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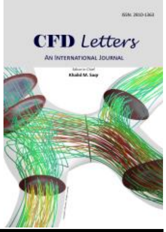


# Effect of Water-based Alumina-copper MHD Hybrid Nanofluid on a Power-law Form Stretching/shrinking Sheet with Joule Heating and Slip Condition: Dual Solutions Study

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**ABSTRACT**

The application of hybrid nanofluid is now being employed to augment the efficiency of heat transfer rates. A numerical study was conducted to investigate the flow characteristics of water-based-alumina copper hybrid nanofluids towards a power-law form stretching/shrinking sheet. This study also considered the influence of magnetic, Joule heating, and thermal slip parameters. This study is significant because it advances our understanding of hybrid nanofluids in the presence of magnetic fields, power-law form stretching/shrinking sheet, and heat transfer mechanisms, providing valuable insights for optimizing and innovating thermal management systems in various industrial applications such as polymers, biological fluids, and manufacturing processes like extrusion, plastic and metal forming, and coating processes. The main objective of this study is to examine the impact of specific attributes, including suction and thermal slip parameters on temperature and velocity profiles. In addition, this exploration examined the reduced skin friction and reduced heat transfer in relation to the solid volume fraction copper and magnetic effects on shrinkage sheet and thermal slip parameter on suction effect. To facilitate the conversion of a nonlinear partial differential equation into a collection of ordinary differential equations, it is necessary to incorporate suitable similarity variables into the transformation procedure. The MATLAB bvp4c solver application is utilized in the conclusion process to solve ordinary differential equations. No solution was found in the sort of when $\epsilon < \epsilon_{ci}$, and $\gamma < \gamma_{ci}$. As the intensity of the Eckert number increases, the temperature profile and boundary layer thickness also increase. The reduced heat transfer rate upsurged in both solutions for solid volume fraction copper for shrinking sheet, while the opposite actions can be noticed in both solutions for thermal slip parameter for suction effect. Finally, the study conducted an analysis to identify two distinct solutions for shrinking sheet and suction zone, while considering different parameter values for the copper volume fractions, magnetic and thermal slip condition effect.

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## 1. Introduction

The boundary layer plays a crucial role in thermal management and heat transfer, controlling the behavior of fluids near solid surfaces, influencing drag and lift powers, facilitating the control and optimization of fluid flow systems, and providing insights into turbulent flows. Its comprehension and manipulation are essential for a wide range of applications and industries, improving design, efficiency, and performance across many engineering specialties. Initially, Sakiadis [1] was the one who came up with the idea of a steady two-dimensional boundary layer on a stretched plane. Crane [2] applied Sakiadis' ideas to flow on two-dimensional linear stretching surfaces and exponential profiles, which allowed for further development and expansion of Sakiadis' research.

Choi *et al.,* [3] came up with the term "nanofluid" in order to describe a mixture that was made by dissolving particles in a liquid that was considered to be a typical or base. Over the last few decades, researchers from a wide range of fields have committed their efforts to the development of increased energy exchange fluids. Enhancing the effectiveness of heat transport has been the key focus of this research. A hybrid nanofluid, which is a novel type of nanofluid, has come into existence as a consequence of this research. A hybrid nanofluid is comprised of numerous types of nanoparticles, in contrast to traditional nanofluids, which are made up of individual nanoparticles from different types (Fadhel *et al.,* [4]). According to Rasool *et al.,* [5], hybrid nanofluids have a number of major benefits. More specifically, these fluids are utilized in a variety of heat transfer applications, including warming plate modifications, heat segments, pipe-formed warmth heat exchangers, minimal channel radiators, etc. The study by Devi and Devi [6] investigated two-dimensional stretch surfaces to increase the heat transmission rate using an aluminum oxide/copper hybrid nanofluid. They were successful in achieving their objective, which was to demonstrate that the efficiency of the rate of heat transfer when compared to nanofluids or conventional fluids is significantly higher. Moreover, Aly and Pop [7] considered the possibility of hybrid nanofluid being produced by a constant flow across a stretched or shrinking plate. Furthermore, a great deal of research has been conducted on nonlinear stretching and shrinking sheets. The surface velocity factor is used to describe the non-linear sheet, whereas the power-law or exponential form is used to describe the sheet. The velocity of the sheet condition is assumed in this study to follow a power law with an exponent of 1/3. Numerous researchers studied power law form, including Bataller [8], Cortell [9], Ferdows *et al.,* [10], Rashidi *et al.,* [11], and Raju *et al.,* [12]. Apart from that, the numerical examination of a nanofluid has been addressed to several parameter-dependent studies utilizing a variety of models from [13-27].

The impact of slip factor on fluid flow structure, particularly for hybrid nanofluids, has not been given a great deal of consideration, according to a comprehensive analysis of the literature that has been presented. Several applications of necessary fluids, such as the polishing of heart regulators, the maintenance of prosthetic heart regulators, and the cleaning of inner cavities, are examples of applications that demonstrate boundary slip factor (Jamil *et al.,* [28]). The magnetic steady flow, including slip conditions for hybrid nanofluids, was investigated by Iftikhar *et al.,* [29]. Additionally, Tshivhi *et al.,* [30] investigated the exponential stretching sheet for a copper nanofluid based on water, taking into account the effects of slip, heat source, magnesium hydroxide, and thermal radiation. The magnetic impact of hybrid nanofluid under slip conditions was investigated by Yan *et al.,* [31]. Moreover, Asghar *et al.,* [32] examined the consequence of slip conditions and heat generation and absorption via magnetic vertical mixed convection shrinking sheet. Additional research, including the incorporation of the slip condition effects are included in [33-37].

Magnetohydrodynamics is known as MHD. It investigates how electrically conducting liquids such as plasmas, ionized gases, and liquid metallic elements behave and interact with magnetic fields. The behaviour of these conducting fluids can be described using MHD, which integrates concepts from the fields of electromagnetism and fluid dynamics. There are many diverse applications for MHD in a variety of fields. It is utilized in the discipline of astrophysics for the purpose of investigating phenomena such as solar flares, star magnetic fields, and the dynamics of cosmic plasma. In the field of engineering, MHD is important for the design of magnetically confined devices, plasma systems for propulsion, and fusion reactors that operate with plasma propulsion (Teh and Asghar [38]). The impacts of radiation on unsteady magnetohydrodynamic hybrid nanofluid flow over a stretching and shrinking sheet were investigated by Lund *et al.,* [39]. Alzabut *et al.,* [40] studied the MHD in an enclosure through convection flow with mathematical analysis. Moreover, Farooq *et al.,* [41] explored the impacts of bioconvection flow of MHD hybrid nanofluid over a stretched surface with buoyancy effect. Additional research on MHD was conducted and assessed employing numerous fluid flow models and characteristics can be seen in these references [42-46].

