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Mathematical Model for MHD Micropolar Fluid with Chemical Reaction towards an Exponential Curved Surface

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ABSTRACT

The current article delineates the aspects of MHD Micropolar fluid flow with chemical reaction towards an exponentially stretchable curved surface. Aspects of heat transmission are delineated by incorporating joule heating and thermal radiation and viscous Dissipation. To frame the mathematical model, curvilinear (r_1, s_1) co-ordinates are employed. Similarity variables are deployed to transfigure the flow modelling PDE's into ODE's. A familiar numerical approach namely Keller box method is operated to resolve the resultant ODE's. Influence of diverse parameters such as material parameter, curvature parameter, magnetic parameter and chemical reaction parameter has been scrutinized via graphical way. Velocity $f'(\eta)$ and Microrotation velocity $g(\eta)$ exhibits unlike nature for magnetic parameter M .Temperature $\theta(\eta)$ exhibits similar response according to variations in M , Rd and Ec .Temperature $\theta(\eta)$ exhibits alike functioning according to variations in k_1, δ and $\Pr$ .Concentration $\phi(\eta)$ exhibits improvement for variations in M and k_1 while it decreases for variations in δ, Sc and kc .An adequate resemblance has been detected with existing results.

1. Introduction

The productive challenges in diverse engineering and science sectors should be resolved owing to their productive applications in day-to-day life, like regulation of temperature in engines, machinery performances, building and etc. The Micropolar fluid class is employed to originate the simulations of smoke or dust particles in nature and gas. Even so, numerous models comprise

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peculiar complex structure with nonlinear nature and complicated to regulate realistically. Due to the increment in relevancy of materials whose functioning can't be delineated with Newtonian law, a novel step in the scrutiny of the dynamical class of the fluid is currently happening. Micropolar fluids are comprised with rotatable micro-components which are able to transform the hydrodynamics of the motion. Numerous works are addressed the prominence of the Micropolar fluids in various sectors. Instances are nuclear reactors, solar energy, army, urinal flow in the kidneys, heating and ventilation, electronic packaging, natural convection of cavities, energy transport and etc (see Ariman *et al.,* [1], Khonsari and Brew [2] and Khonsari [3]).

Eringen [4, 5] instituted the notion of molecular liquids by scrutinizing the inertial features of the sub structural particles. Hassanien and Gorla [6] appraised a mathematical model for thermal transmission in a Micropolar liquid caused by an extendable sheet. Takhar *et al.,* [7] probed the motion of a Micropolar fluid originated by a stretchable plate with mixed convection. Eldabe *et al.,* [8] analyzed magneto Micropolar fluid motion via an extendable sheet by exerting the ChFD (Chebyshev finite difference) approach. A vast amount of literature is available on micro polar fluid. Gangadhar *et al.,* [9-11] explored the characteristics of micro polar fluid flow over various geometries and conditions. Hussanan *et al.,* [12] employed the Cattaneo-Christov form in the energy equation of a MHD Micropolar liquid towards an extendable sheet with variable thickness and observed the attributes of thermal relaxation time. Zadeh *et al.,* [13] worked on the bio convective Micropolar fluid towards a vertically placed extendable sheet and explored the influence of motile microorganisms. Singh *et al.,* [14] focused on the impression of heat source/sink Micropolar liquid via a porous wedge by incorporating chemical reaction and slip constraints. Fatunmbi [15] analyzed the characteristics of a magneto Micropolar liquid towards a nonlinearly stretchable sheet in non-Darcy porous media with slip constraints. Kumar *et al.,* [16] explored the consequences of radiation on MHD Micropolar liquid with slip and melting originated by a porous stretched sheet with exponential stretching by including heat source/sink. Nadeem *et al.,* [17] emphasized the impression of viscoelasticity on slip flow of Maxwell model micropolar liquid by comprising MHD and suction. Fuzang *et al.,* [18] evinced an evaluation on heat transmission by considering slip flow of a Micropolar liquid towards a curve shaped surface. Goud *et al.,* [19- 22] documented the flow of Micropolar liquid originated by a stretching surface and by comprising diverse physical constraints.

The immense utilizations in manufacturing, metallurgical and in industrial sectors compelled researchers to scrutinize the phenomenon involved in the boundary layer flow originated by an extendable surface. Instances are paper production crystal growing and glass blowing. Numerous scientists are enthusiastic in evolving novel machinery and equipment with robust heating (cooling) rates. Crane [23] did the initiation work regarding the fluid motion caused by an expandable plate. In numerable studies are instituted by exerting diverse physical constraints. But entire studies focused only on flat sheets caused by linear stretching and nonlinear stretching. Anyhow, the liquid flow originated by a curve shaped stretchable surface is not disclosed completely. Analysis of liquid flow corresponding to curve shaped surface with linear stretching was instituted by Sajid *et al.,* [24]. Rosca and Pop [25] addressed the unsteady flow of a viscous liquid towards a curve shaped stretchable/shrinkable porous surface with suction. The EMHD flow of a Micropolar liquid via curve shaped extending sheet was disclosed by Naveed *et al.,* [26]. Thermal radiation attributes of an EMHD nano fluid $(Cu-H_2o \,\&\, Ag-H_2o)$ by incorporating heat generation with slip caused by a curve shaped stretching surface was delineated by Abbas *et al.,* [27]. Numerous authors worked on this concept. Recently, Okechi *et al.,* [28] presented an analysis and explored the motion of an incompressible liquid towards a curve shaped surface stretching with an exponential velocity. Hayat *et al.,* [29] addressed the Darcy-Forchheimer flow of a viscous fluid caused by a curve shaped surface with exponential stretching and explained the impact of Brownian motion and thermophoresis. Juwad *et al.,* [30] presented the heat and mass transmission features of MHD nano fluid towards a curve shaped surface with exponential stretching with heat source and activation energy. Qian *et al.,* [31] exhibited the features of a MHD viscous Micropolar fluid through a curve shaped surface with exponential stretching and explained the combined effect of thermal radiation and ohmic heating. Gowda *et al.,* [32] and Saeed *et al.,* [33] addressed the features of hybrid nano fluids towards an exponentially curved stretching surface with diverse conditions. Ramzan *et al.,* [34, 35] and Alotaibi [36] accomplished a numerical evaluation on some nano fluids towards a curve shaped stretching surface by comprising distinct physical situations.

MHD studies have garnered massive engross in exclusive technical sectors due to their prominent role in industrial innovations with their massive applications in biological transportation, nuclear cooling reactors, high temperature plasmas, drying processes, sensors, geophysics etc and also crucial in MHD power generation and earthquake presumption. Maheswari *et al.,* [37], Guled *et al.,* [38], Dharmaiah *et al.,* [39] and Arulmozhi *et al.,* [40] worked on MHD flows with various surfaces and conditions. In recent times, Suneetha *et al.,* [41] and Ravikiran *et al.,* [42] explored some MHD flows by incorporating various physical constraints.

Scrutinizing of chemical reaction is very prominent in bio-engineering and chemical industrial utilizations like processing of food, production of polymers, evaporation, and production of ceramics, energy transmission and drying. In the sectors of metallurgy and chemical engineering, exploration of mass and heat transmission with chemical reaction is very prominent. Bestman [43] initially addressed the impression of chemical reaction on boundary layer flows. Sajid *et al.,* [44] appraised the viscous flow originated by a curve shaped stretched surface. Hayat *et al.,* [45-47] reported the influence of chemical reaction on various fluids with various surfaces and conditions. Latif *et al.,* [48] modeled the radiative Sisko fluid flow towards a curved sheet with mixed convection using RK-45 Fehlberg scheme. Manjunatha *et al.,* [49] made an attempt and explained the influence of chemical reaction on viscous nano material subjected to a curved stretching sheet by comprising Stefan blowing. Harish *et al.,* [50], Jalili *et al.,* [51] explained the role of chemical reaction on Jeffery fluid and Casson fluid by considering different aspects respectively. Reddy *et al.,* [52- 55] reported the influence of chemical reaction on various fluids with various surfaces and conditions. Khan *et al.,* [56] and Azam *et al.,* [57] explored the impression of chemical reaction on Burgers fluid and Casson fluid with diverse geometries.

In prospect of accessible reports, not much attention is given to the flow of Micropolar liquid. The prime intention of the current attempt is to appraise impression of chemical reaction on MHD Micropolar in the existence of joule heating towards a curve shaped surface with exponential stretching. Features of heat transmission are delineated by incorporating and thermal radiation and viscous Dissipation. Numerical simulations are done through novel Keller box method. The gaps in the accessible literature will be furnished with the current attempt. The upshots of this evaluation are relevant to diverse applications like innovative cooling technologies, heat exchangers and advanced thermal management systems. These upshots can be leveraged to enhance heat transfer efficacy and overall system functioning. Finally, authors have to acknowledge the subsequent queries. What are the consequences of curvature factor on fluid profiles? How thermal radiation and Eckert number influences the thermal profile? What is the impression of reaction parameter on concentration?

2. Model Description

An incompressible 2D flow of viscous micro polar fluid is examined. Flow is originated by an exponential stretchable curved surface with stretching velocity $u_w(s_1) = c_1 \exp\left(\frac{s_1}{L_1}\right)$ where $c_1, L_1 > 0$. The orientation of the exponential curved surface is defined by the curvilinear coordinate system (r_1, s_1) with radius R_1 which is exhibited in Figure 1.

