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An Exact Solution of MHD Hybrid Nanofluid over a Stretching Surface Embedded in Porous Medium in the Presence of Thermal Radiation and Slip with Suction

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ABSTRACT

A comprehensive study of magnetohydrodynamics (MHD) hybrid $Cu-Al_2O_3$ nanofluid flow towards a permeable stretching sheet embedded in porous medium is analyzed analytically. The flow problem is mathematical modeled into nonlinear partial differential equations, which are reduced into ordinary differential equations via similarity transformation. The analytical solutions for momentum and energy equations are found. The effects of the porosity on velocity and temperature profiles are analyzed graphically, while the skin friction coefficient and the local Nusselt number are displayed in data tabulation. The results show that increasing the porosity parameter decrease the skin friction coefficient and heat transfer rate.

1. Introduction

Convective heat transfer in a porous medium has been an ongoing research topic worldwide especially in industrial engineering and manufacturing processes. There are several numerical studies to better understand the mechanism of heat transfer with stretching or shrinking effects was discussed in the literature [1-7]. Nanofluid are now the most popular and intriguing research topics being explored by researchers. Choi and Eastman [8] developed this nanofluid when he presented in his study about a heat transfer fluid that may increase thermal conductivity by suspending nanoparticles. Since then, researchers have been working on nanofluids [9,10], which are still being studied today. Hassani *et al.,* [11] have investigated the problem of boundary layer flow of a nanofluid past a stretching sheet analytically by using the Homotopy Analysis Method. Later, Turkyilmazoglu [12] analysed on the MHD slip nanofluid flow for the heat and mass transfer over a permeable stretching or shrinking surface analytically. Rashidi *et al.,* [13] examined the buoyancy and thermal radiation effects in their work on the boundary layer flow of MHD nanofluid via a stretched sheet. Elbashbeshy *et al.*, [14] investigated boundary layer flow past a moving surface of nanofluid in the presence of magnetic field with suction/injection. Stability analysis of stagnation-point flow in a

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nanofluid through a stretching or shrinking sheet in the presence of second-order slip, soret and dufour was studied by Najib *et al.,* [15]. Recently, Ismail *et al.,* [16] came forward to investigate the with MHD and thermal radiation.

Hybrid nanofluid is introduced which is formed by two or more different nanoparticles in the base fluid to enhance thermal conductivity. The characteristics of flow and heat transfer processes past a permeable stretching surface with magnetic field by using hybrid nanofluid ($Cu - Al_2O_3$/water) was introduced by Devi and Devi [17]. Recently, a considerable number of researchers have attempted to investigate hybrid nanofluid flows in boundary layer problem. The effects of radiation on steady flow and heat transport of a $Cu - Al_2O_3$/water hybrid nanofluid across a nonlinear permeable stretching or shrinking surface are being investigated by Waini *et al.,* [18]. Khashi'ie *et al.,* [19] studied Marangoni boundary layer flow over a permeable stretching or shrinking sheet in $Cu\text{-}Al_2O_3$/water. Later, Wahid *et al.,* [20] analyzed the effect of radiation and slip condition on MHD $Cu - Al_20_3$/water nanofluid flow across a stretching surface with suction. To handle the issue, an exact analytical approach is used. The presence of velocity slip decreased the velocity profile while increasing the temperature profile. Khashi'ie *et al.,* [21] investigated the flow properties and heat transfer of a hybrid $Cu - Al_20_3$/water nanofluid related to a radially stretching or shrinking surface using the mutual effects of MHD, suction, and Joule heating. Wahid *et al.,* [22] analyzed the effect of slip velocity on hybrid nanofluid flow over an exponentially stretching or shrinking permeable sheet in the presence of heat generation. Recently, Kho *et al.,* [23] investigated the Homann stagnation point flow and heat transfer of hybrid nanofluids past a permeable radially stretching or shrinking surface.

Therefore, the present investigation deals with the MHD hybrid nanofluid past a stretching surface embedded in porous medium in the presence of thermal radiation with velocity slip and suction. It extends, in fact, the papers by Wahid *et al.,* [20] to the case of porous medium. Numerical results are compared with those of Wahid *et al.,* [20] for special cases and are presented in tables and graphs.