Ohmic heating, often known as Joule heating, is another name for the process of producing heat by passing an electric current through a conductor. Joule heating was utilized in a wide variety of applications, including but not limited to incandescent light bulbs, cartridge heaters, resistance ovens, electric warmers, food processing equipment, soldering irons, electric fuses, electric stoves, and electrical appliances (Yashkun *et al.,* [47]). Hossain and Gorla [48] conducted research on the two-dimensional MHD boundary layer steady flow, taking into account the influence of Joule heating and mixed convection. The effect of Joule heating on the two-dimensional material heat transfer (MHD) steady flow of Burgers' fluid over a stretched surface was investigated by Hayat *et al.,* [49]. Khashi'ie *et al.,* [50] investigated the heat transmission of a two-dimensional MHD steady flow along a radially stretching/shrinking surface by the impact of suction and Joule heating in hybrid nanofluid. Moreover, Shoaib *et al.,* [51] looked into the influence of Joule heating, viscous dissipation, and thermal radiation along a rotating disc in a hybrid nanofluid. Several other studies in hybrid nanofluid with the impacts of Joule heating can be found in [52-56].

Waini *et al.,* [57] investigated heat transfer and hybrid nanofluid steady flow scenarios that used a nonlinear stretching/shrinking surface while it was explored to radiation. However, they did not take into consideration the influence of MHD, Joule heating and thermal slip factor conditions. In spite of this, the current work makes different an effort to fill the gap that was identified in Waini *et al.,* [57] by utilizing the Tiwari and Das [58] model in order to investigate the impact of radiation, heat source/sink, and thermal slip factor. As a result, a novel model of two-dimensional heat transfer hybrid nanofluid flow has been formed. This simulation is based on a power-law structure with an exponent of 1/3 (nonlinear) stretch/shrinking sheet. It incorporates the sensation of MHD, Joule heating and thermal slip factor conditions. For the sake of this particular investigation, the hybrid nanofluid that is being considered is composed of particles of copper and alumina, with water serving as the typical fluid. This research is significant because it advances our understanding of hybrid nanofluids in the presence of magnetic fields, power-law form stretching/shrinking sheet, and heat transfer processes, providing valuable insights for optimizing and innovating thermal management systems in a variety of industrial applications, including polymers, biological fluids, and manufacturing processes such as extrusion, plastic and metal forming, and coating. This main objective of this study to assess the varied effects of solid volume fraction of copper, magnetic field and thermal slip conditions for reduced skin friction and reduced heat transfer in the face of surface stretching/shrinking and suction effect. Furthermore, consideration is given to the velocity and temperature profiles in relation to the suction, and Joule heating effects. Additionally, the computational results of this work are compared to those of previous studies. According to the author's best knowledge, the findings of this study have not been investigated or published by any other researcher, which is what makes the novel possible.

The following research questions focus on the necessity to investigate the fluid flow problem that is presently being provided.

i. What is the effect of MHD?
ii. How many solutions are obtained in this fluid flow problem?
iii. What is the effect of thermal slip parameter?
iv. What is the effect of solid volume fraction alumina ($Al_2O_3$) and copper ($Cu$) are suspended in base fluid water ($H_2O$) for hybrid nanofluid?

## 2. Methodology
### *2.1 Mathematical Model and Formulation*

Figure 1 is an illustration of the steady flow in two dimensions over a sheet that is nonlinear stretching and shrinking with magnetic that appears in hybrid nanofluids. Where the x-axis and the y-axis are Cartesian coordinates, the x-axis is measured together with the sheet, the y-axis is normal to it, and the sheet which is positioned at $y = 0$ in the coordinate system are the entirety of this type of coordinate system. As stated by Jaafar *et al.,* [59], the surface velocity $u_w(x) = (v_f/\ l^{4/3})x^{1/3}$, which is represented by $v_f$, and the characteristic length, which is marked by $l$. Additionally, it is also assumed that the temperature at the surface, which is represented by the symbol $T_w$, does not change. On the other hand, the temperature of the fluid in the far field is maintained at a constant value, which is indicated by the symbol $T_\infty$. In addition, according to Jaafar *et al.,* [59], the term $B(x) = B_0 {v_f}^{1/2}/x^{\frac{1}{3}} l^{\frac{2}{3}}$ represents a uniform magnetic field, with $B_0$ representing the constant strength of the magnetic field.

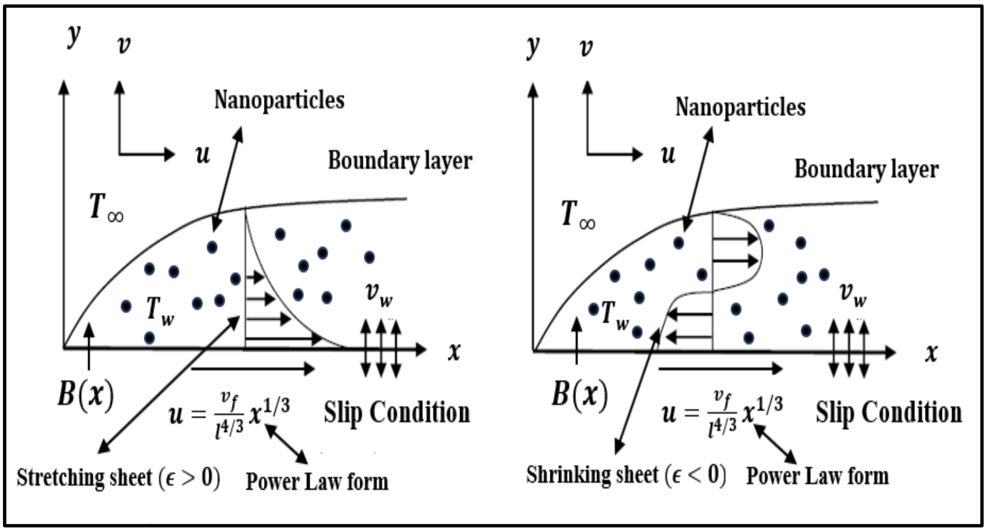


**Fig. 1.** The physical model and coordinate structure

The following equations are the governing equations for fluid flow models of hybrid nanofluids.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{hnf}}{\rho_{hnf}}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma_{hnf}}{\rho_{hnf}}B^2 u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{hnf}}{(\rho c_p)_{hnf}}\frac{\partial^2 T}{\partial y^2}+\frac{\sigma_{hnf}}{(\rho c_p)_{hnf}}B^2u^2 \tag{3}$$

The boundaries are as follows.

$$u=u_w(x)\epsilon,\ v=v_w(x), T=T_w+\chi v_f\frac{\partial T}{\partial y}\text{ as } y=0 \tag{4}$$

$$u\to 0,\ T\to T_\infty,\ \text{as}\ y\to\infty.$$

In taking into consideration the hybrid nanofluid along the $x$ and $y$ axis, the components of the velocities are represented by the letters $u$ and $v$, respectively. The term $T$ is used to represent the temperature of the hybrid nanofluid. Furthermore, $v_w(x)=\frac{-2}{3}\frac{v_f\gamma}{l^{2/3}x^{1/3}}$ means the mass velocity flux. $\gamma$ represents the suction $\gamma>0$ and injection $\gamma<0$ parameter. The surfaces shown by stretching $\epsilon>0$, shrinking $\epsilon<0$, and static $\epsilon=0$ respectively. Additionally, the slip parameter condition is represented by $\chi$.

