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Flow Analysis of Brinkman-Viscoelastic Fluid in Boundary Layer Region of Horizontal Circular Cylinder

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ABSTRACT

This paper examines the flow of Brinkman-viscoelastic fluid in the boundary layer region. The flow over a Horizontal Circular Cylinder (HCC) is investigated theoretically. The proposed model's governing equations, which are partial differential equations (PDEs), are transformed to their simplest form by using the appropriate non-dimensional variables and non-similarity transformation. The numerical computations for the obtained equations are then computed using the Keller-box method (KBM) which is programmed in MATLAB R2019a software. The velocity distribution results, together with the coefficient of skin friction are presented. A comparison study with previously published results is carried out to ensure that the current findings are accurate. It is discovered that the presence of the Brinkman and viscoelastic parameters influences the velocity behaviour of fluid, with a tendency to decrease the velocity distribution of fluid. Furthermore, both parameters have the potential to decrease the skin friction coefficient. The output can be used as a starting point for complex flow problems that occur frequently in engineering applications.

1. Introduction

The analysis of fluid behaviour is occurred in motion or at rest in fluid mechanics. The behaviour of the fluid is critical to comprehend in order to improve the best final product. Many researchers are now interested in studying boundary layer flow because of its numerous applications in fluid mechanics such as aeronautics, ship hydrodynamics, chemical process engineering, and automobile aerodynamics. The majority of boundary layer flow problems have been solved by focusing on the complex fluid known as non-Newtonian fluids such as Jeffrey, Viscoelastic, Williamson, and Burger's fluid. Several works have been done to improve the quality of fluid characteristics, as highlighted by the following researchers [1-5].

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Despite the difficulties of solving complex fluid flow problems, particularly those involving viscosity and elasticity properties in porosity conditions, many experts took on the challenge of solving fluid flow problems involving different geometry. Tonekeboni *et al.,* [6] used similarity solutions of visco-elastic boundary layer flow. They solved the equations using the finite difference method, which they claimed was more accurate. Furthermore, Ahmad *et al.,* [7] used KBM to investigate the flow of viscoelastic fluid across HCC. The authors have incorporated the radiation effect into the fluid. Aziz *et al.,* [8], used KBM to investigate the boundary layer flow around a cylinder surface in viscoelastic fluid. It has been determined that the increment value of the viscoelastic parameter slows the fluid's motion. Jaafar *et al.,* [9] investigated the flow in the same fluid as Aziz *et al.,* [8] by including the viscous dissipation effect over a non-linear stretching sheet. They came to the conclusion that as the viscoelastic parameter increased, so did the coefficient of skin friction. Besides, other reports regarding viscoelastic fluid over a blunt body has been established by several authors [10-12].

Given the importance of boundary layer flow with porosity, a few studies have been carried out by incorporating the viscoelastic fluid into the porous region [13-15]. The Brinkman model, which considers high porosity fluid, is one of the classical models for porous regions. Nazar *et al.,* [16] explored the Brinkman flow model in viscous fluid from a HCC. The coefficient of skin friction decreased as the Brinkman factor was increased. Chamkha *et al.,* [17] then investigated convection flow in porous regions using Darcy's law across a vertical cylinder filled with cold water. They discovered that the density variation in porous regions has a significant impact on fluid flow characteristics. In addition, Shafie *et al.,* [18] examined the phenomenon of fluid flow passing over an oscillating plate using the fractional Brinkman model and the Laplace Transformation to solve. The authors discovered that increasing the Brinkman factor increased fluid velocity. Finally, Flilihi *et al.,* [19] used the shooting method to investigate the flow of the Darcy-Brinkman model through a heated vertical plate. The authors discovered that the more permeable the porosity medium, the greater the velocity profile and frictional force.

Based on the preceding literature, the main goal in this research is to investigate the flow characteristics of a Brinkman-viscoelastic fluid passing over a HCC in a porous region. Salleh *et al.,* [20], who focus solely on boundary layer flow, inspired this study. This problem only considered the continuity and momentum equations, with gravity being ignored. The effect of Brinkman and viscoelastic parameters toward the velocity and coefficient of skin friction is discussed and illustrated graphically.

