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Mixed Convection in a Lid-Driven Cavity in the Presence of Magnetic Field with Sinusoidal Heating

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ABSTRACT

Understanding mixed convection in engineering applications such as heat exchangers, electronics cooling devices, and solar energy collectors have urged researchers to investigate this phenomenon deeper. This study investigates the fluid flow and heat transfer pattern in a two-dimensional (2D) rectangular cavity with sinusoidal heating on the moving top lid numerically. The bottom wall is kept cool while the vertical walls are insulated. The effect of Hartmann number, Ha on the thermal characteristics and fluid flow are analyzed for Richardson number, $Ri = 1$ which indicate mixed convection dominated regime. The governing equations are solved numerically using a SIMPLE algorithm with the finite volume method. The numerical results are displayed in streamlines and isotherms plots. The value of the Nusselt number indicating the heat transfer rate is also discussed. It is found that Ha has a significant effect on the heat transfer process and fluid flow. It can be seen clearly when the value of $Ha = 30$, the rate of heat transfer dropped significantly on the cold wall. Generally, the heat transfer rate decreases with the increase of Ha.

1. Introduction

The research on finding the optimum methods to improve the heat transfer and fluid flow process that reduce the energy lost in a thermal energy system is still ongoing. Researchers and engineers are still trying to evaluate the performance and efficiency of such systems to reduce the cost and increase their effectiveness in such systems. Solar system [1, 2], thermal exchangers, refrigerator, storage technologies [3] and coolants in a nuclear reactor are some examples of mixed convection applications. Mixed convection studies have been done through review papers [4, 5] or deep investigation [6, 7] using various approach. Researchers [8, 9] investigated mixed convection studies through numerical approach and [10] through experiment while [11, 12] through both approaches.

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One of the earlier studies on mixed convection in a driven cavity was done by Iwatsu *et al.,* [13]. It was found that the fluid at the centre and bottom part of the cavity remains stagnant for a higher Richardson number. Then, Khanafer and Chamka [14] investigated the problem of mixed convection in a lid-driven cavity filled with fluid and porous medium. They found that Darcy and Richardson number plays a significant role in convective flow. The importance of using an insulated block inside a cavity to improve the heat transfer process was studied by Biswas *et al.,* [15]. In contrast, Widodo *et al.,* [16] studied the effects of magnetohydrodynamic in mixed convection nanofluid flow at a lower stagnation point on a sliced sphere. Next, mixed convection flow that past a circular cylinder that is placed horizontally was studied by Sharidan *et al.,* [17]. Earlier, Eiyad and Chamka [18] had studied mixed convection flow in a lid-driven cavity with a wavy wall. They found that the average Nusselt number was enhanced or reduced depending on the waviness of the cavity and the Richardson number. Recently, Batool *et al.,* [19] discussed on a numerical investigation of heat and mass transfer in micropolar nanofluids flowing through a lid-driven cavity. The findings of the study indicate that the average Nusselt number and Sherwood number decrease with increasing Hartmann number, aspect ratio, and nanoparticle volume fraction.

The effect of a magnetic field in a lid-driven square cavity filled with a porous medium was investigated by Nayak *et al.,* [20]. MHD mixed convection in a lid-driven rectangular cavity containing a heat-conducting circular block has been examined by Rahman *et al.,* [21] using the FVM method. They found that the Reynolds number and the Prandtl number would change the streamline and the isotherm. In the presence of a magnetic field, the laminar mixed convection in a lid-driven square cavity heated by a corner heater has also been explored by Oztop *et al.,* [22]. Alnaqi *et al.,* [23] investigated the effect of a magnetic field on the convective heat transfer and entropy generation in an inclined square cavity with a heat-conducting fin and thermal radiation. Meanwhile, Maneengam *et al.,* [24] investigated the entropy generation and buoyancy and Lorents forces in a cavity that is filled with porous medium and different shape of obstacles. It is found that the shape of obstacles plays a significant impact to the heat and mass transfer rate. MHD flows along a tiny needle and over a stretching sheet were also observed [25, 26].

Malleswaran *et al.,* [27] and Oztop *et al.,* [28] examined the flow and heat transfer rate impacts of different heating locations in the presence of a magnetic field in a lid-driven cavity. They found that the position of the heat source and the addition of a magnetic field significantly affect the heat transfer rate and fluid flow characteristics. MHD mixed convection in a lid-driven cavity filled with a nanofluid was investigated by Elshehabey and Ahmed [29]. Using Buongiorno's nanofluid model, they investigated a sinusoidal temperature distribution on both vertical walls. Nawafleh *et al.,* [30] examined heat transfer by mixed convection in a lid-driven hollow filled with nanofluids. They considered fluctuating temperature boundary conditions along the sidewalls, as well as sinusoidal temperature distribution. The numerical results revealed that decreasing the Richardson number increases the average Nusselt number. Furthermore, the presence of nanoparticles in pure water improves the transfer of heat.