Additionally, $k_{hnf}$ is equivalent to the thermal conductivity, $\rho_{hnf}$ represents the density, $\mu_{hnf}$ is the dynamic viscosity, $\sigma_{hnf}$ shows electrical conductivity, and $(\rho c_p)_{hnf}$ presents the heat capacity of the hybrid nanofluid. The subscripts $f$, $nf$, $hnf$, $Al_2O_3$, and Cu denote standard fluid, nanofluid, hybrid nanofluid, solid nanoparticle $S1$, and solid nanoparticle $S2$. Table 1 presents the thermophysical properties of the water, alumina and copper nanoparticles, Table 2 shows the thermophysical properties of hybrid nanofluid.

**Table 1**
The thermophysical properties of water, alumina, and copper nanoparticles (Waini *et al.,* [57])

| | $Pr$ | $k$ (W/mK) | $\sigma(S/m)$ | $\rho$ (kg/m$^3$) | $c_p$ (J/kgK) |
|---|---|---|---|---|---|
| Water ($H_2O$) | 6.2 | 0.613 | 0.05 | 997.1 | 4179 |
| Alumina ($Al_2O_3$) | - | 40 | $3.69\times10^7$ | 3970 | 765 |
| Copper (Cu) | - | 400 | $5.96\times10^7$ | 8933 | 385 |

**Table 2**
Thermophysical properties of hybrid nanofluid (Waini *et al.,* [57]).

| Names | Properties |
|---|---|
| Dynamic Viscosity | $\mu_{hnf}=\frac{\mu_f}{(1-\phi_{Cu})^{2.5}(1-\phi_{Al_2O_3})^{2.5}}$ |
| Density | $\rho_{hnf}=(1-\phi_{Cu})\left[(1-\phi_{Al_2O_3})\rho_f+\phi_{Al_2O_3}\rho_{Al_2O_3}\right]+\phi_{Cu}\rho_{Cu}$ |
| Electrical conductivity | $\sigma_{hnf}=\frac{\sigma_{Cu}+2\sigma_{nf}-2\phi_{Cu}(\sigma_{nf}-\sigma_{Cu})}{\sigma_{Cu}+2\sigma_{nf}+\phi_{Cu}(\sigma_{nf}-\sigma_{Cu})}\times(\sigma_{nf})$, where $(\sigma_{nf})=\frac{\sigma_{Al_2O_3}+2\sigma_f-2\phi_{Al_2O_3}(\sigma_f-\sigma_{Al_2O_3})}{\sigma_{Al_2O_3}+2\sigma_f+\phi_{Al_2O_3}(\sigma_f-\sigma_{1Al_2O_3})}\times(\sigma_f)$ |
| Heat capacity | $(\rho c_p)_{hnf}=(1-\phi_{Cu})\left[(1-\phi_{Al_2O_3})(\rho c_p)_f+\phi_{Al_2O_3}(\rho c_p)_{Al_2O_3}\right]+\phi_{Cu}(\rho c_p)_{Cu}$ |
| Thermal conductivity | $k_{hnf}=\frac{k_{Cu}+2k_{nf}-2\phi_{Cu}(k_{nf}-k_{Cu})}{k_{Cu}+2k_{nf}+\phi_{Cu}(k_{nf}-k_{Cu})}\times(k_{nf})$,where $k_{nf}=\frac{k_{Al_2O_3}+2k_f-2\phi_{Al_2O_3}(k_f-k_{Al_2O_3})}{k_{Al_2O_3}+2k_f+\phi_{Al_2O_3}(k_f-k_{Al_2O_3})}\times(k_f)$ |

The following similarity variables are taken into consideration (Waini *et al.,* [57]):

$$u = \frac{\nu_f}{l^{4/3}} x^{1/3} F'(\zeta), v = -\frac{1}{3}\frac{\nu_f}{l^{2/3} x^{1/3}}[2F(\zeta) - \zeta F'(\zeta)], \zeta = y\frac{x^{-1/3}}{l^{2/3}},\ \theta(\zeta) = \frac{T - T_\infty}{T_w - T_\infty}. \quad (5)$$

Eq. (1) is totally fulfilled. Eq. (2) and Eq. (3), as well as the boundary conditions that were taken in Eq. (4), have been expressed into the following ordinary differential equations by the utilization of the similarity variables that are specified by Eq. (5).

$$3\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f} F''' + 2F''F - (F')^2 - 3\frac{\sigma_{hnf}/\sigma_f}{\rho_{hnf}/\rho_f} MF' = 0, \quad (6)$$

$$\frac{3}{Pr(\rho c_p)_{hnf}/(\rho c_p)_f}\left[\frac{k_{hnf}}{k_f}\right]\theta'' + 2\theta' F + \frac{\sigma_{hnf}/\sigma_f}{(\rho c_p)_{hnf}/(\rho c_p)_f} MEc\left(F'^2\right) = 0, \quad (7)$$

$$F(0) = \gamma, F'(0) = \epsilon, \theta(0) = 1 + \beta_T\theta'(0) \quad (8)$$

$$F'(\zeta) \to 0;\ \ \theta(\zeta) \to 0 \text{ as } \ \zeta \to \infty.$$

Moreover, $M = \frac{B_0^2\sigma_f}{\rho_f}$ represents the magnetic parameter, $Pr = \frac{\mu_f (c_p)_f}{k_f}$ is Prandtl number, and $Ec = \frac{u_w{}^2}{T_w - T_\infty (c_p)_f}$ represented by Eckert number. Furthermore, $\beta_T = \chi_1 l^{-1/3}\left(\frac{2\nu_f}{3l}\right)^{0.5}$ expresses thermal slip condition, where the initial amounts of the thermal feature are represented by $\chi_1$.

The local skin friction coefficient $C_F$ and Nusselt number $Nu_x$ are significant physical quantities that are denoted as.

$$C_F = \frac{\mu_{hnf}}{u_w^2\rho_f}\left(\frac{\partial u}{\partial y}\right)_{y=0},\ Nu_x = \frac{x}{(T_w - T_\infty)k_f}\left[-k_{hnf}\left(\frac{\partial T}{\partial y}\right)_{y=0}\right], \quad (9)$$

Through the utilization of Eq. (5) and Eq. (9) and the subsequent terms derived

$$(Re_x)^{1/2} C_F = \frac{\mu_{hnf}}{\mu_f} F''(0),\ \ (Re_x)^{-1/2} Nu_x = -\left[\frac{k_{hnf}}{k_f}\right]\theta'(0), \quad (10)$$

where $Re_x = \frac{x\, u_w}{\nu_f}$ stated the local Reynolds number.

