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Liquid Coating with Variable Thermal Conductivity on a Pipe under Influence of Thermal Radiation and Heat Generation

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ABSTRACT

Flow over a pipe or an elongated cylinder is widely applied in many engineering processes like wire coating and pipe coating. This encourages the present study to examine the fluid flow and heat transfer over a horizontal stretching cylinder with the impact of temperature-reliant thermal conductivity and thermal radiation. The influence of heat generation is also considered. The Carreau rheology model is applied to represent the liquid coating. The similarity technique is used to simplify the developed governing equations and then solved by the homotopy analysis method. The effects of the pertinent parameters such as the thermal conductivity parameter and Weissenberg number on the fluid field and heat transfer are studied by applying the calculated series of analytical solutions, which are scrutinized through graphs and tables. The Nusselt number has a negative function with the radiation and thermal conductivity parameters. Furthermore, the Weissenberg number affects the velocity and temperature profiles differently in conditions $n < 1$ and $n \geq 1$, respectively. The present results are essential in optimizing the pipe coating process.

1. Introduction

Carreau fluid has recently been concerned by plenty of researchers due to its importance in the coating process, polymeric suspensions, food processing, wire coating, and chemical engineering. The Carreau model incorporates the Newtonian model and the power-law model. It was proposed by Carreau [1] to overcome the inadequate power-law constitutive relation while predicting the viscosity with an extremely huge or extremely small shear rate. El Misery *et al*., [2] deliberated the separation flow of a Carreau fluid that is incompressible in a uniform tube through peristaltic motion. A similar problem was considered by Akbar *et al*., [3] by including the impact of mass and thermal transfer. Akbar *et al*., [4] scrutinized the boundary layer stagnation-point flow of Carreau liquid over a shrinking sheet in the presence of a magnetic field. Two different sets of numerical solutions were obtained, and the effect of the shrinking parameter, magnetic number, and Weissenberg number, on the fluid flow was analyzed. Further, Hayat *et al*., [5] elucidated the impact of the Newton

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boundary conditions on a Carreau fluid flow past through a stretched sheet. The authors observed that the velocity and temperature profiles have an opposite behavior under the effect of the power-law index.

Khan [6] emphasized the consequence of nonlinear stretching surface on the Carreau boundary layer fluid flow and heat transmission. Hayat *et al*., [7] discussed the properties of Soret and Brownian motion on the nanofluid with Carreau rheology toward a heated elongating plane. The three-dimensional Carreau fluid flow and heat transmission through a bidirectional elongating sheet were scrutinized by Khan *et al*., [8]. The consequence of irregular thermal radiation was also considered. The authors found that the shear-thickening fluid and shear-thinning fluid provide a conflicting behavior on the velocity distribution for the various value of the Weissenberg number.

Many industrial or manufacturing procedures involve flow over a stretching cylinder such as paper construction, pipe coating, polymer, crude oil refinement, glass fiber production, photographic films, and drawing wire. Generally, the flow over a cylinder is assumed to be axisymmetric, resulting in the governing equations containing a transverse curvature term. This term has a significant influence on the velocity field. In view of this, Wang [9] theoretically discussed a viscous, incompressible, and steady fluid flows over a stretching tube. This pioneering work has inspired some researchers to investigate the Carreau fluid flow which has been induced by a stretching or shrinking tube. Salahuddin [10] elucidated the flow of Carreau fluid toward a stretching cylinder. The impact of the various embedding parameters such as the power-law index, Weissenberg value, and curvature parameter on the fluid movement was discussed. However, in the study, only the shear-thickening fluid is focused on.

Moreover, the heat transfer in Carraeu fluids over a stretched cylinder has relevant applications in sciences and engineering. Thus, Khan *et al*., [11] have studied the impact of the melting heat phenomenon, and the stretching rate on the Carreau liquid flows over a cylinder. The finding shows that the boundary layer thickness of the momentum is getting larger. Salahuddin *et al*.,[12] constructed a study on Carreau nanofluid near a linear enlarging cylinder with the effect of slip flow, chemical reaction, and magnetic field. The authors concluded that the fluid flows have a higher velocity, concentration, and temperature than the fluid through a stretching sheet. Hayat *et al*., [13] studied the consequence of Newtonian heating and thermal radiation on a Carreau fluid movement induced by a stretching plane with a chemical reaction. The authors detected that the temperature distribution is enhanced for larger radiation parameters and smaller Weissenberg numbers. Khan *et al*., [14] highlighted the significance of the magnetic field and Ohmic heating on the thermal and mass transmission of nano-Carreau fluid through a sloping heated elongating cylinder.