In summary, the studies on mixed convection in a lid-driven cavity with sinusoidal heating provided insight into a variety of topics, including varied cavity geometries, nanofluid-filled cavities, porous cavities, and the impacts of magnetic fields. These researches findings provide important insights into the heat transfer and fluid flow behaviour in such systems as per the reviewed studies and considering the previously mentioned industrial uses.

Aside from the earlier investigations, no research has been conducted on a lid-driven, water-filled cavity with a magnetic field and sinusoidal heating on top of the cavity wall. Consequently, this work investigates the influence of magnetic field on mixed convection heat transfer in a lid-driven cavity heated sinusoidally on the top lid. Furthermore, it becomes essential to investigate the magnetic field's impact to gain a comprehensive grasp of the fluid flow pattern. Here, the numerical results for streamlines, isotherms, and the average Nusselt number are graphically illustrated. This research has numerous applications, including fluid heating and cooling in concentrated solar collectors, electronic and electrical machines, food industries, and many others.

2. Methodology

2.1 Mathematical Modelling

The schematic view of the problem under consideration is shown in Figure 1. The horizontal lid-driven cavity is filled with water. The length is denoted as L and the height as H. The aspect ratio is defined as $AR = {}^{L}/_{H} = 2$. The upper wall is moving in a positive way constantly with velocity $u = U_0$. It is heated sinusoidally $T = T_c + (T_h - T_c)\sin\pi\left({}^{x}/_{H}\right)$ while the bottom wall is kept at a constant cold temperature, T_c where $T_h > T_c$. The vertical walls are perfectly insulated. The fluid and flow are considered non-compressible, Newtonian, laminar, steady, and 2-dimensional (2D). Density variation is approximately by the Boussinesq equation. Magnetic field strength B is imposed in the $x-$ direction onto the fluid. Other magnetic fields that present due to the motion of electrically conducting fluid are neglected to reduce the calculation complexity.

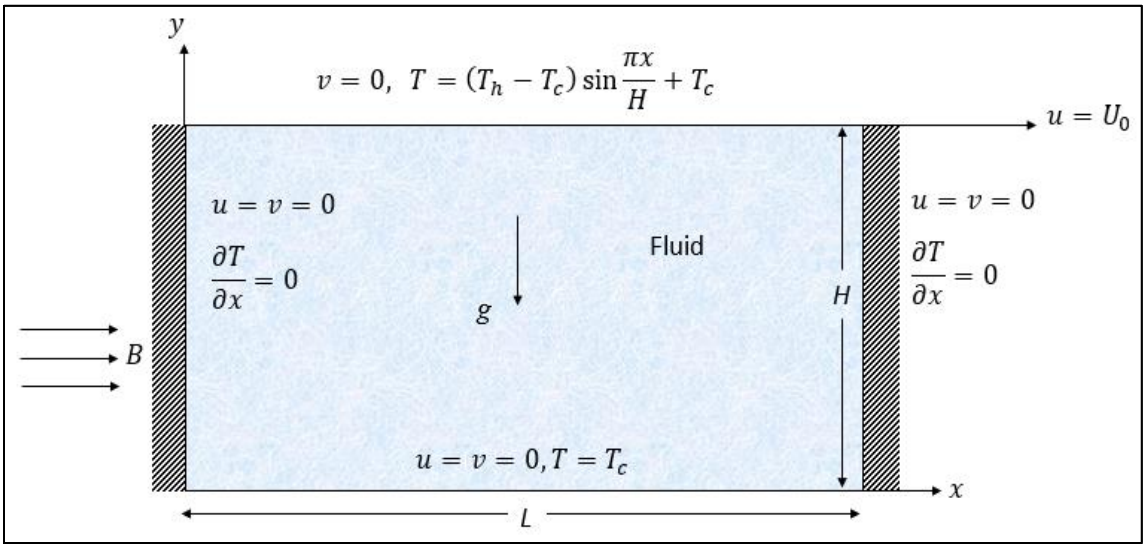


Fig. 1. Physical model of convection in a rectangular cavity together with the coordinate system

Based on the assumptions, the mathematical modelling of the problem can be written in the dimensional form as below [31]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + g\beta(T - T_c) - \frac{\sigma B^2}{\rho}v, \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right), \tag{4}$$

Where u and v is the velocity that acts in the x and $y-$ direction of the Cartesian coordinate, respectively. The parameters p, is fluid pressure, ρ is density and ν is the kinematic viscosity, respectively. The notations β is fluid thermal expansion coefficient, g is gravity, T is temperature, and σ is the electrical conductivity, respectively. The magnetic induction coefficient is denoted as B. The thermal diffusivity, $\alpha = {}^{k}/_{\rho c}$ where k is the thermal conductivity while c is the heat capacity. The boundary conditions are:

$$\begin{aligned}&\text{top wall: } u = U_0, v = 0, T = (T_h - T_c)\sin\frac{\pi x}{H} + T_c,\\ &\text{vertical walls: } u = v = 0, {}^{\partial T}/_{\partial x} = 0,\\ &\text{bottom wall: } u = v = 0, T = T_c.\end{aligned} \tag{5}$$

