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Performance Comparison of Straight, Curved, and Tilted Blades of Pico Scaled Vortex Turbine

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ABSTRACT

In the 26th Conference of the Parties (COP26), United Nations Framework Convention is committed to suppressing the acceleration of climate change due to carbon by applying energy from fossil fuels to New Renewable Energy (NRE). Indonesia targets that in 2060 the coal phase-out will be replaced by renewable energy 2056. As a tropical country with many rivers and many lakes, Indonesia has a water energy potential of 75,000 MW. In some remote areas, the need for electrical power is pico scaled (< 5kW). The vortex turbine was chosen to be suitable for river flow with low water levels, low maintenance and construction costs, as well as being environmentally friendly for the underwater ecosystem. This study aims to compare the performance of the vortex to the differences in the shape of the pico scale blade. This study was carried out with variations in the shape of the straight, tilted, and curved blades. Based on the results of all vortex turbine studies, it can be ascertained that the best blade shape obtained numerically is an inclined blade with a hydraulic efficiency of 36%.

1. Introduction

Electricity is one of the most fundamental things for humans in every country. Based on the Energy Access Outlook 2017 published by the International Energy Agency (IEA), in 2015, 193 countries, including developed and developing countries, have flocked to develop clean and sustainable sources of electrical energy. New Renewable Energy (NRE) is one of the best solutions. The reason is, with the condition of the earth, especially in terms of environmental problems that are getting worse and climate change is increasingly irregular, various countries are trying to find other energy sources.

The UN Climate Change Conference in Glasgow (COP26) from October 31st to November 12th, 2021 committed to suppressing the acceleration of climate change due to carbon by applying energy from fossil fuels to new renewable energy. Indonesia targets 2060 the coal phase-out will be replaced by renewable energy in 2056. [1]

The electricity sector accounts for almost a quarter of global emissions, and the decarbonization of power plants has become a prominent effort to tackle global climate change. The government has

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issued a Business Plan for the Provision of Electricity 2021 – 2030, which mentions an additional 40,600 MegaWatt (MW) of power plant energy. During the 10 years, new and renewable energy obtained a share of 51.6% (20,949.6 MW), larger than fossil energy, which was 48.4%. The installed capacity of Indonesia's electricity generator reaches 73,736 MegaWatt (MW) or 73.74 GigaWatt (GW) until November 2021 [2]. Indonesia has renewable energy that can be used for generating electricity, including wind power, solar, water, geothermal, bioenergy, and tidal currents. The potential of NRE in Indonesia is 417.8 GW, coming from wind power (60.6 GW), solar (207.8 GW), water (75 GW), geothermal (28.9 GW), bioenergy (32.6 GW), and tidal currents (17.9 GW) [1].

In some areas, the existence of electricity is still something of a luxury, such as in the Yapen Islands, Papua, Fatoi Village, Insana District, Central-North Timor Regency, East Nusa Tenggara, and other villages [3, 4]. In general, electricity is used for lighting, communication, information dissemination, and increasing (small) business productivity. Electrical power requirements used are usually not too large (< 5kW). Dragon fruit farmer groups in Fatoi village, Central-North Timor Regency, East Nusa Tenggara, increased the productivity of dragon fruit which previously could only harvest one time a year to three times a year. Dragon fruit trees require sufficient sunlight, which is manipulated by utilizing LED (Light Emitting Diode) lamps.

As a tropical country, Indonesia has a water energy potential of 75,000 MW (PLN study with Nippon Koei in 1983). This is a concern for the government to increase research in the field of micro and pico-hydro to qualify electricity needs in remote areas. As a renewable energy source, pico-hydro has advantages in terms of environmental friendliness and continuous availability compared to conventional power plants such as Combined Cycle Gas Turbine Plant [5]. When compared to geothermal, hydroelectric power plants require much cheaper construction and installation costs and minimal risk of natural damage that will be caused [5–7]. Solar cells powered by sunlight are also an alternative to renewable power plants that are widely used today. This solar power plant is also environmentally friendly. However, the use of solar panels has drawbacks in terms of the stability of electricity generation plus the cost of equipment and installation is more expensive than micro or pico-hydro power plants. Bio Energy is also a renewable alternative energy source, but getting biomass is quite difficult because certain plants do not grow every year, plus the burning process will release greenhouse gasses which are certainly not friendly to the environment [5]. Hydro turbines are generally categorized into two types namely impulse turbines and reaction turbines [8, 9]. The various types of impulse turbines are turbo, Pelton, and cross-flow turbines [7, 10]. In the reaction turbine group, there are Francis and Kaplan turbines as well as vortex flow reaction turbines. The vortex flow reaction turbine has the advantage that it can be built with a small water potential. The vortex turbine has a relatively low head of 0.7 m – 3 m with a flow rate of 50 L/s [11]. A vortex turbine is a type of reaction type hydro turbine with a propeller-shaped blade that utilizes an artificial flow that rotates around a center or is called a vortex. This flow is made intentionally by designing channels and basins in such a way that natural vortices are formed. Vortex flow is defined as a fluid flow that rotates around an axis.