Assumptions and Conditions:

i. MHD micro polar fluid
ii. Buoyancy effect
iii. viscous Dissipation, Joule heating and thermal radiation
iv. Chemical reaction

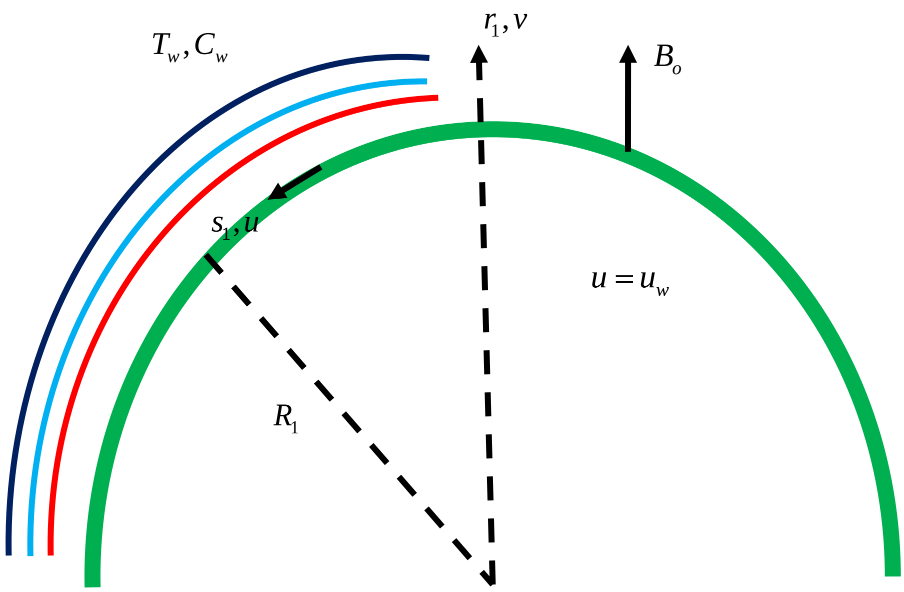

Fig. 1. Flow configuration

Modelling

Flow modelling equations with under above assumptions are given as (see Sajid *et al.,* [24]):

$$\frac{\partial}{\partial r_1}(r_1 + R_1)v + R_1 \frac{\partial u}{\partial s_1} = 0, \tag{1}$$

$$\frac{u^2}{r_1 + R_1} = \frac{\frac{\partial p}{\partial r_1}}{\rho}, \tag{2}$$

$$\left.\begin{aligned}&\frac{u R_1}{R_1+r_1}\frac{\partial u}{\partial s_1}+v\frac{\partial u}{\partial r_1}+\frac{u v}{R_1+r_1}=-\frac{1}{\rho}\frac{R_1}{R_1+r_1}\frac{\partial p}{\partial s_1}\\&\quad+\left(\frac{\mu+k^*}{\rho}\right)\left(\frac{\partial^2 u}{\partial r_1^2}-\frac{u}{(R_1+r_1)^2}+\frac{\frac{\partial u}{\partial r_1}}{R_1+r_1}\right)-\frac{k^*}{\rho}\frac{\partial N}{\partial r_1}-\frac{\sigma B_o^{\,2}}{\rho}u+g\beta_0(T-T_\infty)\end{aligned}\right\},\tag{3}$$

$$\frac{u R_1}{R_1+r_1}\frac{\partial N}{\partial s_1}+v\frac{\partial N}{\partial r_1}=\frac{\gamma^*}{\rho j}\left[\frac{\partial^2 N}{\partial r_1^2}+\frac{\frac{\partial N}{\partial r_1}}{R_1+r_1}\right]+\left(\frac{k^*}{\rho j}\right)\left[2N+\frac{\partial u}{\partial r_1}+\frac{u}{R_1+r_1}\right],\tag{4}$$

$$\left.\begin{aligned}&\rho c_p\left(\frac{u R_1}{R_1+r_1}\frac{\partial T}{\partial s_1}+v\frac{\partial T}{\partial r_1}\right)=k_o\left[\frac{\partial^2 T}{\partial r_1^2}+\frac{\frac{\partial T}{\partial r_1}}{R_1+r_1}\right]-\frac{1}{R_1+r_1}\frac{\partial}{\partial r_1}\left(R_1+r_1\right)q_i\\&\quad+\sigma B_o^{\,2}u^2+\mu\left(\frac{\partial u}{\partial r_1}-\frac{u}{R_1+r_1}\right)^2\end{aligned}\right\},\tag{5}$$

$$\left(\frac{u R_1}{R_1+r_1}\frac{\partial C}{\partial s_1}+v\frac{\partial C}{\partial r_1}\right)=D_B\left[\frac{\partial^2 C}{\partial r_1^2}+\frac{\frac{\partial C}{\partial r_1}}{R_1+r_1}\right]-k_o(C-C_\infty),\tag{6}$$

With the boundary conditions:

$$u-u_w=0,\ v=0,\ N=-m_1\frac{\partial u}{\partial r_1},\ T-T_w=0,\ C-C_w=0 \text{ At } r_1=0,$$

$$u\to 0,\ \frac{\partial u}{\partial r_1}\to 0,\ N\to 0\ T\to T_\infty,\ C\to C_\infty \text{ As } r_1\to\infty.\tag{7}$$

Here $q_i=\frac{4\sigma^*}{3k_1^*}\frac{\partial T^4}{\partial r_1}$.

On employing $u = c_1 \exp\left(\frac{s_1}{L_1}\right) f'(\eta),\ v = -\frac{R_1}{r_1 + R_1}\sqrt{\frac{c_1 \vartheta \exp\left(\frac{s_1}{L_1}\right)}{2L_1}}\left[f(\eta) + \eta f'(\eta)\right],\ \eta = \sqrt{\frac{c_1 \exp\left(\frac{s_1}{L_1}\right)}{2\vartheta L_1}} r_1,$ $p = \rho c_1^2 \exp\left(\frac{2s_1}{L_1}\right) P(\eta),\ N = c_1 \exp\left(\frac{s_1}{L_1}\right)\sqrt{\frac{c_1 \exp\left(\frac{s_1}{L_1}\right)}{2\vartheta L_1}} g(\eta)\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$ (Okechi [28]) the Eq. (3) to Eq. (7) take the following form.

$$\frac{f'^2}{\delta + \eta} = P', \tag{8}$$

$$\left.\begin{array}{l} \frac{4\delta P}{\delta+\eta} + \frac{\eta\delta P'}{\delta+\eta} = (k_1+1)\left(f''' + \frac{f''}{\delta+\eta} - \frac{f'}{(\delta+\eta)^2}\right) + \frac{\delta ff'}{(\delta+\eta)^2} \\ \qquad + \frac{\delta ff''}{(\delta+\eta)} - \frac{(2\delta+\eta)\delta f'^2}{(\delta+\eta)^2} - k_1 g' - M^2 f' + \lambda\theta \end{array}\right\}, \tag{9}$$

$$\left(\frac{k_1}{2}+1\right)\left(g'' + \frac{g'}{\delta+\eta}\right) - k_1\left(2g + f'' + \frac{f'}{\delta+\eta}\right) + \frac{\delta fg'}{\delta+\eta} - \frac{3\delta f' g}{\delta+\eta} = 0, \tag{10}$$

$$\left.\begin{array}{l} (1+Rd)\theta'' + (1+Rd)\frac{\theta'}{\delta+\eta} + \frac{\delta \Pr\left(f\theta' - 2f'\theta\right)}{\delta+\eta} \\ \qquad + \Pr Ec\left(f'' - \frac{f'}{\delta+\eta}\right)^2 + M \Pr Ec(\delta+\eta)\ f'^2 = 0 \end{array}\right\}, \tag{11}$$

$$\phi'' + \frac{\phi'}{\delta+\eta} + \frac{Sc\,\delta}{\delta+\eta} f\phi' - Sc\,kc\,\phi = 0, \tag{12}$$

Now removing pressure term from Eq. (9) we get

$$\left.\begin{array}{l} (k_1+1)\left(f'''' - \frac{f''}{(\delta+\eta)^2} + \frac{2f'''}{\delta+\eta} + \frac{f'}{(\delta+\eta)^3}\right) + \frac{\delta ff'''}{\delta+\eta} - \frac{\delta ff'}{(\delta+\eta)^3} \\ \qquad - \frac{3\delta f' f''}{\delta+\eta} - \frac{3\delta f'^2}{(\delta+\eta)^2} + \frac{\delta ff''}{(\delta+\eta)^2} - \left[k_1 g'' + \frac{k_1 g'}{\delta+\eta}\right] \\ \qquad - \left[M^2 f'' + \frac{M^2 f'}{\delta+\eta}\right] + \left[\lambda\theta' + \frac{\lambda\theta}{\delta+\eta}\right] = 0 \end{array}\right\}, \tag{13}$$

And the boundary conditions are given by:

$$f(0) = 0,\ f'(0) - 1 = 0,\ g(0) + m_1 f''(0) = 0,\ \theta(0) - 1 = 0,\ \phi(0) - 1 = 0,$$

$$f'(\infty)=0,\ f''(\infty)=0,\ g(\infty)=0,\ \theta(\infty)=0,\ \phi(\infty)=0 \tag{14}$$

The values of the parameters which are raised in the Eq. (8) to Eq. (14)

$$\gamma^{*}=\mu\left(\frac{k_1}{2}+1\right)j,\ M^{2}=\frac{\sigma B_0^2 L_1}{\rho c_1},\ \delta=\sqrt{\frac{u_w}{2\vartheta L_1}}R_1,$$
$$\lambda=\frac{g\beta_0(T_w-T_\infty)L_1}{c_1u_w^2},\ Rd=\frac{16\sigma^{*}T_\infty^3}{3kk^{*}},\ Ec=\frac{2L_1u_w^2}{c_p(T_w-T_\infty)},$$
$$Sc=\frac{v}{D_B},\ kc=\frac{2k_oL_1}{c_1}$$

2.1 Skin Friction Coefficient

$$C_f=\frac{\tau_{r_1s_1}}{\rho u_w^2}=\frac{(\mu+k^{*})\left(\frac{\partial u}{\partial r_1}-\frac{u}{r_1+R_1}+\delta N\right)_{at\ \eta=0}}{\rho u_w^2},$$