The impact of the curvature parameter, power-law index, and Weissenberg number on the rate of heat transfer was discussed. Gangadhar *et al*., [15] presented the effect of temperature jump, and slip boundary conditions on the MHD Carreau Cattaneo-Christov heat fluid over a stretching cylinder. The reflection of the velocity and temperature profile for the various value of the Biot number was graphically displayed and analyzed. Gopal and Kishan [16] numerically elaborated the unsteady flow of Carreau fluid past a shrinking cylinder. The impact of the unsteadiness, thermal relaxation parameter, and Wiesenberger number on the distribution of fluid motion and heat was analyzed. Recently, Song *et al*., [17] analyzed the impact of the melting heat phenomenon and bioconvection on a nano-Carreau fluid flow driven by a nonuniform stretching cylinder. Akram *et al*., [18] acknowledged the significance of the abnormal heat generation on Carreau fluid that contains nanoparticles. Besides, the authors have also considered the same configuration as concerned by Song *et al*., [17]. The effect of the nonlinear stretching and shrinking rate of a cylinder on an MHD non-Newtonian fluid was discussed by Kardri *et al*., [19]. The influence of the heat source and viscous

dissipation have been considered, the author found that the shear stress and heat transmission are enhanced by the curvature of the cylinder.

So far, all the aforementioned literature has treated the thermal conductivity of the ambient fluid to be constant. However, in real-world circumstances, such properties demand variable characteristics. The thermal conductivity of liquid metals is approximated to be directly proportional to the temperature from 0 °F to 400 °F [20-25]. Recently, Khan *et al*., [26] investigated the effect of combined electrical and MHD fields on the flowing of nano-Carreau fluid passing a stretching oscillatory porous surface in the existence of temperature-dependent thermal conductivity. The authors observed that a high conduction process is produced when the parameter for thermal dependence conductivity is increased. Furthermore, Abbas *et al*., [27] acknowledged the characteristic of the MHD Carreau fluid with variable thermal conductivity under the influence of variable viscosity toward a permeable sheet that is stretched. The heat transport with convective heat flux was discussed. The characteristics of the velocity, concentration, and heat fields under the impact of diverse fluid parameters such as the suction parameter, Weissenberg number, Lewis number, and magnetic parameter were analyzed graphically. Recently, the effect of the modification in thermal conductivity on a bioconvection non-Newtonian fluid movement was studied by Yin *et al*., [28]. The flow was assumed across a stretching tube, and the results revealed that the variable thermal conductivity parameter increases the rate of heat exchange. Furthermore, Nabwey *et al*., [29] investigated the bioconvection of the nanofluid with Carreau rheology behavior. The impact of the sloping stretchable cylinder and the variable conductivity in thermal on the fluid properties was discussed. More relevant investigations about the significance of the inconstant thermal conductivity that is dependent on the temperature can be found in Hayat *et al*., [30], Jain *et al*., [31], Salahuddin *et al*., [32], Malik *et al*., [33], Rangi *et al*., [34], Ewis [35].

To the best of the author's knowledge, the research on the impact of stretching rate, heat generation, and thermal radiation on the fluid field and heat transmission of a Carreau fluid passing a cylinder under the influence of temperature-reliant thermal conductivity has not been conducted in the literature yet. Hence, the objective of the current investigation is to explore the behavior of the heat transport and Carreau fluid flow through a stretching cylinder in attendance of temperature-dependent conductivity, heat source, and thermal radiation. Due to the effectiveness of the HAM in solving highly non-linear differential equations, HAM is applied in the current study. The developed Mathematica solver by Liao [36-38], which is named BVPh2.0, is utilized to compute the solutions.

