runner Conical tank	[13]
Ratio between the chamber diameter and the outlet diameter (D/d) Inlet channel slope angle	CFD	2.5 60°	No runner Standard tank	[14]
Outlet diameter	CFD and experimental	0.14 D – 0.18 D	No runner Standard tank	[15, 16]
Number of blades in the runner	CFD and experimental	12	Standard tank	[17]
Thickness of runner blades (according to geometry)	CFD and experimental	Blade thickness	Standard tank	[18]
Blade profile in the runner	CFD	Curved profile	Standard tank	[19, 20]
Multi-stage runner	Experimental	Multi-stage runner	Conical tank	[21, 22]
Economic evaluation	Experimental	Economic feasibility	Runner with 4 blades Standard tank	[23]
Review	Review	Companies that install GVT worldwide	-	[24]

Currently, there is a lack of knowledge regarding the ideal geometry for the runner in a gravitational vortex turbine that would maximize efficiency, as no optimal geometry has been reported in numerical or experimental studies. Additionally, there is no clear indication of the suitable design for the runner or the optimal number of blades that would result in the best performance. The methodology proposed by the authors of previous studies is also unclear, and the mathematical models and numerical configurations used are not well-documented. One significant gap in the literature is the lack of reported turbine efficiency, as it is unclear how it is calculated or not reported at all. Although some studies have examined runner configuration, particularly the number of blades, they have not taken full advantage of the design parameters. Therefore, the runner design is a parameter that should be studied to determine its impact on the GVT performance. Thus, the GVT efficiency can be significantly increased with an ideal tank-rotor configuration.

The objective of the present study consists of comparing the torque generated by three rotors for the gravitational vortex turbine. The three rotors were selected according to it performance as a turbine.

2. Methodology

To characterize the vortex phenomena, there are numerous mathematical models as shown in Table 2. Therefore, these models do not agree with both the experimental and numerical results at GVT turbines, especially at the tangential velocity. The governing equations for the unsteady, viscous and vortex formation's turbulent flow are the continuity and Navier Stokes equations described in Eq. (12) and Eq. (13), respectively. Those equations were resolved using a commercial ANSYS® software. The turbulence model was set up as recommended S. Mulligan, *et al.,* [8], with total time of 15 seconds (to establish the water vortex) and a time-step of 0.001s, guaranteeing a courant number of 1 and a water velocity inlet of 0.1 m/s. The turbine height is 0.3m and chamber diameter is 0.3m.

Table 2
Mathematical models developed to characterize the vortex generated inside the GVT and governing equation

Mathematical models to characterize the vortex formed		
Article	**Tangential velocity equation**	**Equation**
[8]	$v_\theta(r) \propto 1/r$	(1)
[25]	$v_\theta(r) = \frac{\Gamma}{2\pi r}$	(2)
[26]	$v_\theta(r) = \frac{\Gamma}{2\pi}\left(\frac{r}{({r_c}^4 + r^4)^{\frac{1}{2}}}\right)$	(3)
[27]	$v_\theta(r) = \frac{\Gamma}{2\pi}\left(\frac{r}{({r_c}^2 + r^2)}\right)$	(4)
[28]	$v_\theta(r) = \frac{\Gamma}{2\pi}\left(\frac{2r}{({r_c}^2 + 2r^2)}\right)$	(5)
[29]	$V_\theta(r) = \frac{\Gamma}{2\pi r}\left[1 - \exp\left(-\frac{1}{4}\frac{v_z}{Hv}r^2\right)\right]$	(6)
[30]	$v_\theta(r) = \omega r = \frac{\Gamma}{2\pi}\frac{r}{r_c^2}$	(7)
	$v_\theta(r) = \frac{\Gamma}{2\pi r} = \omega\frac{r_c^2}{r}$	(8)
[31]	$v_\theta(\mathrm{r}) = \frac{EC}{2\pi r}\left(\mathrm{e}^{\frac{-\mathrm{Er}^2}{2\pi}}\right)$	(9)
[32]	$v_\theta(\mathrm{r}) = \frac{(\Gamma_\infty)(r_c)}{\sqrt{[8({\mathrm{r_c}}^2)(\mathrm{g})(\pi^2)(H-h) + {\Gamma_\infty}^2]}}$	(10)
[33]	$v_\theta(r) = \frac{\Gamma d\sqrt{2(g)(H+h)}}{2(\pi)(\mathrm{r})}$	(11)
Governing system equations		
Continuity	$\frac{\partial v_r}{\partial r} + \frac{\partial v_z}{\partial Z} + \frac{v_r}{r} = 0$	(12)
Navier Stokes	$\frac{\partial \bar{u}}{\partial t} + \bar{u}\cdot(\nabla\bar{u}) = \frac{-1}{\rho}\nabla p + v\nabla^2\bar{u} + \bar{g}$	(13)