2. Formulation of the Problem

The details of the methods and the approach utilized will be described in this section, and the steps will begin with the formulation of the mathematical model. The mathematical model in the form of partial differential equations will be reduced to ordinary differential equations utilizing similarity transformations. After that, the ordinary differential equations will be solved analytically using the exact method to obtain the final form of the exact solution for the fields of velocity and temperature. The Maple programmed will be used to help with the computations and to obtain the results. The physical problem is illustrated in Figure 1.

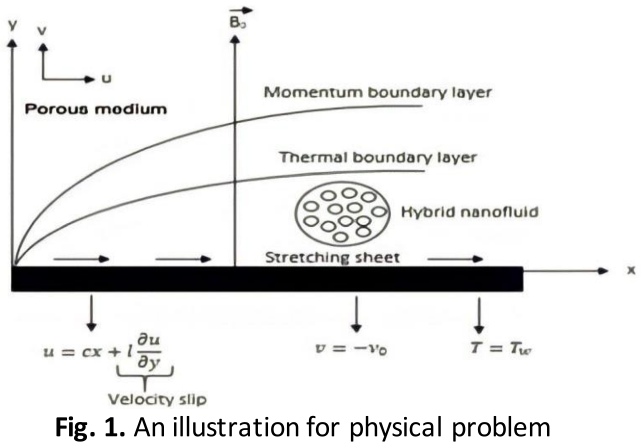

Fig. 1. An illustration for physical problem

As part of the Wahid *et al.,* [20] extension, various changes and enhancements are made to the model by adding porous medium as well as addressing the issue analytically rather than numerically, as shown in earlier research. The model's governing equations are given by

$$u_y + v_y = 0, \tag{1}$$

$$uu_x + vu_y = -\frac{\sigma_{hnf}}{\rho_{hnf}} B_0^2 + v_{hnf} u_{yy} - v_{hnf}\frac{u}{k}, \tag{2}$$

$$uT_X + vT_y = \frac{k_{hnf}}{(\rho Cp)_{hnf}} T_{yy} - \frac{1}{(\rho Cp)_{hnf}} q_{r_y}, \tag{3}$$

As well as the boundary conditions are defined by

$$u = u_w(x) = cx + \iota u_y, \qquad v = v_0, \qquad T = T_{w,} \tag{4}$$

$$u \to 0, u_y \to 0, \qquad T \to T_\infty, \qquad \text{as} \qquad y \to \infty,$$

where u and v are the velocity components in x - and y - directions respectively. Then, σ is the fluid conductivity, B_0 for uniform magnetic field, T for temperature and C_p is the specific heat at constant pressure. $v = \frac{\mu}{\rho}$ is the kinematic viscosity, ρ is the fluid density, μ is the coefficient of fluid viscosity, c denotes the constant stretching rate, T_∞ denotes the ambient fluid temperature, T_w is the constant surface temperature, ι defines the slip parameter and x_L stands for the characteristic length.

Due of the difficulties of directly solving the partial differential Eq. (2) and Eq. (3), similarity variables are utilized to reduce the partial differential equations into ordinary differential equations. The similarity transformations are,

$$u = cxf'(\eta), \qquad v = -\sqrt{cv_f} f(\eta),$$

$$\eta = \left(\frac{c}{v_f}\right)^{\frac{1}{2}} y, \qquad \theta(\eta) = \frac{T - T_w}{T_w - T_\infty}, \tag{5}$$

Eq. (2) and Eq. (3) are now reduced and become

$$f''' = Mf'(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}\frac{\sigma_{hnf}}{\sigma_f} -$$

$$(f'^2 - ff'')\left(\frac{\phi_2\rho_{s2}}{\rho_f} + (1-\phi_2)\left[(1-\phi_2)\rho_f + \frac{\phi\rho_{s1}}{\rho_f}\right]\right)(1-\phi_2)^{-2.5}(1-\phi_2)^{-2.5} + \frac{\mu_f}{\rho_f ck} f'. \tag{6}$$

$$\theta'' = -\left(\frac{k_f}{k_{hnf}} + \frac{1}{R_d}\right)\left(\frac{(\rho Cp)_{s2}}{(\rho Cp)_f}\phi_2 + (1-\phi_2)\left[\phi_1\frac{(\rho Cp)_{s1}}{(\rho Cp)_f} + (1-\phi_1)\right]\right) P_r f\theta'. \tag{7}$$