On simplification we get $\sqrt{\frac{L_1}{2s_1}}\mathrm{Re}_s^{0.5}C_{fs}=(k_1+1)\left(f''(0)-\frac{f'(0)}{\delta}\right)$. (15)

2.2 Couple Stress on Surface

$$C_{s_1}=\frac{M_w}{\mu ju_w}=\frac{\gamma\frac{\partial N}{\partial r_1}_{at\,r_1=0}}{\mu ju_w},$$

On simplification, $2L_1s_1\mathrm{Re}_s^{-1}C_{s_1}=(\frac{k_1}{2}+1)g'(0)$. (16)

2.3 Nusselt Number

$$Nu_x=\frac{s_1q_w}{k_o(T_w-T_\infty)},\ q_w=-k_o(1+Rd)\frac{\partial T}{\partial r_1}\Big|r_1=0\ ,\quad \sqrt{\frac{2L_1}{s_1}}Nu_{s_1}\mathrm{Re}_{s_1}^{\frac{-1}{2}}=-(1+Rd)\theta'(0) \tag{17}$$

2.4 Sherwood Number

$$Sh_x=\frac{s_1q_m}{D_B(C_w-C_\infty)},\ q_m=-D_B\frac{\partial C}{\partial r_1}\Big|r_1=0\ ,\quad \sqrt{\frac{2L_1}{s_1}}Sh_{s_1}\mathrm{Re}_{s_1}^{\frac{-1}{2}}=-\phi'(0) \tag{18}$$

3. Numerical Interpretation

In the process modelling, Majority part of the physical mechanisms are transfigured into an intricate system of non-linear ODE. But, in usual it is not an easy task to acquire the exact solution to these equations analytically. In contrast numerical methods will proffer precise, efficient and effectual solutions. We employed Keller box method (see ref [58]) to acquire the solution for the system of non-linear ODE to analyze the consequences of various pertinent parameters of the flow. In the light of efficiency and stability, KBM established as a as a robust tool to resolve nonlinear ODE. In the computation process, a uniform step size of $\eta = 0.01$ is employed and solutions are acquired with an error tolerance of 10^{-6} which provides an accuracy of 4 decimal places. KBM comprises the subsequent steps.

Step 1: reduction of ODE system in to 1st order ODE System
Step 2: Initialization of central differences to achieve difference Equations
Step 3: introduction of Newton's scheme to linearize the resulting equations
Step 4: Initialization of matrix tridiagonal System

On applying $p = f', q = p' = f'', w = q' = p'' = f'''$ $g = u, g' = u' = v$ $\theta = g, \theta' = t$ and $\phi = h, \phi' = h' = n$ the equations from [10-13] can be written as:

$$\left.\begin{aligned} &(k_1+1)\left(w' - \frac{q}{(\delta+\eta)^2} + \frac{2w}{\delta+\eta} + \frac{p}{(\delta+\eta)^3}\right) + \frac{\delta\, fw}{\delta+\eta} - \frac{\delta\, fp}{(\delta+\eta)^3} \\ &\qquad - \frac{3\delta\, pq}{\delta+\eta} - \frac{3\delta\, p^2}{(\delta+\eta)^2} + \frac{\delta\, fq}{(\delta+\eta)^2} - k_1\left[v' + \frac{v}{\delta+\eta}\right] \\ &\qquad - M^2\left[q + \frac{p}{\delta+\eta}\right] + \lambda\left[t + \frac{g}{\delta+\eta}\right] = 0 \end{aligned}\right\}, \tag{19}$$

$$\left(\frac{k_1}{2}+1\right)\left(v' + \frac{v}{\delta+\eta}\right) - k_1\left(2u + q + \frac{p}{\delta+\eta}\right) + \frac{\delta\, fv}{\delta+\eta} - \frac{3\delta\, pu}{\delta+\eta} = 0, \tag{20}$$

$$\left.\begin{aligned} &[1+Rd]t' + (1+Rd)\frac{t}{\delta+\eta} + \frac{\delta \Pr\left(f\,t - 2pg\right)}{\delta+\eta} + \Pr Ec\left(q - \frac{p}{\delta+\eta}\right)^2 \\ &\qquad + M \Pr Ec\,(\delta+\eta)\; p^2 = 0 \end{aligned}\right\}, \tag{21}$$

$$n' + \frac{n}{\delta+\eta} + \frac{Sc\,\delta}{\delta+\eta}\, fn - Sc\,kc\,h = 0, \tag{22}$$

The resulting boundary conditions are

$$\begin{aligned} &f(0)=0,\ p(0)-1=0,\ u(0)+m_1\,q(0)=0,\ g(0)-1=0,\ h(0)-1=0, \\ &p(\infty)=0,\ q(\infty)=0,\ u(\infty)=0,\ g(\infty)=0,\ h(\infty)=0\ . \end{aligned} \tag{23}$$

Now consider the central differences as:

$$\frac{f_j^i - f_{j-1}^i}{\omega_j} = 0.5(p_j^i + p_{j-1}^i),\ \frac{p_j^i - p_{j-1}^i}{\omega_j} = 0.5(q_j^i + q_{j-1}^i),\ \frac{q_j^i - q_{j-1}^i}{\omega_j} = 0.5(w_j^i + w_{j-1}^i),$$
$$\frac{u_j^i - u_{j-1}^i}{\omega_j} = 0.5(v_j^i + v_{j-1}^i),\ \frac{g_j^i - g_{j-1}^i}{\omega_j} = 0.5(t_j^i + t_{j-1}^i),\ \frac{h_j^i - h_{j-1}^i}{\omega_j} = 0.5(n_j^i + n_{j-1}^i).$$

On exerting above central differences, Eq. (19) – Eq. (23) becomes as

$$\left.\begin{aligned}&(k_1+1)\left(w_j^i - w_{j-1}^i - \frac{\omega_j q_{j-1/2}^i}{(\delta+\eta)^2} + \frac{2\omega_j w_{j-1/2}^i}{\delta+\eta} + \frac{\omega_j p_{j-1/2}^i}{(\delta+\eta)^3}\right) + \frac{\omega_j \delta\, f_{j-1/2}^i w_{j-1/2}^i}{\delta+\eta}\\ &\quad - \frac{\omega_j \delta\, f_{j-1/2}^i p_{j-1/2}^i}{(\delta+\eta)^3} - \frac{3\omega_j \delta\, p_{j-1/2}^i q_{j-1/2}^i}{\delta+\eta} - \frac{3\omega_j \delta\, {p_{j-1/2}^i}^2}{(\delta+\eta)^2}\\ &\quad + \frac{\delta\omega_j\, f_{j-1/2}^i q_{j-1/2}^i}{(\delta+\eta)^2} - k_1\left[v_j^i - v_{j-1}^i + \frac{\omega_j v_{j-1/2}^i}{\delta+\eta}\right]\\ &\quad - M^2\omega_j\left[q_{j-1/2}^i + \frac{p_{j-1/2}^i}{\delta+\eta}\right] + \lambda\omega_j\left[t_{j-1/2}^i + \frac{g_{j-1/2}^i}{\delta+\eta}\right] = F_{j-1/2}\end{aligned}\right\}, \tag{24}$$

$$\left.\begin{aligned}&\left(\frac{k_1}{2}+1\right)\left(v_j^i - v_{j-1}^i + \frac{\omega_j v_{j-1/2}^i}{\delta+\eta}\right) - k_1\,\omega_j\left(2u_{j-1/2}^i + q_{j-1/2}^i + \frac{p_{j-1/2}^i}{\delta+\eta}\right) + \frac{\omega_j \delta\, f_{j-1/2}^i v_{j-1/2}^i}{\delta+\eta}\\ &\quad - \frac{3\omega_j \delta\, p_{j-1/2}^i u_{j-1/2}^i}{\delta+\eta} = G_{j-1/2}\end{aligned}\right\}, \tag{25}$$

$$\left.\begin{aligned}&(1+Rd)(t_j^i - t_{j-1}^i) + \omega_j(1+Rd)\frac{t_{j-1/2}^i}{\delta+\eta} + \frac{\omega_j \delta \Pr\left(f_{j-1/2}^i t_{j-1/2}^i - 2p_{j-1/2}^i g_{j-1/2}^i\right)}{\delta+\eta}\\ &\quad + \omega_j \Pr Ec\left(q_{j-1/2}^i - \frac{p_{j-1/2}^i}{\delta+\eta}\right)^2 + \omega_j M \Pr Ec\,(\delta+\eta)\ {p_{j-1/2}^i}^2 = 0\end{aligned}\right\}, \tag{26}$$

$$n_j^i - n_{j-1}^i + \frac{\omega_j n_{j-1/2}^i}{\delta+\eta} + \frac{Sc\,\delta\omega_j}{\delta+\eta}\, f_{j-1/2}^i n_{j-1/2}^i - Sc\,kc\,\omega_j\, h_{j-1/2}^i = T_{j-1/2}, \tag{27}$$

The resulting boundary conditions are

$$f_o^i = 0,\ p_o^i = 0,\ u_o^i + m_1 q_o^i = 0,\ g_o^i = 0,\ h_o^i = 0,$$
$$p_J^i = 0,\ q_J^i = 0,\ u_J^i = 0,\ g_J^i = 0,\ h_J^i = 0. \tag{28}$$