Where V_ϑ, V_r and V_z are tangential, radial, and axial velocity respectively, Γ is circulation, r is water radius, r_c is the air core radius, v is kinetic viscosity, C and E are constants, g is gravity acceleration, H is vortex height, and h is pointing height. Where:

$$\Gamma = \oint_L \vec{v}\, dl \quad (14)$$

Where $\vec{v}$ is the velocity field and *L* is the vertical axis at the surface. However, Stoke's theorem expresses the previous equation with a rotational velocity field.

$$\Gamma=\iint_A(\nabla_X \vec{v})\cdot dA \quad (15)$$

Where A is the surface area, and the rotational velocity field ($\nabla_x \vec{v}$) is equal to vector field vorticity (*Ω*), Eq. (16) is expressed as:

$$\Gamma=\iint A\ \Omega dA \quad (16)$$

For the Navier Stokes equation, ū represents the velocity vector and it is defined in Eq. (17), and *v* reduces the partial derivation in each component (*x*, *y*, *z*) and it is explained in Eq. (18).

$$u=(u,y,w) \quad (17)$$

$$\nabla=\partial/\partial x,\partial/\partial y,\partial/\partial z \quad (18)$$

To exemplify the variables in Table 2**,** Figure 3 shows the physical behavior of a water particle (wp) inside the GVT chamber.

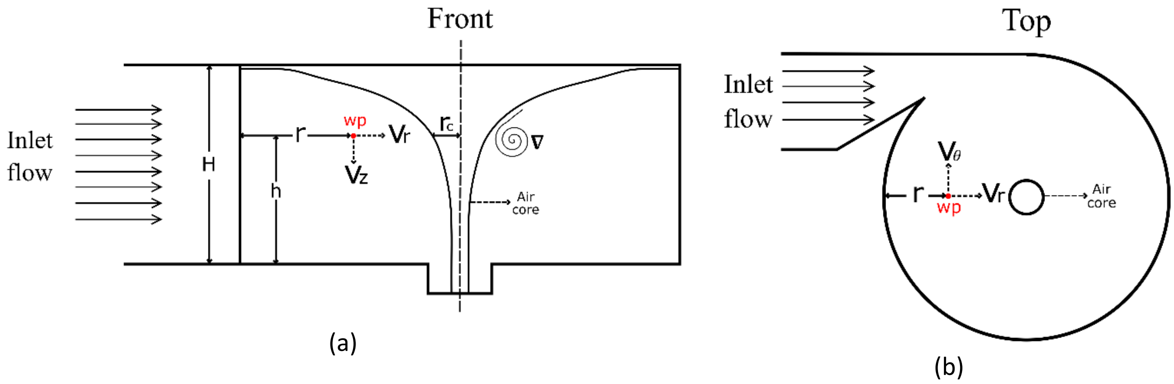

Fig. 3. Velocity profiles and variables for wp inside the GVT chamber: (a) Front view; (b) Top view [34]

3. Results

Figure 4 shows the mesh independence of the rotor chamber configuration for the three rotors at an angular velocity of 25 rpm. This angular velocity was selected as some companies have claimed that it results in the highest generated torque as mentioned in previous study [40, 41]. An approximately equal number of elements were employed for all three configurations, ensuring similarity in the results. The simulations began with around 2E5 elements and were increased up to 2E6 elements. Results indicated that there was a variation of less than 5% between the number of elements and the outcome for each configuration as mentioned in previous study [35–39]. Therefore, a mesh of about 4E5 elements was selected, with the rotary domain having a mesh of about 3E5 elements.

