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Analysis of Tangent Hyperbolic over a Vertical Porous Sheet of Carreau Fluid and Heat Transfer

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ABSTRACT

The purpose of this study is to investigate the boundary layer of Carreau fluid and heat transfer over an exponentially stretching plate derived in a vertical porous with variable surface thermal flux. The partial differential equations that represent the momentum equation and heat equation are commuted into nonlinear ODEs by applying similarity transformations and results found numerically. The impact of several emerging dimensionless parameters labelled the Weissenberg number (We), the power-law index (n), Velocity slip (Sf), Thermal jump (St), and Prandtl number (Pr) on the velocity profile and heat transfer on the boundary layer are showed in detail. In more detail, also the influence of physical parameters on local skin friction and Sherwood number are studied. The shooting method with the explicit technique is used to find the solution and all results are illustrated graphically and numerically. We noted that by increasing power index, radiation parameter and velocity slip, the velocity profile increases, and the temperature profile decreases. Furthermore, it is deduced that rising the thermal radiation parameter reduces the local Nusselt number.

1. Introduction

Non-Newtonian fluid properties in several models have been studied by many researchers because of the absence of Newton's law of viscosity. Carreau fluid model is a common model of non-Newtonian viscosity model (for large and very small shear rates) and is a combination of Newtonian and power-law properties. Within the past few decades, there is a lot of work done by researchers on boundary layer flow and heat exchange models via stretching surfaces. The reason behind this immense favor is the pivotal role played by stretching surfaces in numerous technological processes and engineering applications. They also discussed the non-Newtonian fluid flow characteristics by incorporating pertinent physical effects. Research of a laminar stream and thermic edge substrate

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over a stretched plate in a viscous fluid flow is interesting and important because of its branches of applications and applicants on several theological processes, sciences, and engineers like polymer, and thermal oil recovery, diet processing, and slurry transportation.

A kind of non-Newtonian fluid flow formula has been suggested in the literature keeping in its applications of their multiple theological aspects. In non-Newtonian fluid flow, the fundamental relationship that combines stress and rate of strain is nonlinear compared with the momentum equation which is generally fine for Newtonian fluid flow. Non-Newtonian fluid phenomena appear in many types of mechanical processes, chemistry, and substance engineering. A lot of non-Newtonian mathematical models are complicated with some form of modification to the equation of fluid flow like as power-law fluids, viscoelastic fluids containing Johnson-Segalman liquid flow [1], Walters-B short memory models [2], and differential Reiner-Rivlin models [3]. The non-Newtonian fluid flow foreside of the temperature profile is an interesting research study because of its connection with polymer processing and biotechnology [4,5]. The porosity effects along with the mixed convection time-dependent boundary layer flow were discussed by Harris *et al.,* [6]. The flow pattern of micropolar liquid caused by linearly elongated accompanied by variably surfaced thermal distribution was taken by Ibrahim and Hassanien [7]. Channeled boundary layer flow when both the suction and injection are considered was reported by Temam and Wang [8]. The stability analysis for convectional flow was accomplished by Mureithi and Mason [9]. The time-dependent stagnant motion of micropolar liquid was intrigued by Lok *et al.,* [10] stagnant viscid flow over a stretchable surface was incorporated by Nazar *et al.,* [11]. The current advancement in the dynamics of the flow regime of numerous liquids under the Prandtl boundary layer approach can be retrieved in refs. [12-16]. MHD Flow for various types of fluids has been studied such as double stratified micropolar fluid [17], Jeffery Hamel flow with suction/injection [18], viscoelastic nanofluid flow [19], the unsteady flow second-grade nanofluid [20], and micropolar fluid with a Buoyancy effect [21].

The velocity slip occurs when the fluid flow is unattached to a solid boundary layer, which is a phenomenon that has been observed under specific circumstances [22]. The slip fluid flow knot of the laminar edge substrate is of significant practical use. Micro-channels today's turbo-machinery technologies are vastly focused on the cooling of electronic machines, micro heat exchange regimes, and a lot of other processes. In the case, that the trait value of the fluid flow system is not large, nor the pressure of flow is tiny, slip flow occurs when the trait value of the fluid flow regime is disposed to the average free route of the molecular, continuum physics is no more observed. If the flow rate at a solid-fluid interface is zero, then the fluid temperature near the solid walls is equivalent to the solid layers. The fluids representing boundary slip discover usages in technology for example, in the polishing of artificial heart valves and internal cavities. The velocity slip effects are important to certain industrial heat transfer problems, and the fabrication of the fluid dynamic machine. The significant analysis of laminar slip-flow of thermal exchange including uniform wall temperature was investigated by Sparrow and Lin [23] and Inman [24]. Larralde *et al.,* [25] and Spillane [26] investigated the affection of slip boundary conditions of momentum and thermal transfer in circular tubes. Yu and Ameel [27] obtain an analytical solution on forced convection in isolux rectangular microchannels with thermal slip. Further important studies of slip flow on a microcylinder were reported by Crane and McVeigh [28]. The solutions to other viscous fluid flow remarkably as same as to boundary layer flow, like as Poiseuille, Couette, and Rayleigh flow, appeared to a shift in thermal exchange and shear stress [29]. Akbar *et al.,* [30] and Gaffar *et al.,* [31] presented numerical fluid flow solutions with tangent hyperbolic heat transfer on a stretching sheet. Furthermore, comprehensive reviews on the effects of slip boundary layer flow are found in [32-36]. Atif *et al.,* [37] discussed the effect of variable thermal conductivity and thermal radiation on MHD tangent hyperbolic fluid past a stretching sheet. Recently, Shahzad *et al.,* [38] presented their numerical study for an unsteady tangent hyperbolic nanofluid flow pas a wedge in the presence of suction or injection.