For the purpose of linearization, exert Newton's approach as

$f_j^{i+1} = \delta f_j^i + f_j^i,\ p_j^{i+1} = \delta p_j^i + p_j^i,\ q_j^{i+1} = \delta q_j^i + q_j^i,\ w_j^{i+1} = \delta w_j^i + w_j^i,\ u_j^{i+1} = \delta u_j^i + u_j^i,\ v_j^{i+1} = \delta v_j^i + v_j^i,$
$g_j^{i+1} = \delta g_j^i + g_j^i,\ t_j^{i+1} = \delta t_j^i + t_j^i,\ h_j^{i+1} = \delta h_j^i + h_j^i,\ n_j^{i+1} = \delta n_j^i + n_j^i.$

We get

$$(\delta f_j - \delta f_{j-1}) - 0.5\omega_j \left(\delta p_j + \delta p_{j-1}\right) = (\Re_1)_{j-1/2}, \tag{29}$$

$$(\delta p_j - \delta p_{j-1}) - 0.5\omega_j \left(\delta q_j + \delta q_{j-1}\right) = (\Re_2)_{j-1/2}, \tag{30}$$

$$(\delta q_j - \delta q_{j-1}) - 0.5\omega_j \left(\delta w_j + \delta w_{j-1}\right) = (\Re_3)_{j-1/2}, \tag{31}$$

$$(\delta u_j - \delta u_{j-1}) - 0.5\omega_j \left(\delta v_j + \delta v_{j-1}\right) = (\Re_4)_{j-1/2}, \tag{32}$$

$$(\delta g_j - \delta g_{j-1}) - 0.5\omega_j \left(\delta t_j + \delta t_{j-1}\right) = (\Re_5)_{j-1/2}, \tag{33}$$

$$(\delta h_j - \delta h_{j-1}) - 0.5\omega_j \left(\delta n_j + \delta n_{j-1}\right) = (\Re_6)_{j-1/2}, \tag{34}$$

$$\left.\begin{aligned}&(A_1)_{j-1/2}\delta w_j + (A_2)_{j-1/2}\delta w_{j-1} + (A_3)_{j-1/2}\delta f_j + (A_4)_{j-1/2}\delta f_{j-1} + (A_5)_{j-1/2}\delta p_j \\ &+ (A_6)_{j-1/2}\delta p_{j-1} + (A_7)_{j-1/2}\delta q_j + (A_8)_{j-1/2}\delta q_{j-1} + (A_9)_{j-1/2}\delta t_j + (A_{10})_{j-1/2}\delta t_{j-1} \\ &+ (A_{11})_{j-1/2}\delta g_j + (A_{12})_{j-1/2}\delta g_{j-1} + (A_{13})_{j-1/2}\delta v_j + (A_{14})_{j-1/2}\delta v_{j-1} = (\Re_7)_{j-1/2}\end{aligned}\right\}, \tag{35}$$

$$\left.\begin{aligned}&(B_1)_{j-1/2}\delta v_j + (B_2)_{j-1/2}\delta v_{j-1} + (B_3)_{j-1/2}\delta u_j + (B_4)_{j-1/2}\delta u_{j-1} + (B_5)_{j-1/2}\delta f_j \\ &+ (B_6)_{j-1/2}\delta f_{j-1} + (B_7)_{j-1/2}\delta p_j + (B_8)_{j-1/2}\delta p_{j-1} + (B_9)_{j-1/2}\delta q_j + (B_{10})_{j-1/2}\delta q_{j-1} = (\Re_8)_{j-1/2}\end{aligned}\right\}, \tag{36}$$

$$\left.\begin{aligned}&(C_1)_{j-1/2}\delta t_j + (C_2)_{j-1/2}\delta t_{j-1} + (C_3)_{j-1/2}\delta g_j + (C_4)_{j-1/2}\delta g_{j-1} + (C_5)_{j-1/2}\delta f_j + (C_6)_{j-1/2}\delta f_{j-1} \\ &+ (C_7)_{j-1/2}\delta p_j + (C_8)_{j-1/2}\delta p_{j-1} + (C_9)_{j-1/2}\delta q_j + (C_{10})_{j-1/2}\delta q_{j-1} = (\Re_9)_{j-1/2}\end{aligned}\right\}, \tag{37}$$

$$\left.\begin{aligned}&(D_1)_{j-1/2}\delta n_j + (D_2)_{j-1/2}\delta n_{j-1} + (D_3)_{j-1/2}\delta f_j + (D_4)_{j-1/2}\delta f_{j-1} \\ &\qquad + (D_5)_{j-1/2}\delta h_j + (D_6)_{j-1/2}\delta h_{j-1} = (\Re_{10})_{j-1/2}\end{aligned}\right\}, \tag{38}$$

Where

$$(A_1)_{j-1/2} = 1 + \frac{\omega_j}{\delta + \eta} + \frac{0.5\omega_j \delta f_{j-1/2}}{(\delta + \eta)(1 + k_1)},\ (A_2)_{j-1/2} = -1 + \frac{\omega_j}{\delta + \eta} + \frac{0.5\omega_j \delta f_{j-1/2}}{(\delta + \eta)(1 + k_1)},$$

$$(A_3)_{j-1/2} = \frac{0.5\omega_j \delta w_{j-1/2}}{(1 + k_1)(\delta + \eta)} - \frac{0.5\omega_j \delta p_{j-1/2}}{(1 + k_1)(\delta + \eta)^3} + \frac{0.5\omega_j \delta q_{j-1/2}}{(1 + k_1)(\delta + \eta)^2},\ (A_4)_{j-1/2} = (A_3)_{j-1/2},$$

$$(A_5)_{j-1/2} = \frac{0.5\omega_j}{(\delta + \eta)^3} - \frac{0.5\omega_j \delta f_{j-1/2}}{(\delta + \eta)^3 (1 + k_1)} - \frac{3(0.5)\omega_j \delta q_{j-1/2}}{(1 + k_1)(\delta + \eta)} - \frac{3(0.5)\omega_j \delta p_{j-1/2}}{(1 + k_1)(\delta + \eta)^2} - \frac{0.5\omega_j M^2 w_{j-1/2}}{(\delta + \eta)(1 + k_1)},$$

$$(A_6)_{j-1/2}=(A_5)_{j-1/2},\ (A_7)_{j-1/2}=-\frac{0.5\omega_j}{(\delta+\eta)^2}+\frac{0.5\omega_j\delta f_{j-1/2}}{(\delta+\eta)^2(1+k_1)}-\frac{3(0.5)\omega_j\delta p_{j-1/2}}{(1+k_1)(\delta+\eta)}-\frac{0.5\omega_j M^2}{(1+k_1)},$$

$$(A_8)_{j-1/2}=(A_7)_{j-1/2},\ (A_9)_{j-1/2}=\frac{0.5\lambda\omega_j}{(1+k_1)},\ (A_{10})_{j-1/2}=(A_9)_{j-1/2},\ (A_{11})_{j-1/2}=\frac{0.5\lambda\omega_j}{(\delta+\eta)(1+k_1)},$$

$$(A_{12})_{j-1/2}=(A_{11})_{j-1/2},\ (A_{13})_{j-1/2}=-\frac{k_1}{k_1+1}-\frac{0.5k_1\,\omega_j}{(\delta+\eta)(1+k_1)},\ (A_{14})_{j-1/2}=\frac{k_1}{k_1+1}-\frac{0.5k_1\,\omega_j}{(\delta+\eta)(1+k_1)},$$

$$(B_1)_{j-1/2}=1+\frac{0.5\omega_j}{(\delta+\eta)}+\frac{0.5\omega_j\delta f_{j-1/2}}{(\delta+\eta)(1+k_1/2)},\ (B_2)_{j-1/2}=-1+\frac{0.5\omega_j}{(\delta+\eta)}+\frac{0.5\omega_j\delta f_{j-1/2}}{(\delta+\eta)(1+k_1/2)},$$

$$(B_3)_{j-1/2}=-\frac{k_1\,\omega_j}{(1+k_1/2)}-\frac{3(0.5)\omega_j\delta p_{j-1/2}}{(\delta+\eta)(1+k_1/2)},\ (B_4)_{j-1/2}=(B_3)_{j-1/2},\ (B_5)_{j-1/2}=\frac{0.5\omega_j\delta v_{j-1/2}}{(\delta+\eta)(1+k_1/2)},$$

$$(B_6)_{j-1/2}=(B_5)_{j-1/2},\ (B_7)_{j-1/2}=-\frac{0.5k_1\,\omega_j}{(\delta+\eta)(1+k_1/2)}-\frac{3(0.5)\omega_j\delta u_{j-1/2}}{(\delta+\eta)(1+k_1/2)},\ (B_8)_{j-1/2}=(B_7)_{j-1/2},$$

$$(B_9)_{j-1/2}=-\frac{0.5\omega_j k_1}{(1+k_1/2)},\ (B_{10})_{j-1/2}=(B_9)_{j-1/2},\ (C_1)_{j-1/2}=1+Rd+\frac{0.5\omega_j(1+Rd)}{(\delta+\eta)}+\frac{0.5\omega_j\delta\Pr f_{j-1/2}}{(\delta+\eta)},$$

$$(C_2)_{j-1/2}=-1-Rd+\frac{0.5\omega_j(1+Rd)}{(\delta+\eta)}+\frac{0.5\omega_j\delta\Pr f_{j-1/2}}{(\delta+\eta)},\ (C_3)_{j-1/2}=-\frac{\omega_j\delta\Pr p_{j-1/2}}{(\delta+\eta)},\ (C_4)_{j-1/2}=(C_3)_{j-1/2},$$

$$(C_5)_{j-1/2}=\frac{0.5\omega_j\delta\Pr t_{j-1/2}}{(\delta+\eta)},\ (C_6)_{j-1/2}=(C_5)_{j-1/2},$$

$$(C_7)_{j-1/2}=-\frac{\omega_j\delta\Pr g_{j-1/2}}{(\delta+\eta)}-\frac{\omega_j\Pr Ec\,p_{j-1/2}}{(\delta+\eta)^2}-\frac{\omega_j\Pr Ec\,q_{j-1/2}}{(\delta+\eta)}+0.5\omega_j M\Pr Ec(\delta+\eta)p_{j-1/2},$$