Then, the dimensionless variables are introduced [32]:

$$\begin{aligned}&X = \frac{x}{H}, \quad Y = \frac{y}{H}, \quad U = \frac{u}{U_0}, \quad V = \frac{v}{U_0}, \quad \theta = \frac{T - T_c}{T_h - T_c}, \quad Gr = \frac{g\beta(T_h - T_c)H^3}{\nu^2}\\ &Pr = \frac{\nu}{\alpha}, \quad P = \frac{p}{\rho U_0^2}, \quad Re = \frac{U_0 H}{\nu}, \quad Ha^2 = \frac{B^2 H^2 \sigma}{\rho\nu},\end{aligned} \tag{6}$$

where θ is temperature, P is pressure, Gr is Grashof number, Re is Reynold number, Pr is Prandtl number and Ha^2 is Hartmann number, respectively. The governing equations Eq. (1) – (5) with boundary conditions in Eq. (6) are transformed into a dimensionless form using the Eq. (6). The resulting governing equations in the dimensionless form are:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{7}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right), \tag{8}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + \frac{Gr}{Re^2}\theta - \frac{Ha^2}{Re}V, \tag{9}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{RePr}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right), \tag{10}$$

with dimensionless boundary conditions as follows:

$$\begin{aligned}&\text{top wall: } U = U_0, V = 0, \theta = \sin\pi X,\\ &\text{vertical walls: } U = V = 0, {}^{\partial\theta}/_{\partial X} = 0,\\ &\text{bottom wall: } U = V = 0, \theta = 0.\end{aligned} \tag{11}$$

2.2 Numerical Approach

The dimensionless Eq. (7) – (11) are solved using the finite volume method (FVM) [26]. The resulting algebraic equations are solved iteratively by the semi-implicit method for pressure linked equation (SIMPLE) algorithm and tridiagonal matrix algorithm (TDMA). The iterations for calculating U, V, and θ continues until the convergence criteria in Eq. (12) is fulfilled. This study uses a staggered grid system to store the velocity components halfway between the scalar storage locations. A non-uniform mesh system captures the rapid changes of all variables involved, especially near the walls.

$$\varepsilon = \frac{\sum_{i,j}\left|\zeta_{i,j}^{k+1} - \zeta_{i,j}^{k}\right|}{\sum_{i,j}\left|\zeta_{i,j}^{k+1}\right|} \leq 10^{-7}, \tag{12}$$

where ε is the tolerance, i and j are the grid points in x and $y-$ direction, respectively while ζ is any of the computed variables and k is the iteration number. To investigate the heat transfer rate, the Nusselt number is calculated in the following manner:

$$Nu = \frac{1}{L/_H}\int_0^H -\left(\frac{\partial\theta}{\partial Y}\right)_{Y=0,1} dX. \tag{13}$$

2.3 Grid Independence Test

A numerical experiment was performed on different grid resolutions to check the independency of the solution. The procedure is adapted into the FORTRAN90 programming language. Several grid size distributions ranging from 50 to 120 grid sizes for the horizontal axis were run on Asus, AMD Ryzen 5 CPU @ 2.10 GHz. The size of the vertical axis is half the size of the horizontal axis. The Nusselt number along the top wall were compared as in Figure 2. The calculation was done for $Ri = 1.0$, $Ha = 30$, $Pr = 7.0$ and fixed Gr value that is 10^4. The grid size of 110×55 was chosen as the grid for this study because it is sufficiently accurate. Next, the validation process was done against the previous research of Iwatsu *et al.,* [13] and Khanafer and Chamka [14] as shown in Table 1. The present study also performed another numerical validation with Al-Salem *et al.,* [33] as in Figure 3. It is found that all the results of comparison on streamlines, isotherms and vertical velocity profiles are in good agreement. However, a slight difference in the analysis outcomes is observed due to the grid resolution and discretization schemes used in numerical simulations, which can affect the accuracy and convergence of the results.

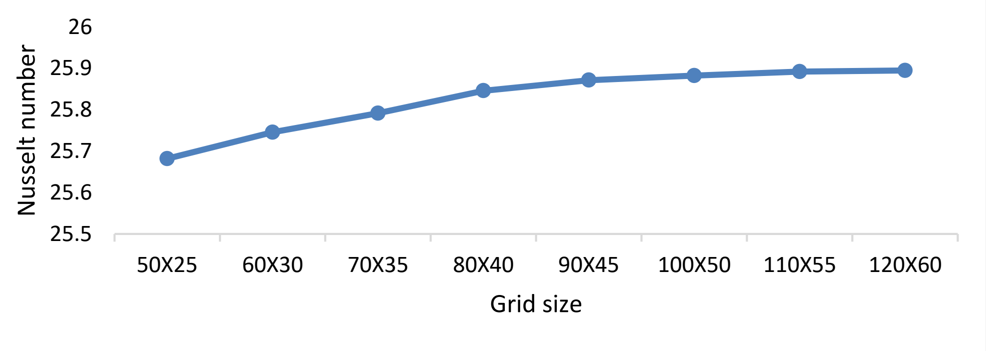


Fig. 2. Average Nusselt number on top lid and the grid size for $Ri = 1.0$ and $Ha = 30$