Some generalized Newtonian rheological models usually describe the blood viscosity and chemical materials processing applications: the Carreau model, the Carreau-Yasuda model, etc. Motivated by various importance of non-Newtonian fluid and previous effort in the literature, the current study aims to analyze the non-Newtonian Carreau fluid flow and heat transfer of a tangent hyperbolic forward to a vertical. The dimensionless equations related to non-dimensional boundary layer conditions form a strongly nonlinear, coupled boundary layer problem. Then in the next section, the dimensionless form of the fundamental equations is solved using Shooting method. Further, a result and discussion section has been presented precisely with graphical illustration and eventually a concluding section has been depicted briefly.

2. Mathematical Model

A steady laminar fluid of a viscous incompressible fluid flow and thermal transition tangent hyperbolic fluid towards a vertical porous plate, including radiation in two-dimension, are considered. Both the plate and tangent hyperbolic fluid with the same temperature is maintained initially. Directly they are grown to a thermal $T_W \rightarrow T_1$, the thermal surrounded of the fluid which remains without change [39, 40]. The mass equation, momentum equation, and thermal transfer with the boundary layer approximations, one can be written as:

$$u_x + v_y = 0, \tag{1}$$

$$uu_x + vu_y = v(1-n)u_{yy} + \frac{(3n-3)\Gamma^2}{2} vu_{yy}u_y^2 + gB^2(x)(T - T_\infty), \tag{2}$$

$$uT_x + vT_y = \frac{K}{\rho c_p} T_{yy} + \frac{\mu}{\rho c_p} u_y^2 + \frac{\sigma B_0^2 u^2}{\rho c_p} - \frac{1}{\rho c_p}(q_r)_y, \tag{3}$$

where u is the velocity constituent on the x-axis, v is the velocity constituent on the y-axis, and T is the fluid temperature.

The symbols v, g, ρ, $B_0(x)$ and Γ are representing kinematic viscosity, gravity, density material fluid parameter, and constant magnetic field strength, respectively. The parameters K and c_p are specific heat capacity, thermal diffusivity. q_r is the radiative heat flux, defined as:

$$q_r = -\frac{4\sigma^*}{3k^*}\left(4TT_\infty^3 - 3T_\infty^4\right). \tag{4}$$

The boundary conditions subject to velocity and temperature field are:

$$\begin{aligned} &u = N_0 u_y,\ v = 0,\ T = T_w + K_0 T_y \text{ at } y = 0, \\ &u \to 0,\ T \to T_\infty \text{ at } y \to \infty, \end{aligned} \tag{5}$$

In the case of $N_0 = K_0 = 0$, we get no-slip conditions. By using stream function $u = \psi_y$ and $v = -\psi_x$, where

$$\xi = \frac{ax}{\nu} Gr^{-1/4},\ \eta = \frac{y}{x} Gr^{1/4},\ \psi = 4\nu Gr^{1/4}\left(f\left(\xi,\eta\right) + 0.25\xi \right),$$
$$g\left(\xi,\eta\right) = \frac{T - T_\infty}{T_w - T_\infty},\ r = \frac{\nu}{\kappa},\ Gr = \frac{g\beta\left(T_w - T_\infty\right)x^3}{4\nu^2}, \tag{6}$$

and applying transformations, the continuity equation is automatically satisfied, and the following coupled partial differential equations are introduced

$$\left(1-n\right) f_{\eta\eta\eta} + \left(3f + \xi\right) f_{\eta\eta} - 2 f_\eta^2 + nWef_{\eta\eta}^2 f_{\eta\eta\eta} + g = \xi\left(f_\eta f_{\eta\xi} - f_{\eta\eta} f_\xi \right), \tag{7}$$

$$\frac{1}{\Pr}\left(1 + \frac{4}{3}R\right) g_{\eta\eta} + \left(3f + \xi\right) g_\eta + fg_\eta - f_\eta g + Ec\left(f_{\eta\eta}^2 + Mf_\eta^2 \right) = \xi\left(f_\eta g_\xi - g_\eta f_\xi \right), \tag{8}$$

with related boundary conditions

$$f\left(0\right) = 0,\ \ f_\eta\left(\eta\right) = S\ ff_{\eta\eta}\left(0\right),\ \ g\left(\eta\right) = 1 + Stg_\eta\left(0\right),$$
$$f_\eta\left(\infty\right) \to 0,\ \ g\left(\infty\right) \to 0, \tag{9}$$