Thus, the new boundary conditions are

$$f = s, \; f' = 1 + Lf''(0), \qquad \theta = 1,$$

$$f' \to 0, \qquad f'' \to 0, \qquad \text{at} \qquad \eta \to \infty, \tag{8}$$

These parameters can be detailed by using the definitions given below

$$R_d = \frac{-16\sigma * T_\infty^3}{3k*}, \qquad M = \frac{\sigma B_0^2}{\rho_f c}, \qquad P_r = \frac{\mu C_p f}{k}, \qquad L = \ell\sqrt{\frac{c}{v_f}}, \qquad K = \frac{v_f}{\rho_f c k} f', \tag{9}$$

where R_d, M, P_r, L and K are the thermal radiation, magnetic, Prandtl number, velocity slip and porosity parameter, accordingly.

Table 1
Nanoparticles and fluid thermophysical properties

Physical properties	Water	Al_2O_3	Cu
$\rho(kg/m^3)$	997.1	3970	8933
$C_p(J/kgK)$	4179	765	385
$k(W/mK)$	0.6130	40	400

Table 2
Mono and hybrid nanofluid thermophysical properties

Properties	Nanofluid	Hybrid nanofluid
Density	$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s$	$\rho_{hnf} = (1-\phi_2)\begin{bmatrix}(1-\phi_1)\rho_f \\ +\phi_1\rho_{s1}\end{bmatrix} + \phi_2\rho_{s2}$
Heat Capacity	$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s$	$(\rho C_p)_{hnf} = (1-\phi_2)\begin{bmatrix}(1-\phi_1)(\rho C_p)_f \\ +\phi_1(\rho C_p)_{s1}\end{bmatrix} + \phi_2(\rho C_p)_{s2}$
Dynamic viscocity	$\frac{\mu_{nf}}{\mu_f} = \frac{1}{(1-\phi_1)^{2.5}}$	$\frac{\mu_{hnf}}{\mu_f} = \frac{1}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}}$
Thermal conductivity	$\frac{k_{nf}}{k_f} = \left[\frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}\right]$	$\frac{k_{hnf}}{k_{bf}} = \left[\frac{k_{s2} + 2k_{bf} - 2\phi_2(k_{bf} - k_{s2})}{k_{s2} + 2k_{bf} + \phi_2(k_{bf} - k_{s2})}\right]$ where: $\frac{k_{bf}}{k_f} = \left[\frac{k_{s1} + 2k_f - 2\phi_1(k_f - k_{s1})}{k_{s1} + 2k_f + \phi_1(k_f - k_{s1})}\right]$

3. Solution of the Problem

Following Crane [24], regarding the conditions of $f(\eta)$ in Eq. (8), the momentum Eq. (6) has an exponential solution such that

$$f(\eta) = s + \frac{1}{C}(1 - e^{-z\eta}), \tag{10}$$

where C is a constant to be determined. By substituting Eq. (10) into Eq. (6) and after some manipulations, we then obtain

$$zC-\frac{MC}{z}(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}\frac{\sigma_{hnf}}{\sigma_f}+\frac{2v_f}{v_{hnf}}-\frac{v_f c}{zck}=0. \tag{11}$$

Now, we need to find the value of the positive root z by solving the above algebraic equation with the aided from Maple software.

