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Temperature Dependent Viscosity Effect on Oscillatory Mode of Darcy-Rayleigh Convection in a Double Diffusive Binary Viscoelastic Fluid Saturated Anisotropic Porous Layer

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ABSTRACT

An anisotropic porous layer saturated with a viscoelastic double diffusive binary fluid is examined numerically for the onset of Darcy-Rayleigh convection. From below, the system is heated, while from above, it is cooled. The temperature-dependent viscosity was added to the double diffusive binary fluid, and the Galerkin expansion method was used to determine the critical Darcy-Rayleigh number. When their values are raised, the impacts of strain retardation, thermal anisotropy parameter and Dufour number slow the production of heat transfer and stabilise the system. When the values are increased, the stress relaxation, Darcy-Prandtl, mechanical anisotropy parameters, temperature dependent viscosity and Soret number accelerate the heat transfer process in convection, which destabilises the system.

1. Introduction

The viscoelastic property in a non-Newtonian fluid has given rise to interest in different fields such as science, engineering, and technology. This property exhibit both viscous and elastic characteristics when thermal motion occurs. The dissipation of the fluid characterizes the viscous nature and the energy characterized by the fluid's elastic response [1]. Shear stress is not proportional to the rate of deformation in a non-Newtonian fluid and does not follow Newton's viscosity law. It is interesting to study this kind of fluid as it can be applied in medicine where the blood modeling as a non-Newtonian fluid may help to understand the human body and improve health techniques, in the n food industry such as in the processing of ketchup and jam, extraction of rice bran protein or maybe in creating body vest for military use [2]. There are also unusual patterns

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of instability in viscoelastic fluids that are not predicted or observed in Newtonian flow. Mahat *et al.,* [3] studied the viscoelastic nanofluid in a linear circular cylinder.

Double diffusive convection was studied in a porous medium due to its importance in geophysics where groundwater usually contains salts in solution and hence both thermal expansion and solute concentration variations can produce variations in density. This phenomenon was explored by Horton and Rogers [4], proved experimentally by Morrison *et al.,* [5], and explained thoroughly by Parts [6]. Hirata *et al.,* [7] also did an experimental study on the small scaler process occurring in the global climate system where a natural double diffusive was considered under-ice melt.

Later, research was done in a more complex system where the system is saturated by an anisotropic fluid. Anisotropic is found in numerous systems in the industry and in nature. The properties of an isotropic material are uniform in all directions but in anisotropy, the properties of the material have dependent direction. Anisotropy can also be a characteristic of synthetic porous materials such as pellets used in chemical engineering as well as insulating fiber materials. Recent research into convective flow through anisotropic porous media has been documented by Filippi E and T [8], Swamy *et al.,* [9] and Rees and Storesletten [10]. The current trends on understanding the anisotropic show the relevance of understanding the characteristics of this porous layer. Nield [11] studied on a constant viscosity in a two-component fluid saturating a porous medium taking only Darcy resistance into consideration while Patil and Rudraiah [12] use the Brinkman model. The Darcy Law's were introduced by Darcy [13] and then the extension of the law is the Brinkman model introduced by Brinkman [14]. Later, Patil and Vaidyanathan [15] extended the study by to a two-component fluid saturating a porous medium where the viscosity varies with temperature, using Darcy and Brinkman models. Both models were compared and they conclude that Darcy model is more stable than the Brinkman Model, since the effects of concentration and temperature dependent viscosity on the critical Rayleigh number is small. The Darcy's Law replaced the nonlinear Navier-Stokes equations, which describe the velocities in the bulk fluid experiment with a linear set of velocity equations. The addition of a porous medium would eliminate a nonlinearity inherent in the binary fluid bulk system and thus provide a step forward to understand more complicated, previous binary fluid bulk results.

The viscoelastic flows through porous media have been the subject of theoretical interest in the industry. Kim *et al.,* [16] conduct a theoretical thermal instability analysis in a porous layer saturated with viscoelastic fluid. They found that overstability is a preferred mode for a certain range of parameters, and when the elastic parameters are used, a supercritical and stable bifurcation takes the form of the onset of convection. The initiation of thermal convection in a horizontal porous layer saturated with viscoelastic fluid using linear theory was studied by Yoon *et al.,* [17]. To examine the effects of relaxation times, a simple constitutive model is used and it is shown that oscillatory instabilities can be set before stationary modes. Bertola and Cafaro [18] use a dynamic system approach to study the instability theory of the viscoelastic fluid saturating a horizontal porous layer. Equilibrium points and their stability are expressed as a function of the relaxation and retardation parameters. Laroze *et al.,* [19] study the convection in a rotating binary viscoelastic which contributes to DNA withdrawal due to the problem's complexity. Changes in parameter values can induce a large threshold and affect the frequency variations. Malashetty *et al.,* [20] also study the binary fluid where the fluid is saturated by an isotropic porous layer. When analytically deriving the stationary and oscillatory convection, it is shown that competition between the processes of thermal, solute diffusion, and viscoelasticity exists in the oscillatory case rather than stationary.