The first study about vortex was conducted by a previous study, an Austrian naturalist. Experience as a ranger made Victor a fluid mechanics engineer. Water is channeled into an egg-shaped channel, where it naturally creates a phenomenon known as a "free vortex," which can be used to generate power. An American scientist named Brown patented the "Vertical Vortex" process for harnessing water energy using vertical vortices. Brown's design, in contrast to Victor's, uses a rectangular channel with vertical walls attached to a guide plate to determine the rotational velocity of the resulting flow. As a result, energy is produced when water is discharged through the drain side and the vertical wall rotates, turning an electric generator. At its peak in 2004, an Austrian engineer, Franz Zoelterer patented his vortex turbine. The Franz apparatus operates on the same principles as

Brown's, but the channel is shaped differently to better take use of the tangential flow velocity. However, Franz's work on what parameters are needed in the manufacture of vortex turbines and their channels is not published and the efficiency of the turbine is not known with certainty Mulligan *et al.,* [12] conducted analytical and experimental studies on the effect of vortex rotation on the geometry of the created channel. Mulligan created 12 channel types with different geometric parameters (r_{in} /b). It was found that the rotational field or the number of vortex rotations (Nv) is highly dependent on the geometrical approach [12]. In addition, this rotational field has been shown to have a relationship with the amount of discharge generated (Discharge Number). In addition, Mulligan [13] also concluded that to obtain a large output power the ratio between the diameter of the basin and the diameter of the exhaust side (orifice) should be 14% - 18% [13]. In addition to Mulligan, Wanchat and Suntivarakorn conducted a study on the effect of the basin structure on the formed vortex [14, 15] The study carried out is to determine what parameters are in the formation of the vortex and the configuration of the vortex itself. They concluded that these parameters are the water level at the inlet, the diameter of the exhaust side (orifice), the configuration of the channel, and the condition at the inlet side. In addition, it was also found that the channel with a cylindrical shape with a flow rectifier plate on the inlet side is the best configuration for producing a large tangential velocity so that the resulting vortex becomes stronger. Dhakal *et al.,* [16, 17] also conducted a comparative study of the resulting efficiency between a conical basin (Conical Basin) and a cylindrical basin (Cylindrical Basin) with the optimum blade placement position. It was concluded that a blade with a conical channel (Conical Basin) provides greater efficiency than a cylindrical channel (Cylindrical Basin), namely 36.84% (Conical Basin) and 27.75% (Cylindrical Basin). This is because the conical basin (Conical Basin) provides an even velocity distribution compared to the cylindrical basin (Cylindrical Basin). In this study, an analysis of various blade shapes with fixed height and discharge positions on the vortex turbine was carried out to analyze the effect of blade shape on the efficiency of the pico scale vortex turbine.

2. Methodology

2.1 Blade

The blade is one of the main parts of a vortex turbine where the water flowing as a vortex of energy will be captured as kinetic energy which will later be channeled to the shaft. The shape of the profile and the dimensions of the blades are very influential, especially on the relative speed produced which affects the performance of the resulting turbine.