$$(C_8)_{j-1/2}=(C_7)_{j-1/2},\ (C_9)_{j-1/2}=-\frac{\omega_j\Pr Ec\,p_{j-1/2}}{(\delta+\eta)}+\omega_j\Pr Ec\,q_{j-1/2},\ (C_{10})_{j-1/2}=(C_9)_{j-1/2},$$

$$(D_1)_{j-1/2}=1+\frac{0.5\omega_j}{(\delta+\eta)}+\frac{0.5\omega_j\delta\,Sc\,f_{j-1/2}}{(\delta+\eta)},\ (D_2)_{j-1/2}=-1+\frac{0.5\omega_j}{(\delta+\eta)}+\frac{0.5\omega_j\delta\,Sc\,f_{j-1/2}}{(\delta+\eta)},$$

$$(D_3)_{j-1/2}=\frac{0.5\omega_j\delta\,Sc\,n_{j-1/2}}{(\delta+\eta)},\ (D_4)_{j-1/2}=(D_3)_{j-1/2},\ (D_5)_{j-1/2}=-\frac{0.5\omega_j Sc\,kc}{2},\ (D_6)_{j-1/2}=(D_5)_{j-1/2},$$

$$\left.\begin{aligned}(\Re_7)_{j-1/2}=\;&w_{j-1}-w_j+\frac{\omega_j q_{j-1/2}}{(\delta+\eta)^2}-\frac{2\omega_j w_{j-1/2}}{(\delta+\eta)}-\frac{\omega_j p_{j-1/2}}{(\delta+\eta)^3}-\frac{\omega_j\delta(fw)_{j-1/2}}{(k_1+1)(\delta+\eta)}+\frac{\omega_j\delta(fp)_{j-1/2}}{(k_1+1)(\delta+\eta)^3}\\&+\frac{3\delta\omega_j(pq)_{j-1/2}}{(k_1+1)(\delta+\eta)}+\frac{3\delta\omega_j p_{j-1/2}^2}{(k_1+1)(\delta+\eta)^2}+\frac{M^2\omega_j q_{j-1/2}}{(k_1+1)}+\frac{M^2\omega_j p_{j-1/2}}{(k_1+1)(\delta+\eta)}\\&-\frac{\delta\omega_j(fq)_{j-1/2}}{(k_1+1)(\delta+\eta)^2}-\frac{\lambda\omega_j t_{j-1/2}}{(k_1+1)}-\frac{\lambda\omega_j g_{j-1/2}}{(k_1+1)(\delta+\eta)}\\&+\frac{k_1}{k_1+1}(v_j-v_{j-1})+\frac{k_1\,\omega_j v_{j-1/2}}{(k_1+1)(\delta+\eta)}\end{aligned}\right\},$$

$$\left.\begin{aligned}(\Re_8)_{j-1/2} &= v_{j-1} - v_j - \frac{\omega_j v_{j-1/2}}{(\delta+\eta)} + \frac{k_1\,\omega_j}{1+k_1/2}\left(2u_{j-1/2} + q_{j-1/2} + \frac{p_{j-1/2}}{\delta+\eta}\right)\\ &\quad - \frac{\delta\omega_j(fv)_{j-1/2}}{(1+k_1/2)(\delta+\eta)} + \frac{3\delta\omega_j(pu)_{j-1/2}}{(1+k_1/2)(\delta+\eta)} + \frac{3\delta\omega_j p_{j-1/2}^2}{(k_1+1)(\delta+\eta)^2}\\ &\quad + \frac{M^2\omega_j q_{j-1/2}}{(k_1+1)} + \frac{M^2\omega_j p_{j-1/2}}{(k_1+1)(\delta+\eta)}\end{aligned}\right\},$$

$$\left.\begin{aligned}(\Re_9)_{j-1/2} &= (1+Rd)(t_{j-1} - t_j) - \frac{\omega_j(1+Rd)t_{j-1/2}}{(\delta+\eta)} - \frac{\delta\Pr\omega_j(ft)_{j-1/2}}{(\delta+\eta)}\\ &\quad + \frac{2\delta\Pr\omega_j(pg)_{j-1/2}}{(\delta+\eta)} + \frac{Ec\Pr\omega_j(p)^2{}_{j-1/2}}{(\delta+\eta)^2} + \frac{2Ec\Pr\omega_j(pq)_{j-1/2}}{(\delta+\eta)}\\ &\quad - Ec\Pr\omega_j(q)^2{}_{j-1/2} - Ec\Pr M\omega_j(\delta+\eta)(p)^2{}_{j-1/2}\end{aligned}\right\},$$

$$(\Re_{10})_{j-1/2} = n_{j-1} - n_j - \frac{\omega_j n_{j-1/2}}{(\delta+\eta)} - \frac{\delta Sc\omega_j(fn)_{j-1/2}}{(\delta+\eta)} + Sc\,kc\,\omega_j h_{j-1/2}$$

The boundary conditions are

$$\begin{aligned}&\delta f_o = 0,\ \delta p_o = 0,\ \delta u_o = 0,\ \delta g_o = 0,\ \delta h_o = 0,\\ &\delta p_J = 0,\ \delta q_J = 0,\ \delta u_J = 0,\ \delta g_J = 0,\ \delta h_J = 0\ .\end{aligned} \tag{39}$$

The reduced block tridiagonal scheme follows as:

$$\begin{bmatrix}[E_1][G_1] & & \\ [F_2][E_2][G_2] & & \\ & \cdots\cdots & \\ & \cdots\cdots & \\ & & [F_{j-1}][E_{j-1}][G_{j-1}]\\ & & [F_j][E_j]\end{bmatrix}\begin{bmatrix}[\delta_1]\\ [\delta_2]\\ \cdots\cdots\\ \cdots\cdots\\ [\delta_{j-1}]\\ [\delta_j]\end{bmatrix} = \begin{bmatrix}[\Re_1]\\ [\Re_2]\\ \cdots\cdots\\ \cdots\cdots\\ [\Re_{j-1}]\\ [\Re_j]\end{bmatrix}, \tag{40}$$

$$E_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{-\omega_j}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & \frac{-\omega_j}{2} & 0 & 0 & 0 & 0 & \frac{-\omega_j}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{-\omega_j}{2} & 0 & 0 & 0 & 0 & \frac{-\omega_j}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{-\omega_j}{2} & 0 & 0 & 0 & 0 & \frac{-\omega_j}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{-\omega_j}{2} & 0 & 0 & 0 & 0 & \frac{-\omega_j}{2} \\ A_8 & A_2 & A_{14} & A_{10} & 0 & A_3 & A_1 & A_{13} & A_9 & 0 \\ B_{10} & 0 & B_2 & 0 & 0 & B_5 & 0 & B_1 & 0 & 0 \\ C_{10} & 0 & 0 & C_2 & 0 & C_5 & 0 & 0 & C_1 & 0 \\ 0 & 0 & 0 & 0 & D_2 & D_3 & 0 & 0 & 0 & D_1 \end{bmatrix}$$

Where

$$E_j = \begin{bmatrix} \frac{-\omega_j}{2} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & \frac{-\omega_j}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & \frac{-\omega_j}{2} & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & \frac{-\omega_j}{2} & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & \frac{-\omega_j}{2} & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & \frac{-\omega_j}{2} \\ A_6 & A_8 & 0 & A_{12} & 0 & A_3 & A_1 & A_{13} & A_9 & 0 \\ B_8 & B_{10} & B_4 & 0 & 0 & B_5 & 0 & B_1 & 0 & 0 \\ C_8 & C_{10} & 0 & C_4 & 0 & C_5 & 0 & 0 & C_1 & 0 \\ 0 & 0 & 0 & 0 & D_6 & D_3 & 0 & 0 & 0 & D_1 \end{bmatrix},$$

$$F_j = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{-\omega_j}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-\omega_j}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-\omega_j}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-\omega_j}{2} \\ 0 & 0 & 0 & 0 & 0 & A_4 & A_2 & A_{14} & A_{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & B_6 & 0 & B_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_6 & 0 & 0 & C_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & D_4 & 0 & 0 & 0 & D_2 \end{bmatrix}, G_j = \begin{bmatrix} \frac{-\omega_j}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & \frac{-\omega_j}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ A_5 & A_7 & 0 & A_{11} & 0 & 0 & 0 & 0 & 0 & 0 \\ B_7 & B_9 & B_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ C_7 & C_9 & 0 & C_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & D_5 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Now exert LU factorization to tridiagonal System to obtain the unknown vector δ .

4. Results and Discussions

The forthcoming paragraphs are assigned to illustrate impression diverse variables on liquid flow by exerting graphical depiction. The Figure 2 is explicates the impression of $M(=0.0,0.5,1.0,2.0)$ on velocity $f'(\eta)$ .Intensification of M inducts Lorentz force which instigates resistance for the flow. Consequently velocity $f'(\eta)$ decelerates.