where Sf denotes the dimensionless velocity slip, and St is the thermal jump parameters. The interesting physical quantities which are the local Nusselt number and the Sherwood number are given as:

$$0.25Gr^{-3/4}C_f = \left(1-n\right) f_{\eta\eta}\left(\xi,0\right) + \frac{nWe}{3}\left(f_{\eta\eta}\left(\xi,0\right)\right)^3, \tag{10}$$

$$Gr^{-1/4}N_u = -g_\eta\left(\xi,0\right). \tag{11}$$

In case the lower stagnation point $\xi = 0$, a coupled strong nonlinear ordinary differential equations related to their boundary conditions can be rewritten as

$$\left(1-n\right) f_{\eta\eta\eta} + 3ff_{\eta\eta} - 2 f_\eta^2 + nWef_{\eta\eta}^2 f_{\eta\eta\eta} + g = 0, \tag{12}$$

$$\frac{1}{\Pr}\left(1 + \frac{4}{3}R\right) g_{\eta\eta} + 4fg_\eta - f_\eta g + Ec\left(f_{\eta\eta}^2 + Mf_\eta^2 \right) = 0, \tag{13}$$

subject to boundary conditions in Eq. (9).

A couple of ordinary differential Eq. (12) and Eq. (13) together with boundary conditions in Eq. (9) are solved numerically using the shooting method. The non-dimensional quantities are defined as:

$$\Pr = \frac{c_p \mu}{\kappa},\ Ec = \frac{\nu Gr^{3/4}}{c_p\left(T_w - T_\infty\right)},\ M = \frac{\sigma B_0^2 x^2}{\rho \nu Gr^{1/2}},\ We = \frac{\sqrt{2}\Gamma^2 \nu Gr^{3/4}}{x^2},$$
$$R = \frac{4\sigma^* T_0^3}{\kappa k^*},\ Sf = \frac{N_0 Gr^{1/4}}{a},\ St = \frac{k_0 Gr^{1/4}}{a}. \tag{14}$$

where Pr, Ec, M, We, R, Sf and St are Prandtl number, Eckert number, magnetic field, Weissenberg number, radiation parameter, velocity slip, and thermal jump, respectively. The Weissenberg number has been reported in some previous works. A critical Weissenberg number was reported by Ng and Hartnett [41] for solutions containing Separan AP-273. A similar study was reported by Kwack and Yi [42]. Weissenberg numbers have been determined for various pipe diameters, solutes, and solvent chemistry by Kwack and Hartnett [43,44].

3. Numerical Solution

To solve and investigate favorable physical parameters and numbers, the Eq. (12) and Eq. (13) with boundary conditions in Eq. (9) are converted into first-order ordinary differential equations by letting:

$$\omega_1 = f,\ \omega_2 = f',\ \omega_3 = f'',\ \omega_4 = g,\ \omega_5 = g',$$

which gives

$$\begin{bmatrix}\omega_1'\\ \omega_2'\\ \omega_3'\\ \omega_4'\\ \omega_5'\end{bmatrix} = \begin{bmatrix}\omega_2\\ \omega_3\\ -\dfrac{3\omega_1\omega_3 - 2\omega_2^2 + \omega_4}{(1-n) + nWe\omega_3^2}\\ \omega_5\\ -\dfrac{4\omega_1\omega_5 - \omega_2\omega_4 + Ec\left(\omega_3^2 + M\omega_2^2\right)}{\dfrac{1}{\Pr}\left(1 + \dfrac{4}{3}R\right)}\end{bmatrix} \tag{15}$$

and the corresponding initial conditions are

$$\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\\ \omega_4\\ \omega_5\end{bmatrix} = \begin{bmatrix}0\\ Sf\,\omega_3\\ \varphi_1\\ 1 + St\omega_5\\ \varphi_2\end{bmatrix} \tag{16}$$

MATHEMATICA software 11.3 is used to solve numerically Eq. (15) and Eq. (16) by applying the shooting technique with truncation error less than 10^{-6}. The effect of non-dimensional physical parameters is illustrated graphically and the interesting parameter local Nusselt number $f_{\eta\eta}(0)$ and

Sherwood number $g_\eta(0)$ showed numerically and all affection of parameters are presented in Table 1.

4. Results and Discussion

In this section, the results are graphically showed on the velocity profiles $f'(\eta)$ and heat transfer rate $g(\eta)$ for different interesting physical parameters illustrated, such as power index (n), Weissenberg number (We), thermal jump (St), velocity slip (Sf), radiation parameter (R), Prandtl number Pr, Magnetic parameter (M), Eckert number (Ec), skin friction number and local Nusselt number.

Figure 1 describes the Efficacy of power index n on the velocity profiles $f'(\eta)$. Here, it is noticeable that expansion in power index n increases the velocity of the fluid and skin friction number contrary to the local Nusselt number that is decreased. Figure 2, velocity profiles value and local Nusselt number grow in the order of increasing Weissenberg number while skin friction number declined. From Figure 3, it is concluded that for higher values of thermal jump St, the value of the velocity profile, skin friction number, and local Nusselt number is down. Figure 4, illustrates that for enhancing the various value of velocity slip Sf, the magnitude of the velocity profile and skin friction number increased conversely local Nusselt number decreased.