Following Aly and Ebaid [25], analytical solution for temperature field can be obtained by using the following transformation as

$$t=e^{-z\eta} \tag{12}$$

Then, the following relations are also introduced

$$\frac{d}{d\eta}\theta=-zt\frac{d}{dt}\theta, \qquad \frac{d^2}{d\eta^2}=z^2\left[t^2\frac{d^2}{dt^2}\theta+t\frac{d}{dt}\theta\right]. \tag{13}$$

Upon simplification, the new transformed equation is obtained as below

$$t\theta''(t)+(n-mt)\theta'(t)=0, \tag{14}$$

With boundary conditions

$$\theta(0)=0, \qquad \theta(1)=1. \tag{15}$$

Then, by solving Eq. (14) with the aided from Maple software, we get the exact solution for temperature field function as below,

$$\theta(\eta)=\frac{(-t^{-n}(-m)^{-n}n(-mt)^n\Gamma(-n)+t^{-n}(-m)^{-n}n(-mt)^n\Gamma(-n,-mt)-t^{-n}(-m)^{-n}e^{mt})}{((-m)^{-n}(\Gamma(-n,-m)(-m)^n n-(-m)^n\Gamma(-n)n-e^m))}, \tag{16}$$

where

$$n=1+\left(\left(\frac{k_f}{k_{hnf}}\left[(1-\phi_2)\left[(1-\phi_1)+\phi_1\frac{(\rho Cp)_{s1}}{(\rho Cp)_f}\right]+\phi_2\frac{(\rho Cp)_{s2}}{(\rho Cp)_f}\right)\right]P\right)\left(-\frac{s}{z}-\frac{1}{z(z+Lz^2)}\right),$$

$$m=\left(\left(\frac{k_f}{k_{hnf}}\left[(1-\phi_2)\left[(1-\phi_1)+\phi_1\frac{(\rho Cp)_{s1}}{(\rho Cp)_f}\right]+\phi_2\frac{(\rho Cp)_{s2}}{(\rho Cp)_f}\right)\right]P\right)\left(\frac{1}{z(z+Lz^2)}\right),$$

$$P=\frac{Pr}{(1+R_d)}.$$

Following Devi and Devi [17], the skin friction coefficient and the local Nusselt number are given by

$$Re_x^{\frac{1}{2}}C_{fx}=f''(0)\frac{1}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}}, \tag{17}$$

$$Re_x^{\frac{1}{2}} Nu_f = -\frac{k_{hnf}}{k_f}\theta'(0), \tag{18}$$

where $Re_x = u_w x/v$ is the local Reynold's number.

4. Results and Discussion

The boundary layer flow of $Cu - Al_2 0_3$ in a water-based hybrid nanofluid flow over a permeable stretching surface embedded in porous medium are studied in the presence of thermal radiation and slip condition. Analytical approach is applied to solve the differential equations of the velocity and temperature. We will therefore proceed to evaluate and discuss the graphical and tabulation of results obtained. In the present work, the Prandtl number is fixed at $P_r = 6.135$.

Physical impact of slip parameter L, solid volume fraction(ϕ_1 for $Al_2 0_3$ and ϕ_2 for Cu), suction parameter s, magnetic parameter M and porosity K on velocity field of fluid profile $f'(\eta)$ is depicted through Figure 2 to Figure 7. It is found that $f'(\eta)$ decreases with an increase in L, ϕ_1, s, M and K where the momentum layer thickness declines and tend asymptotically to zero as the distance increase from the boundary. The behavior of porosity K is demonstrated in Figure 7. With increasing K, the resistance to the fluid motion increases and hence velocity decreases.

Influence of L, ϕ_1, s, M and R_d on temperature distribution $\theta(\eta)$ is depicted in Figure 8 to Figure 15. It is revealed from Figure 8 and Figure 9 that temperature distribution $\theta(\eta)$ increases with increasing values of L and R_d parameters. In the absence of a slip condition, the slip enables the flow to pass through it, causing the flow of the fluid to slow down and the temperature of the fluid to rise. Furthermore, the presence of R_d increases the heat flux from the sheet, which raises the temperature distribution and also the thickness of the thermal boundary layer. Figure 11 shows thermal boundary layer decreases as the strength of M increases. However, as ϕ_2 increases, $\theta(\eta)$ increases as displayed in Figure 13. Figure 14 shows the influence of the P_r on the temperature distribution can be seen, in which the increasing value of P_r decreases the temperature distribution and associated thermal layer thickness. The effects of K on the temperature profile $\theta(\eta)$ are shown in Figure 15. For increase values of the porosity parameters K, temperature profile $\theta(\eta)$ increase. Porosity parameter K grows a resistance force that enhance thermal boundary layer thickness.