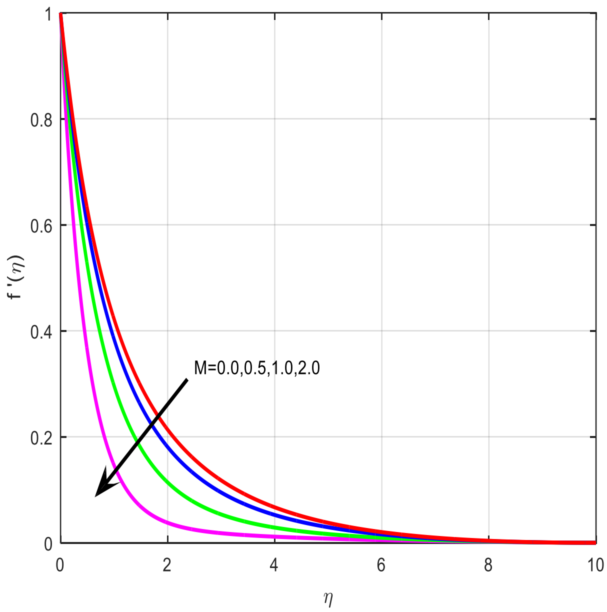

Fig. 2. Variations in $f'(\eta)$ caused by M

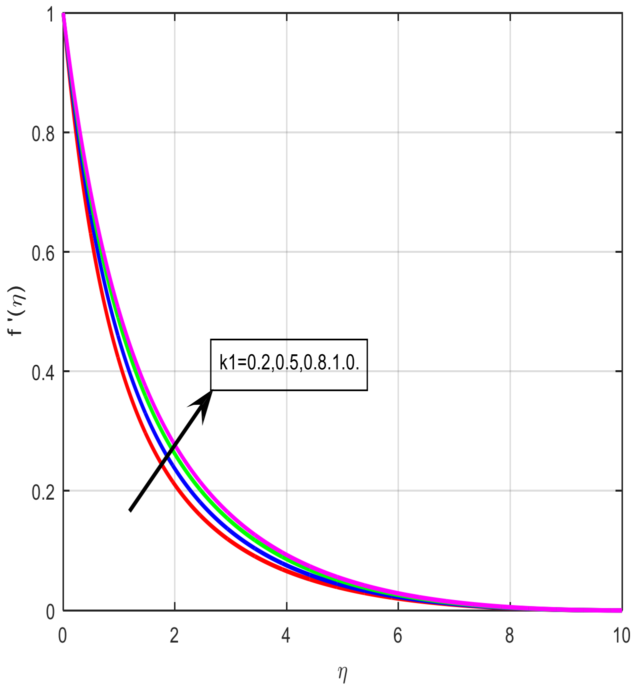

Fig. 3. Variations in $f'(\eta)$ caused by k_1

Figure 3 is made to demonstrate the distinction of $k_1(0.2, 0.5, 0.8, 1.0)$ on velocity $f'(\eta)$. Liquid velocity magnifies for intensified values of k_1. Actually for enhancement corresponding to k_1, micro concentration of liquid improves and velocity $f'(\eta)$ expands. Figure 4 evokes the decrement of $f'(\eta)$ corresponding to variations in curvature parameter $\delta\left(=1.0, 2.0, 3.0, 4.0\right)$. With augmentation in curvature parameter δ, semi diameter of the sheet reduces which effects the particles associated with the sheet and also stretching velocity. Hence, decrement in motion. Radius of curvature is very prominent in physics as it has a vital role in the designing of mirrors and spherical lenses. Also, it is vital in mathematics due to its utilizations in differential geometry. For instance; Cesaro equation and 3 part equation in bending of beams. While designing tracks for trains, engineers have to ensure that track curvature which provides safe and secured ride for specific velocities of trains. In the design of highways, especially at sharp turning points curvature is vital. Selection of curvature has to be large to provide sustainable control to drivers on highways. Impression of mixed convection parameter $\lambda\left(=0.0, 0.5, 1.0, 2.0\right)$ on $f'(\eta)$ is plotted in Figure 5. Intensification in λ slowdowns the viscous force and hence amplification occurs in velocity.

Figure 6 delineates the disparities in Micro-rotation velocity $g(\eta)$ for several values of magnetic parameter $M(=0.0, 0.5, 1.0, 2.0)$.The Micro-rotation velocity exhibits upsurge nature only within reach of the stretchable sheet, intersecting profile nature distant from the sheet and exhibits deterioration as addressed in Figure 6. The deterioration in $g(\eta)$ distant from the sheet prevails since M is associated to Lorentz force which instigates resistance to liquid particles. Consequently, $g(\eta)$ declines. Figure 7 plots more values of $k_1\left(=0.2, 0.5, 0.8, 1.0\right)$ on Micro-rotation velocity $g(\eta)$. It is spotted that $g(\eta)$ enriches for inflated values of k_1. Physically, improvement in material parameter k_1 give rise to vortex viscosity which fabricates rotation in liquid particles. Thus, Micro-rotation velocity $g(\eta)$ magnifies. Figure 8 is assigned to perceive the impression of Micro-rotation parameter$\left(m_1 = 0.5, 1.0, 1.5, 2.0\right)$. It is anticipated that for amplified values of m_1 the micro-rotation velocity magnifies. Influence of $\delta\left(=1.0, 2.0, 3.0, 4.0\right)$ on $g(\eta)$ is delineated in Figure 9. Micro-rotation velocity escalates for strengthened values of δ. It is due to semi diameter of the surface escalates and amplifies $g(\eta)$.