The physical configuration viscoelastic double diffusive binary fluid layer saturated in an anisotropic porous layer with temperature dependent viscosity is considered. Due to the viscoelastic flows, the stress relaxation, λ_1 and the strain retardation, λ_2 were introduced. When only the λ_1 is taken into account, the modal is known as the Maxwell model as done by Wang and Tan [1] where the Maxwell model is applied to double diffusive convection in an isotropic porous medium.

Hilt *et al.,* [21] include the temperature dependent effect in a binary fluid but they only study the physical part where they consider the separation ratio effect. They introduce a dimensionless quantity, Γ which is the viscosity difference between the upper and lower boundaries. Results show that when a positive separation ratio (positive Soret number) in a larger Γ increases, there will be a discontinuous shift in the critical Rayleigh number and when a negative separation ratio (negative Soret number) becomes higher and Γ is moderate, it also shows a discontinuous transition from an oscillatory to a stationary instability. However, when a larger Γ is used, the transition remains continuous. Khan *et al.,* [22] also studied the effect in a nanoliquid and the impacts of heat transfer by forced convection were studied by Rebhi *et al.,* [23]. In an experimental binary mixture (water and ionic fluid), Rodr´ıguez and Brennecke [24] analyzed the combined temperature and composition dependence of both density and viscosity and Khan *et al.,* [25] studied the effects in a Williamson fluid flow. Lu and Chen [26] study on the onset of double-diffusive convection in a single direction binary solution and shows that a decrease in temperature will improve the stability of the convection. This particular study of temperature dependent viscosity in a binary fluid has been overlooked whereas this effect is important to understand the instability of convection in any system [27]. This builds our interest to explore and seeks the knowledge to understand more about the binary fluid system.

2. Mathematical Formulation

Viscoelastic and anisotropy parameters affect the onset criterion for oscillatory convection. The continuity equation from the mass conservation is

$$\nabla \cdot u = 0 \tag{1}$$

The momentum equation is using the amended Darcy law for the viscoelastic fluid of the Oldroyd type by Malashetty *et al.,* [20] which is

$$\left(1 + \overline{\lambda}_1 \frac{\partial}{\partial t}\right)\left\{\frac{\rho_0}{\varepsilon}\frac{\partial u}{\partial t} + \nabla p - \nabla \cdot [\mu(\nabla u + \nabla u^T) - \rho g]\right\} = \frac{\mu}{K}\left(1 + \overline{\lambda}_2 \frac{\partial}{\partial t}\right) \cdot u \tag{2}$$

where t is the dimensionless time, ρ is the density, ε is the porosity, p is the pressure, μ is the dynamic viscosity, g is the gravity, and K is the permeability tensor. Meanwhile, $\overline{\lambda}_1$ is the relaxation time depending on viscoelasticity and the $\overline{\lambda}_2$ is the retardation time due to the action of the porous matrix. The relaxation parameter, $\overline{\lambda}_1$ is a dimensionless number used in rheology to characterize the properties of fluid and material. A smaller value of $\overline{\lambda}_1$ define the more fluidity of the material. A diluted polymeric solution is confined between the range of [0.1,2].
The energy equation follows Nield and Kuznetsov [28]

$$\rho_0 c\left[\frac{\partial T}{\partial t} + (u \cdot \nabla)T\right] = \kappa_t \nabla^2 T + \rho c D_{TS} \nabla^2 S \tag{3}$$

and for solute concentration equation is in the form

$$\frac{\partial S}{\partial t} + (u \cdot \nabla)S = \kappa_s \nabla^2 S + D_{ST} \nabla^2 T \tag{4}$$

where c is the fluid specific heat (at constant pressure),$\kappa_t$is the thermal diffusivity, κ_s is the mass diffusivity, DTS is the Soret diffusivity, DST is the Dufour diffusivity, S is the solute concentration and T is the temperature. Soret effect (thermo-diffusion) is the mass diffusion induced by the temperature gradient and Dufour effect (diffusion-thermo) is the heat transfer induced by the concentration gradient.