It is observed that from Figure 8, Figure 11, and Figure 12, the effect of the nondimensional parameters n, St, and Sf have the same effect of velocity profile on heat transfer rate, but Weissenberg number We has the reverse effect on heat transfer in a contrary manner of velocity profile as shown in Figure 5. Physically Weissenberg number We is the ratio of the relaxation time of the fluid and the specific process time. It grows the thickness of fluid and that is why velocity of the fluid depreciates. By increasing the value of Weissenberg number (We), the reduction time upsurges which impedes the drift of fluid and henceforth velocity condenses. From Figure 6, that radiation parameter R has a positive effect on heat transfer rate, skin friction number, and local Nusselt number, which means that increasing the value of R, increases the heat transfer of fluid flow, skin friction number, and local Nusselt number whereas Prandtl number Pr, has opposite manner effect on heat transfer, skin friction number and local Nusselt number as explained in Figure 7.

Finally in Figure 9 and Figure 10, they illustrate the influence of Magnetic field M and Eckert number Ec on heat transfer rate. It is observed that for the high value of the non-dimensional parameters, the temperature rate will grow. Skin friction number and local Nusselt number increase in case of increasing Magnetic field M and Eckert number Ec as shown in Table 1. Physically, the magnetic field creates an obstruction force on the fluid flow and these kinds of power are called Lorentz force which impedes the movement of fluid.

Table 1
Present numerical results of local Nusselt number and Nherwood number in various physical parameters

n	*We*	*Pr*	*Ec*	*M*	*R*	*Sf*	*St*	$f''(0)$	$g'(0)$
0.2	0.3	0.71	0.1	1	1	0.5	1		
0								0.17047	0.67521
0.2								0.18699	0.67054
0.7								0.23988	0.65927
0.9								0.32739	0.6485
	0							0.18729	0.67048
	2							0.18533	0.67088
	4							0.18350	0.67125
	6							0.18179	0.67160
		0						0.08577	0.888889
		0.71						0.05585	0.69068
		1.1						0.05059	0.65596
		2						0.04358	0.60850
			0.1					0.17832	0.67292
			0.6					0.18210	0.68399
			1					0.18531	0.69354
			2					0.19412	0.72054
				0				0.18642	0.66898
				1				0.18699	0.67054
				2				0.18756	0.67212
				3				0.18814	0.67372
					0			0.15187	0.59496
					1			0.18699	0.67054
					1.5			0.19736	0.69249
					2			0.20548	0.70966
						0.1		0.055858	0.69068
						0.3		0.13525	0.67809
						0.5		0.18621	0.67072
						0.8		0.23588	0.66387
							0.5	0.20861	0.79534
							1	0.18699	0.67054
							1.5	0.17120	0.58478
							2.5	0.1491	0.47244