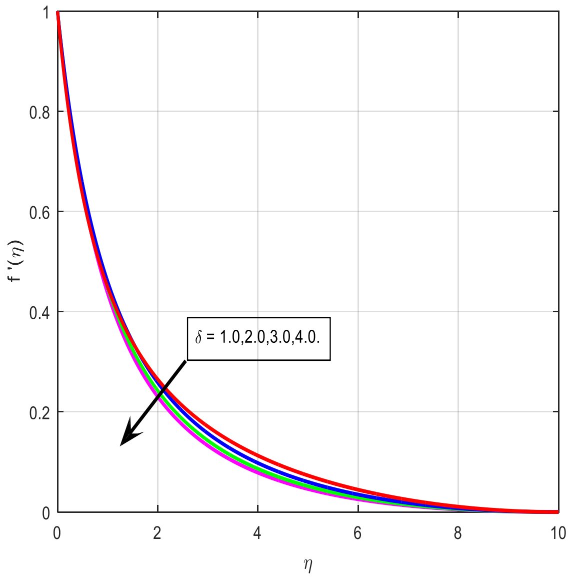

Fig. 4. Variations in $f'(\eta)$ caused by δ

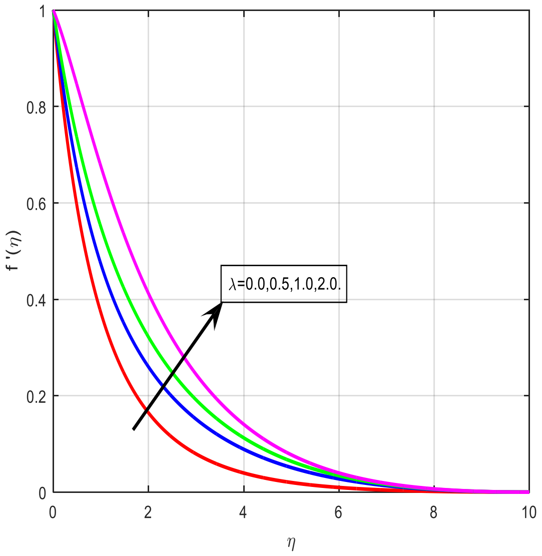

Fig. 5. Variations in $f'(\eta)$ caused by λ

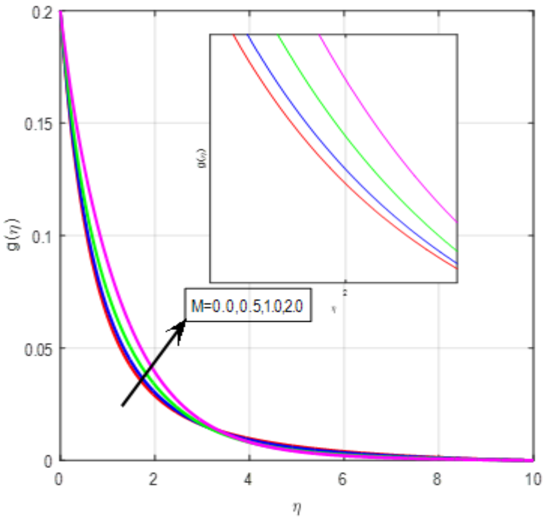

Fig. 6. Variations in $g(\eta)$ caused by M

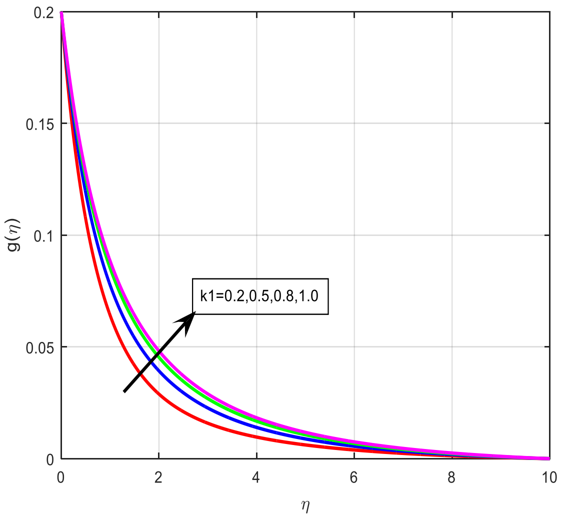

Fig. 7. Variations in $g(\eta)$ caused by k_1

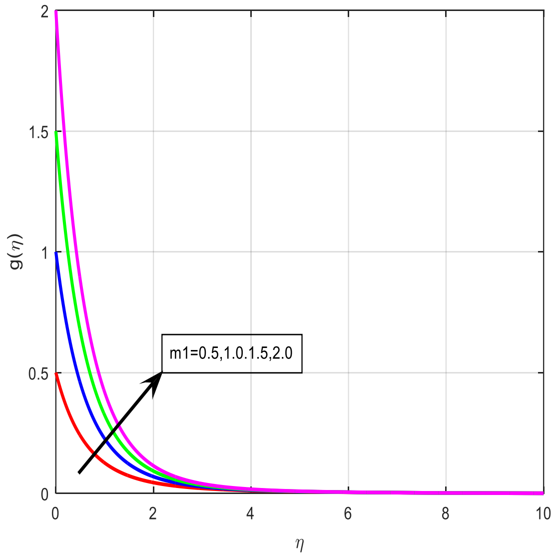

Fig. 8. Variations in $g(\eta)$ caused by m_1

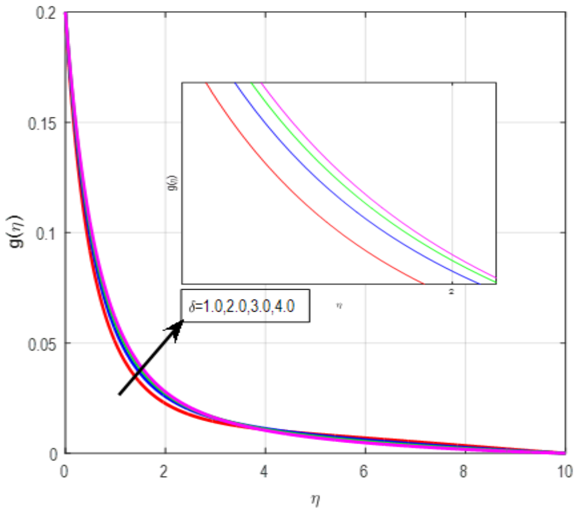

Fig. 9. Variations in $g(\eta)$ caused by δ

Figure 10 represents Outcomes of magnetic parameter $M(=0.0,0.5,1.0,2.0)$ on temperature $\theta(\eta)$. Parameter M relies on Lorentz force which imparts additional heat in the liquid and with that amplification occurs in temperature $\theta(\eta)$. The variations of $k_1\left(=0.2, 0.5, 0.8, 1.0\right)$ on temperature $\theta(\eta)$ are displayed in Figure 11. Here we perceived that temperature $\theta(\eta)$ exhibits deterioration for material parameter k_1. Actually, by strengthening material parameter k_1 a drop occurs in viscosity which also depreciates the productivity of resistance among the particles. This depreciation in resistance decelerates heat productivity also and with that temperature decelerates. Identical response is spotted for curvature parameter $\delta\left(=1.0, 2.0, 3.0, 4.0\right)$ on temperature which is imparted in Figure 12.

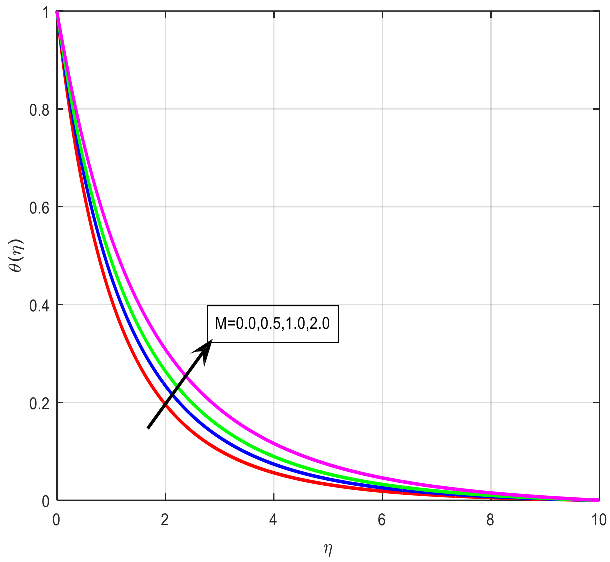

Fig. 10. Variations in $\theta(\eta)$ caused by M

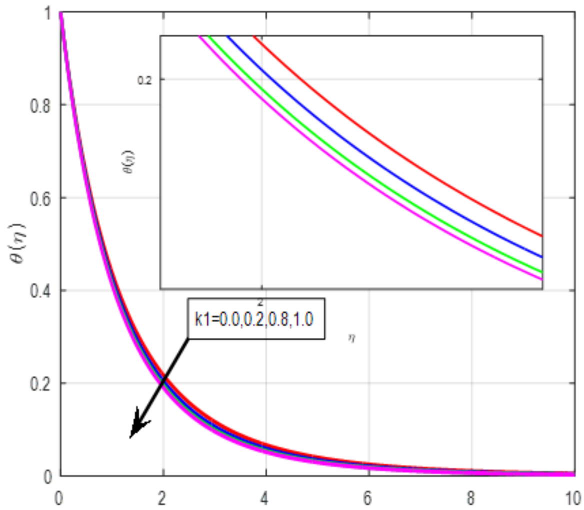

Fig. 11. Variations in $\theta(\eta)$ caused by k_1

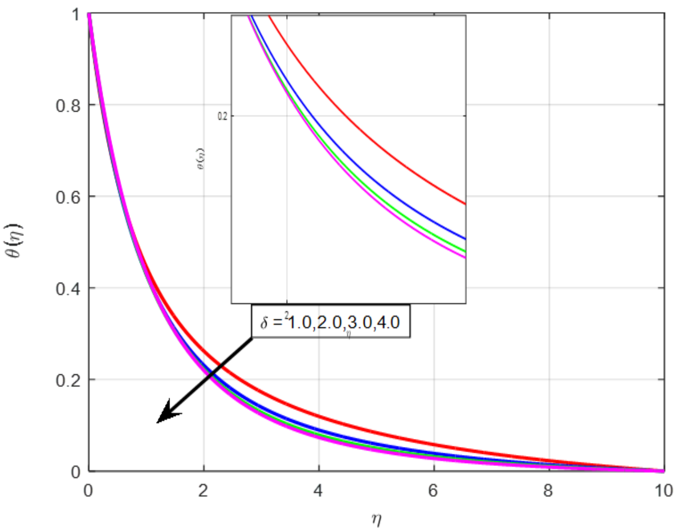

Fig. 12. Variations in $\theta(\eta)$ caused by δ

Figure 13 displays the influence of $\lambda(=0.0,0.5,1.0,2.0)$ on temperature $\theta(\eta)$. A decrement is discerned in the behaviour of $\theta(\eta)$. Figure 14 is designated to address the functioning of $Ec(=0.0,0.3,0.6,1.0)$ temperature $\theta(\eta)$ .by augmenting Ec a drop occurs in viscous force with that temperature decelerates. Basically, Eckert number decelerates the stress involved in viscous liquids by transmuting kinetic energy as internal heat.The enhancement in the Eckert number upsurges the internal heat, which boosts the thermal profile.

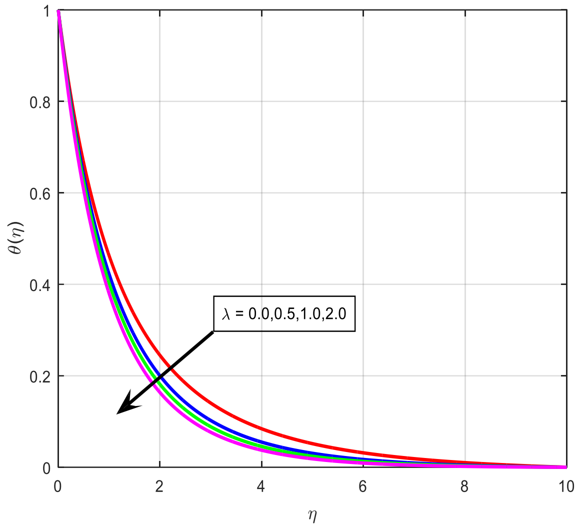

Fig. 13. Variations in $\theta(\eta)$ caused by λ

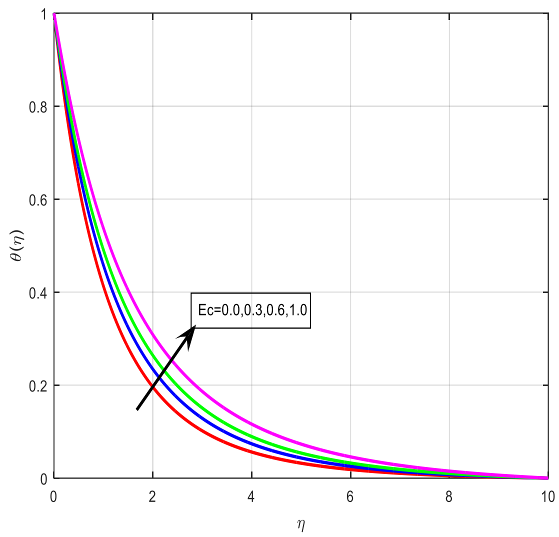

Fig. 14. Variations in $\theta(\eta)$ caused by Ec

Similar functioning is spotted for radiation parameter $Rd(=0.0,0.5,1.0,2.0)$ on temperature $\theta(\eta)$ which is manifested in Figure 15. Augmentation Rd means reduction in absorption coefficient; so, temperature amplifies. Liberation of thermal radiation as electromagnetic waves is due to environmental temperature. This thermal effect magnifies the conduction phenomena of the liquid. Consequently, an increment in thermal distribution is perceived. Figure 16 specifies that temperature $\theta(\eta)$ decelerates for variations in Prandtl number $\Pr\left(=1.0,2.0,3.0,4.0\right)$.This fall in temperature takes place due to the drop in thermal conduction and thermal boundary layer. The possible selection of a tiny Prandtl number implicates that the liquid has an extreme thermal conductivity in relevant to its momentum diffusivity, which enables heat transfer efficiently. This activity is vital to refine heat dissipation attributes, which are much prominent in applications. For instance, engine cooling systems, where rapid and effectual heat evacuation is recommended for extreme performance and longevity.

Figure 17 is dedicated to impart the effect of magnetic parameter $M(=0.0,0.5,1.0,2.0)$ on concentration $\phi(\eta)$. From figure it is revealed that $\phi(\eta)$ upsurges for diverse values of M.