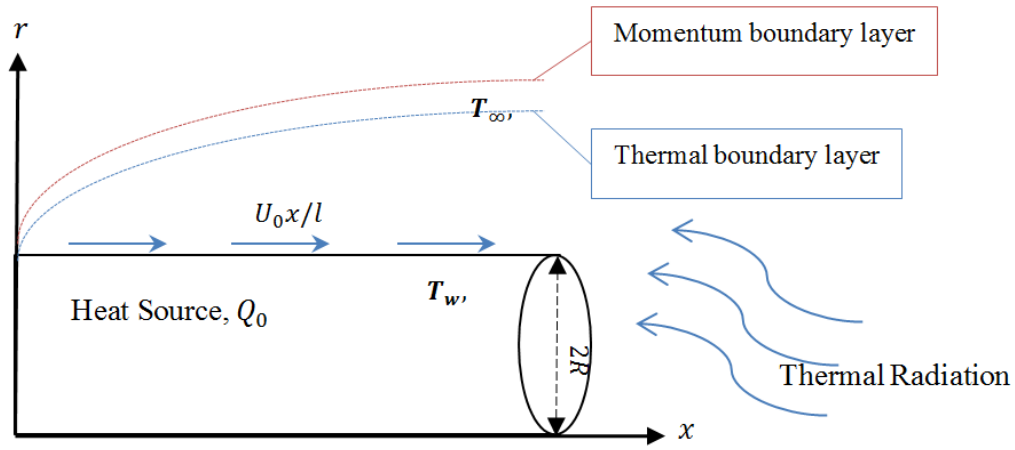

Fig. 1. Schematic diagram of fluid flow over a stretching pipe under the effect of heat source and thermal radiation

2. Methodology

2.1 Mathematical Formulation

An incompressible axisymmetric flow of steady two-dimensional Carreau fluid toward an elongating cylinder is studied in the present research. The *r*-axis is determined along the radial direction and the *x*-axis is measured along the axial direction, as presented in Figure 1. A cylinder with radius *R* and *l* characteristic length is considered. U_0 is the reference velocity, and the cylinder is stretched with a velocity $U_0 x/l$. T_w is the temperature at the surface of the cylinder and T_∞ is assumed to be the ambient temperature. Carreau fluid model is applied to model the rheology of the fluid flow. The Cauchy stress tensor for Carreau fluid is written as [3, 5, 10, 12]

$$\tau_{ij} = \mu \mathbf{R}, \tag{1}$$

With

$$\frac{\mu - \mu_\infty}{\mu_0 - \mu_\infty} = \left[1 + (\Gamma\chi)^2\right]^{\frac{n-1}{2}}. \tag{2}$$

Where the first kind Rivlin-Erickson tensor $\mathbf{R}$, the infinite-shear-rate viscosity μ_∞, the viscosity of the shear rate μ, the power-law index n, the viscosity of the zero-shear rate μ_0, and the material time constant Γ. χ is the shear rate and is expressed as

$$\chi = \sqrt{\frac{1}{2}\Delta} = \sqrt{\frac{1}{2}\sum_i\sum_j \chi_{ij}\chi_{ij}} = \sqrt{\frac{1}{2}tr\left(R^2\right)}, \tag{3}$$

where the second invariant strain tensor is denoted by Δ. The current study deliberates the case for $\eta_\infty = 0$ and $\Gamma\chi < 1$, in the constitutive Equation (1) thus by binomial expansion, Equation (2) become

$$\mu = \mu_0\left[1 + \frac{n-1}{2}(\Gamma\chi)^2\right]. \tag{4}$$

Substitute Equation (4) into (1), and the stress tensor can be exhibited as

$$\tau_{ij} = \mu_0\left[1 + \frac{n-1}{2}(\Gamma\chi)^2\right]\mathbf{R}, \tag{5}$$

The governing equations of the fluid with the Carreau model are developed by utilizing the boundary layer approximation and are represented as

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial x} = 0, \tag{6}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = \upsilon\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{3\Gamma^2(n-1)}{2}\left(\frac{\partial u}{\partial r}\right)^2\frac{\partial^2 u}{\partial r^2} + \frac{\Gamma^2(n-1)}{2r}\left(\frac{\partial u}{\partial r}\right)^3\right), \tag{7}$$

where the kinematic viscosity is denoted by υ. The considered boundary conditions are

$$\begin{aligned} &u = \frac{U_0 x}{l}, \quad v = 0, \quad at \quad r = R, \\ &u \to 0, \quad as \quad r \to \infty. \end{aligned} \tag{8}$$