The basic state of the fluid is quiescent and is given by

$$u_b = (u, v, w) = (0,0,0), T = T_b(z), \mu_b = \mu_0 f(z), p = p_b(z), \rho = \rho_b(z), S = S_b(z) \tag{5}$$

where the subscript b denotes the basic state. Then, the system is perturbed with the following form

$$(u, p, \rho, T, S) = [u_b(z) + u', p_b(z) + p', \rho_b(z) + \rho', T_b(z) + T', S_b(z) + S'] \tag{6}$$

where the primes quantities indicate the perturbed variables and are assumed to be small. Substituting Eq. (6) into Eq. (1)- Eq. (4) with the basic state solution, we obtained

$$\rho' = \rho_0[\alpha_t T' - \alpha_s S'] \tag{7}$$

By operating the curl twice on Eq. (2) and eliminating p', we obtained the nondimensional equation

$$\left(1 + \overline{\lambda_1}\frac{\partial}{\partial t}\right)\left[\frac{\mathrm{Da}}{\varepsilon \mathrm{Pr}_d}\frac{\partial}{\partial t} + \nabla^2 w\text{-f}(z)\nabla^4 w\text{-}2\frac{\partial f(z)}{\partial z}\nabla^2\left(\frac{\partial w}{\partial z}\right) - \frac{\partial^2 f(z)}{\partial z^2} \times (\nabla^2 w + 2\nabla_h^2 w)\right.$$
$$\left. -\left(1 + \overline{\lambda_1}\frac{\partial}{\partial t}\right) Ra_d \nabla_1^2 T + Rs_d \nabla_1^2 S\right] + \left(1 + \overline{\lambda}_2\frac{\partial}{\partial t}\right)\left(\nabla_1^2 + \frac{1}{\varepsilon}\frac{\partial^2}{\partial z^2}\right) w = 0 \tag{8}$$

where $Pr_d = \frac{\varepsilon Pr}{Da}$ is the Darcy-Prandtl number,$Ra_d = \frac{RaK_z}{d^2}$ is the Darcy-Rayleigh number, and $Rs_d = \frac{RsK_z}{d^2}$ is the solutal Darcy-Rayleigh number.

Substitute the normal mode of Eq. (9) into Eq. (3), Eq. (4) and Eq. (8),

$$(w', T', S') = [W(z), \Theta(z), \Phi(z)]e^{i(\alpha_x x + \alpha_y y) + i\omega t} \tag{9}$$

we have

$$(1 + \lambda_1\omega)\left[\frac{\omega}{\mathrm{Pr}_d}(D^2 - \alpha^2)W - \overline{f}(D^2 - \alpha^2)^2 W - D^2\overline{f}(D^2 - \alpha^2)W - 2D\overline{f}(D^2 - \alpha^2)DW\right.$$
$$\left. - Ra_d\alpha^2\Theta + Rs_d\alpha^2\Phi\right] + (1 + \lambda_2\omega)\left(\frac{1}{\xi}D^2 - \alpha^2\right)W = 0 \tag{10}$$

$$W + (D^2 - \eta\alpha^2)\Theta + Df(D^2 - \alpha^2)\Phi = 0 \tag{11}$$

$$W + Sr(D^2 - \alpha^2)\Theta + Le(D^2 - \alpha^2)\Phi = 0 \tag{12}$$

where $\lambda_1 = \frac{Dz}{d^2}\bar{\lambda}_1$ is the relaxation parameter, $\lambda_2 = \frac{Dz}{d^2}\bar{\lambda}_2$ is the retardation parameter. Both boundaries were set to be isosolutal, isothermal and free-free representing the lower-upper boundaries.

$$W = D^2W = \Theta = \Phi = 0 \tag{13}$$

Then, by using the boundary conditions (13), we solved Eq. (10), Eq. (11) and Eq. (12) respectively. By adopting the Galerkin techniques to find an approximate eigenvalue solution, we extract the Darcy-Rayleigh number, Ra_d , from

$$\begin{vmatrix} a_{11} b_{11} c_{11} \\ d_{11} e_{11} f_{11} \\ g_{11} h_{11} i_{11} \end{vmatrix} = 0 \tag{14}$$