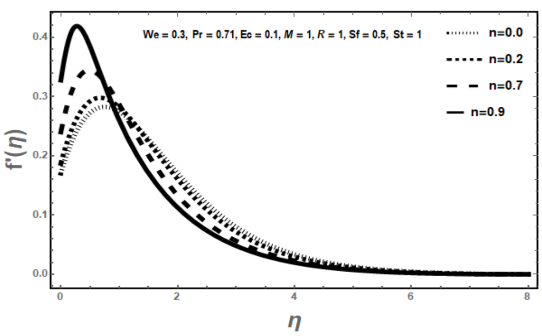

Fig. 1. Effect of power indexes n on velocity $f'(\eta)$

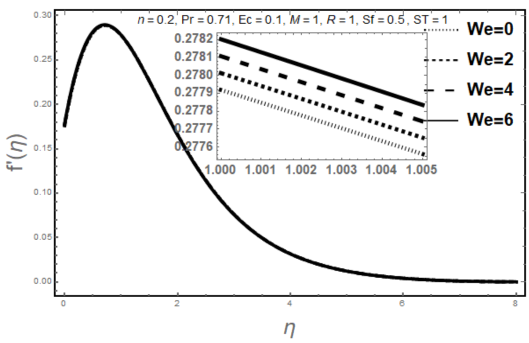

Fig. 2. Effect of Weissenberg number (We) on velocity $f'(\eta)$

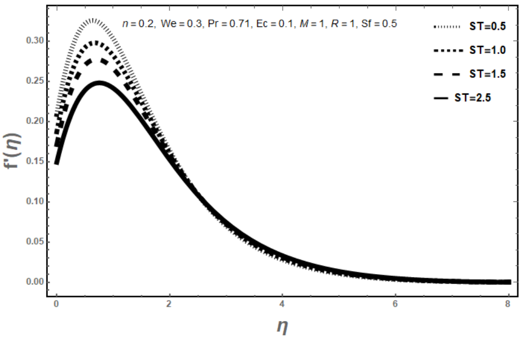

Fig. 3. Effect of Thermal jump (ST) on velocity $f'(\eta)$

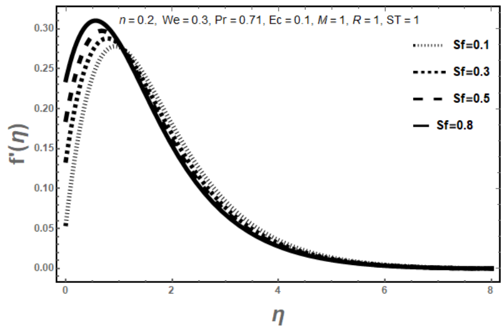

Fig. 4. Efficacy of Velocity slip (Sf) on velocity $f'(\eta)$

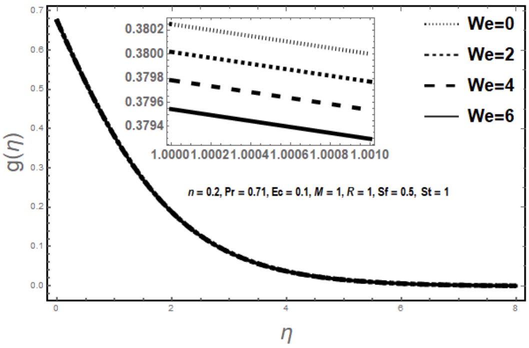

Fig. 5. Efficacy of Weissenberg number (We) on $g(\eta)$

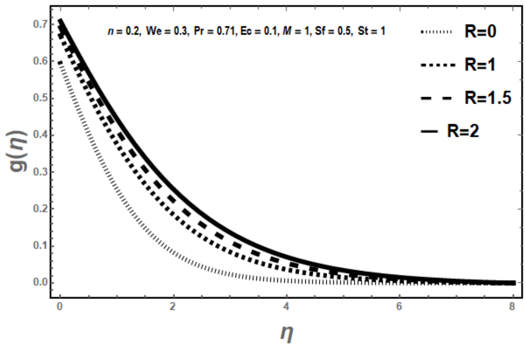

Fig. 6. Efficacy of Radiation parameter (R) on $g(\eta)$

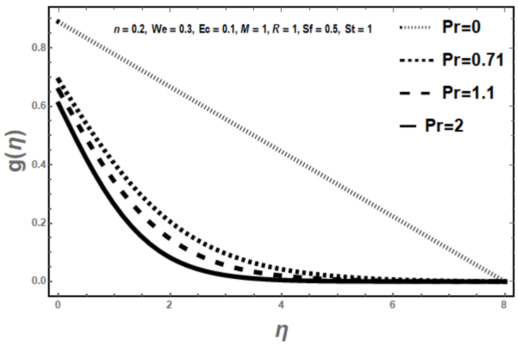

Fig. 7. Efficacy of Prandtl number (Pr) on $g(\eta)$

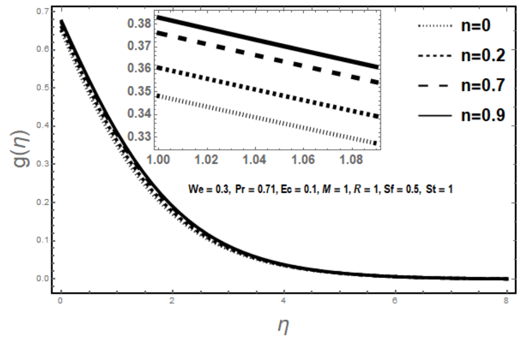

Fig. 8. Efficacy of power index (n) on $g(\eta)$

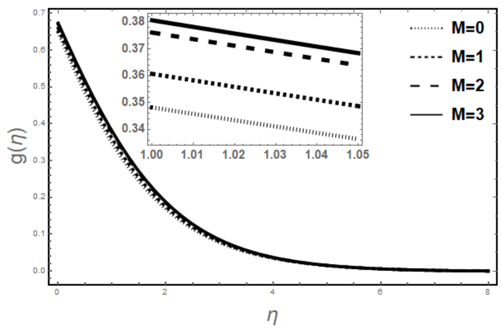

Fig. 9. Efficacy of magnetic field on $g(\eta)$

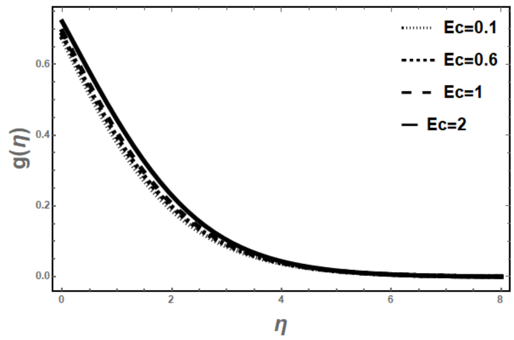

Fig. 10. Efficacy of Eckert on $g(\eta)$

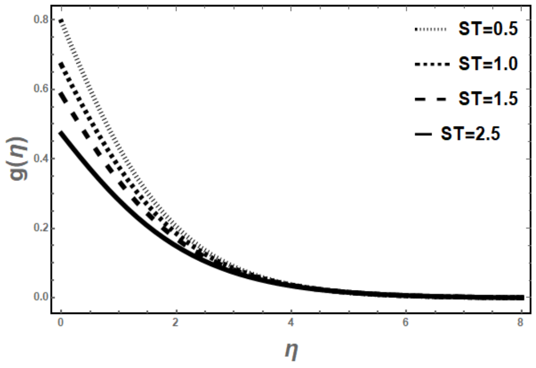

Fig. 11. Efficacy of velocity slip on $g(\eta)$

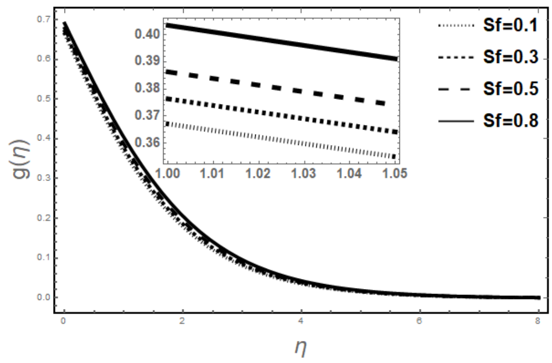

Fig. 12. Efficacy of thermal jump on $g(\eta)$

5. Conclusions

MHD boundary layer two-dimensional fluid flow for Carreau fluid model over a vertical porous sheet with radiation investigated numerically. Moreover, influences for different magnetics of physical numbers and parameters are discussed on the velocity profile $f'(\eta)$, heat transfer $g(\eta)$, local skin friction coefficient $f''(0)$ and local Nusselt number $g'(\eta)$. The main results are as follows:

I. Power index, Weissenberg number, and velocity slip are increasing coefficients on the velocity profile of Carreau fluid whereas thermal jump has a reverse effect on the velocity profile.
II. Power index, Radiation parameter, Magnetic field, Eckert number, and velocity slip are increasing the coefficient of heat transfer rate, but Weissenberg number, Prandtl number, and thermal jump are decreased the coefficient of heat transfer rate.
III. Power index, Eckert number, Radiation parameter, Magnetic field, and velocity slip grow skin friction number while other parameters have the contrary minor.
IV. Weissenberg number, Eckert number, Radiation parameter, and Magnetic field have a positive effect on local Nusselt number and other parameters have a negative effect.

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References