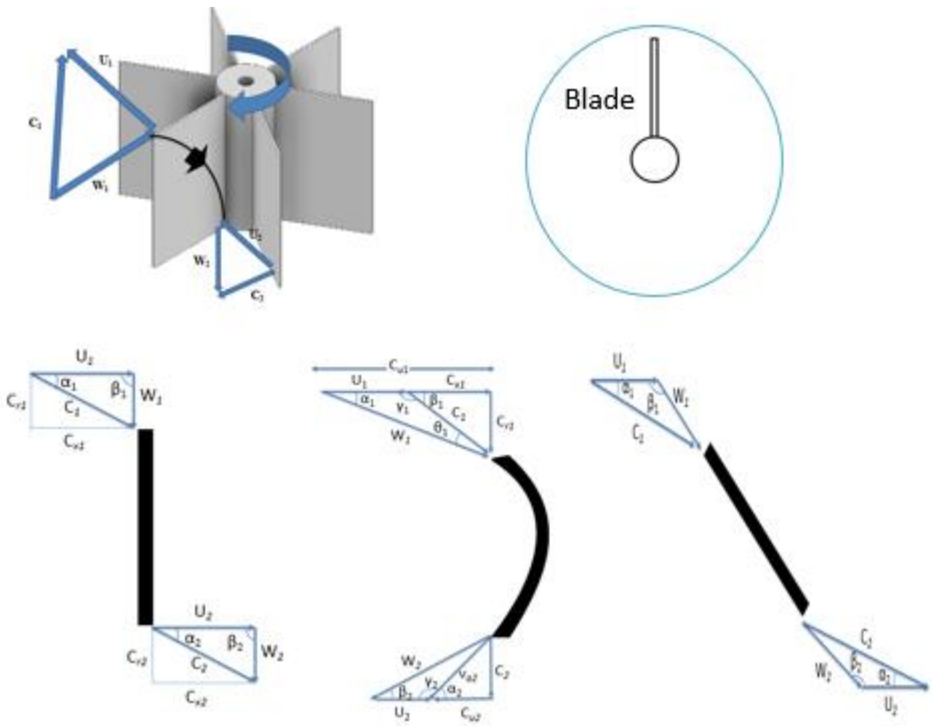


Fig. 1. Velocity Triangle

Description:
U = tangential speed of rotor blade
W = fluid velocity relative to the rotor blades
C = absolute velocity of the fluid

Here are some things to consider in making a speed triangle:

i. The rotor blade speed, U, always has a direction tangential to the blade rotation direction. The instantaneous relative velocity of the fluid, W, is always tangential to the fluid flow line flowing between the two blades. In other words, W is also tangential to the blade profile.
ii. The absolute velocity of the fluid, C, is the vector sum between U and W so that C = U + W. That way, knowing the two values of the velocity component, we can get the value of the other velocity component.
iii. The fluid and blade angles are determined by comparison with the blade tangential velocity, U. for a more complete calculation related to the speed triangle, this paper is not discussed.

2.2 Basin Geometry

Basin is one of the main parts of a vortex turbine which is a container where water flows to form a vortex that will hit the turbine so that the turbine rotates. Basin has an important role in vortex formation. According to Mulligan and Casserly, in a cylindrical basin, the optimum vortex strength is found in the basin-nozzle diameter ratio range of 14-18% for places with low and high heads.

Figure 2 is a display of the shape of the vortex turbine basin used in this study where from the top view it can be seen that the flow enters through the inlet after it enters the basin circle which will make the water flow form a vortex then the flow hits the blades and exits through the outlet channel below.