Where

$$a_{11} = (1+\lambda_1\omega)\frac{\omega}{P\,r}\langle(DW)^2\rangle - \alpha^2\langle W^2\rangle - \langle(D^2W)^2\rangle - 2\alpha^2\langle(DW)^2\rangle + \alpha^4\langle W^2\rangle$$
$$- 2B\langle(D^2W)DW\rangle - \alpha^2\langle DW^2\rangle + B^2\langle(DW)^2\rangle + \alpha^2\langle W^2\rangle + (1+\lambda_1\omega)\frac{1}{\xi}\langle(DW)^2\rangle$$
$$- \alpha^2\langle W^2\rangle,$$
$$b_{11} = -\alpha^2 Ra_d\langle W\Theta\rangle,$$
$$c_{11} = \alpha^2\frac{Rs}{Le}\langle W\Phi\rangle,$$
$$d_{11} = \langle W\Theta^2\rangle,$$
$$e_{11} = -\omega\langle\Theta^2\rangle + \langle(D\Theta)^2\rangle - \eta\alpha^2\langle\Theta^2\rangle,$$
$$f_{11} = Df(\langle D\Theta\rangle\langle D\Phi\rangle - \alpha^2\langle\Theta\Phi\rangle),$$
$$g_{11} = \langle W\Phi\rangle,$$
$$h_{11} = Sr(\langle D\Theta\rangle\langle D\Phi\rangle - \alpha^2\langle\Theta\Phi\rangle),$$
$$i_{11} = -\omega\langle\Phi^2\rangle + \frac{1}{Le}[\langle(D\Phi)^2\rangle - \alpha^2\langle\Phi^2\rangle],$$

and < …> represent the integration from z = 0 to z = 1.

In the case when both boundaries are isosolutal, isothermal and the lower-upper boundaries are set to be free-free, the respective chosen trial functions to get the critical Darcy-Rayleigh number, Ra_{dc} are

$$W = \Theta = \Phi = sin(z\pi) \tag{15}$$

Oscillatory stability mode is obtained by separating the eigenvalue equations into the real and imaginary parts. Therefore, we obtained the following expression for the thermal Darcy-Rayleigh number, Ra_d in the form

$$Ra_d = \Delta_1 + i\omega_i\Delta_2 \tag{16}$$

For oscillatory onset $\Delta_2 = 0(\omega_i \neq 0)$and this gives a dispersion relation of the form (on dropping the subscript *i*)

$$b_1(\omega^2)^2 + b_2(\omega^2)^2 + b_3 = 0. \tag{17}$$

Then,

$$Ra_d^{osc} = \alpha_0(\alpha_1 + \omega^2\alpha^2). \tag{18}$$

To obtain the oscillatory onset neutral solutions, the steps are as follows: first, the number of positive equation solutions Eq. (17) should be calculated. If there is none, then there can be no oscillating instability. If there are two, then the minimum (over α^2) of Eq. (18) with ω^2 given by Eq. (17) gives the oscillatory Darcy-Rayleigh number, Ra_d^{osc}. Since Eq. (17) is quadratic in ω^2, it can result in more than one positive value of ω^2 for fixed values of the parameters B, Sr, Df, Le, Rs, ξ ,η, λ_1 and λ_2. The numerical solution obtained in this study gives only one positive value of ω^2 which signify that only one positive oscillatory solution exists. To evaluate the influence of oscillatory convection on the onset of convection, various values of physical parameters is substitute for $\omega^2(> 0)$from (18).

3. Results and Discussion

The onset of Darcy-Rayleigh convection in a viscoelastic double diffusive binary fluid layer saturated in an anisotropic porous with temperature-dependent viscosity is investigated. Only the oscillatory (over-stability) curves presented in this research. Hence, we used Ra_d instead of Ra_{osc} to represent the Darcy-Rayleigh number obtained in this chapter. Figure 1 and Figure 2 shows the additional parameters involved in the case of viscoelastic fluid which are the stress relaxation, λ_1 and the strain retardation, λ_2 parameters. It is shown that as the stress relaxation, λ_1 increased, the critical Darcy-Rayleigh number decreased. In this figure, other parameters were set to be Le = 2, Rs = 100, ξ = 0.5, η = 0.3, B = 0.3, Sr = Df = 0.005, λ_2 = 0.1 and Pr_d = 10. As for the strain retardation, λ_2, the onset of convection increased as the value increased. Figure 3 shows the effect of Darcy-Prandtl number, Pr_d when the Prandtl number is increased. We use Pr_d = 10 as the fixed value in other figures as this is the typical value in a dilute DNA suspension as stated by Kolodner (1998). In these three figures, besides the anisotropic case, we also represent the isotropic case where η = ξ = 1.