## 3. Results

The numerically based bvp4c solver in MATLAB is applied to achieve a numerical solution of the higher-order nonlinear ordinary differential equations (ODEs) specified in Eq. (6) – Eq. (7), with the boundary conditions stated in Eq. (8). The bvp4c solver is a finite difference procedure that utilizes the three-stage Lobatto IIIA formula. The MATLAB software's bvp4c function is coded. The function handle @Tiwari_ode, into which the Eq. (6) – Eq. (7) are coded, is contained in the solver's syntax, sol = bvp4c (@Tiwari_ode, @Tiwari_bc, solinit, options). After that, the function handle @Tiwari_bc is coded including the boundary condition Eq. (8). The points of the initial approximation and initial mesh of the solution at mesh points are coded in solinit, though the integration parameter is optional for the options. The solver is executed, and its findings are presented in the form of numerical solutions and graphical representations. When a different initial guess value in the intended solinit offers other solutions that satisfy the boundary conditions, dual solution happens to the fluid flow problem. In order to create the two solutions depicted in the figures, the initial guesses for reduced skin friction $F''(0)$ and reduced heat transfer $-\theta'(0)$ were utilised separately. Therefore, the temperature and velocity profiles obeyed the boundary condition $\zeta \to \infty$. In order to demonstrate that the approach is reliable, the conclusions that are now being observed are assessed by applying the knowledge that was gathered from previous studies. Hence, Table 3 explains the comparison conclusions of $F''(0)$ for different suction $\gamma$ parameter values corresponding to $\phi_{Al_2O_3} = \phi_{Cu} = \beta_T = R = 0, Pr = 6.2$, and $\epsilon = 1.0$. In addition, we assess the results of $F''(0)$ that were obtained by the work of Cortell [9], Ferdows *et al.,* [10]. Rashidi *et al.,* [11], and Waini *et al.,* [57]. Furthermore, Table 4 provides an evaluation of the results of $-\theta'(0)$ in accordance with the equation $\phi_{Al_2O_3} = \phi_{Cu} = \beta_T = R = 0, Pr = 6.2$, and $\epsilon = 1.0$. This evaluation is based on the findings of Cortell [9], Ferdows *et al.,* [10], Rashidi *et al.,* [11], Raju *et al.,* [12], and Waini *et al.,* [57]. The trustworthiness of the bvp4c method code was verified by comparing the resulting values of reduced skin friction $F''(0)$ and reduced heat transfer $-\theta'(0)$ with those determined by Cortell [9], Ferdows *et al.,* [10], Rashidi *et al.,* [11], Raju *et al.,* [12], and Waini *et al.,* [57]. According to Table 3 and 4, the results that were obtained are in excellent agreement with the results that were published. This indicates that the bvp4c method code is consistent and, as a result, may be utilized with confidence to solve the present research that is being investigated.

**Table 3**
Comparison values of the reduced skin friction $F''(0)$ for pure fluid $\phi_{Al_2O_3}$ = $\phi_{Cu}$ = $M = Ec = \beta_T = \epsilon = 1.0$, and $Pr$=6.2

| $\gamma$ | Cortell [9] | Ferdows *et al.*, [10] | Rashidi *et al.,* [11] | Waini *et al.,* [57] | Present study |
|---|---|---|---|---|---|
| 0.75 | 0.984417 | -0.984439 | -0.98444 | -0.98443 | -0.98443 |
| 0.5 | -0.873627 | -0.873643 | -0.87364 | -0.87364 | -0.87364 |
| 0.25 | - | - | - | - | -0.77116 |
| 0 | -0.677647 | -0.677648 | -0.67765 | -0.67764 | -0.67764 |
| -0.25 | - | - | - | - | -0.59351 |
| -0.5 | -0.518869 | -0.518869 | -0.51889 | -0.51886 | -0.51886 |
| -0.75 | -0.453521 | -0.453523 | -0.45354 | -0.45352 | -0.45352 |

**Table 4**
Comparison values of the reduced heat transfer $-\theta'(0)$ for pure fluid $\phi_{Al_2O_3}$ = $\phi_{Cu}$ = $= Ec = \beta_T = \epsilon = 1.0$, and $Pr$=6.2

| $\gamma$ | Cortell [9] | Ferdows *et al.,* [10] | Rashidi *et al.,* [11] | Raju *et al.,* [12] | Waini *et al.,* [57] | Present study |
|---|---|---|---|---|---|---|
| -0.5 | 0.398946 | 0.39895 | 0.399084 | 0.39908 | 0.39910 | 0.39909 |
| -0.25 | - | - | - | - | - | 0.56696 |
| 0 | 0.764355 | 0.76437 | 0.764352 | 0.76435 | 0.76435 | 0.76435 |
| 0.25 | - | - | - | - | - | 0.98709 |
| 0.5 | 1.230766 | 1.23095 | 1.230791 | 1.23079 | 1.23079 | 1.23079 |

The velocity and temperature profiles are presented in Figure 2 - Figure 4. The displays of velocity profile $F'(\zeta)$ and temperature profile $\theta(\zeta)$ against suction $\gamma = 2.1, 2.2$ and 2.3 are portrayed in Figure 2 – Figure 3, under numerous values of specific parameters such as $\phi_{Al_2O_3} = \phi_{Cu} = 0.01, \beta_T = M = 0.01, \epsilon = -1, Ec = 0.01, Pr = 6.2$ respectively. According to Figure 2, it is evident that increasing the amount of suction $\gamma$ results in an increase in the $F'(\zeta)$ profile of both solutions. Further, it is worth noting that when the magnitude of the suction $\gamma$ effect is increased, the value of $\theta(\zeta)$ in both solutions decreases, as demonstrated in Figure 3. When the suction $\gamma$ values increase, the wall shear stress decreases, the fluid strength increases, and the surface gradient of velocity decreases. This is because of the physical mechanism that occurs.