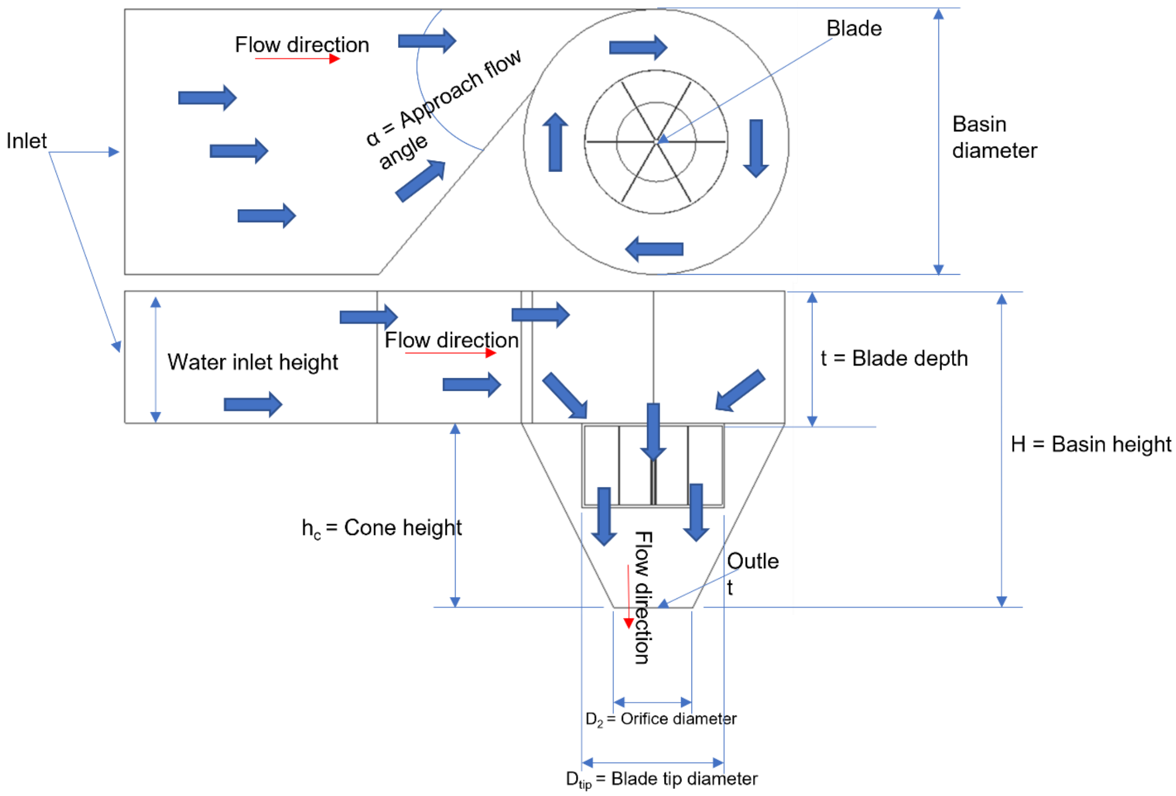


Fig. 2. Channel and Basin Geometry

Table 1
Parameter Values of the Channel and Basin

Parameter	Value
Basin Diameter	1000 mm
Blade Tip Diameter	500 mm
Orifice Diameter	300 mm
Approach Flow Angle	40^0
Blade Depth	270 mm
Hub Diameter	115 mm

2.3 Simulation Setup

Basin is one of the main parts of a vortex turbine which is a container where water flows to form a vortex that will hit the turbine so that the turbine rotates. Basin has an important role in vortex formation

The numerical analysis in this study uses Computational Fluid Dynamics (CFD). The CFD software used is Ansys Fluent 18.1. This numerical method is carried out through three main stages, namely pre-processing, solver, and post-processing.

The simulation was carried out by setting D_{ip}/D_1 with a value of 0.5 and the position of the turbine depth at H1 = 270 mm because the previous study resulted in maximum efficiency at this location and magnitude. The results obtained will be compared with the analytical and analysis of the maximum results.

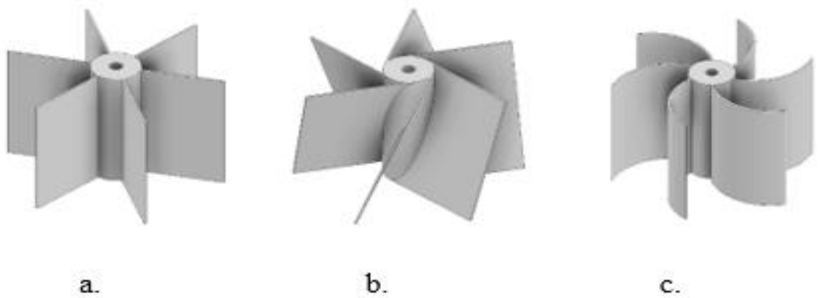


Fig. 3. Design (a) Straight blade (b) Tilted blade and (c) Curved blade

In a reaction turbine, the flow of fluid about the turbine blades is followed by a significant drop in static pressure and a change in the relative velocity of the fluid across the turbine blades whereas in an impulse turbine such effects are negligible. So the geometry of the fluid turbine provides torque on the rotor in the direction of its rotation. The proper blade shape is essential for producing the maximum of available fluid conditions. Therefore, 3 blade profiles were tested in this study to see the hydraulic performance of each blade. Figure 3 is the design of the 3 blade profiles used in this study made at the inventor with a D_{ip} size of 0.5 meters and a D_{hub} of 0.115 meters with a shaft hole of 0.03 meters.

The boundary conditions at the inlet were determined at a speed of 0.22 m/s and the conditions at the outlets were set for a static pressure of 1 atm. The blade rotation is set at a rotating speed from 10 to 130 rpm clockwise. Water flow is considered incompressible. SST k-ω used definite curvature correction in descending and rotating pressure gradientsb [18]. Because SST k-ω combines the calculations of k-ω near the wall and k-ω in free flow.