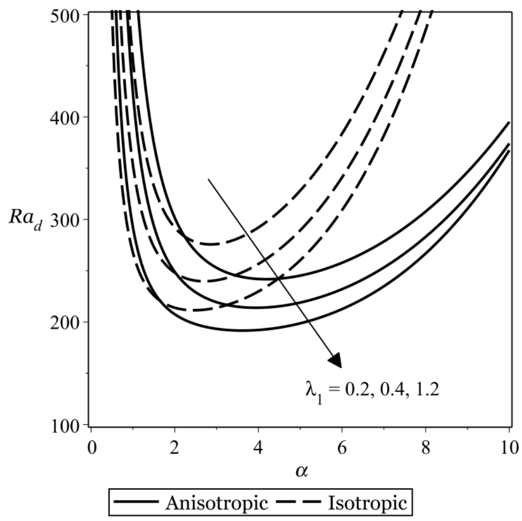

Fig. 1. Variation of Ra_d with α for different values of λ_1

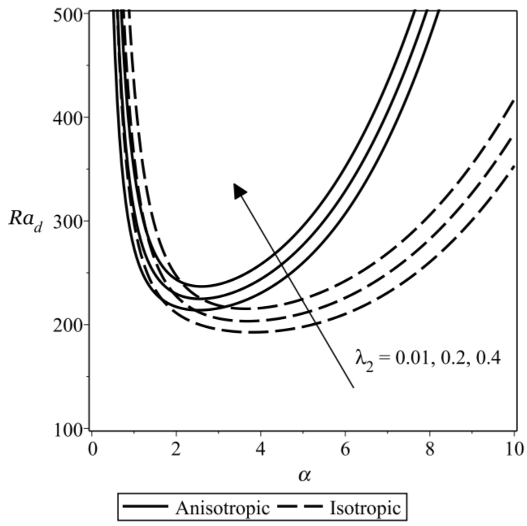

Fig. 2. Variation of Ra_d with α for different values of λ_2

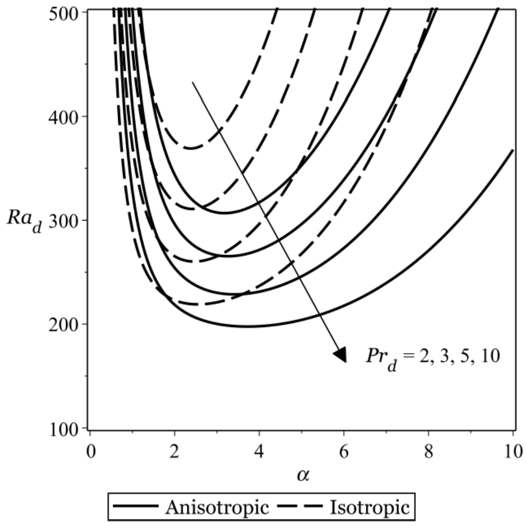

Fig. 3. Variation of Ra_d with α for different values of Pr_d

Figure 4 and Figure 5 show the effect of the anisotropic porous medium parameter where the characteristics of the convection is similar as shown in the stationary case. In other words, in the exchange of the equilibrium regime, the heat transfer characteristics are similar to those of the Newtonian case. As the mechanical anisotropy, ξ increased, the marginal stability curves shift downward and as the thermal anisotropy, η increased, the marginal stability curves shift upward. In Figure 4, η = 0.3 and in Figure 5, ξ = 0.5 while the other paramaters are Le = 2, Rs = 100, B = 0.3, Sr = Df = 0.005, λ_1 = 0.8 and Pr_d = 10. Figure 6 shows the variarion of Ra_d with α for different values of B with Le = 2, Rs = 100, ξ = 0.5, η = 0.3, Sr = Df = 0.005, λ_1 = 0.8, λ_2 = 0.1 and Pr_d = 10. As B increases, the values of Ra_{dc} decrease showing that B has the effect of destabilizing the system.