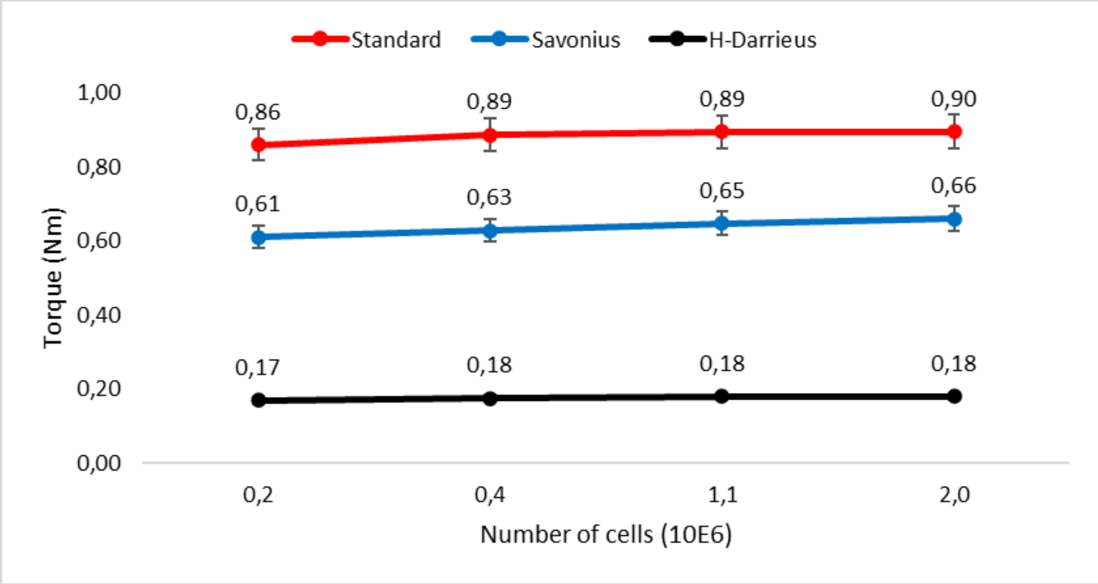

Fig. 4. Mesh independence for the 25 RPM case

Figure 5 represents the behavior of the three rotors at 4 different angular velocity. In this figure it can be seen how the generated torque decreases as the rpm increases. Being the standard rotor with the highest performance over the Savonius and H-Darrieus runners. At an angular velocity of 25 rpm, the maximum torque generated by the gravitational vortex turbine was observed. The standard rotor produced 0.89 Nm, while the Savonius and H-Darrieus rotors generated 0.52 Nm and 0.17 Nm, respectively. The order of the rotors in terms of the least variation in generated torque between the angular speeds of 25 and 100 rpm was the same. The difference between the standard rotor and the other two rotors ranged from 11% to 18%. The Savonius rotor generated 40% less torque than the standard rotor, while the H-Darrieus rotor generated 79% less torque than the standard rotor.

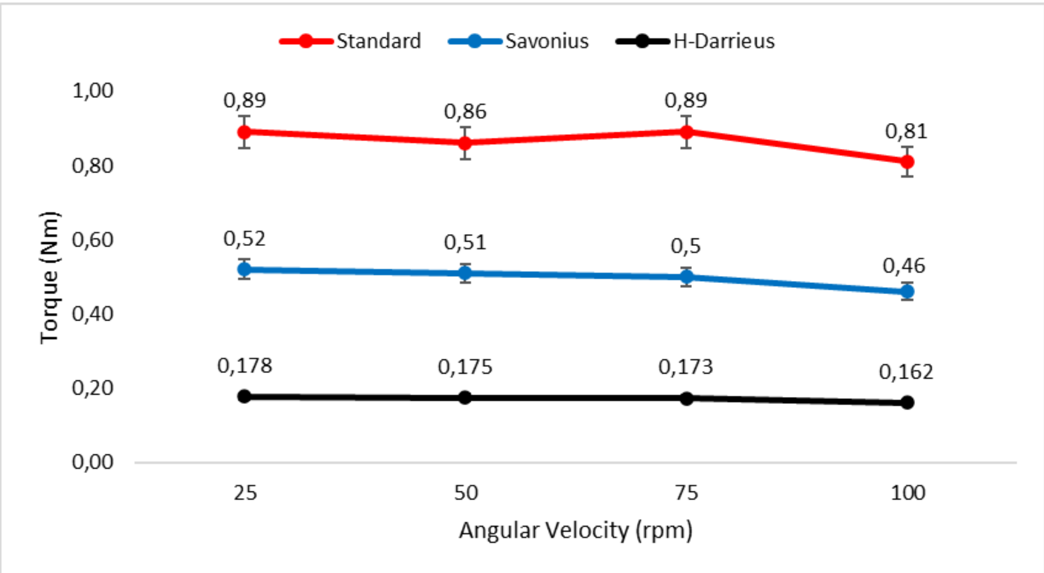

Fig. 5. Torque variation vs angular velocity for each case

The main difference between the rotors lies in the type of force generated on the blade surface, which is mainly drag force. The larger the perpendicular area of the blade facing the fluid, the greater the pressure and the generated torque. The standard and Savonius rotors mainly rely on this type of force, whereas the H-Darrieus rotor performs better when there is a greater lift force involved as mentioned V. Patel, *et al.,* [36, 42].