Table 1
Comparisons of minimum and maximum values of velocities at the mid-sections of the cavity between the present study and previous works

	Velocities	**Present study**	**Iwatsu *et al.,* [13]**	**Khanafer and Chamka [14]**
$\boldsymbol{Re = 100}$	U_{min}	-0.2049	-0.2122	-0.2037
	U_{max}	1.0000	1.000	1.000
	V_{min}	-0.2328	-0.2506	-0.2448
	V_{max}	0.1673	0.1765	0.1699
$\boldsymbol{Re = 400}$	U_{min}	-0.3023	-0.3197	-0.3099
	U_{max}	1.000	1.0000	1.0000
	V_{min}	-0.4219	-0.4459	-0.4363
	V_{max}	0.2802	0.2955	0.2866

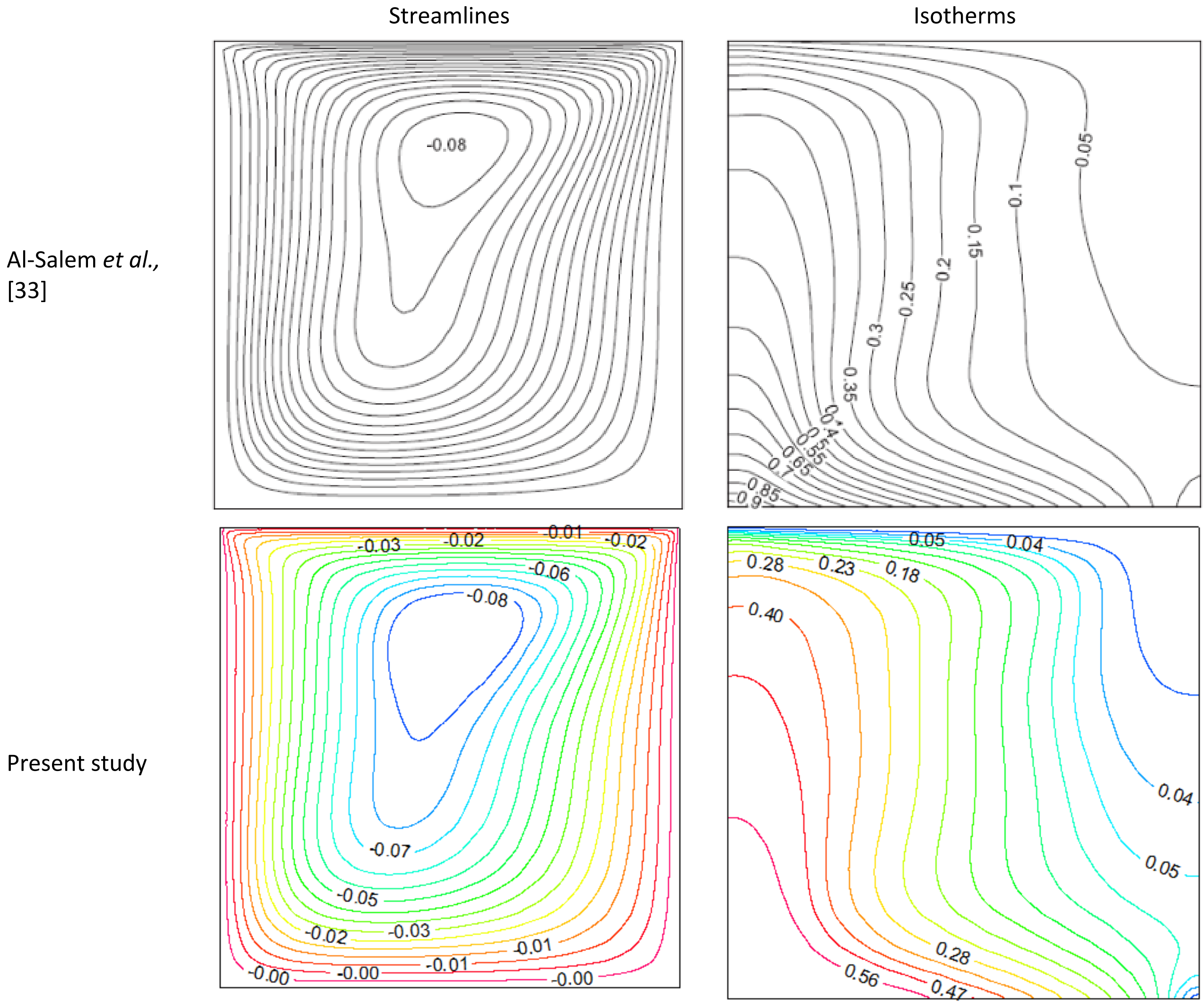


Fig. 3. Comparison of the present study with Al-Salem *et al.,* [33]