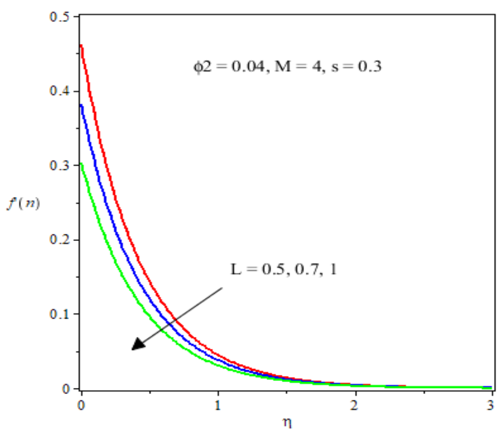

Fig. 2. Velocity profile for L

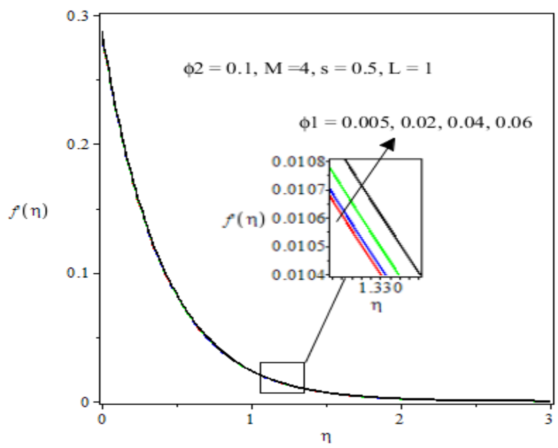

Fig. 3. Velocity profile for ϕ_1

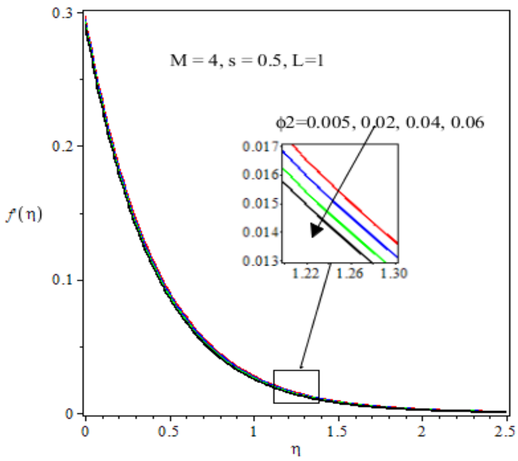

Fig. 4. Velocity profile for ϕ_2

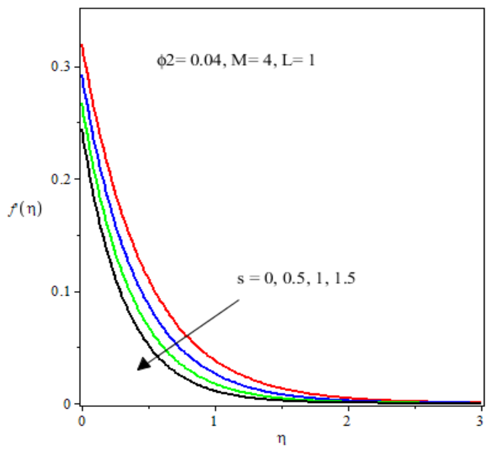

Fig. 5. Velocity profile for s

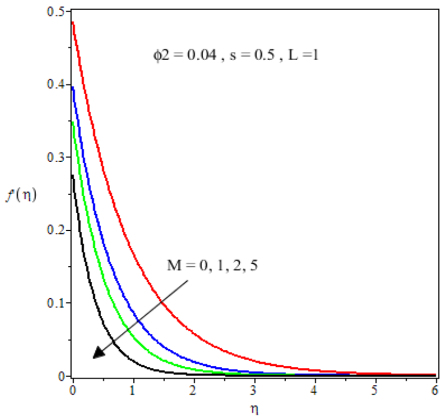

Fig. 6. Velocity profile for M

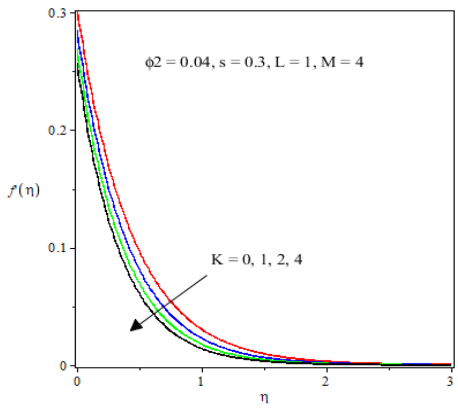

Fig. 7. Velocity profile for K

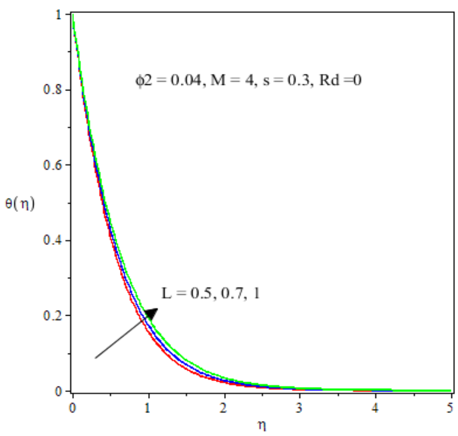

Fig. 8. Temperature profile for L

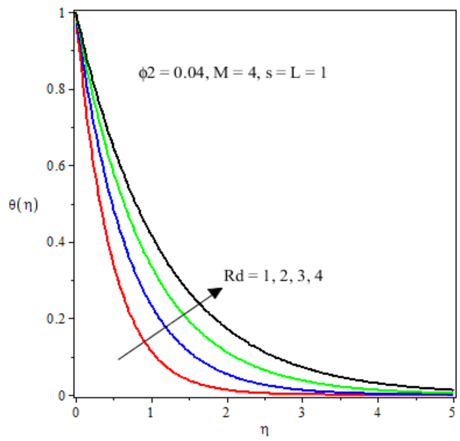

Fig. 9. Temperature profile for R_d

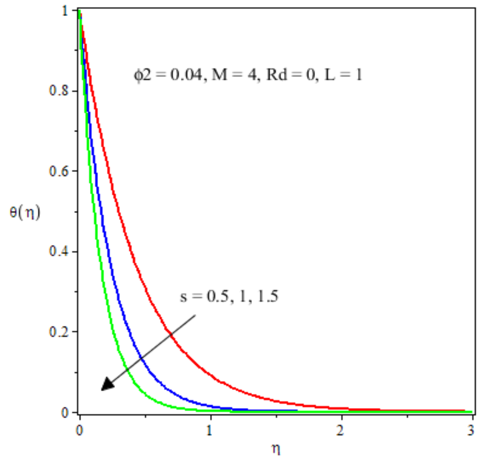

Fig. 10. Temperature profile for s