2. Methodology

This section discusses research methodology. The boundary layer approximation is used to present the basic governing models. Using non-dimensional variables and non-similarity transformations, the conversion and derivation of the governing model are reduced into a less complicated form of PDEs. Figure 1 shows summaries of the problem methodology used in this study.

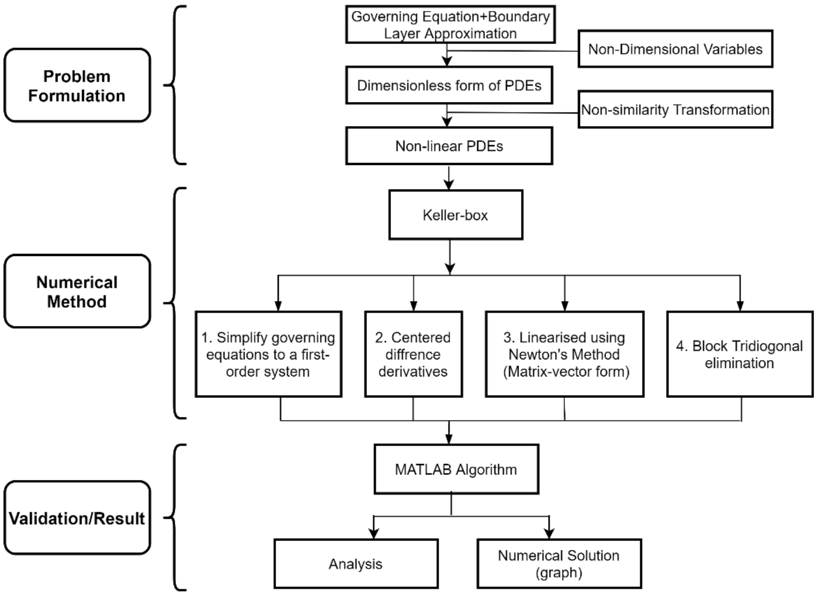

Fig. 1. Problem methodology

2.1 Mathematical Formulation

Consider the steady boundary layer flow past a HCC of radius a embedded in a porous region. The problem is to analyse the flow characteristics of a Brinkman-viscoelastic fluid in a boundary layer region using the continuity and momentum equations. According to Figure 2, the coordinate of $\bar{x}$ and $\bar{y}$ are calculated along the cylinder's surface. The ambient velocity, $\frac{1}{2}U_\infty$ is assumed to be as-cending as it passes over the cylinder. The gravity in momentum equation can be ignored in the case of flow analysis.

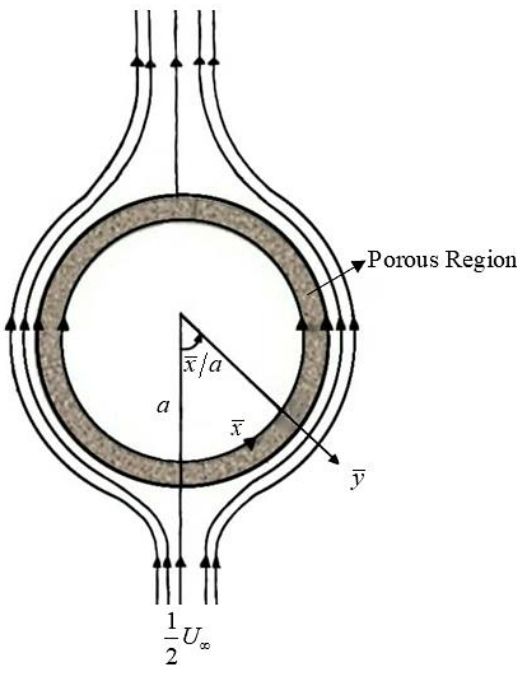

Fig. 2. General physical flow

The governing boundary layer equations consists of continuity and momentum equations are introduced as follows