[1] Hayat, T., S. A. Shehzad, M. Qasim, and S. Obaidat. "Radiative flow of Jeffery fluid in a porous medium with power law heat flux and heat source." *Nuclear Engineering and Design* 243 (2012): 15-19. https://doi.org/doi:10.1016/J.NUCENGDES.2011.11.005
[2] Prasad, Vallampati R., Buddakkagari Vasu, Bég Osman Anwar, and Rana Parshad. "Unsteady free convection heat and mass transfer in a Walters-B viscoelastic flow past a semi-infinite vertical plate: a numerical study." *Thermal Science* 15, no. suppl. 2 (2011): 291-305. https://doi.org/10.2298/TSCI101102002P
[3] Tripathi, Dharmendra, O. Anwar Bég, and Jose Luis Curiel-Sosa. "Homotopy semi-numerical simulation of peristaltic flow of generalised Oldroyd-B fluids with slip effects." *Computer methods in biomechanics and biomedical engineering* 17, no. 4 (2014): 433-442. http://dx.doi.org/10.1080/10255842.2012.688109
[4] Wilson, Laurie L., R. Alex Speers, and Marvin A. Tung. "Yield stresses in molten chocolates." *Journal of texture studies* 24, no. 3 (1993): 269-286. http://dx.doi.org/10.1111/J.1745-4603.1993.TB01284.X

[5] Hossain, M. Anwar, Khalil Khanafer, and Kambiz Vafai. "The effect of radiation on free convection flow of fluid with variable viscosity from a porous vertical plate." *International Journal of Thermal Sciences* 40, no. 2 (2001): 115-124. https://doi.org/10.1016/S1290-0729(00)01200-X

[6] Harris, S. D., D. B. Ingham, and I. Pop. "Unsteady mixed convection boundary-layer flow on a vertical surface in a porous medium." *International journal of heat and mass transfer* 42, no. 2 (1999): 357-372. https://doi.org/10.1016/S0017-9310(98)00209-9

[7] Ibrahim, F. S., and I. A. Hassanien. "Mixed convection boundary layer flow of a micropolar fluid on a horizontal flat plate with power law variation in surface temperature." *International journal of thermal sciences* 39, no. 3 (2000): 360-373. https://doi.org/10.1016/S1290-0729(00)00221-0

[8] Temam, Roger, and Xiaoming Wang. "Boundary layers in channel flow with injection and suction." *Applied mathematics letters* 14, no. 1 (2001): 87-91. https://doi.org/10.1016/S0893-9659(00)00117-8

[9] Mureithi, Eunice W., and David P. Mason. "On the stability of a forced-free boundary layer flow with viscous heating." *Fluid dynamics research* 31, no. 1 (2002): 65. https://doi.org/10.1016/S0169-5983(02)00088-6

[10] Lok, Yian Yian, Norsarahaida Amin, and Ioan Pop. "Unsteady boundary layer flow of a micropolar fluid near the rear stagnation point of a plane surface." *International Journal of Thermal Sciences* 42, no. 11 (2003): 995-1001. https://doi.org/10.1016/S1290-0729(03)00079-6