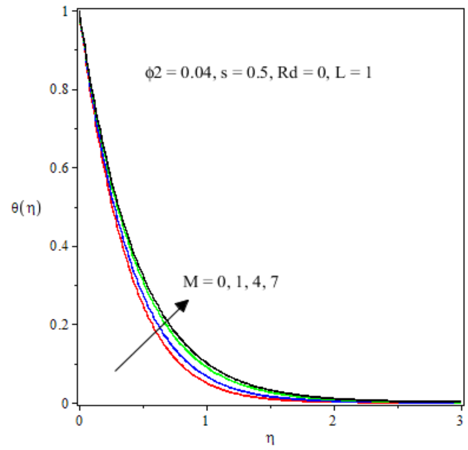

Fig. 11. Temperature profile for M

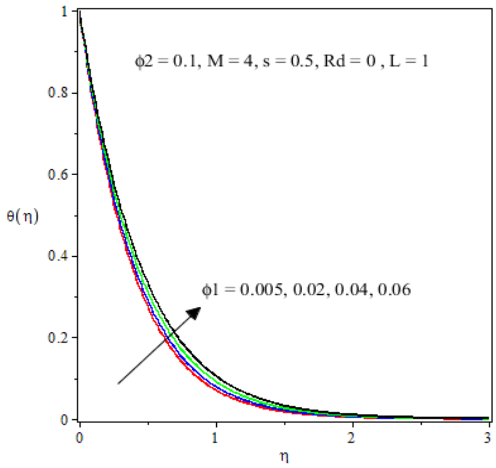

Fig. 12. Temperature profile for ϕ_1

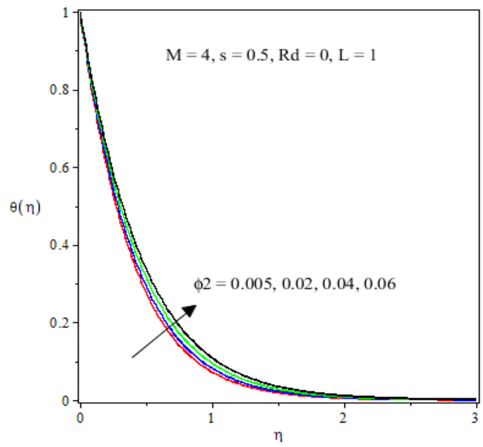

Fig. 13. Temperature profile for ϕ_2

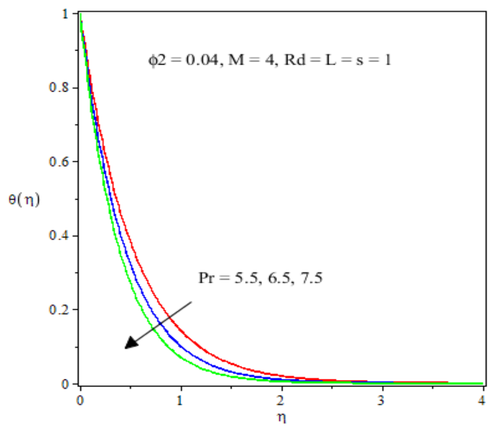

Fig. 14. Temperature profile for P_r

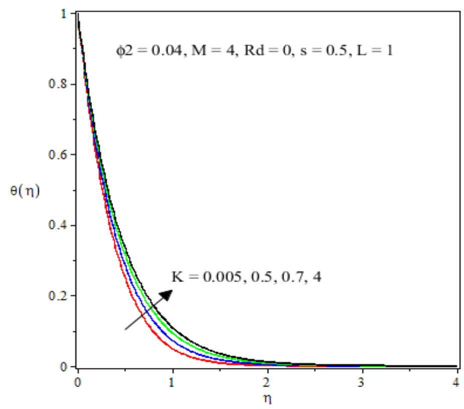

Fig. 15. Temperature profile for K

The skin friction coefficient is shown as data in Table 3 and Table 4, correspondingly. When a hybrid nanofluid is used instead of a mono nanofluid, the skin friction coefficient values are substantially lower. Table 3 shows that the existence of the porosity parameter K reduces the value of the skin friction coefficient. Additionally, the inclusion of L parameter put the value of the skin friction coefficient in increasing. Therefore, the increase in the value of ϕ_2, the parameter M and s will reduces the skin friction coefficient in both situations of hybrid nanofluid and mono nanofluid. Similarly, as shown in Table 4, as ϕ_2 increase, the local Nusselt number increases for both the hybrid and mono nanofluid. Since it is known that the hybrid nanofluid may enhance thermal performance, it is clear that the values of the heat transfer coefficient are greater for the hybrid nanofluid when compared to mono nanofluid.