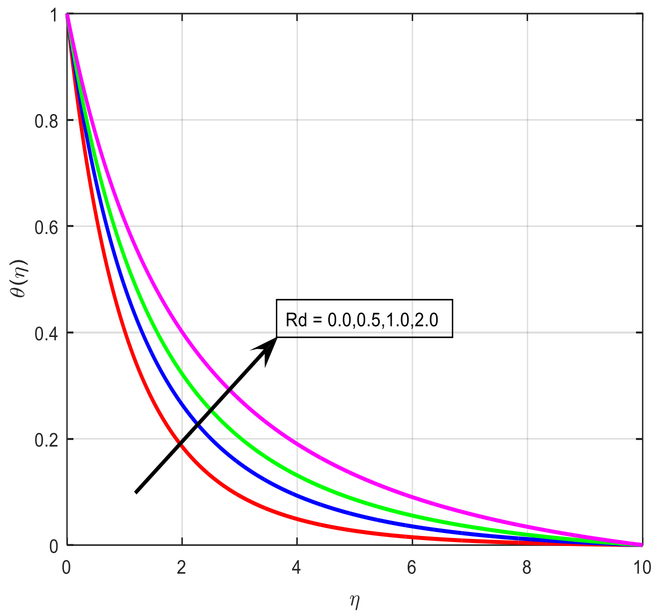

Fig. 15. Variations in $\theta(\eta)$ caused by Rd

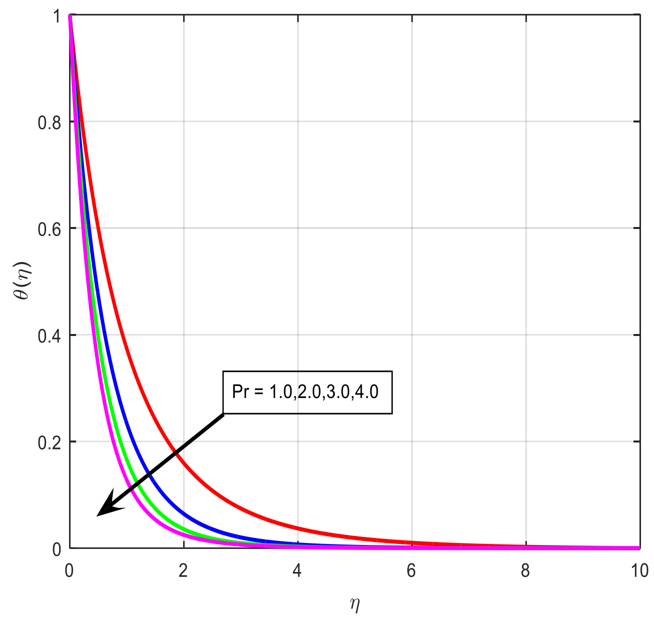

Fig. 16. Variations in $\theta(\eta)$ caused by $\Pr$

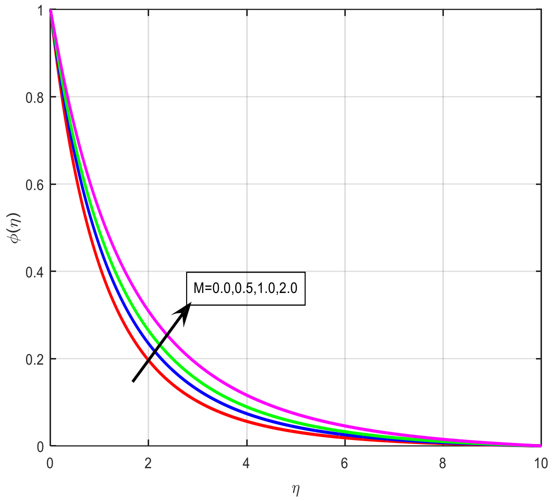

Fig. 17. Variations in $\phi(\eta)$ caused by M

Influence of $\delta\left(=1.0, 2.0, 3.0, 4.0\right)$ on concentration $\phi(\eta)$ is discussed in Figure 18. A slight decrement is detected in concentration profiles. Impression of Sc on concentration is addressed in Figure 19.

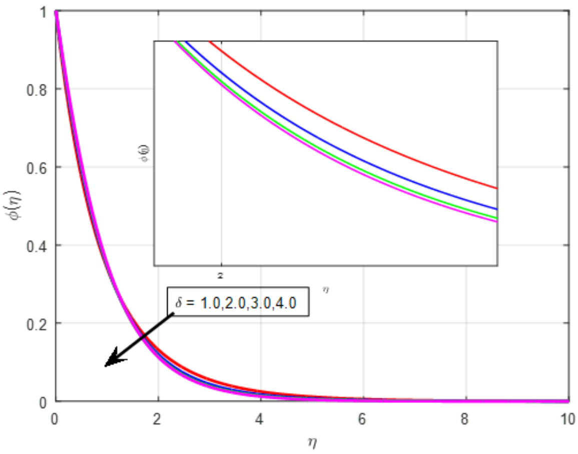

Fig. 18. Variations in $\phi(\eta)$ caused by δ

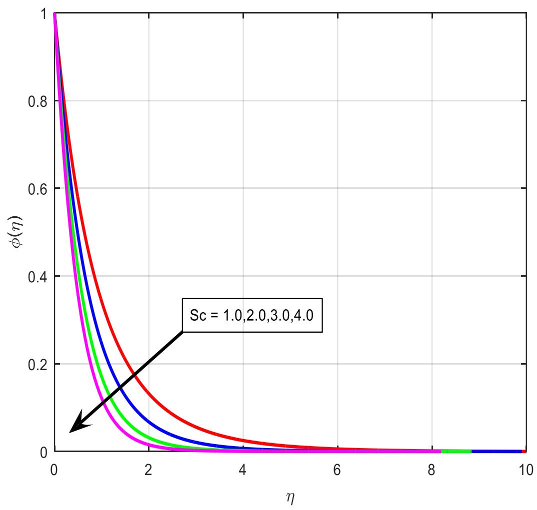

Fig. 19. Variations in $\phi(\eta)$ caused by Sc

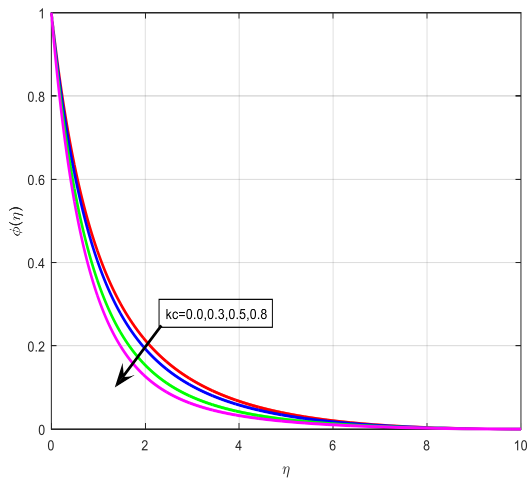

Fig. 20. Variations in $\phi(\eta)$ caused by kc

Up surged values of $Sc\left(=1.0, 2.0, 3.0, 4.0\right)$ declines the mass transfer. High Sc produces high viscous diffusion and hence improvement in molecular motion. Hence, the concentration $\phi(\eta)$ decreases. Schmidt number has more prominence in thermal and chemical engineering. It is a primary widget in evolution of developed gas turbine combustors. Prediction of adequate temperature at the exit and combustor wall has a vital role for gas turbines since temperature may decelerate the longevity of combustor and the connected turbine. Figure 20 explicates the aspects of chemical reaction parameter $kc\left(=0.0, 0.3, 0.5, 0.8\right)$ on concentration $\phi(\eta)$. It is portrayed that mass transmission declines for applied chemical reaction parameter kc . Physically, for robust chemical reaction, a decrement takes place in reactant species. Thus, concentration decelerates for enhanced chemical reaction parameter kc .

5. Validation

Table 1 is outlined to signify the comparative upshots of the proposed effort with Okechi *et al.,* [19] .In this table, we manifested the upshots of Okechi *et al.,* [19] with the present effort by exerting Keller Box method (KBM). An impressive standard in compatibility is manifested with this resemblance.

Table 1
Comparative study of Skin Friction Coefficient with the existing reports distinct values of δ

Skin friction coefficient		
δ	Okechi *et al.,* [28]	Present analysis(KBM)
5	1.4196	1.419131
10	1.3467	1.346583
20	1.3135	1.313414
30	1.3028	1.302746
40	1.2975	1.297479
50	1.2944	1.294339
100	1.2881	1.288102
200	1.2850	1.285005
1000	1.2826	1.282537
∞	1.2818	1.281922

6. Conclusions

A mathematical model is established to explore the nature of Micropolar fluid via stretching of exponential curved surface. The Keller Box solution is exhibited for the initiated problem. Impressions of flow parameters like MHD parameter, curvature parameter and reaction parameter on liquid profiles have been probed and exhibited as Figures. The significant observationsof proposed effort are delineated below:

i. The superior curvature factor decelerates the radii of curved surface and deteriorates velocity, temperature and concentration curves.
ii. Extensive Magnetic factor decelerates the liquid's velocity contour while enhancing its energy and concentration contours whereas opposite trend for Buoyancy parameter.
iii. Refining in material parameter accelerates vortex viscosity which initiates rotation in liquid particles. Thus, amplification in Micro-rotation velocity.
iv. It is anticipated that for amplified values of Micro-rotation parameter the micro-rotation velocity magnifies.
v. By refining the radiation and dissipation factors, amplification occurs in thermal profile.
vi. Grater reaction factor declines mass transmission consequently decay in concentration curves.
vii. In additional, the proposed work can be extended by incorporating porous media, heat source and soret and dufour effects along with diverse boundary constraints.

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