Figure 6 shows the water streamlines within the chamber for each rotor from a top view. It can be seen how in the standard and Savonius rotors a turbulence is generated between the blades. On the other hand, the H-Darrieus rotor does not present this phenomenon. This generated turbulence generates a higher pressure on the blade because it is increasing its velocity in the formed vortex and when it collides with the blade it transforms the kinetic energy of the fluid into mechanical energy with the axis. This figure also shows that the negative torque, which is the main problem that occurs in the Savonius as A. Kumar, *et al.,* [35] turbine, does not occur in the GVT. This is because there is no fluid to find from the incoming blade to create a pressure or counterflow.

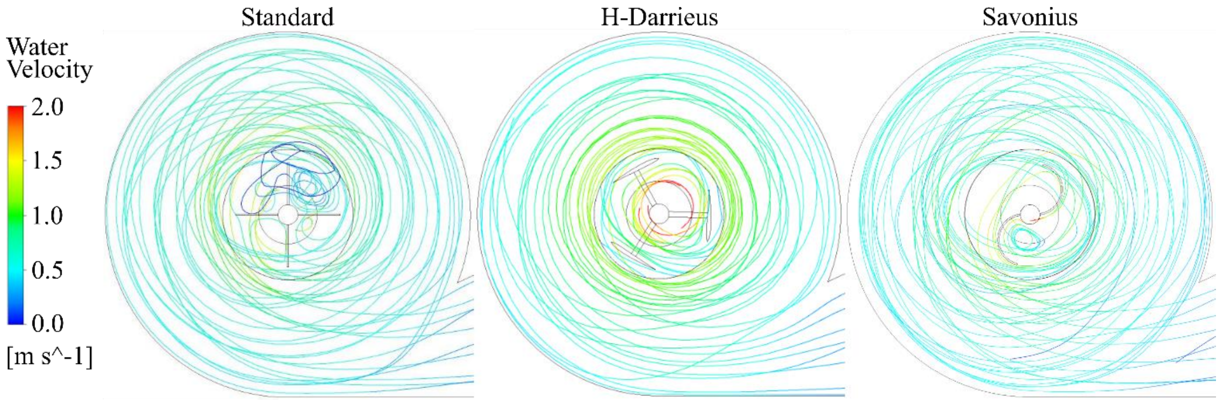

Fig. 6. Top view for water velocity streamlines inside the chamber

To exemplify what was said above, Figure 7 represents the pressure generated in the rotors by the water. In this figure both in the Savonius rotor and in the standard rotor, a pressure is generated on the face perpendicular to the fluid of the blade, while in the H-Darrieus this pressure is generated in the blade support with the rotor axis. It is worth mentioning that once the water vortex is formed inside the tank, it deforms once it collides with the rotor blade. This is because its velocity drops drastically to 0 m/s and therefore it leaves the turbine directly.

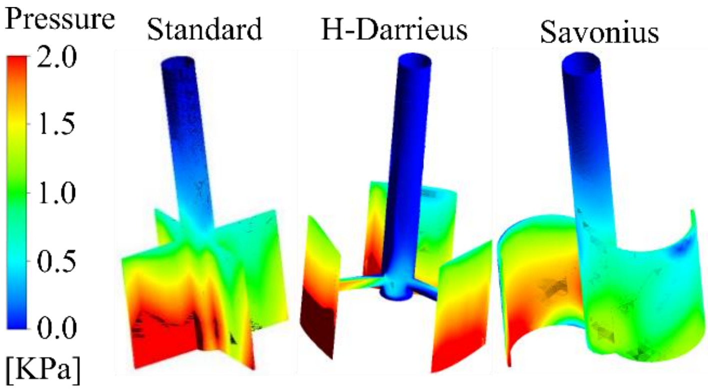

Fig. 7. Water pressure contours in blades runners

The image shown in Figure 8 illustrates the distribution of air inside the chamber for each rotor. The figure demonstrates that both the standard and Savonius rotors contain a volume fraction of air in the upper part of the chamber, whereas the H-Darrieus rotor has this fraction only in the tank region. This suggests that the GVT equipped with an H-Darrieus rotor is entirely filled with fluid, particularly in the inlet channel. This flooding of the GVT is attributed to the fluid losing all its velocity upon collision with the blade, resulting in it exiting the chamber immediately. As a result, the fluid stays in the chamber longer in the GVT with an H-Darrieus rotor, resulting in flooding.