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0, \tag{1}$$

$$\frac{\mu}{K}\bar{u}=\frac{\mu}{\phi}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}+\bar{u}_e\frac{\partial \bar{u}_e}{\partial \bar{x}}+k_0\left[\bar{u}\frac{\partial^3 \bar{u}}{\partial \bar{x}\partial \bar{y}^2}+\bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3}-\frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}}+\frac{\partial \bar{u}}{\partial \bar{x}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right], \tag{2}$$

with the appropriate boundary condition

$$\begin{aligned}&\bar{v}=0,\quad \bar{u}=0, \qquad \text{at} \quad \bar{y}=0,\\ &\bar{u}\to\bar{u}_e(\bar{x}),\quad \bar{v}\to 0,\quad \text{as} \quad \bar{y}\to\infty,\end{aligned} \tag{3}$$

where $\bar{u}$ and $\bar{v}$ are the velocity components in $\bar{x}-$ and $\bar{y}-$directions, respectively. Besides, $\bar{u}_e(\bar{x})$ is considered as external flow moving in upward direction passing the cylinder. Note that μ is defined as dynamic viscosity, K is determined as permeability of porous region, ϕ is porosity of porous region and k_0 is referred to viscoelasticity.

2.1.1 Non-dimensional variables

The following non-dimensional variables adopted from Tham *et al.,* [21], are introduced in order to simplify the governing Eq. (1) and Eq. (2).

$$\begin{aligned}&x=\bar{x}/a,\quad y=Pe^{1/2}(\bar{y}/a),\quad u=\bar{u}/U_\infty,\quad v=Pe^{1/2}(\bar{v}/U_\infty),\\ &u_e(\bar{x})=\bar{u}_e(\bar{x})/U_\infty,\end{aligned} \tag{4}$$

where $Pe=U_\infty a/\alpha_m$ is a modified Péclet number in porous region.

Substituting Eq. (4) for Eq. (1) and Eq. (2), and then differentiating with respect to y in the momentum equation, gives

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{5}$$

$$\frac{\partial u}{\partial y}=\Gamma\frac{\partial^3 u}{\partial y^3}+k_1\begin{bmatrix} u\dfrac{\partial^4 u}{\partial x\partial y^3}+\dfrac{\partial^3 u}{\partial x\partial y^2}\dfrac{\partial u}{\partial y}+v\dfrac{\partial^4 u}{\partial y^4}+\dfrac{\partial^3 u}{\partial y^3}\dfrac{\partial v}{\partial y}\\ -\dfrac{\partial u}{\partial y}\dfrac{\partial^3 u}{\partial x\partial y^2}-\dfrac{\partial^2 u}{\partial x\partial y}\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\partial u}{\partial x}\dfrac{\partial^3 u}{\partial y^3}+\dfrac{\partial^2 u}{\partial y^2}\dfrac{\partial^2 u}{\partial x\partial y}\end{bmatrix}, \tag{6}$$

and the boundary conditions Eq. (3) become

$$\begin{aligned} &u = 0, \quad v = 0, \qquad \text{at} \quad y = 0, \\ &u \to u_e, \quad v \to 0, \quad \text{as} \quad y \to \infty, \end{aligned} \tag{7}$$

where $\Gamma = \dfrac{Da}{\phi} Pe$ is Brinkman parameter and $k_1 = \dfrac{k_0 K U_\infty Pe}{\mu a^3}$ is the viscoelastic parameter.

2.1.2 Non-similarity transformation

Then, the non-similarity transformation is introduced to find the solutions of Eq. (5) to Eq. (7) as below

$$\psi = x\, f(x, y), \quad u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \tag{8}$$

Eq. (5) is automatically satisfied after several steps. Then, Eq. (6) and Eq. (7) comes to be

$$\frac{\partial f}{\partial y} = \Gamma \frac{\partial^3 f}{\partial y^3} + k_1 \begin{bmatrix} x \dfrac{\partial f}{\partial y} \dfrac{\partial^4 f}{\partial x \partial y^3} + \dfrac{\partial f}{\partial y} \dfrac{\partial^3 f}{\partial y^3} - x \dfrac{\partial f}{\partial x} \dfrac{\partial^4 f}{\partial y^4} \\ - f \dfrac{\partial^4 f}{\partial y^4} - x \dfrac{\partial^2 f}{\partial y^2} \dfrac{\partial^3 f}{\partial x \partial y^2} - \dfrac{\partial^2 f}{\partial y^2} \dfrac{\partial^2 f}{\partial y^2} \\ + x \dfrac{\partial^2 f}{\partial x \partial y} \dfrac{\partial^3 f}{\partial y^3} + \dfrac{\partial f}{\partial y} \dfrac{\partial^3 f}{\partial y^3} \end{bmatrix} + \frac{\sin x}{x}, \tag{9}$$

$$\begin{aligned} &f(x, 0) = 0, \quad \frac{\partial f}{\partial y}(x, 0) = 0, \qquad \text{at } y = 0, \\ &\frac{\partial f}{\partial y}(x, \infty) \to \frac{\sin x}{x}, \quad \frac{\partial^2 f}{\partial y^2}(x, \infty) \to 0, \quad \text{as } y \to \infty. \end{aligned} \tag{10}$$