3. Results

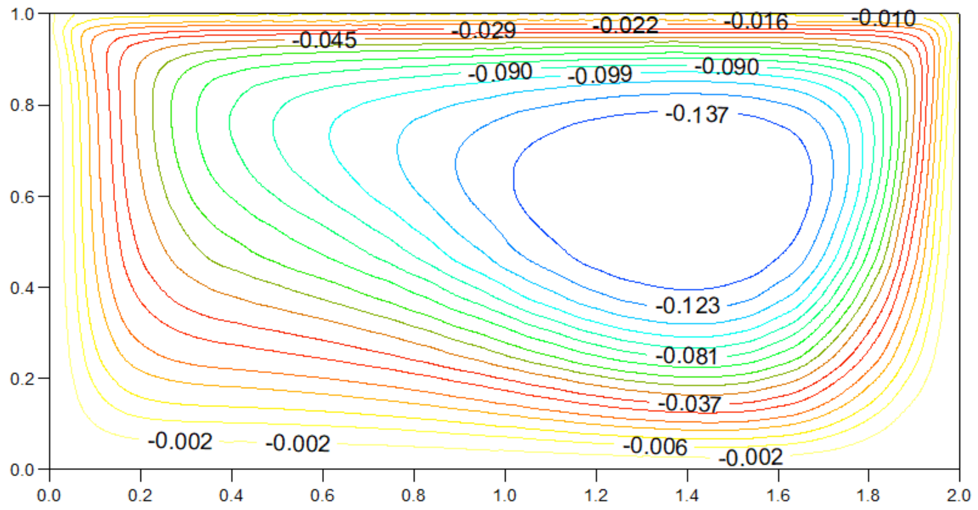
Mixed convection heat transfer and fluid flow for a lid-driven horizontal cavity with the presence of a magnetic field while the top lid was heated sinusoidally. The bottom wall is kept constant at a cold temperature. Both the vertical walls are kept insulated. In the present study, the fluid chosen is water with $Pr = 7.0$. The controlling parameters is *Ha*. The Reynolds and Grashof number are kept constant at 100 and 10 000, respectively. This study focuses on $Ri = 1.0$, which mixed convection as the dominant convection that happened in the cavity. The values for Ha are 0, 10, 30 and 60. The numerical results are presented in the form of streamlines which show the path of a particle moving. And isotherms which show the variation of temperature.

The streamlines for different values of Ha are shown in Figure 4. When $Ha = 0$, a primary clockwise rotating eddy at the right side of the cavity. Rotating means the fluid flows towards the bottom wall and returns to the top wall. This flow indicated the shear effect is dominant due to the movement of the top wall from left to right in constant velocity. Thus, it pushed the fluid down, and the temperature difference of the fluid also existed and pushed back the fluid upward.

As the magnetic field strength values increased to 10, the recirculating eddy tended to move towards the top lid, and the eddy grew smaller. When the magnetic field strength increases to 30, two recirculating eddies exist. The primary eddy was recirculating in a clockwise direction and situated near the top wall, while the secondary eddy flowed in a counter-clockwise direction near the bottom wall. The primary eddy exists due to the movement of the moving lid on top, and the secondary recirculating eddy exists due to the temperature difference between the top and bottom wall. At this stage, the magnetic field effect started to become significant.

As the magnetic field strength increased to $Ha = 60$, the primary eddy became smaller, and the secondary eddy grew bigger. The magnetic field effect has become significant at this stage. The fluid in the cavity flows in two main directions due to the shear effect and temperature difference. Note that the secondary eddy has become more extensive, which indicates that the flow strength has become weaker as the *Ha* increases.

Figure 4 depicts the development of the flow pattern inside the cavity as the magnetic field strength (Ha) increases. It illustrates the effect of the magnetic field on the rotation and magnitude of the recirculating eddies, as well as the varying strengths of the flow components due to shear and temperature differences.