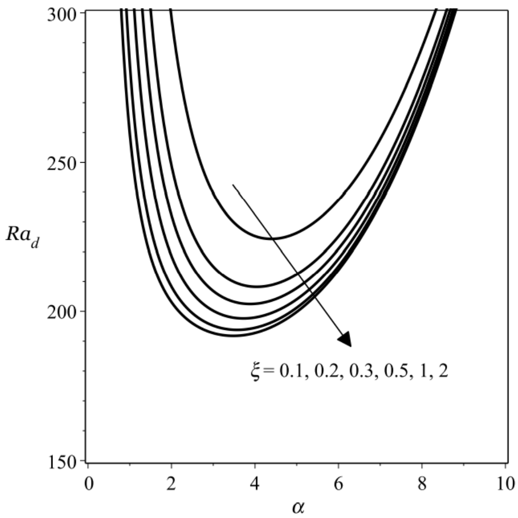

Fig. 4. Variation of Ra_d with α for different values of ξ

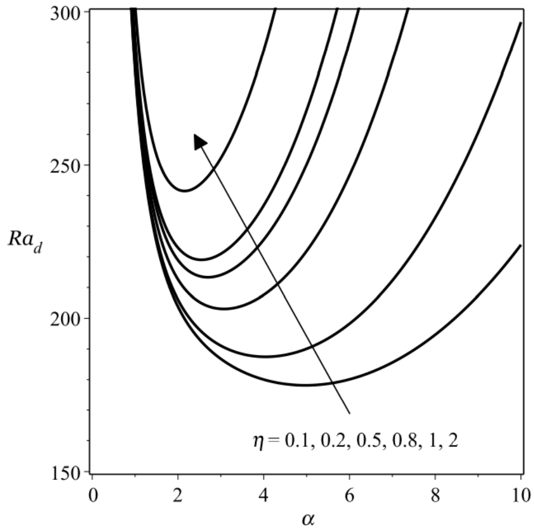

Fig. 5. Variation of Ra_d with α for different values of η

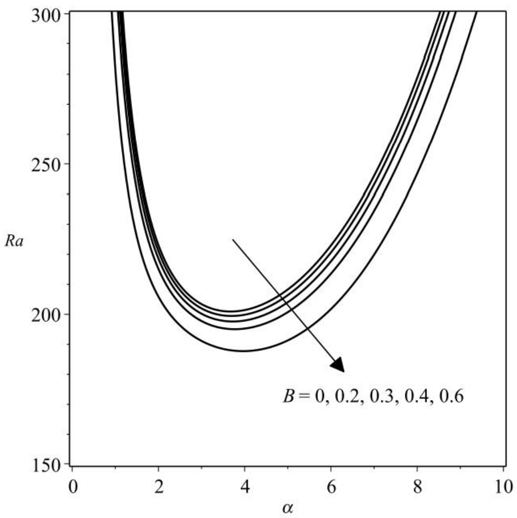

Fig. 6. Variation of Ra_{dc} with α for different values of B

Figures 7 - 12 concerned on various parameters on the critical Darcy-Rayleigh number, Ra_{dc}. Figure 7 reveals the effects of strain retardation, λ_2 to the critical Rayleigh number, Ra_c with different values of stress relaxation, λ_1. It shows that as λ_2 increases, Ra_c will increases. We also note that as the he gradient is steeper when the value of λ_1 is lower. This indicate that λ_2 has more significant effect in a lower λ_1 compare to a higher value of λ_1. Figure 8 shows the variation of Ra_{dc}. with λ_2 for different values of Prandtl number, Pr_d. Ra_{dc}. will increase as λ_2 increased and Ra_{dc}. decreases as Pr_d increased.

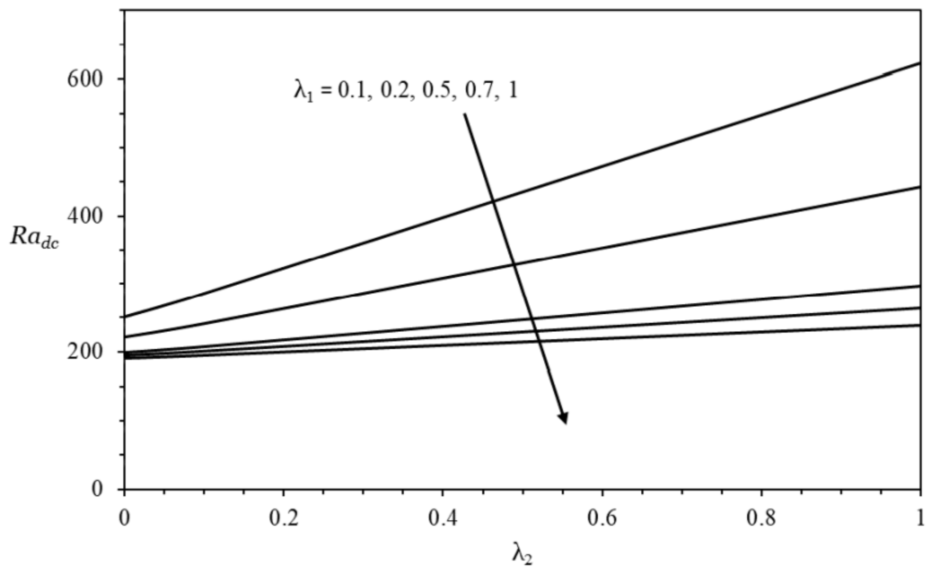

Fig. 7. The effect of λ_2 on the stability of the Ra_{dc} for different values of λ_1