Table 3
The skin friction coefficient $Re_x^{\frac{1}{2}}C_{fx}$

M	s	L	K	Skin friction coefficient $Re_x^{\frac{1}{2}}C_{fx}$					
				Cu/water			$Cu-Al_2 0_3$/water		
				Devi and Devi [14]	Wahid *et al.,* [17]	Present	Devi and Devi [14]	Wahid *et al.,* [17]	Present
				-2.547303	-2.547304	-2.547309	-3.335263	-3.335264	-3.335362
				-2.691472	-2.691472	-2.691447	-3.503772	-3.503772	-3.503743
4				-2.889322	-2.889322	-2.889417	-3.735916	-3.735915	-3.735784
				-3.094195	-3.094195	-3.094130	-3.977340	-3.977340	-3.977325
0	0.5			-1.582096	-1.582070	-1.582090	-1.973957	-1.973910	-1.973823
1				-2.007264	-2.007264	-2.007346	-2.554136	-2.554136	-2.554034
4		0	0	-2.889322	-2.889322	-2.889417	-3.735916	-3.735915	-3.735784
9				-3.90473	-3.904729	-3.904709	-5.084042	-5.084043	-5.083983
	0			-2.538415	-2.538415	-2.538582	-3.310532	-3.310532	-3.310643
	0.5			-2.889322	-2.889322	-2.889417	-3.735916	-3.735915	-3.735784
	1			-3.281808	-3.281809	-3.281891	-4.208702	-4.208701	-4.208627
				-3.712996	-3.712997	-3.712903	-4.726000	-4.725999	-4.726001
	1.5	0.5			-1.362856	-1.362856		-1.759935	-1.759935
		1			-0.841327	-0.841327		-1.089710	-1.089710
4			0.05			-3.724198			-4.740701
		0	1			-3.929516			-5.009331
			2			-4.131067			-5.273206
			4			-4.499730			-5.754837

Table 4
The local Nusselt number $Re_x^{\frac{1}{2}} Nu_f$

ϕ_2	M	s	L	R_d	K	Nusselt number $Re_x^{\frac{1}{2}} Nu_f$			
						Cu/water		$Cu - Al_2 0_3$/water	
						Wahid*et al.,* [17]	Present	Wahid*et al.,* [17]	Present
0.005						3.966019	3.966013	4.088189	4.008172
0.02						3.974342	3.974339	4.018690	4.018686
0.04	4					3.985379	3.985363	4.032618	4.032625
0.06			0			3.996439	3.996439	4.046564	4.046558
	0	0.5				4.132245	4.132239	4.242133	4.242141
	1			0		4.081782	4.081769	4.168261	4.168269
	4					3.985379	3.985363	4.032618	4.032625
	9					3.888602	3.888595	3.902394	3.902390
		0			0	1.531486	1.531443	1.635823	1.635791
		0.5				3.985379	3.985363	4.032618	4.032625
		1				6.705389	6.750309	6.705389	6.705387
0.04						9.638725	9.638723	9.498253	9.498248
			0.5			9.340092	9.340092	9.144460	9.144460
	4	1.5	1			9.265850	9.265852	9.054644	9.054645
				1		4.668692	4.668692	4.565752	4.565752
				2		3.127858	3.127858	3.059237	3.059237
		0.5			0.05		3.368617		3.332278
					1		3.339689		3.297525
				0	2		3.316210		3.269588
					4		3.280475		3.227130

5. Conclusions

The boundary layer flow and heat transport of a magnetohydrodynamic hybrid $Cu - Al_2 0_3$/water nanofluid over a permeable stretched sheet embedded in porous medium with thermal radiation and slip effects are developed and analyzed. The nonlinear partial differential equations are reduced into ordinary differential equations via similarity transformation. The boundary layer problem is solved using an exact analytical approach and can be concluded that:

I. Increasing the porosity parameter causes the increasing of temperature profile and decreasing of velocity profile.
II. The existence of porosity causes the reducing of both skin friction coefficient and the local Nusselt number.
III. The increment of velocity slip causes the velocity profile to decrease and rise the temperature profile.
IV. The presence of thermal radiation causes the increase of temperature profile and the local Nusselt number.
V. Increasing the porosity parameter decreases the skin friction and local Nusselt number.

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