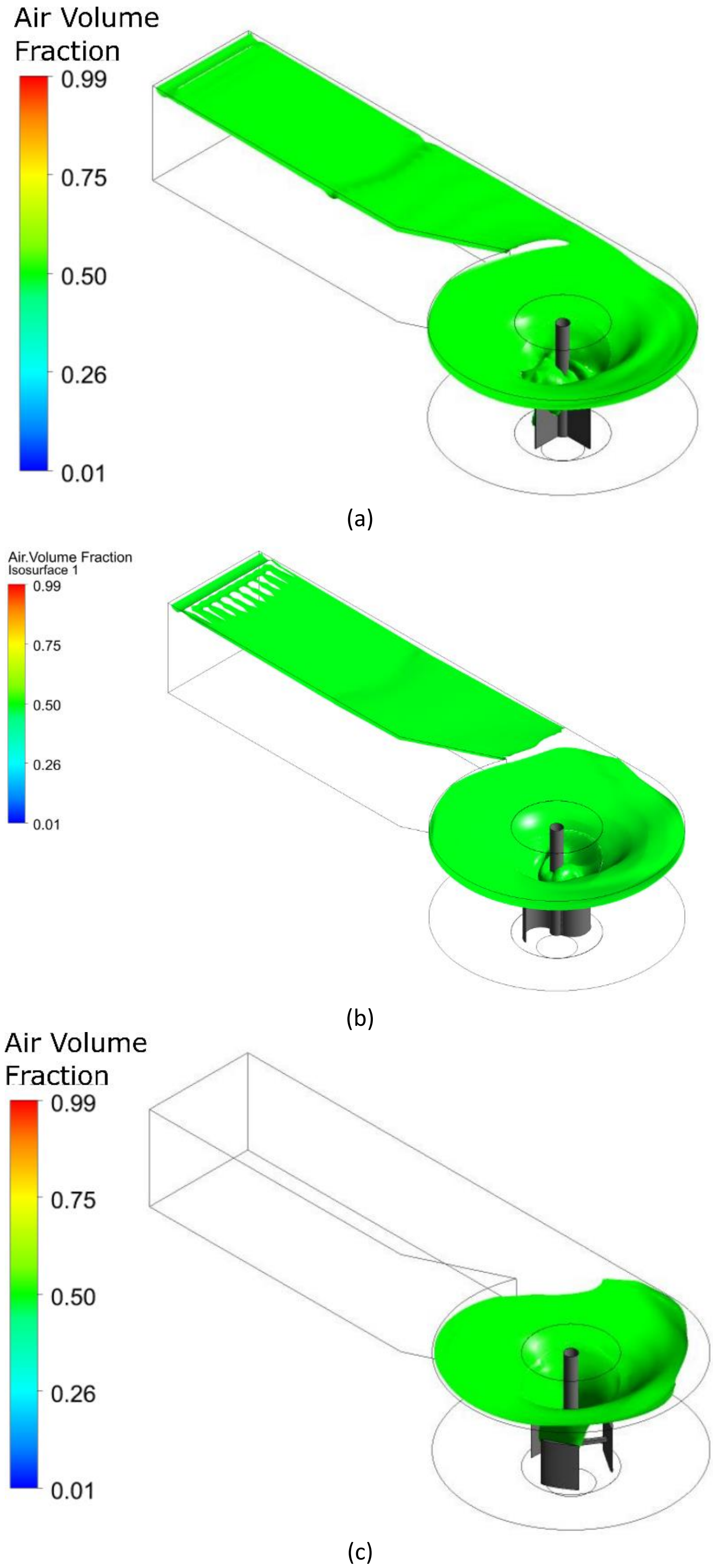

Fig. 8. Air volume fraction for GVT (a) Standard (b) Savonius (c) H-Darrieus

4. Conclusions

A numerical comparison of torque generated by three different rotors was made for a gravitational vortex turbine with a ratio of 18% between the diameter of the tank and the outlet hole. The three selected rotors were: H-Darrieus, Savonius and standard, defined as a rotor with 4 straight blades. The study was carried out with the ANSYS ® software and its ICEM and CFX modules.