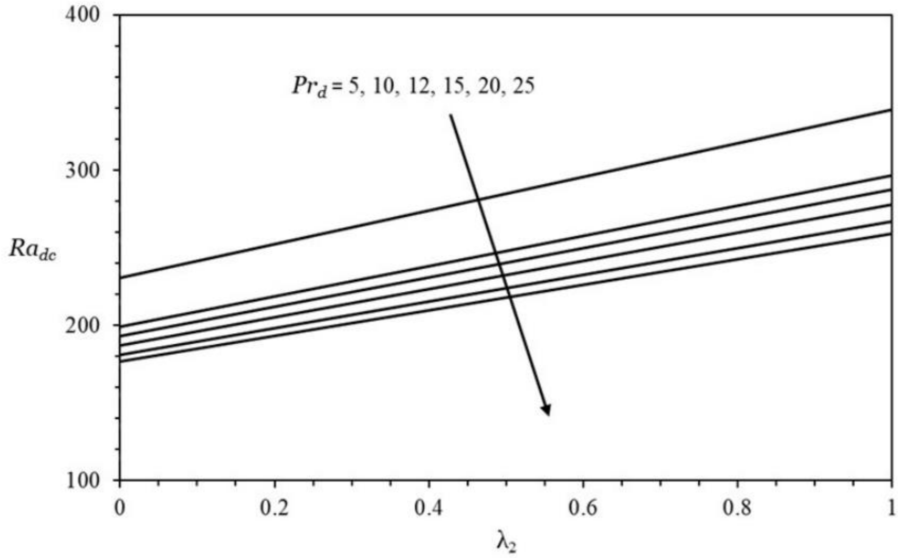

Fig. 8. The effect of λ_2 on the stability of the Ra_{dc} for different values of Pr_d

Figure 9 presents the variations of stress relaxation, λ_1 versus the critical Darcy-Rayleigh number, Ra_{dc} for different values of thermal anisotropy parameter, ξ. As ξ increased, Ra_{dc} increased making the system becomes more destabilize and as λ_1 increased, Ra_{dc} decreased. When $\lambda_2 = 0$, the system where viscoelastic fluid considered is reduced to a Maxwell fluid where only λ_1 is considered. We note that viscoelastic have higher stability compared to the Maxwell fluid system. Figure 10 shows the variation of temperature dependent viscosity, B and η to the onset of convection where when both parameters increased, Ra_{dc} will decrease. The figure also represents the isotropic case where ξ = η = 1. The last two figures, 11 and 12 shows the coupled effect of double diffusive where Soret parameter destabilize the system and the Dufour parameter stabilizes the system.

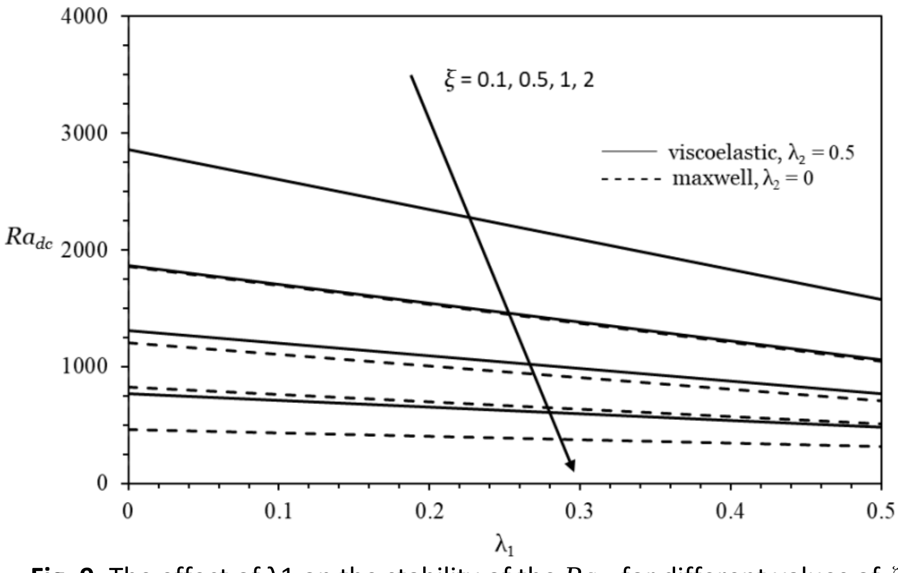

Fig. 9. The effect of λ1 on the stability of the Ra_{dc} for different values of ξ