The energy equation of the Carreau fluid in the presence of temperature generation and thermal radiation on the stretching tube is given by

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(\alpha r \frac{\partial T}{\partial r}\right) - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial r} + \frac{Q_0\left(T - T_\infty\right)}{\rho c_p}, \tag{9}$$

with the density ρ, the thermal conductivity α, the thermal radiation q_r, the specific heat c_p, and the heat source Q_0. Equation (9) is subjected to boundary conditions as

$$\begin{aligned} &T = T_w, \quad at \quad r = R, \\ &T \to T_\infty, \quad as \quad r \to \infty. \end{aligned} \tag{10}$$

The Rosseland approximation is applied to approximate the derivative of the thermal radiation. Following [28, 31, 39, 40], we have

$$\frac{\partial q_r}{\partial r} = \frac{-16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial r^2}. \tag{11}$$

Where σ^* defines the Stefan-Boltzmann constant and the mean absorption coefficient is denoted by k^*. Substitutes Equation (11) into Equation (9), thus obtaining

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(\alpha r \frac{\partial T}{\partial r}\right) - \frac{1}{\rho c_p}\frac{-16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial r^2} + \frac{Q_0\left(T - T_\infty\right)}{\rho c_p}. \tag{12}$$

In the range between -17.78°C to 204.444°C, the thermal conductivity is linearly proportional to the temperature and can be assumed as $\alpha = \alpha_\infty(1 + \varepsilon\theta)$ [30, 32, 34]. α_∞ is the constant thermal conductivity at $\eta \to \infty$ and ε is the small quantity.

2.2. Similarly Transformation

Equations (12) and (6)-(7) are transformed into ordinary differential equations by utilizing the subsequent similarity variables[10, 32]

$$\eta = \sqrt{\frac{U_0}{\upsilon l}}\left(\frac{r^2 - R^2}{2R}\right), \quad \theta = \frac{T - T_w}{T_w - T_\infty}, \quad \psi = \sqrt{\frac{\upsilon U_0}{l}} xRf\left(\eta\right). \tag{13}$$

Substitute Equations (13) into Equations (6), (7), and (12). Equation (6) is automatically satisfied, and Equations (7) and (12) reduce to

$$\frac{3(n-1)}{2}We^2(1+2\lambda\eta)\left(\lambda f''^3+(1+2\lambda\eta)f''^2f'''\right)+2\lambda f''+ (1+2\lambda\eta)f'''+\frac{(n-1)}{2}We^2\lambda(1+2\lambda\eta)\left(f''\right)^3+ff''-\left(f'\right)^2=0, \tag{14}$$

$$(1+2\lambda\eta)(1+\varepsilon\theta+\frac{4}{3}Rd)\theta''+\left(2\lambda+\Pr f+2\lambda\varepsilon\theta+\frac{4}{3}\lambda Rd\right)\theta' +(1+2\lambda\eta)\left(\theta'\right)^2\varepsilon+\Pr Q\theta=0, \tag{15}$$

with boundary conditions

$$f(\eta)=0,\quad \theta(\eta)=1,\quad f'(\eta)=1,\quad at\quad \eta=0,$$
$$f'(\eta)\to 0,\quad \theta(\eta)\to 0,\quad as\quad \eta\to\infty. \tag{16}$$

where the prime indicates differentiation with respect to η, the curvature parameter λ, *We* is the Weissenberg number, the heat generation parameter *Q*, the Prandtl number is defined by Pr, and the radiation number *Rd*. These parameters are defined by

$$\lambda=\frac{1}{R}\sqrt{\frac{\upsilon l}{U_0}},\qquad We=\Gamma x\sqrt{\frac{U_0^{\,3}}{\upsilon l^3}},\qquad Q=\frac{lQ_0}{\rho c_pU_0},$$
$$\Pr=\frac{\upsilon}{\alpha_\infty},\qquad Rd=\frac{4\sigma^*T_\infty^3}{3k^*\alpha_\infty\rho c_p}. \tag{17}$$

Re_x is the Reynold number. The skin friction coefficient ($\frac{1}{2}C_fRe_x^{1/2}$) and the Nusselt number ($NuRe_x^{-1/2}$) are represented as

$$\frac{C_f\,\mathrm{Re}_x^{1/2}}{2}=f''(0)+\frac{n-1}{2}We^2\left[f''(0)\right]^3, \tag{18}$$

$$\frac{Nu}{\mathrm{Re}_x^{1/2}}=-\theta'(0). \tag{19}$$

Where $\sqrt{Re_x}=\sqrt{\frac{x^2U_0}{\upsilon l}}$.