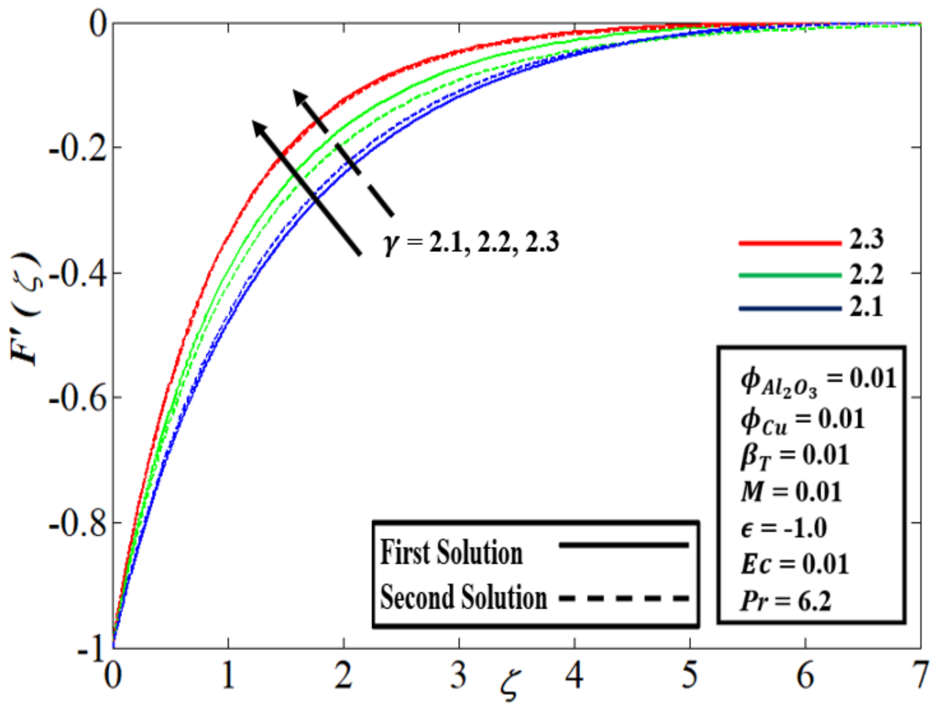


**Fig. 2.** Velocity $F'(\zeta)$ profile of different values of suction $\gamma$

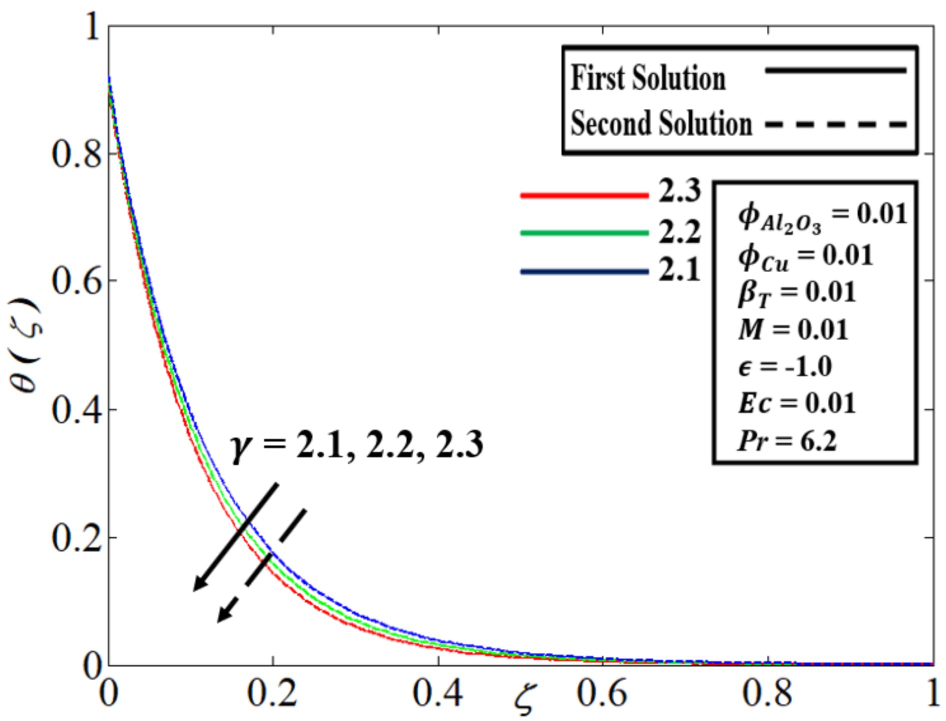


**Fig. 3.** Temperature $\theta(\zeta)$ profile of different values of suction $\gamma$

The temperature profile $\theta(\zeta)$ represents the effect of the Eckert numbers $Ec = 0.0, 0.5$, and 1.0 for a range of different parameters, including $\phi_{Al_2O_3} = \phi_{Cu} = 0.01, \beta_T = M = 0.01, \gamma = 2.2, \epsilon = -1$, $Pr = 6.2$ as shown in Figure 4. Because the Eckert number $Ec$ is not included in Eq. (2) and since it does not have any influence on $F'(\zeta)$, it is required to just consider $\theta(\zeta)$ for the purpose of this inquiry. The results presented in Figure 4 indicate that when the quantity of $Ec$ increases, there is a corresponding improvement in the value of $\theta(\zeta)$ for both solutions. It may be deduced from the fact that when the Eckert number increases, the kinetic energy of the fluid flow is greater than the thermal energy of the flow. Greater combination of fluids and convective heat transmission are both promoted by the increased kinetic energy concentration. As a result, the heat energy is carried throughout the system in a more efficient manner, which ultimately results in an overall improvement in the $\theta(\zeta)$.