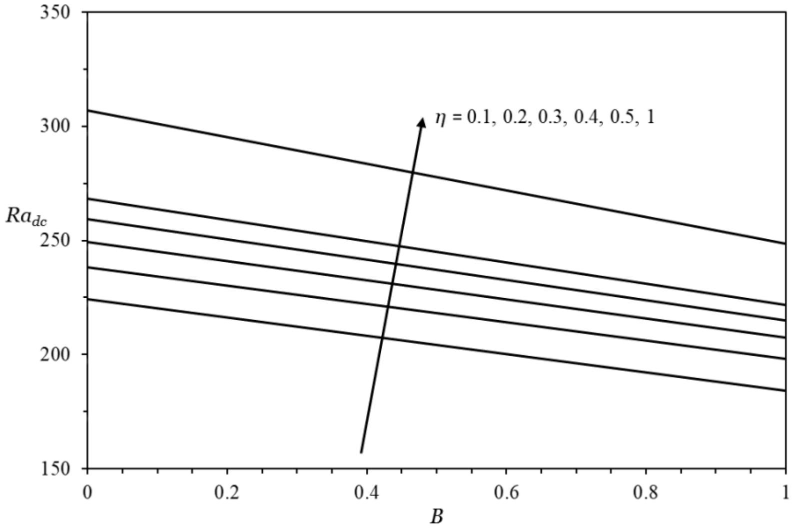

Fig. 10. The effect of B on the stability of the Ra_{dc} for different values of η

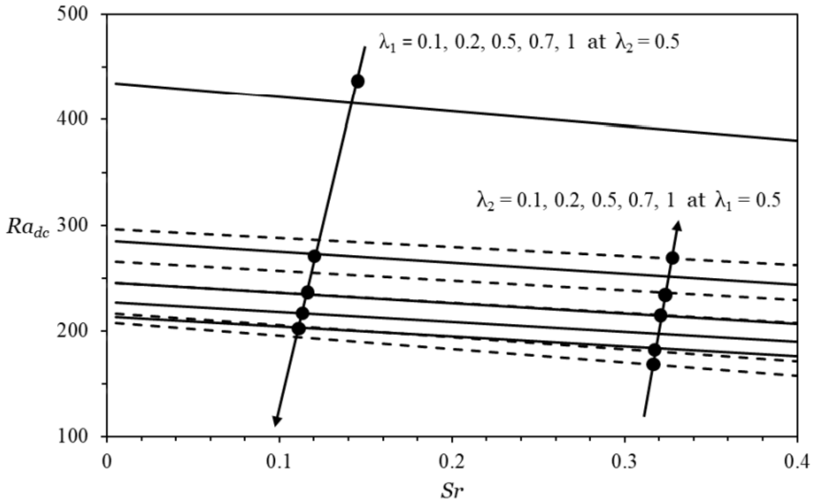

Fig. 11. The effect of Sr on the stability of the Ra_{dc} for different values of λ_1 and λ_2

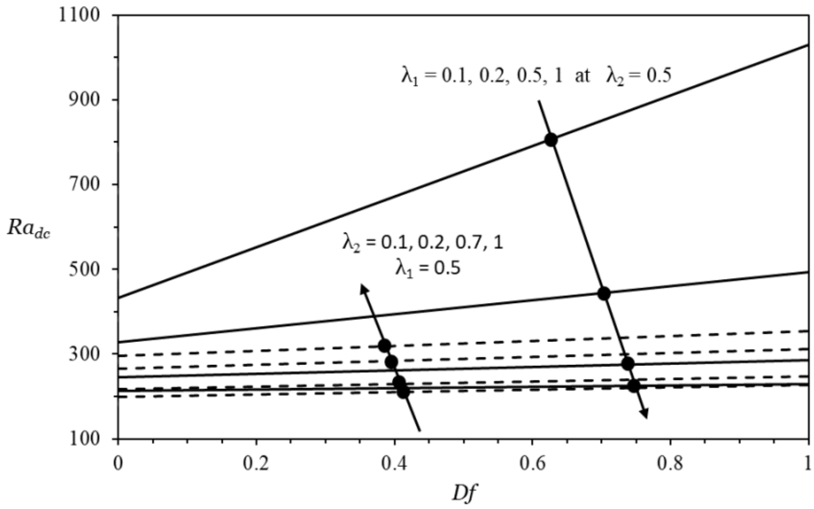

Fig. 12. The effect of D f on the stability of the Ra_{dc} for different values of λ_1 and λ_2

4. Conclusions

The findings from the mathematical model indicate that when the strain retardation, thermal anisotropy, and the Dufour number are increased, the system becomes stabilizing and slowing down the formation of heat transfer. While the system becomes unstable and the heat transfer mechanism in convection accelerated quickly as the stress relaxation, Darcy-Prandtl, mechanical anisotropy, temperature dependent viscosity, and Soret parameter grow increased.

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