2.3. Homotopy Analysis Method

The homotopy technique for determining the solutions of the fluid movement and heat exchange of the Carreau fluid toward an elongating horizontal cylinder is demonstrated. Heat generation and thermal radiation are included in the energy equation. Besides, the impact of the variable thermal

conductivity has also been taken into consideration. The guessed and selected initial estimates and the linear operator are

$$f_0(\eta) = -(e^{-\eta} - 1), \qquad \theta_0(\eta) = e^{-\eta}, \tag{20}$$

$$L_f = f''' + f'', \qquad L_\theta = \theta'' + \theta'. \tag{21}$$

The above auxiliary linear operator has the following properties:

$$L_f\left(a_1 + a_2\eta + a_3 e^{-\eta}\right) = 0, \qquad L_\theta\left(a_4 + a_5 e^{-\eta}\right) = 0. \tag{22}$$

Where $a_1, a_2, a_3, a_4,$ and a_5 are arbitrary constants. The *k*th deformation equations for Equations (14) and (15) are

$$L_f\left\{f_k(\eta) - \chi_k f_{k-1}(\eta)\right\} - \hbar_f R_{k-1}\left(N_f\left\{f(\eta:q)\right\}\right) = 0. \tag{23}$$

$$L_\theta\left\{\theta_k(\eta) - \chi_k \theta_{k-1}(\eta)\right\} - \hbar_\theta R_{k-1}\left(N_\theta\left\{f(\eta:q), \theta(\eta:q)\right\}\right) = 0. \tag{24}$$

$\hbar_\theta\ (\neq 0)$ and $\hbar_f\ (\neq 0)$ are the control convergence parameter for the energy and momentum equation, respectively. N_f and N_θ are nonlinear operators. q is the embedding parameter, and

$$\chi_k = \begin{cases} 0, & k \le 1, \\ 1, & k > 1, \end{cases} \qquad R_{k-1} = \frac{1}{(k-1)!}\frac{\partial^{k-1}}{\partial q^{k-1}}\bigg|_{q=0},$$

with,

$$\begin{aligned}
&R_{k-1}\left(N_f\left\{f(\eta:q)\right\}\right) = \\
&\left[\frac{3(n-1)}{2}We^2(1+2\lambda\eta) + \frac{(n-1)}{2}We^2(1+2\lambda\eta)\right]\left[\sum_{l=0}^{k-1} f''_{k-1-l}\sum_{j=0}^{l} f''_{l-j} f''_l\right] + \\
&(1+2\lambda\eta)^2\frac{3(n-1)}{2}We^2\left[\sum_{l=0}^{k-1} f''_{k-1-l}\sum_{j=0}^{l} f''_{l-j} f'''_l\right] + \\
&2\lambda f''_{k-1} + (1+2\lambda\eta) f'''_{k-1} + \left[\sum_{l=0}^{k-1} f_{k-1-l} f''_l\right] - \left[\sum_{l=0}^{k-1} f'_{k-1-l} f'_l\right].
\end{aligned} \tag{25}$$

$$\begin{aligned}
&R_{k-1}\left(N_\theta\left\{f(\eta:q), \theta(\eta:q)\right\}\right) = \\
&(1+2\lambda\eta)\left(1+1\frac{1}{3}Rd\right)\theta''_{k-1} + 2\left(1+\frac{2}{3}Rd\right)\lambda\theta'_{k-1} + \\
&Q\,\mathrm{Pr}\,\theta_{k-1} + (1+2\lambda\eta)\varepsilon\left[\sum_{l=0}^{k-1}\theta'_{k-1-l}\theta'_l\right] + \\
&(1+2\lambda\eta)\varepsilon\left[\sum_{l=0}^{k-1}\theta''_l\theta_{k-1-l}\right] + 2\lambda\varepsilon\left[\sum_{l=0}^{k-1}\theta_{k-1-l}\theta'_l\right] + \mathrm{Pr}\left[\sum_{l=0}^{k-1} f_{k-1-l}\theta'_l\right].
\end{aligned} \tag{26}$$