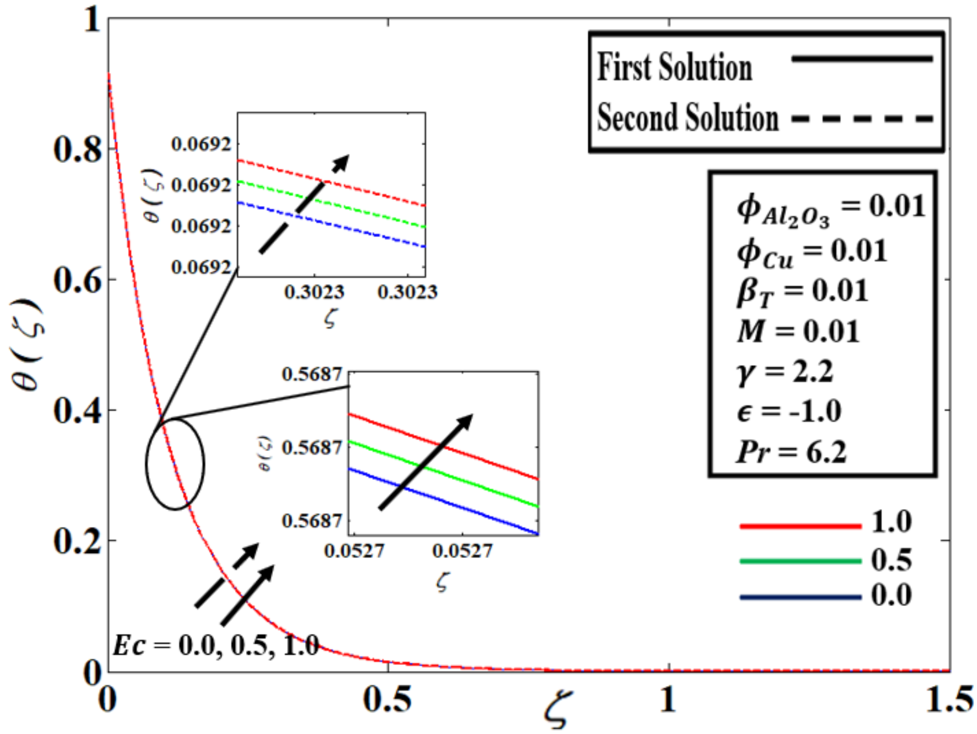


**Fig. 4.** Temperature $\theta(\zeta)$ profile of different values of Eckert number $Ec$

The variation of reduced skin friction $F''(0)$, and reduced heat transfer $-\theta'(0)$ is demonstrated in Figure 5 – Figure 6 by the appearance of multiple parameter amounts. These parameters include $\phi_{Al_2O_3} = 0.01, \beta_T = 0.01, \gamma = 3.0, Ec = M = 0.01$, $Pr = 6.2$ with three amounts of the solid volume fraction copper $\phi_{Cu} = 0.01, 0.03$ and $0.05$ versus stretching/shrinking sheet $\epsilon$. The possibility of dual solutions for $\epsilon_{ci} < \epsilon < -2$ is explored and found to be feasible. Besides, when $\epsilon < \epsilon_{ci}$, there is no favorable comportment solutions, and a unique solution is observed as $\epsilon \geq -2$. It is worth noting that the dual solutions achieved a shrinking ϵ region only. The critical point is denoted as $\epsilon_{ci}$, since it represents the critical point between the first and second solutions. It is important to note the happening that as $\phi_{Cu} = 0.01$, the quantity created for $\epsilon_{c1} = -2.1560$, subsequently, 3% of $\phi_{Cu}$ being amalgamated, and the quantity of $\epsilon_{c2} = -2.2542$. Also, the value of $\epsilon_{c3} = -2.3356$ revealed to enhance as 5% of the solid volume fraction $\phi_{Cu}$ being contained in hybrid nanofluid. Figure 5 demonstrates that the $F''(0)$ increased as the quantity of $\phi_{Cu}$ enhanced, while the opposite tendency was observed when $\epsilon > 0$. Furthermore, Figure 6 reveals an increasing behavior pattern for the $-\theta'(0)$ in both solutions, when the ability of $\phi_{Cu}$ is increased. As the quantity of solid volume fraction $\phi_{Cu}$ increases, the critical point values of $\epsilon_{ci}$ decrease, which results in the elongation of the boundary layer separation. This is a physical phenomenon that occurs.