$Ha = 0$

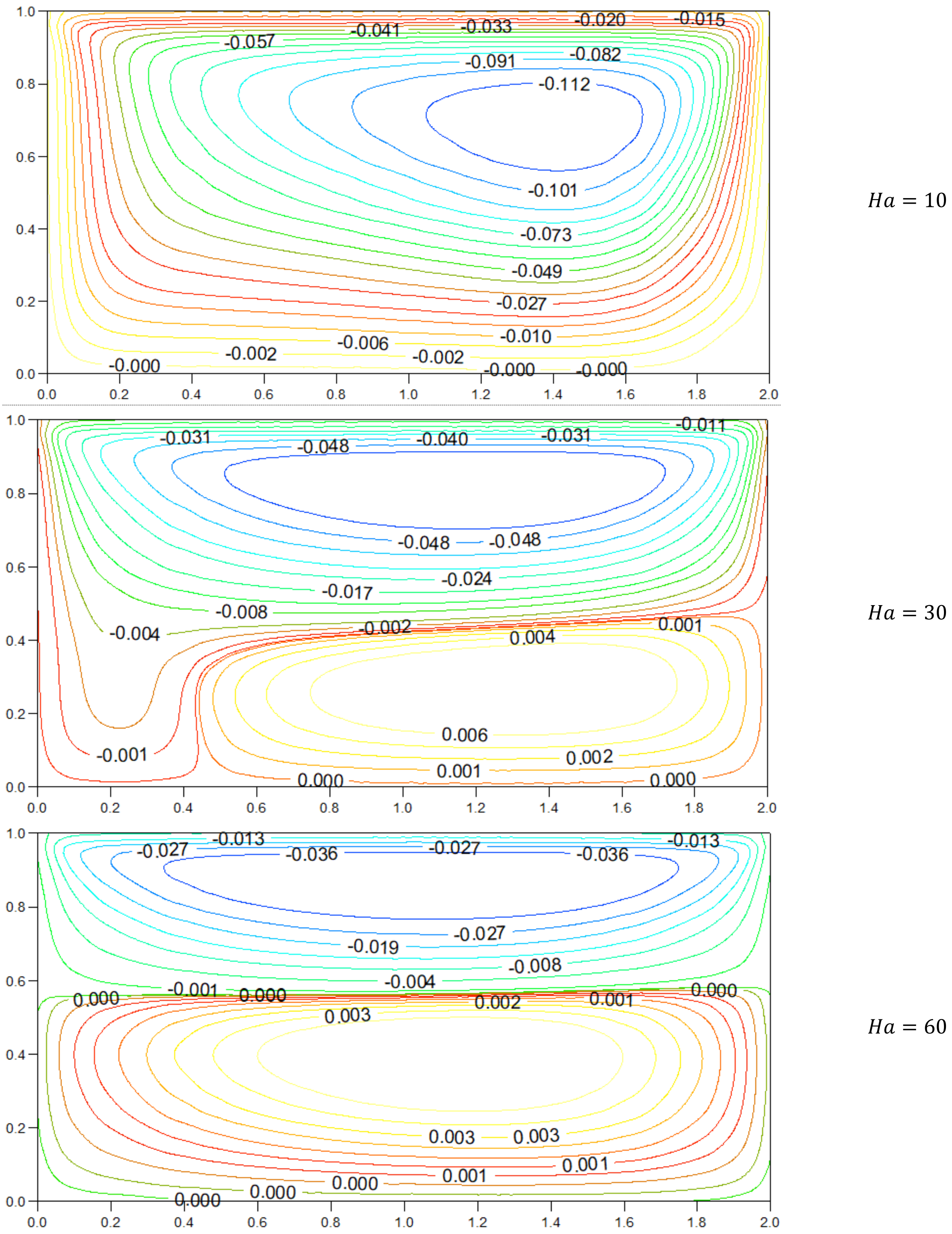


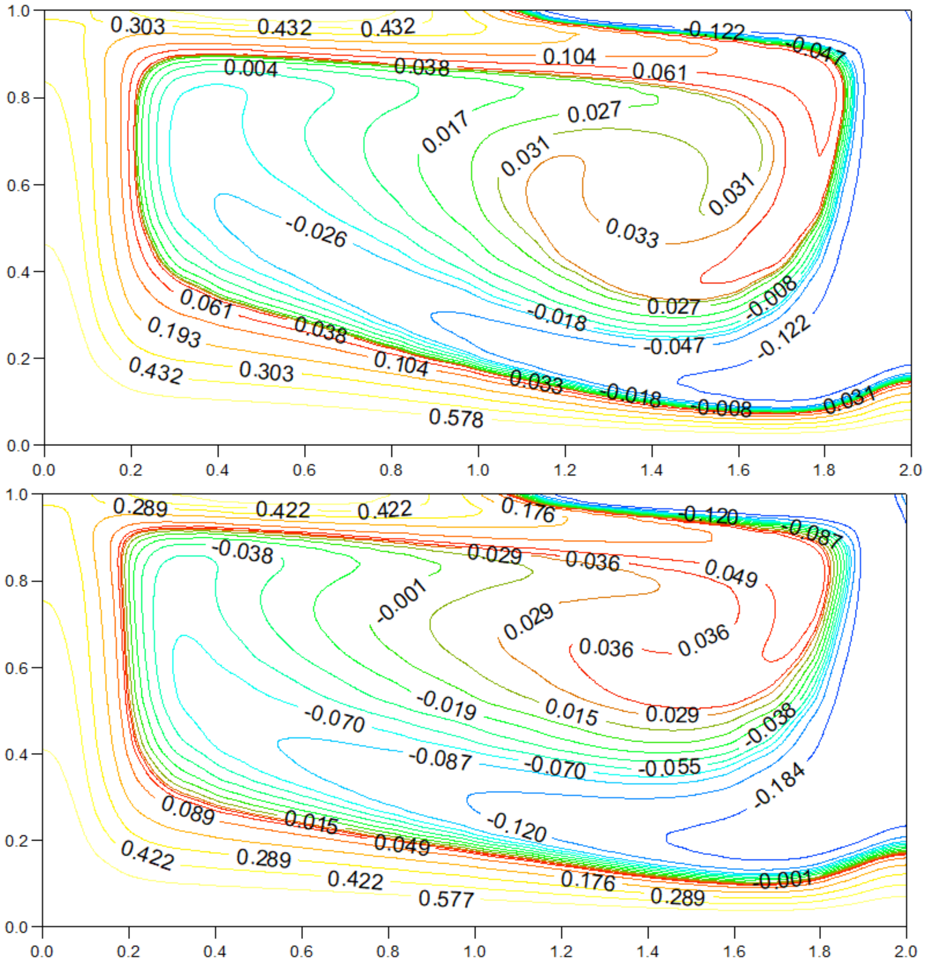
Fig. 4. Variation of streamlines for vary Ha