The established BVPh 2.0 (a Mathematica package) by [38] has been employed to calculate θ_k and f_k for $k \geq 1$ and then the solutions of the governing equations. The convergence control parameters $\hbar_f$ and $\hbar_\theta$ are introduced to assure the convergence of the series solutions. The average residual error technique calculates the optimal *k*th-order approximation convergence control parameters. The residual error for $\hbar_f$ and $\hbar_\theta$ are defined as

$$E_k^f(\hbar_f) = \frac{1}{Z+1}\sum_{i=0}^{Z}\left[N_f\left(\sum_{j=0}^{k} f_j(\eta_i)\right)\right]^2, \qquad E_k^\theta(\hbar_\theta) = \frac{1}{Z+1}\sum_{i=0}^{Z}\left[N_\theta\left(\sum_{j=0}^{k} \theta_j(\eta_i)\right)\right]^2. \tag{27}$$

$$E_k^{Total}(\hbar_f, \hbar_\theta) = E_k^f(\hbar_f) + E_k^\theta(\hbar_\theta). \tag{28}$$

Where the step size δη, *Z* is an integer and $\eta_i = i(\delta\eta)$. For further information, the explanation and application of the HAM in different fields can be referred to in the book by [38]. the optimal values at the *k*th-order approximation of the convergence control ($\hbar_f$ and $\hbar_\theta$) are calculated by the minimum of the total error $E_k^{Total}(\hbar_f, \hbar_\theta)$.

3. Results and Discussions

The optimal convergence control parameters of momentum and energy equation for λ = *We* = 0.1, *Pr* =0.7, ε = *Q* = 0.2, *Rd* = 1, and *n* = 1.2, from 1*st* order up to the 6*th*- order of estimation is showed in Table 1 respectively. Table 1 demonstrates that the total error reduces to 7.77228×10^{-3} from 3.62064×10^{-2} as increasing the order of approximations. The 6*th*-order optimal approximations convergence-control parameters are $\hbar_f = -1.48981$ and $\hbar_\theta = -0.557282$.

Table 1 shows the series of analytical solutions converging in $-1.48981 \leq \hbar_f \leq -0.809868$ for velocity profile and $-0.557282 \leq \hbar_\theta \leq -0.323611$ for temperature profile. For generality, the 3*rd* order of approximation of $\hbar_f = -1.29224$ and $\hbar_\theta = -0.47300$ (see Table 1) is chosen to calculate the results for the following analysis. In this way, the profile and total error residual against the order of calculation of the series solution for λ = *We* = 0.1, ε = *Q* = 0.2, *Pr* =0.7, *Rd* = 1, and *n* = 1.2, are exhibited in Figure 2 and Table 2 respectively. The total residual error is indeed diminishing as the order of calculation increases (see Table 2 and Figure 2).