2.4 Grid Convergence Study

The independence test was carried out to ensure that the number of meshes (elements) and the time to be used did not affect the computational results or were independent. The independence test method used in this study is the grid convergence index (GCI) [19]. GCI analysis was carried out to report the results of grid convergence using the Richardson extrapolation concept (static approach), where this approach was used to predict the exact value. The exact value is a guessed value if the computation uses a discrete space towards zero (Δx→0). The equation used to analyze GCI is as follows:

$$GCI_{12}=F_s\left|\frac{1}{\tau_{fine}}\frac{\tau_{medium}-\tau_{fine}}{r_{12}{}^{pn}-1}\right| \times 100\% \tag{1}$$

Fs is the safety factor with a value of 1.25, and r is the grid refinement ratio (Roache, 1998, 1997). Determining r:

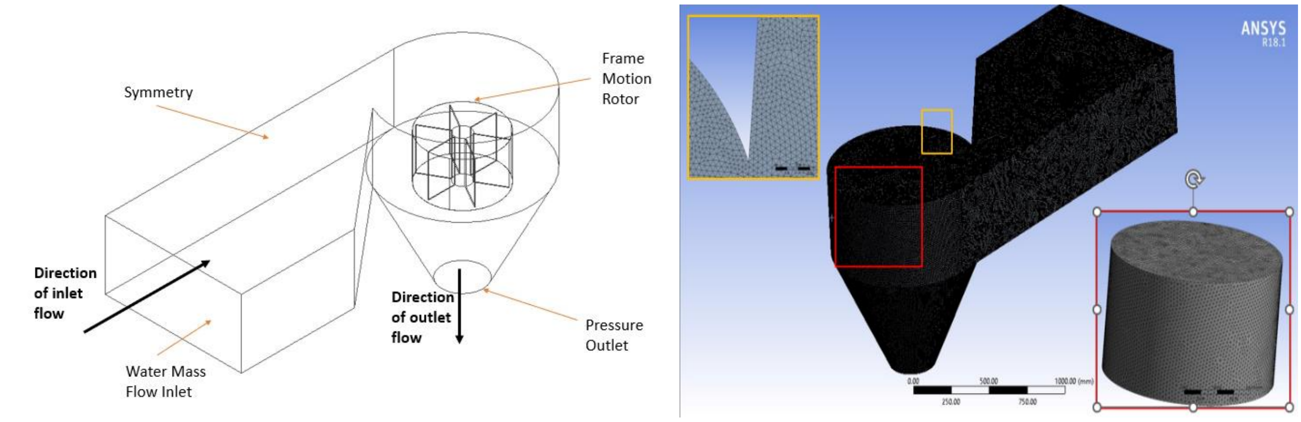
$$r_{12}=(\left(\frac{M_{fine}}{M_{medium}}\right)^{0.5} \tag{2}$$

M is the number of meshes, before GCI is performed the order of observed convergence (p) is one of the variables used to determine the exact value.

$$P_{n+1}=\ln\left[\left(\frac{\tau_{coarse}-\tau_{medium}}{\tau_{medium}-\tau_{fine}}\,(r_{12}{}^{pn}-1)\right)+r_{12}{}^{pn}\right]/ln(r_{12}.r_{23}) \tag{3}$$

Usually, the initial value of the p guess is 2 (Roache, 1998). After the p-value is known, continue to determine the exact value using Richardson's extrapolation method:

$$\tau_{exact} = \tau_{fine} - \left(\frac{\tau_{medium} - \tau_{fine}}{{r_{12}}^{pn} - 1}\right) \tag{4}$$



(a) (b)

Fig. 4. (a) Geometry and boundary condition (b) visualization of 2,478,135 elements

2.5 Governing Equation

In this study, the type of turbulent flow used is SST (Shear-Stress Transport) K-e and k-w by considering accuracy and reliability. This type of turbulent flow has the following basic equation:

for $k - \epsilon$:

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho \omega u_i)}{\partial x_1} = \frac{\partial}{\partial x_j}\left(\Gamma_k \frac{\partial k}{\partial x_j}\right) + G_k + Y_k + S_k \tag{5}$$

for $k - \omega$:

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho \omega u_i)}{\partial x_1} = \frac{\partial}{\partial x_j}\left(\Gamma_\omega \frac{\partial \omega}{\partial x_j}\right) + G_\omega + Y_\omega + S_\omega + D_\omega \tag{6}$$

where Γ_k and Γ_ω represent the effective diffusivity of turbulent kinetic energy (k) and (ω), $G_k$and G_ω represent the generation of turbulent electrical energy (k) and (ω) caused by velocity gradient, Y_k and Y_ω are dissipation values of turbulent kinetic energy (k) and (ω), D_ω represents the modified cross-diffusion term between $k - \omega$ with $k - \epsilon$ and S_k and S_ω is turbulent kinetic energy (k) and (ω) humidification in free surface flow caused by the magnitude of the velocity gradient in the section. two fluid interfaces.