[11] Nazar, Roslinda, Norsarahaida Amin, Diana Filip, and Ioan Pop. "Unsteady boundary layer flow in the region of the stagnation point on a stretching sheet." *International journal of engineering science* 42, no. 11-12 (2004): 1241-1253. https://doi.org/10.1016/j.ijengsci.2003.12.002

[12] Cai, Chunpei. "Near continuum boundary layer flows at a flat plate." *Theoretical and Applied Mechanics Letters* 5, no. 3 (2015): 134-139. https://doi.org/10.1016/j.taml.2015.03.005

[13] Afify, Ahmed A., and Nasser S. Elgazery. "Effect of a chemical reaction on magnetohydrodynamic boundary layer flow of a Maxwell fluid over a stretching sheet with nanoparticles." *Particuology* 29 (2016): 154-161. https://doi.org/10.1016/j.partic.2016.05.003

[14] Khan, Masood, and Muhammad Azam. "Unsteady boundary layer flow of Carreau fluid over a permeable stretching surface." *Results in Physics* 6 (2016): 1168-1174. https://doi.org/10.1016/j.rinp.2016.11.035

[15] Rahimi, J., D. D. Ganji, M. Khaki, and Kh Hosseinzadeh. "Solution of the boundary layer flow of an Eyring-Powell non-Newtonian fluid over a linear stretching sheet by collocation method." *Alexandria Engineering Journal* 56, no. 4 (2017): 621-627. https://doi.org/10.1016/j.aej.2016.11.006

[16] Liu, Lin, and Fawang Liu. "Boundary layer flow of fractional Maxwell fluid over a stretching sheet with variable thickness." *Applied Mathematics Letters* 79 (2018): 92-99. https://doi.org/10.1016/j.aml.2017.10.008

[17] Khan, Ansab Azam, Khairy Zaimi, Suliadi Firdaus Sufahani, and Mohammad Ferdows. "MHD flow and heat transfer of double stratified micropolar fluid over a vertical permeable shrinking/stretching sheet with chemical reaction and heat source." *Journal of Advanced Research in Applied Sciences and Engineering Technology* 21, no. 1 (2020): 1-14. https://doi.org/10.37934/araset.21.1.114

[18] Hamrelaine, Salim, Fateh Mebarek-Oudina, and Mohamed Rafik Sari. "Analysis of MHD Jeffery Hamel flow with suction/injection by homotopy analysis method." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 58, no. 2 (2019): 173-186.

[19] Mahat, Rahimah, Muhammad Saqib, Imran Ulah, Sharidan Shafie, and Sharena Mohamad Isa. "MHD Mixed Convection of Viscoelastic Nanofluid Flow due to Constant Heat Flux." *Journal of Advanced Research in Numerical Heat Transfer* 9, no. 1 (2022): 19-25.

[20] Shafique, Ahmad, Muhammad Ramzan, Zaib Un Nisa, Mudassar Nazar, and Hafeez Ahmad. "Unsteady magnetohydrodynamic flow of second grade Nanofluid (Ag-Cu) with CPC fractional derivative: Nanofluid." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 97, no. 2 (2022): 103-114. https://doi.org/10.37934/arfmts.97.2.103114

[21] Bakar, Fairul Naim Abu, and Siti Khuzaimah Soid. "MHD Stagnation-Point Flow and Heat Transfer Over an Exponentially Stretching/Shrinking Vertical Sheet in a Micropolar Fluid with a Buoyancy Effect." *Journal of Advanced Research in Numerical Heat Transfer* 8, no. 1 (2022): 50-55.

[22] Yoshimura, Ann, and Robert K. Prud'homme. "Wall slip corrections for Couette and parallel disk viscometers." *Journal of Rheology* 32, no. 1 (1988): 53-67. https://doi.org/10.1122/1.549963

[23] Sparrow, E. M., and S. H. Lin. "Laminar heat transfer in tubes under slip-flow conditions." (1962): 363-369. https://doi.org/10.1115/1.3684399

[24] Inman, Robert M. *Heat transfer for laminar slip flow of a rarefied gas between parallel plates with unsymmetrical wall heat flux*. National Aeronautics and Space Administration, 1964.

[25] Larrodé, Francisco Ezquerra, Christos Housiadas, and Yannis Drossinos. "Slip-flow heat transfer in circular tubes." *International Journal of Heat and Mass Transfer* 43, no. 15 (2000): 2669-2680. https://doi.org/10.1016/S0017-9310(99)00324-5

[26] Spillane, Sandra B. "A Study of Boundary Layer fFlow with No-slip and Slip Boundary Conditions." (2008). https://doi.org/10.21427/D7CG6Q

[27] Yu, Shiping, and Timothy A. Ameel. "Slip flow convection in isoflux rectangular microchannels." *J. Heat Transfer* 124, no. 2 (2002): 346-355. https://doi.org/10.1115/1.1447932

[28] Crane, L. J., and A. G. McVeigh. "Slip flow on a microcylinder." *Zeitschrift für angewandte Mathematik und Physik* 61, no. 3 (2010): 579-582. https://doi.org/10.1007/s00033-009-0019-x