At the lowest stagnation point ($x \approx 0$) of the cylinder, Eq. (9) and Eq. (10) results in the succeeding ordinary differential equations as follows

$$f' - \Gamma f''' - k_1 \left[2 f f''' - f f^{(iv)} - \left(f'' \right)^2 \right] - 1 = 0, \tag{11}$$

$$\begin{aligned} &f(0) = 0, \quad f'(0) = 0, \\ &f'(\infty) \to 1, \quad f''(\infty) \to 0. \end{aligned} \tag{12}$$

The dimensionless local skin friction coefficient can be written as follows

$$C_{fl} Pe^{1/2} / \Pr = x f''. \tag{13}$$

According to Harris *et al.,* [22], the exact solution for Eq. (11) when $\Gamma \neq 0$ can be written in the following form

$$f(y) = y + \frac{\Gamma}{\gamma + \sqrt{\Gamma}}\left(e^{-\frac{y}{\Gamma}} - 1\right), \quad (14)$$

with its second derivative,

$$f''(y) = \frac{1}{\gamma + \sqrt{\Gamma}}, \quad (15)$$

It is worth to mention that, at the surface of $y = 0$, Eq. (15) with $\gamma = 0$ becomes

$$f''(0) = \frac{1}{\sqrt{\Gamma}}. \quad (16)$$

2.2 Numerical Method

In this study, KBM is used to obtain the numerical results of Eq. (11) under the boundary conditions Eq. (12). Figure 1 shows the four stages of KBM. The numerical algorithms and computations are carried out using the MATLAB R2019a tool. The current solutions are achieved by selecting the finite boundary layer thickness, $y_\infty = 8$ and step size, $\Delta y = 0.02$ based on satisfying the boundary conditions Eq. (12). The parameters of Γ and k_1 in Eq. (11) are computed to determine the flow of fluid velocity, as displayed in Figures 3 and 4. Eq. (13) are used to summarise the local skin friction in tabular and graphical form. The fixed values of $k_1 = 8$ for various Γ and $\Gamma = 0.1$ for various k_1 are chosen to carry out the graphical solution shown in Figures 3–6.

2.3 Validation Procedure

The current results are compared to the numerical solution of Nazar *et al.,* [16] and exact Eq. (16) in the absence of k_1. Table 1 compares the current model at the stagnation point to the existing equation with a few limited cases. Table 2 shows the results for $f''(0)$ are validated with previous literature. The comparison values in Table 2 show that the numerical results are close to the exact values, implying that the numerical algorithm proposed in this study is precise.

Table 1
Comparative model at stagnation point

Author	Model (momentum)	Limiting cases
Current	$f' - \Gamma f''' - k_1\left[2ff''' - ff^{(iv)} - (f'')^2\right] - 1 = 0$	$k_1 = 0, \Gamma \neq 0$
Nazar *et al.,* [16]	$f' - \Gamma f''' - 1 - \lambda\theta = 0$	$\Gamma \neq 0, \lambda = 0$
Harris *et al.,* [22]	$f' - \Gamma f''' - 1 - \lambda\theta = 0$	Exact solution $\Gamma \neq 0, \lambda = 0$

Table 2
Comparison values of $f''(0)$ at stagnation point for several Γ with $k_1 = 0$

Γ	Exact Eq. (16)	Nazar *et al.,* [16]	Current
0.1	3.1622	3.1623	3.1622
0.2	2.2360	2.2361	2.2360
0.3	1.8257	1.8257	1.8257
0.4	1.5811	-	1.5813
0.5	1.4142	-	1.4147

3. Results and Discussion

The viscoelastic parameter, k_1 is assigned as non-negative value in this study because the focus of the investigation is on determining the existence of viscosity and elasticity characteristics. The Brinkman parameter is characterised as $\Gamma > 0$ due to the variation in porosity. Therefore, for the entire analysis, values for the Brinkman parameter and the viscoelastic parameter are selected from the intervals $0.1 \le \Gamma \le 3$ and $8 \le k_1 \le 23$, respectively. Figures 3 and 4 demonstrate that an increment in k_1 and Γ leads to a decrease in fluid flow velocity. This occurred because the presence of viscoelastic properties causes fluid to resist movement. This result also suggests that the Brinkman factor, which determined the porosity of porous medium, increases the fluid's drag force and consequently slows the velocity profile.