3. Results

3.1 Independence Test Result

The mesh independence test was done using Richardson extrapolation [19]. The test achieved a result of mesh which were 747954, 1083267, and 2478135 elements Torque at each number of elements was calculated to analyze the appropriate number of elements with the GCI calculation method. The GCI calculation results are shown in Table 2 where the best number of elements is 2478135.

Table 2
Mesh Independence Test

Number of Mesh Element	Torque	Grid Spacing	r	p	GCI
747954	32.54	1.82	-	3.55	-
1083267	23.86	1.51	1.20	5.02	9.77 %
2478135	22.03	1	1.51	6.21	0.22 %

3.2 Turbine Performance by Numerical

The turbine performance shown in this study is the power and efficiency of the turbine. Turbine power (Pin and Pout) and efficiency are obtained based on the following equation:

$$P_{in} = \rho g Q H \tag{7}$$

$$P_{out} = \tau\omega \tag{8}$$

$$\eta = \frac{P_{out}}{P_{in}} x100\% \tag{9}$$

Based on Figure 5 comparison graph of hydraulic efficiency with rotational rotation speed in the simulation, it is found that the inclined blade profile has the highest efficiency of 36%, for straight and curved blade profiles the maximum efficiency is 33% and 30%, respectively.

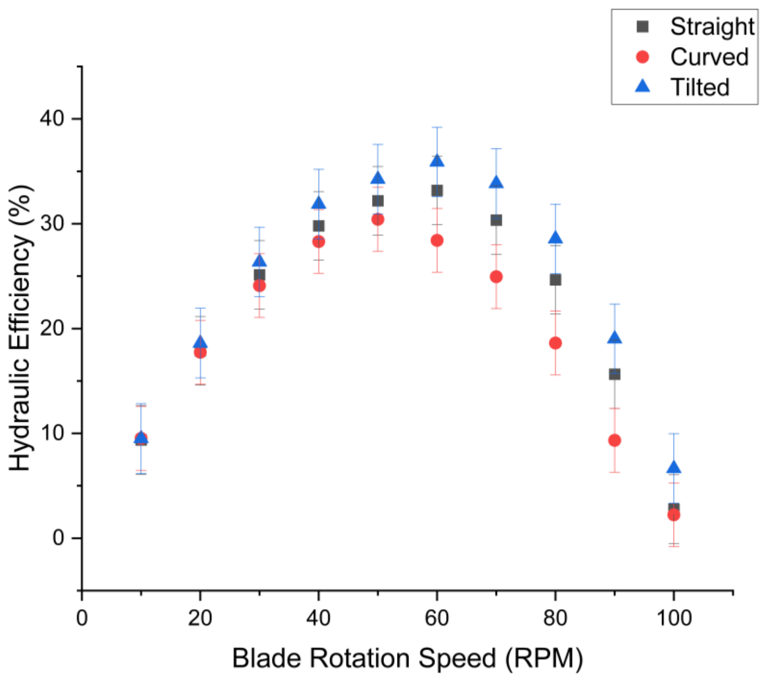


Fig. 5. Hydraulic Efficiency Comparison Chart (η) with rotational speed (n) in the simulation

Based on Figure 6 and Figure 7 it is shown that the comparison of the pressure contours of the three blades side view where there is a pressure drop when the fluid hits the blade. The distribution of pressure on the inclined blade looks more even and wider, which according to the theory of pressure on the vortex is lower as it approaches the center. The pressure distribution shown above shows that the pressure distribution area on the inclined blade is more stable and can be assumed as a vortice flow formed. So that it can be said that the inclined blade profile type is superior to the formation of vortex flow compared to the other two blades.