[29] Wuest, W. "Boundary layers in rarefied gas flow." *Progress in Aerospace Sciences* 8 (1967): 295-352. https://doi.org/10.1016/0376-0421(67)90006-1

[30] Akbar, Noreen Sher, S. Nadeem, R. Ul Haq, and Z. H. Khan. "Numerical solutions of magnetohydrodynamic boundary layer flow of tangent hyperbolic fluid towards a stretching sheet." *Indian journal of Physics* 87 (2013): 1121-1124. https://doi.org/10.1007/s12648-013-0339-8

[31] Gaffar, S. Abdul, V. Ramachandra Prasad, and O. Anwar Bég. "Computational analysis of magnetohydrodynamic free convection flow and heat transfer of non-Newtonian tangent hyperbolic fluid from a horizontal circular cylinder with partial slip." *International Journal of Applied and Computational Mathematics* 1 (2015): 651-675. https://doi.org/10.1007/s40819-015-0042-x

[32] Amanulla, C. H., S. Saleem, Abderrahim Wakif, and M. M. AlQarni. "MHD Prandtl fluid flow past an isothermal permeable sphere with slip effects." *Case Studies in Thermal Engineering* 14 (2019): 100447. https://doi.org/10.1016/j.csite.2019.100447

[33] Amanulla, C. H., Abderrahim Wakif, Zoubair Boulahia, M. Suryanarayana Reddy, and N. Nagendra. "Numerical investigations on magnetic field modeling for Carreau non-Newtonian fluid flow past an isothermal sphere." *Journal of the Brazilian Society of Mechanical Sciences and Engineering* 40 (2018): 1-15. https://doi.org/10.1007/s40430-018-1385-0

[34] Amanulla, C. H., Abderrahim Wakif, Zoubair Boulahia, Syed Fazuruddin, and S. Noor Mohammed. "A study on non-Newtonian transport phenomena in MHD fluid flow from a vertical cone with Navier slip and convective heating." *Nonlinear Engineering* 8, no. 1 (2019): 534-545. https://doi.org/10.1515/nleng-2018-0065

[35] Amanulla, C. H., N. Nagendra, and M. Suryanarayana Reddy. "Numerical simulations on magnetohydrodynamic non-Newtonian nanofluid flow over a semi-infinite vertical surface with slip effects." *Journal of Nanofluids* 7, no. 4 (2018): 718-730. https://doi.org/10.1166/jon.2018.1499

[36] Amanulla, C. H., N. Nagendra, and M. Suryanarayana Reddy. "MHD flow and heat transfer in a williamson fluid from a vertical permeable cone with thermal and momentum slip effects: a mathematical study." *Frontiers in Heat and Mass Transfer (FHMT)* 8 (2017). http://dx.doi.org/10.5098/hmt.8.40

[37] Atif, S. M., S. Hussain, and M. Sagheer. "Effect of viscous dissipation and Joule heating on MHD radiative tangent hyperbolic nanofluid with convective and slip conditions." *Journal of the Brazilian Society of Mechanical Sciences and Engineering* 41 (2019): 1-17. https://doi.org/10.1007/s40430-019-1688-9

[38] Shahzad, U., M. Mushtaq, S. Farid, K. Jabeen, and R. M. A. Muntazir. "A Numerical Approach for an unsteady tangent hyperbolic nanofluid flow past a wedge in the presence of suction/injection." *Mathematical Problems in Engineering* 2021 (2021): 1-15. https://doi.org/10.1155/2021/8653091

[39] Ismael, Hajar F. "Carreau-Casson fluids flow and heat transfer over stretching plate with internal heat source/sink and radiation." *Int. J. Adv. Appl. Sci. J* 6, no. 2 (2017): 81-86. https://doi.org/10.21833/ijaas.2017.07.003

[40] Ismael, Hajar F., and N. M. Arifin. "Flow and heat transfer in a maxwell liquid sheet over a stretching surface with thermal radiation and viscous dissipation." *JP J Heat Mass Transf* 15 (2018): 847-66. http://dx.doi.org/10.17654/HM015040847

[41] Ng, K. S., and J. P. Hartnett. "Effects of mechanical degradation on pressure drop and heat transfer performance of polyacrylamide solutions in turbulent pipe flow." *Studies in Heat Transfer* 297 (1979): 307.

[42] Kwack, E. Y., and Cho Yi. "Turbulent heat transfer in circular tube flows of viscoelastic fluids." (1982). https://doi.org/10.1007/BF01322804

[43] Kwack, E. Y., and J. P. Hartnett. "Effect of diameter on critical Weissenberg numbers for polyacrylamide solutions in turbulent pipe flow." *International Journal of Heat and Mass Transfer* 25, no. 6 (1982): 797-805. https://doi.org/10.1016/0017-9310(82)90092-8

[44] Kwack, E. Y., and J. P. Hartnett. "Effect of solvent chemistry on critical Weissenberg numbers." *International Journal of Heat and Mass Transfer* 25, no. 9 (1982): 1445-1450. https://doi.org/10.1016/0017-9310(82)90138-7