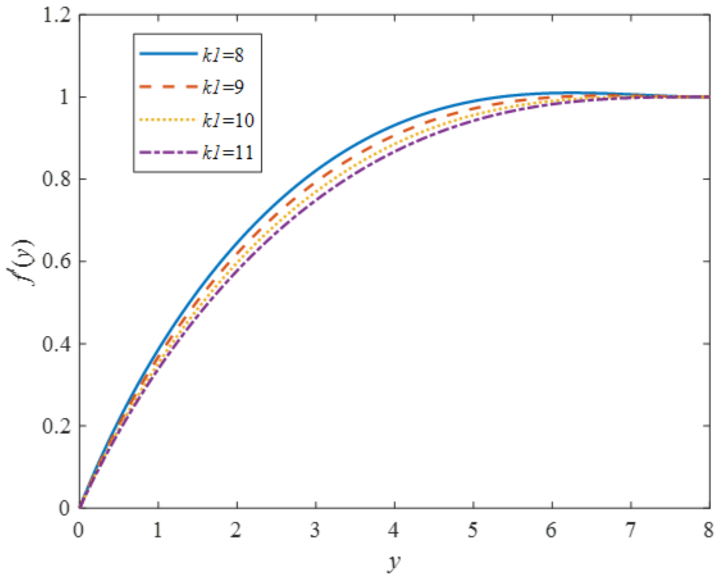

Fig. 3. Performance of viscoelastic, k_1 on velocity, $f'(y)$

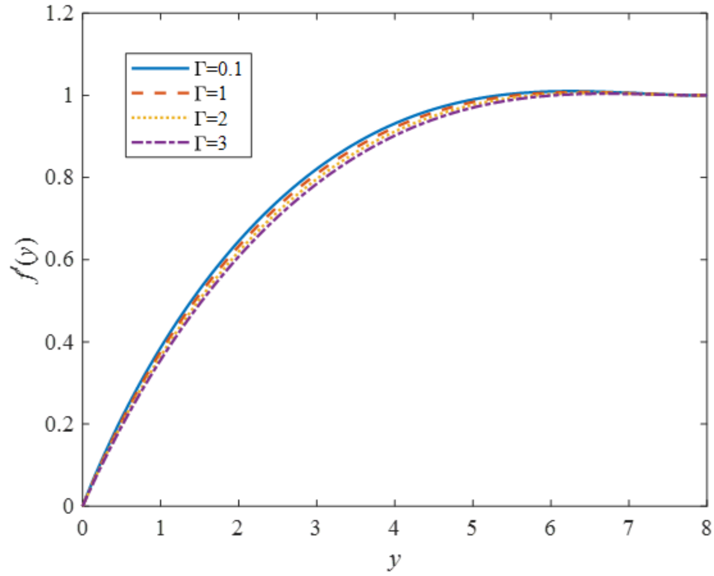

Fig. 4. Performance of Brinkman, Γ on velocity, $f'(y)$

Table 3 displays the effect of various Brinkman and viscoelastic parameters on the coefficient of local skin friction at $x=1$. It can be seen that when Γ and k_1 are increased, the rate of skin friction decreases. According to the flow analysis, the total percentage decrease in skin friction coefficient for Γ within $0.1 \le \Gamma \le 3$ is 19.13% and for k_1 within $8 \le k_1 \le 11$ is 14.22%. Figures 5 and 6 portray the graph of the skin friction coefficient toward Brinkman and the viscoelastic parameters. The increase of k_1 and Γ allows for a reduction in skin friction. This is due to the existence of a solid matrix in the fluid. Furthermore, according to the numerical analysis, Figure 5 clearly demonstrates that the visco-elastic properties delayed the separation of the boundary layer within the range $0 \le x \le \pi$ whereas variations in the Brinkman factor had no effect on flow separation.