Figure 5 shows the variation of isotherms with different values of Ha. When no magnetic field strength is imposed on the cavity, the isotherms illustrate a steep temperature gradient near the top wall and along the right side of the bottom wall. This is due to the shear force and the sinusoidal heating of the top lid that nominated the flow. The temperature of the bottom fluid is hotter as the hot fluid is pushed towards the bottom cavity.

As the magnetic field strength increased, the steep temperature gradient gradually decreased. The temperature profile was mixed with hot and cold fluid due to the sinusoidal heating, especially at the centre of the cavity. When $Ha = 30$, the isotherms were stratified horizontally near the right wall and at the centre of the cavity. The temperature profile was divided into two sections, hot and cold fluid. This indicated that the magnetic field significantly affects the temperature profile, where the temperature distribution seems more relaxed. This is due to the reduction of convection. Conduction seems to emerge at this stage, where the heat transfer and fluid flow are slower.

When $Ha = 60$, the isotherms were almost horizontally stratified, especially near the bottom wall. This indicates that conduction happened at the bottom of the cavity. The fluid is hot at the bottom wall and in some regions at the top wall. This is due to sinusoidal heating at the top wall while conduction happens at the bottom of the cavity.

The significant occurrences in Figure 5 illustrate the impact of the magnetic field on the temperature distribution, the transition between convection and conduction, and the stratification of hot and cold fluid regions within the lid-driven cavity.