Rotor design is a parameter that drastically affects the performance of the GVT. For this, parameters such as the design and number of blades must be considered.

The rotor that displayed superior performance was the standard rotor equipped with straight blades. This can be attributed to the larger blade count in comparison to the Savonius rotor, as well as the blade's orientation which maximizes the perpendicular surface area exposed to the fluid. These characteristics result in a higher-pressure generation on the rotor blades, leading to an increased torque output.

The primary issue with the Savonius turbine is its negative torque, which significantly hampers its performance. However, within the GVT, the rotational fluid movement within the chamber eliminates this torque issue for the Savonius rotor. Further research is encouraged in the scientific community regarding the use of the Savonius rotor in the GVT, exploring parameters such as blade quantity and radius variation.

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References

[1] Ramiro. Ortiz Flórez and Eduardo. Machado Hernández, *Pequeñas centrales hidroeléctricas*. McGraw-Hill Interamericana, 2001. Accessed: Mar. 04, 2018.
[2] First, Renewables. *What is the difference between micro, mini and small hydro*. 2016.
[3] Zotlöterer, Franz. *Gravitational Water Vortex Power Plants*. 2003.
[4] Li, Hai-feng, Hong-xun Chen, Zheng Ma, and Yi Zhou. "Formation and influencing factors of free surface vortex in a barrel with a central orifice at bottom." *Journal of hydrodynamics* 21, no. 2 (2009): 238-244. https://doi.org/10.1016/S1001-6058(08)60141-9
[5] Shabara, H. M., O. B. Yaakob, Yasser M. Ahmed, A. H. Elbatran, and Muhammad SM Faddir. "CFD validation for efficient gravitational vortex pool system." *Jurnal Teknologi* 74, no. 5 (2015). https://doi.org/10.11113/jt.v74.4648
[6] Nishi, Yasuyuki, Ryouta Suzuo, Daichi Sukemori, and Terumi Inagaki. "Loss analysis of gravitation vortex type water turbine and influence of flow rate on the turbine's performance." *Renewable Energy* 155 (2020): 1103-1117. https://doi.org/10.1016/j.renene.2020.03.186
[7] MBasri, Muhammad Hasan, and Ainun Nasuki. "Water Discharge Management Based on Open and Closed Cylinders in the Gravitation Water Vortex Power Plant." *JEEE-U (Journal of Electrical and Electronic Engineering-UMSIDA)* 5, no. 1 (2021): 22-36. https://doi.org/10.21070/jeeeu.v5i1.1008
[8] Mulligan, Sean, John Casserly, and Richard Sherlock. "Experimental and numerical modelling of free-surface turbulent flows in full air-core water vortices." *Advances in Hydroinformatics: SIMHYDRO 2014* (2016): 549-569. https://doi.org/10.1007/978-981-287-615-7_37
[9] Wanchat, Sujate, and Ratchaphon Suntivarakorn. "Preliminary design of a vortex pool for electrical generation." *Advanced Science Letters* 13, no. 1 (2012): 173-177. https://doi.org/10.1166/asl.2012.3855
[10] Dhakal, Sagar, Ashesh B. Timilsina, Rabin Dhakal, Dinesh Fuyal, Tri R. Bajracharya, Hari P. Pandit, Nagendra Amatya, and Amrit M. Nakarmi. "Comparison of cylindrical and conical basins with optimum position of runner: Gravitational water vortex power plant." *Renewable and Sustainable Energy Reviews* 48 (2015): 662-669. https://doi.org/10.1016/j.rser.2015.04.030