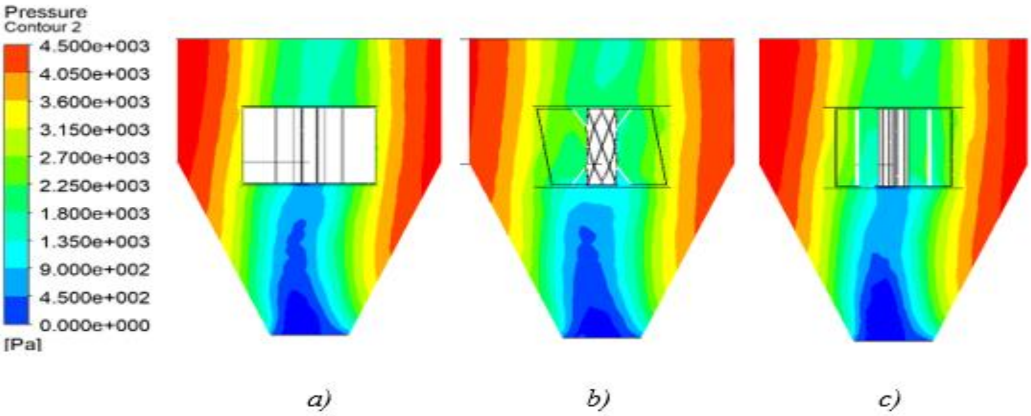


Fig. 6. Side View of the Pressure Contour (a) Straight Blade (b) Tilted Blade and (c) Curved Blade

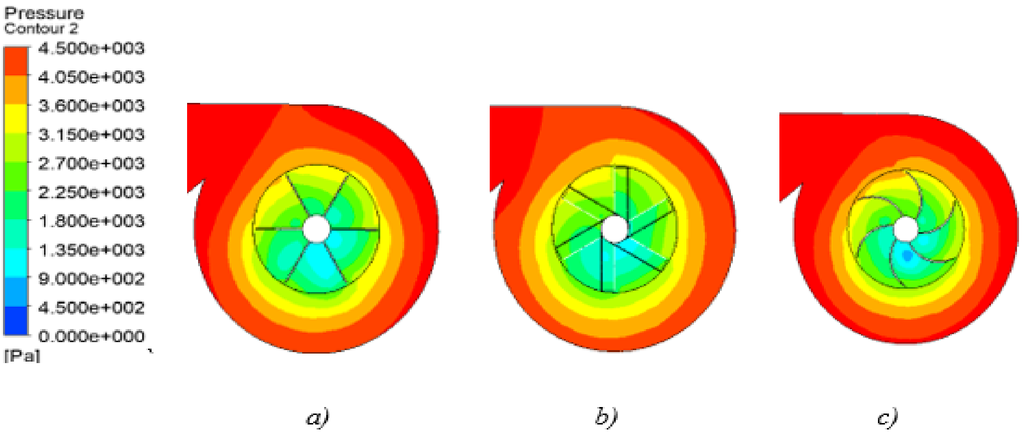


Fig. 7. Top View of the Pressure Contour (a) Straight Blade (b) Tilted Blade and (c) Curved Blade

It can be seen in Figure 8 and Figure 9 that the visualization of the velocity contour on the three-blade profiles shows the velocity distribution. The velocity at the outlet plane tends to be higher due to the conical shape of the basin while at the center of the vortex the flow velocity tends to be low. The speed when hitting the blade becomes smaller, this shows that the blade absorbs kinetic energy well.

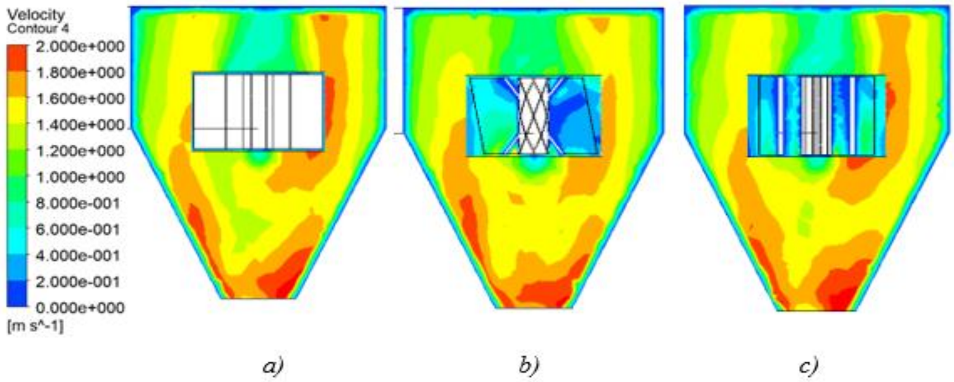


Fig. 8. Top View of the Pressure Contour a) Straight Blade (b) Tilted Blade and (c) Curved Blade