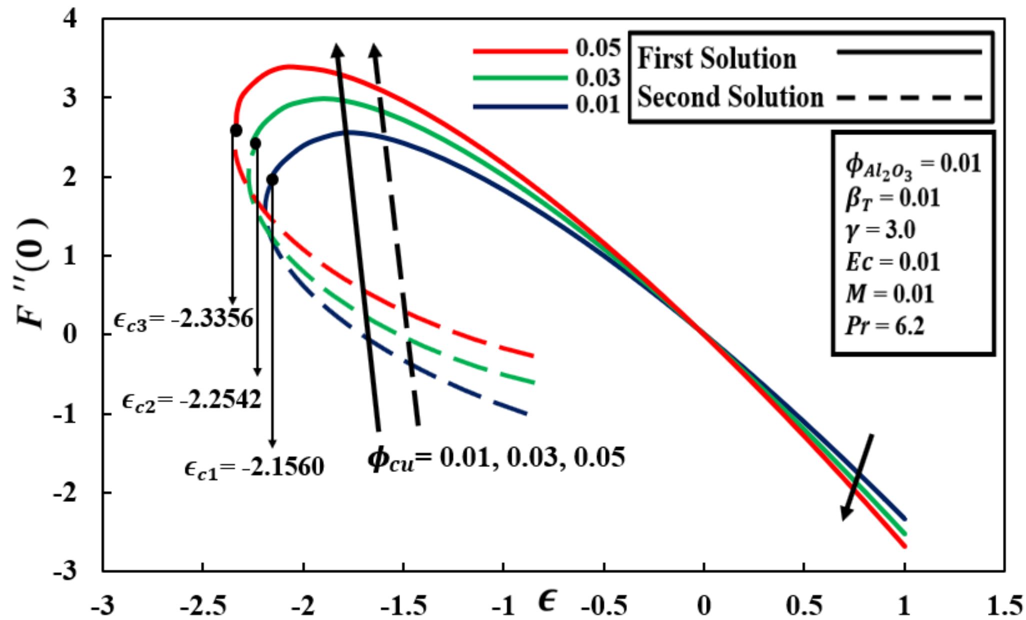


**Fig. 5.** Variation of $F''(0)$ against $\epsilon$ for different values of $\phi_{Cu}$

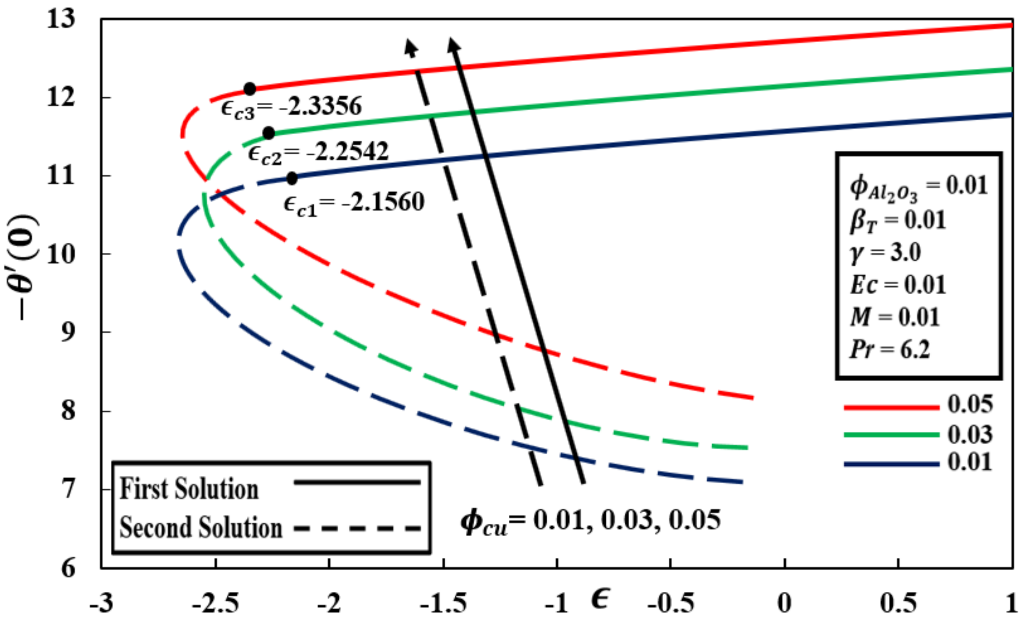


**Fig. 6.** Variation of $-\theta'(0)$ against $\epsilon$ for different values of $\phi_{Cu}$

The variation of $F''(0)$, and $-\theta'(0)$ is shown in Figure 7 – Figure 8 by the appearance of different parameter amounts such as $\phi_{Al_2O_3} = \phi_{Cu} = 0.01, \beta_T = 0.01, \gamma = 3.0, Ec = 0.01,\ Pr = 6.2$ with quantity of the magnetic parameter $M = 0.01, 0.05$ and 1.0 against stretching/shrinking sheet $\epsilon$. Three critical points are obtained in the zone of shrinking surface such as $\epsilon_{c1} = -2.1560$, $\epsilon_{c2} = -2.1897$ and $\epsilon_{c3} = -2.2160$. As depicted in Figure 7, the reduced skin friction $F''(0)$ exhibits an upsurge in first solution, while it declines in second solution. This indicates that the rate of mobility experienced a significant decrease as the value of $M$ grew. Furthermore, the same pattern that is observed in Figure 8, denoted as reduced heat transfer $-\theta'(0)$, suggests that the increase in the occurrence of the first solution and the decline in second solution have an impact on the increase in the entire amount of $M$. Physically, this is because the Lorentz force that was produced by the utilization of the magnetic field made it feasible for the transportation mechanism to be more resistant to damage.

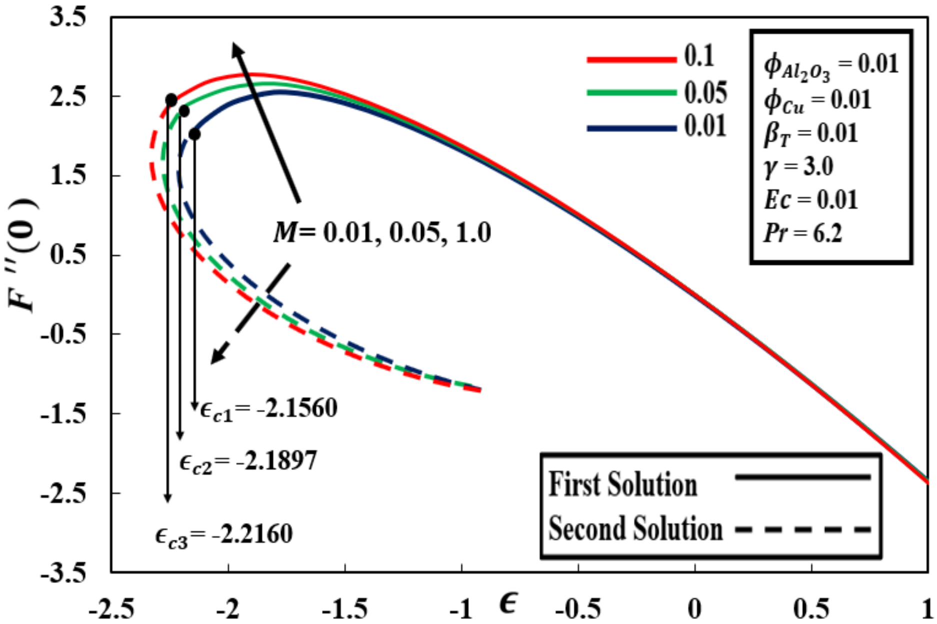


**Fig. 7.** Variation of $F''(0)$ against $\epsilon$ for different values of $M$

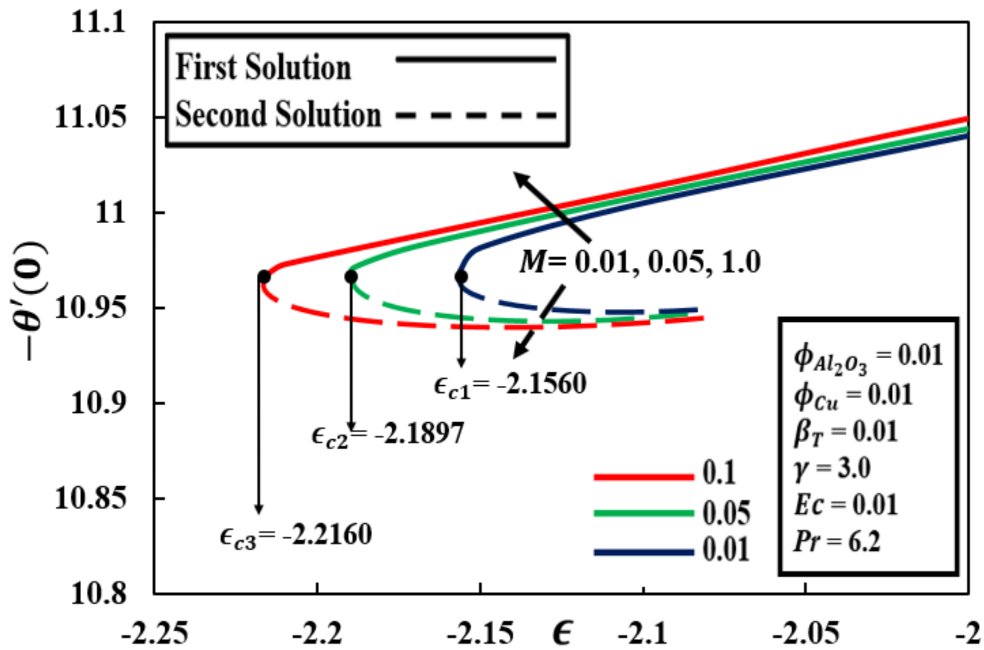


**Fig. 8.** Variation of $-\theta'(0)$ against $\epsilon$ for different values of $M$

The variation impact of the thermal slip parameter $\beta_T = 0.01, 0.0, 3, 0.05$ on the $F''(0)$, and $-\theta'(0)$ against the suction $\gamma$ parameter is described in Figure 9 – Figure 10 for various different parameters, and consist of $\phi_{Al_2O_3} = \phi_{Cu} = 0.01, M = 0.01, \epsilon = -1, Ec = 0.01, \ Pr = 6.2$

respectively. $\gamma_{ci}$ represents the critical point of $\gamma$, and it is situated at the point where the first and the second solution link to one another. When the value of $\gamma < \gamma_{ci}$, there are no alternatives solutions. The observation of two distinct solutions is conceivable, contingent upon the value of the thermal slip parameter $\beta_T$. Both solutions begin close to the critical point, which becomes $\gamma_{c1} = 1.8033$, $\gamma_{c2} = 1.7291$, and $\gamma_{c3} = 1.6581$. Additionally, when the thermal slip parameter $\beta_T$ is augmented, $F''(0)$ and $-\theta'(0)$ declined significantly in both solutions. This suggests that the presence of a thermal slip factor can impede the movement of water-based hybrid nanofluids. To include thermal slip in a fluid flow model, it is necessary to modify the governing equations for fluid flow and heat transfer to consider the impact of slip at the interface between the fluid and solid.