[11] Sánchez, Alejandro Ruiz, Jorge Andrés Sierra Del Rio, Angie Judith Guevara Muñoz, and José Alejandro Posada Montoya. "Numerical and experimental evaluation of concave and convex designs for gravitational water vortex turbine." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 64, no. 1 (2019): 160-172.
[12] Dhakal, Sagar, Ashesh Babu Timilsina, Rabin Dhakal, Dinesh Fuyal, Tri Ratna Bajracharya, and Hari Prasad Pandit. "Effect of dominant parameters for conical basin: Gravitational water vortex power plant." In *Proceedings of IOE graduate conference*, vol. 5, p. 381. 2014. doi: 10.13140/RG.2.1.1455.7843.
[13] Sreerag, S. R., C. K. Raveendran, and B. S. Jinshah. "Effect of outlet diameter on the performance of gravitational vortex turbine with conical basin." *International Journal of Scientific & Engineering Research* 7, no. 4 (2016): 457-463.
[14] Rehman, Wajiha, Masooma Ijaz, and Asma Munir. "Designing of micro gravitational vortex turbine's vortex pool." In *ASME Power Conference*, vol. 57618, p. V002T12A002. American Society of Mechanical Engineers, 2017.
[15] Wanchat, Sujate, Ratchaphon Suntivarakorn, Sujin Wanchat, Kitipong Tonmit, and Pongpun Kayanyiem. "A parametric study of a gravitation vortex power plant." *Advanced Materials Research* 805 (2013): 811-817. https://doi.org/10.4028/www.scientific.net/AMR.805-806.811
[16] Maika, Nosare, and Syed MS Wahid. "Implementing gravitational vortex hydro power plant: Case study." *Journal of The Institution of Engineers (India): Series C* 102 (2021): 1565-1570. https://doi.org/10.1007/s40032-021-00754-z
[17] Dhakal, Subash, Susan Nakarmi, Pikam Pun, Arun Bikram Thapa, and Tri Ratna Bajracharya. "Development and Testing of Runner and Conical Basin for Gravitational Water Vortex Power Plant." *Journal of the Institute of Engineering* 10, no. 1 (2014). https://doi.org/10.3126/jie.v10i1.10895
[18] Wichian, Pongsakorn, and Ratchaphon Suntivarakorn. "The effects of turbine baffle plates on the efficiency of water free vortex turbines." *Energy Procedia* 100 (2016): 198-202. https://doi.org/10.1016/j.egypro.2016.10.165
[19] Dhakal, Rabin, T. R. Bajracharya, S. R. Shakya, B. Kumal, Sam Williamson, K. Khanal, S. Gautam, and D. P. Ghale. "Computational and experimental investigation of runner for gravitational water vortex power plant." In *2017 IEEE 6th International Conference on Renewable Energy Research and Applications (ICRERA 2017): Proceedings of a meeting held 5-8 November 2017, San Diego, California, USA*, pp. 365-373. Institute of Electrical and Electronics Engineers (IEEE), 2018. https://doi.org/10.1109/ICRERA.2017.8191087
[20] Saleem, Abdul Samad, Taqi Ahmad Cheema, Rizwan Ullah, Sarvat Mushtaq Ahmad, Javed Ahmad Chattha, Bilal Akbar, and Cheol Woo Park. "Parametric study of single-stage gravitational water vortex turbine with cylindrical basin." *Energy* 200 (2020): 117464. https://doi.org/10.1016/j.energy.2020.117464
[21] Ullah, Rizwan, Taqi Ahmad Cheema, Abdul Samad Saleem, Sarvat Mushtaq Ahmad, Javed Ahmad Chattha, and Cheol Woo Park. "Performance analysis of multi-stage gravitational water vortex turbine." *Energy Conversion and Management* 198 (2019): 111788. https://doi.org/10.1016/j.enconman.2019.111788
[22] Ullah, Rizwan, Taqi Ahmad Cheema, Abdul Samad Saleem, Sarvat Mushtaq Ahmad, Javed Ahmad Chattha, and Cheol Woo Park. "Preliminary experimental study on multi-stage gravitational water vortex turbine in a conical basin." *Renewable Energy* 145 (2020): 2516-2529. https://doi.org/10.1016/j.renene.2019.07.128
[23] Guzmán, Vladimir J. Alzamora, Julie A. Glasscock, and Ferris Whitehouse. "Design and construction of an off-grid gravitational vortex hydropower plant: A case study in rural Peru." *Sustainable Energy Technologies and Assessments* 35 (2019): 131-138. https://doi.org/10.1016/j.seta.2019.06.004
[24] Sierra, Jorge, Alejandro Ruiz, Angie Guevara, and Alejandro Posada. "Gravitational Vortex Turbines as a Renewable Energy." *International Journal of Fluid Machinery and Systems* 13, no. 4 (2020): 704-717. https://doi.org/10.5293/IJFMS.2020.13.4.704
[25] Einstein, Hans Albert, and Huon Li. "Steady vortex flow in a real fluid." *Proc. Heat Transfer and Fluid Mechanics Institute, Stanford University* (1951): 33-43.
[26] Vatistas, G. H., S. Lin, and C. K. Kwok. "Theoretical and experimental studies on vortex chamber flows." *AIAA journal* 24, no. 4 (1986): 635-642. https://doi.org/10.2514/3.9319
[27] Rosenhead, Louis. "The spread of vorticity in the wake behind a cylinder." *Proceedings of the Royal Society of London. Series A, Containing papers of a mathematical and physical character* 127, no. 806 (1930): 590-612. https://doi.org/10.1098/rspa.1930.0078
[28] JHite Jr, John E., and Walter C. Mih. "Velocity of air-core vortices at hydraulic intakes." *Journal of Hydraulic Engineering* 120, no. 3 (1994): 284-297. https://doi.org/10.1061/(ASCE)0733-9429(1994)120:3(284)
[29] Odgaard, A. Jacob. "Free-surface air core vortex." *Journal of Hydraulic Engineering* 112, no. 7 (1986): 610-620. https://doi.org/10.1061/(ASCE)0733-9429(1986)112:7(610)
[30] Rankine, William John Macquorn. *A manual of applied mechanics*. Charles Griffin and Company, 1872.
[31] Burgers, Johannes Martinus. "A mathematical model illustrating the theory of turbulence." *Advances in applied mechanics* 1 (1948): 171-199. https://doi.org/10.1016/S0065-2156(08)70100-5