Table 3
Numerical values of $C_f Pe^{1/2} / \Pr$ for various values of k_1 and Γ at $x=1$

Γ	k_1	$C_f Pe^{1/2} / \Pr$
0.1	8	0.003319
1		0.003024
2		0.002828
3		0.002684
0.1	8	0.003319
	9	0.003138
	10	0.002982
	11	0.002847

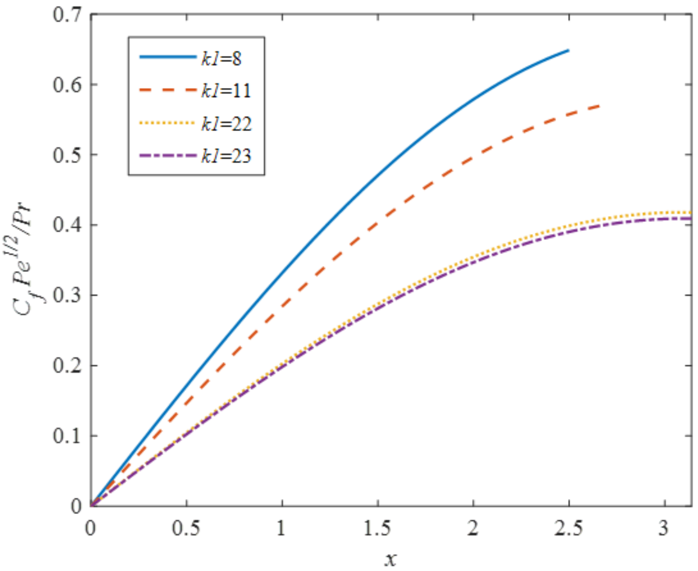

Fig. 5. Performance of viscoelastic, k_1 on skin friction, $C_f Pe^{1/2} / \Pr$

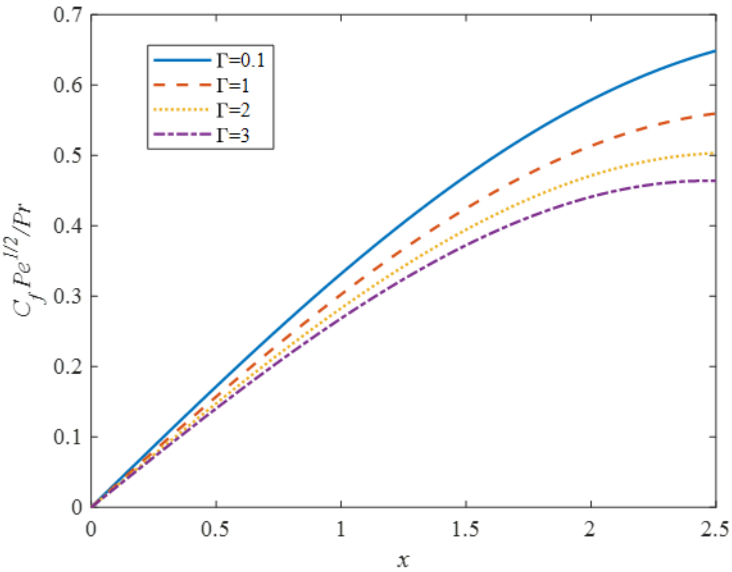

Fig. 6. Performance of Brinkman, Γ on skin friction, $C_f Pe^{1/2} / \Pr$

4. Conclusions

This research focuses on the boundary layer flow analysis over HCC in porous region. The Brinkman model with the viscoelastic properties is adopted to model the problem. The problem is transformed from PDEs to dimensionless PDEs using the appropriate variables as shown in Figure 1. The PDEs are solved numerically using KBM and the MATLAB R2019a software. The proposed model is validated with previously published work. Conclusively, the Brinkman and viscoelastic parameters influenced the fluid flow characteristics. It is observed that increasing the Brinkman and viscoelastic parameters reduces the fluid velocity. By increasing both parameters, the skin friction coefficient decreases. It was also discovered that the flow encounters a separation boundary layer after $x = 2.5$ for the variation of Brinkman parameter.

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