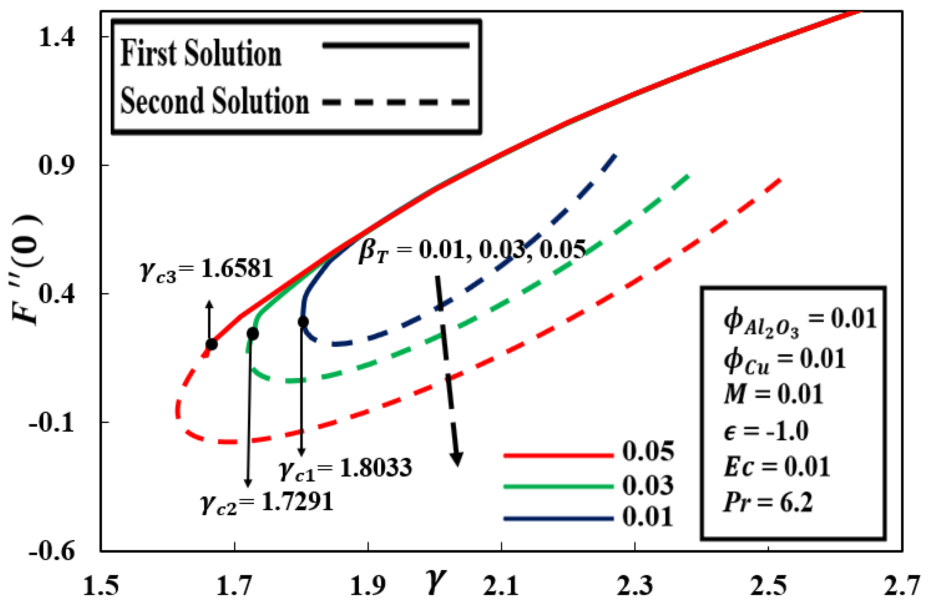


**Fig. 9.** Variation of $F''(0)$ against $\gamma$ for different values of $\beta_T$

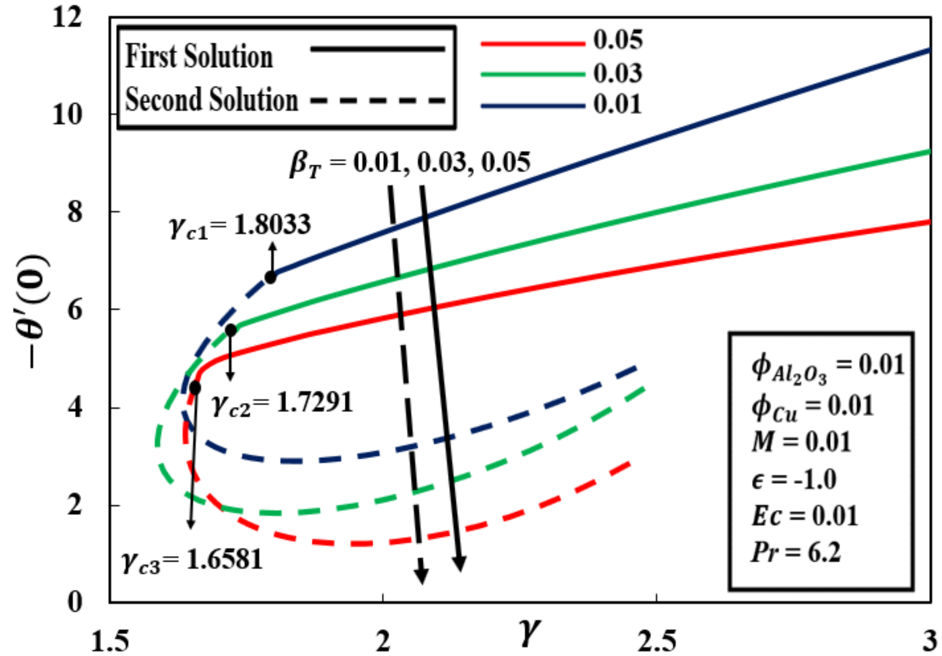


**Fig. 10.** Variation of $-\theta'(0)$ against $\gamma$ for different values of $\beta_T$

## 4. Conclusions

In this study, the bvp4c solver was used in conjunction with the MATLAB computational framework to investigate the dual solution with effect of water-based alumina-copper MHD hybrid nanofluid by Joule heating and slip condition though accounting for power-law form stretching and shrinking layers. The modeling technique that is mentioned in this work has been demonstrated to be accurate by coding that was executed while the bvp4c algorithm was functioning. A set of higher-order ordinary differential equations (ODEs) and accompanying boundary conditions are generated from the governing partial differential equations (PDEs), which are then illustrated graphically as well as numerically. The main objective of this study is to explore the behavior of $F''(0)$, and $-\theta'(0)$ for thermal slip factor against the suction effect and the behavior of solid volume fraction copper and magnetic parameter against stretching/shrinking sheet. In addition, the present study has also included the temperature and velocity profile hybrid nanofluid flow that correlate to the influence of suction effect and Eckert number. The significant results obtained from this study are as follows.

i. The viability of a dual solution has been demonstrated by the fact that a suitable set of parameters has been constructed.
ii. Water based hybrid nanofluid flows until a critical point $\epsilon < \epsilon_{ci}$, $\gamma < \gamma_{ci}$, have a unique solution for shrinking and suction effects.
iii. The temperature profile and thickness of boundary layer is boosted when rising the intensity of Eckert number.
iv. Additionally, temperature profile and thickness of boundary is reduced when rising the strength of suction effect. The reduced heat transfer rate enhanced in both solutions for solid volume fraction copper against shrinking sheet, but the reverse actions can be seen in both solutions for thermal slip parameter for suction effect.

The improved thermal conductivity of the alumina-copper hybrid nanofluid can be utilized in heat exchangers to enhance cooling efficiency in power plants, and chemical processing units. The study involves the application of magnetic fields (MHD), which is crucial for controlling and manipulating the flow and heat transfer characteristics of conducting fluids. This has practical implications in industries such as metallurgy, nuclear reactors, and cooling of electronic devices. Understanding the behaviour of power-law stretching and shrinking surfaces is vital in the extrusion and polymer processing industries, helping to optimize the production of plastic films, sheets, and fibers. The study's exploration of slip conditions and Joule heating is particularly relevant for microfluidic devices, which are used in biomedical applications, chemical analysis, and miniaturized lab-on-a-chip technologies. The current research can be expanded in the future to incorporate more complexity and factors. Future research directions may include such as, offering more realistic boundary conditions, includes Riga and cylindrical plates and studying various types of nanoparticles.

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