$Ha = 0$

$Ha = 10$

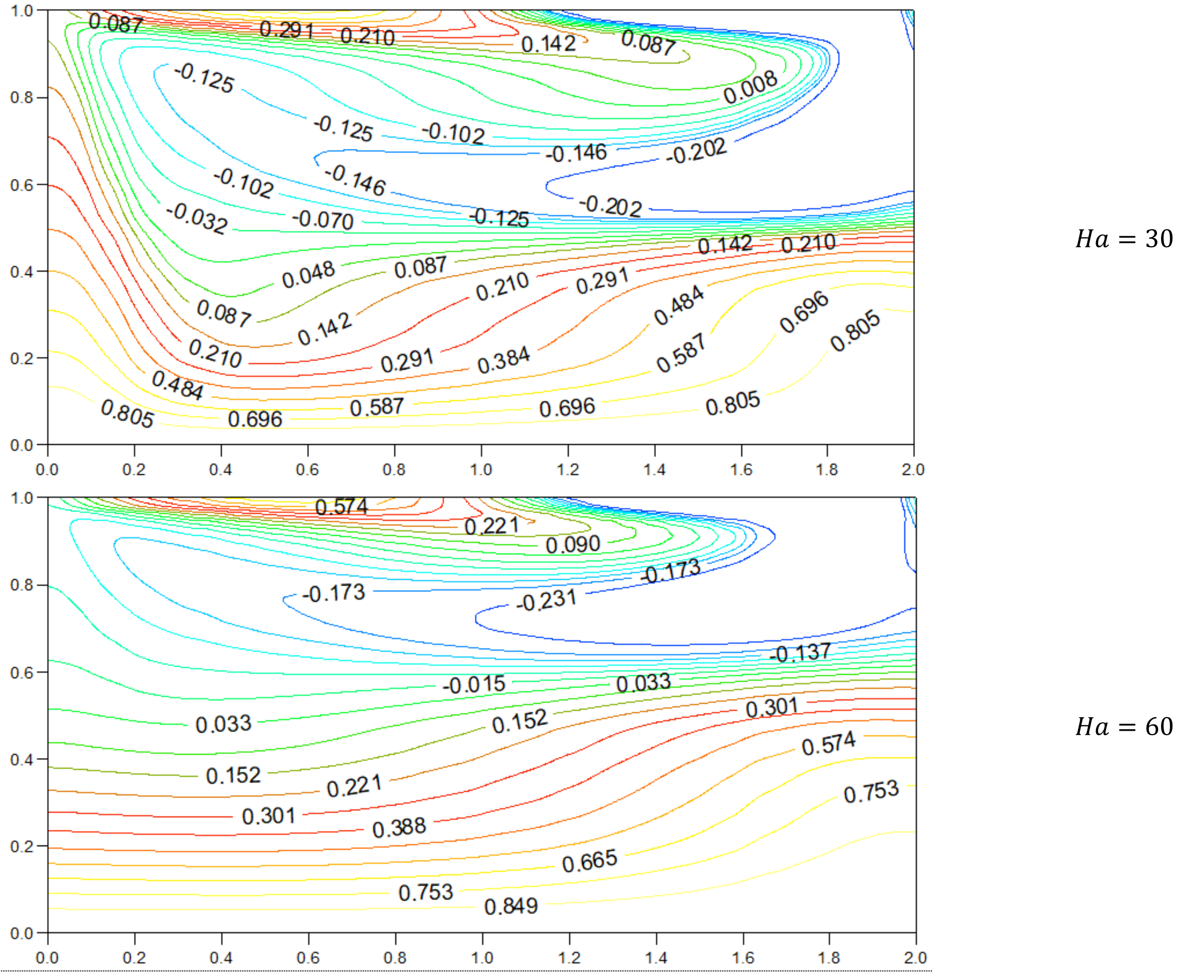


Fig. 5. Variation of isotherms for vary Ha

The variation of the Nusselt number for the top and bottom walls with various Ha can be seen in Figure 6. The Nusselt number for the bottom wall decreased as the Ha value increased. This shows that magnetic field presence has affected the heat transfer process near the bottom wall significantly. However, the opposite pattern of the Nusselt number for the top wall can be seen compared to the bottom wall. As the Ha values increased, the Nusselt number also increased. But, a slight decrement of the Nusselt number value is found when $Ha = 10$. This is due to the sinusoidal heating effect on the heat transfer process. This opposite situation happened because, in the presence of a magnetic field, fluid motion is influenced by the Lorentz force, which is a force exerted on a charged particle in motion within a magnetic field. Higher heat transfer rates can come from the top lid because turbulence flow can break up the thermal barrier layer close to the heated surface and encourage heat transfer from the surface to the bulk fluid.

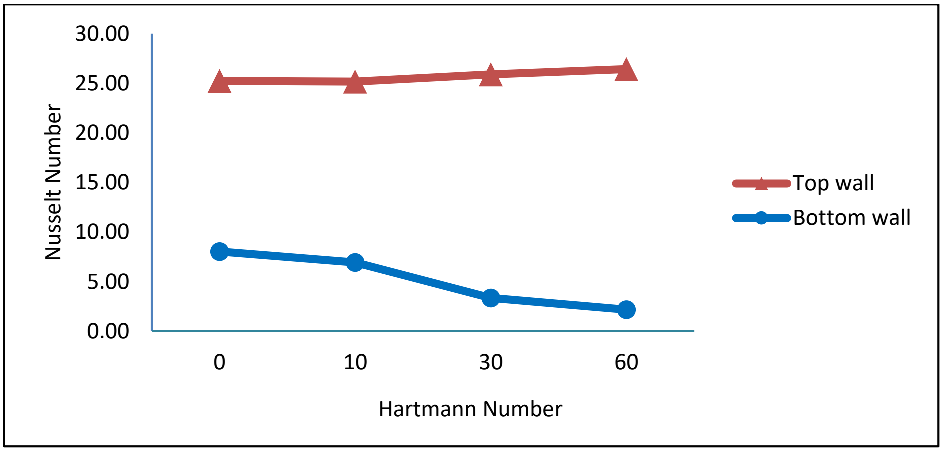


Fig. 6. Variation of Nusselt number for vary Ha

4. Conclusions

The present paper investigated the mixed convection phenomenon in a lid-driven horizontal cavity filled with water subjected to sinusoidal heating on the top wall while moving in the positive direction and influenced by a magnetic field. The governing equations were solved using the finite volume method, implemented in the FORTRAN90 programming language. The numerical results, depicted as streamlines and isotherms, revealed the significant impact of the magnetic field on the fluid flow and heat transfer process within the cavity. Mainly, the study focused on the Nusselt number as an indicator of heat transfer rate. Here are the main findings of the study:

i. The magnetic field significantly influences the fluid flow and heat transfer process in the lid-driven horizontal cavity.
ii. An increase in the Hartmann number (Ha) results in a decrease in the Nusselt number at the bottom wall, indicating suppressed heat transfer near the bottom wall due to the magnetic field effect.
iii. However, the heat transfer rate increases at the top wall due to the presence of forced convection and turbulence flow, which disrupts the thermal barrier and promotes enhanced heat transfer.
iv. The periodic nature of the sinusoidal heating contributed to variations in the temperature distribution and consequently influenced the heat transfer characteristics.

Overall, this study provides valuable insights into the intricate relationship between mixed convection, magnetic field, and sinusoidal heating in a lid-driven horizontal cavity filled with water. The findings contribute to a better understanding of heat transfer phenomena in similar systems, offering opportunities for optimizing heat transfer processes in various engineering applications. Further investigations could explore the effects of different magnetic field strengths and frequencies of sinusoidal heating and consider additional geometries and boundary conditions to gain deeper insights into the underlying mechanisms and optimize heat transfer performance in lid-driven cavities.

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