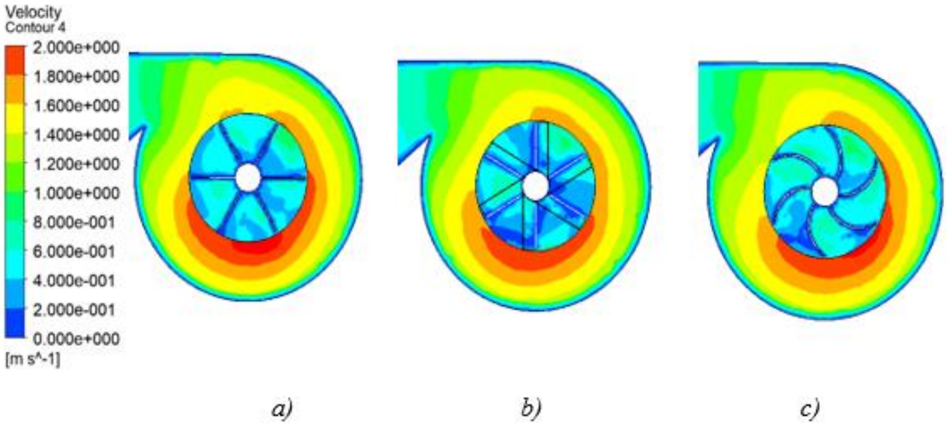


Fig. 9. Top View of the Speed Contour (a) Straight Blade (b) Tilted Blade and (c) Curved Blade

Figure 10 is a visualization of the pressure contour when the blade hits the flow. In the tilted blade, the top of the blade first gets pressure from the flow that is passed down the blade so that the pressure below near the lowest shaft can be said to be good at absorbing flow pressure. While the curved blade receives the most pressure, as seen from the red colour which is more on the blade because of its curved shape, a lot of flow accumulates between the blades which creates turbulence in it so it creates an obstacle that affects the resulting rotation.

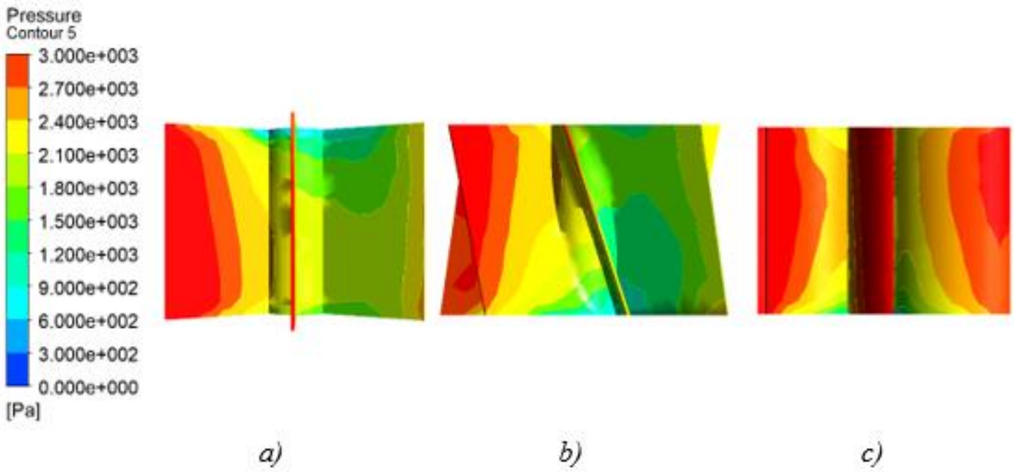


Fig. 10 Pressure on Blades (a) Straight Blade (b) Tilted Blade, and (c) Curved Blade

In case of varying submergence of the turbine, this study gives similar results with the study of Dhakal *et al.,* [16, 17] which showed matches the theoretical concepts of impacts of free jets on different profiled blades as the tilted profile blade is best due to the presence of larger surface area to impinge the flow. Moreover, the tiltted profile blade has less losses form the streamflow as it is matching the velocity angle of the vortex better than the straight and curved one.

4. Conclusion

Based on the study process that has been carried out numerically related to the ratio of 3 blades on the vortex turbine, it can be concluded that:

i. The maximum hydraulic efficiency of this pico hydro scale vortex turbine is achieved by using an tilted blade profile has the highest efficiency of 36%, for straight and curved blade profiles the maximum efficiency is 33% and 30%.
ii. The tilted blade type is suitable for use on pico scale vortex turbines because it is the best at absorbing air energy so as to produce the lowest and most optimal efficiency.

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