[32] Rahman, M., Tan Jian Hong, Raymond Tang, Ling Leh Sung, and Fadzlita Binti Mohd Tamiri. "Experimental study the effects of water pressure and turbine blade lengths & numbers on the model free vortex power generation system." *International Journal of Current Trends in Engineering & Research (IJCTER)* 2, no. 9 (2016): 13-17.

[33] Marian, Marius Gheorghe, Tudor Sajin, and Abdelkrim Azzouz. "Study of micro hydropower plant operating in gravitational vortex flow mode." *Applied Mechanics and Materials* 371 (2013): 601-605. https://doi.org/10.4028/www.scientific.net/AMM.371.601

[34] Sánchez, Alejandro Ruiz, Angie Guevara Muñoz, Jorge Andrés Sierra Del Rio, and Jose Alejandro Posada Montoya. "Numerical comparison of two runners for gravitational vortex turbine." *Engineering Transactions* 69, no. 1 (2021): 3-17. https://doi.org/10.37934/cfdl.14.8.111

[35] Kumar, Anuj, and Rajeshwer Prasad Saini. "Performance parameters of Savonius type hydrokinetic turbine–A Review." *Renewable and Sustainable Energy Reviews* 64 (2016): 289-310.

[36] Patel, Vimal, T. I. Eldho, and S. V. Prabhu. "Performance enhancement of a Darrieus hydrokinetic turbine with the blocking of a specific flow region for optimum use of hydropower." *Renewable Energy* 135 (2019): 1144-1156. https://doi.org/10.1016/j.rser.2016.06.005

[37] Ceballos, Y. Castañeda, M. Cardona Valencia, Diego Hincapie Zuluaga, J. Sierra Del Rio, and S. Vélez García. "Influence of the number of blades in the power generated by a Michell Banki Turbine." *International Journal Of Renewable Energy Research IJRER* 7, no. 4 (2017): 1989-1997. https://doi.org/10.1016/j.renene.2018.12.074

[38] Roache, Patrick J., Kirti N. Ghia, and Frank M. White. "Editorial policy statement on the control of numerical accuracy." *Journal of Fluids Engineering* 108, no. 1 (1986): 2. https://doi.org/10.1115/1.3242537

[39] ANSYS, "Courant number," in *ANSYS HELP*, 2019.

[40] F. Zotlöterer, "Zotloterer Smart-Energy-System."

[41] Turbulent;, "Turbulent micro hydropower," 2019.

[42] Beltran-Urango, D., J. L. Herrera-Díaz, J. A. Posada-Montoya, L. Castañeda, and J. A. Sierra-del Rio. "Generación de Energía Eléctrica Mediante Vórtices Gravitacionales." *MEMORIAS EXPO TECNOLOGIAS 2016, Medellin, Antioquia* (2016): 90-107.