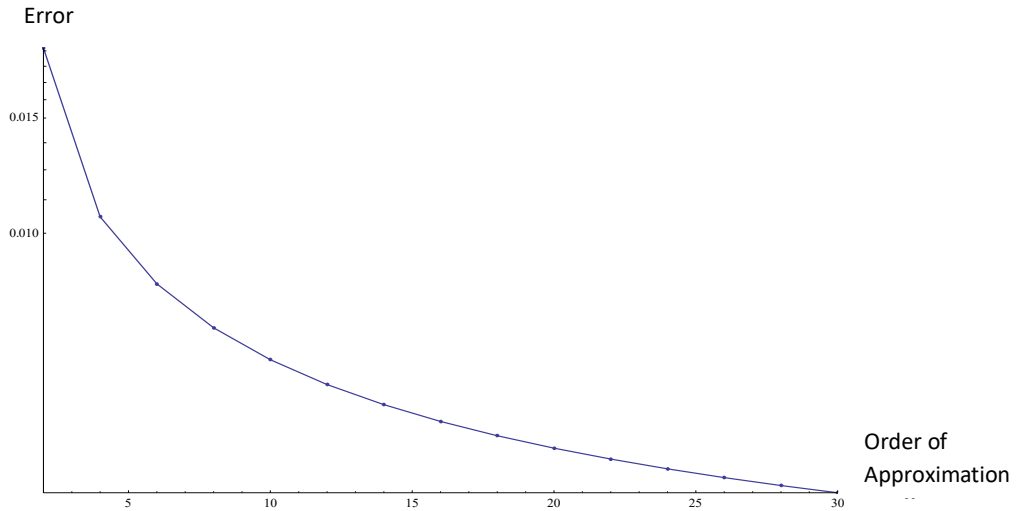

Fig. 2. The total error profile with $\hbar_f = -1.27994$ and $\hbar_\theta = -0.481381$ for λ = *We* = 0.1, ε = *Q* =0.2, *Rd* = 1, *Pr* =0.7, and *n* = 1.2.

Table 3 displays that the current results are in outstanding agreement with the former numerical results. For different values of λ in limiting cases, Table 3 compares the obtained solutions in a published journal paper with the present computed skin friction coefficient to validate the determined series solution. Furthermore, Table 4 presents that the present solutions for the local Nusselt number are well in agreement with the output given in [41] and [42]. The comparison for the local Nusselt number $-\theta'(0)$ is made for various magnitudes of *Pr* at *n* = 1 and *We* =*λ*= *Rd* = *ε* = *Q* =0. This has enhanced our confidence in the solutions determined in the present study.

Table 1
*k*th- orders of optimal approximation value of $\hbar_f$ and $\hbar_\theta$ for *λ* = *We* = 0.1, ε = *Q* = 0.2, *Rd* = 1, *Pr* =0.7, and *n* = 1.2

k	E_k^{Total}	$\hbar_f$	$\hbar_\theta$
1	3.62064×10^{-2}	-0.809868	-0.323611
3	1.31479×10^{-2}	-1.29224	-0.47300
6	7.77228×10^{-3}	-1.48981	-0.557282

Table 2
The results of the square residual errors of the *k*th-orders series solutions for *Pr* =0.7, *λ* = *We* = 0.1, *ε* = *Q* = 0.2, *Rd* = 1, and *n* = 1.2

k	E_k^f	E_k^θ	E_k^{Total}	time-consuming (s)
10	8.39599×10^{-6}	6.39686×10^{-3}	6.40526×10^{-3}	104.845
20	4.40212×10^{-6}	4.68824×10^{-3}	4.69264×10^{-3}	2154.28
30	3.00318×10^{-6}	4.00734×10^{-3}	4.01034×10^{-3}	14720.7

Table 3
The results of the $C_f\sqrt{Re_x}$ in [11], [15], [43], and the present result for a range of values of curvature number *λ* with *n* = 1 and *We* = 0

λ	[11]	[43]	[15]	Present
0.1	-1.03698	-1.03698	-1.03698	-1.03989
0.3	-1.11117	-1.11114	-1.11115	-1.11881
0.7	-1.25705	-1.25701	-1.25702	-1.27297
1.0	-1.45337	-1.36387	-1.36387	-1.38641

Table 4
The results of the $Nu/\sqrt{Re_x}$ in [41], [42] and the present result for a range of values of *Pr* with *n* = 1 and *λ* = *Q* =*We* = *Rd* = *ε* =0

Pr	[41]	[42]	Present
1	0.5832	0.5830	0.5835
10	2.3080	2.3080	2.3095

Figure 3 depicts an increment in the velocity when a higher number of *λ*, the curvature parameter, is applied. This is because the cylinder radius is negatively proportional to the curvature parameter *λ*. A bigger curvature number *λ* gives a smaller diameter of the cylinder, consequently reducing the touching region of the fluid with the cylinder surface. Hence, the resistive force generated by the surface of the tube reduces the fluid velocity.

Additionally, the curvature *λ* has caused the decline of the temperature at the exterior of the tube, as described in Figure 4, since less conduction of heat from the tube to the ambient fluid is allowed due to the narrow contact surface. While the *λ* increases the temperature distribution in the region far away from the pipe (cylinder).

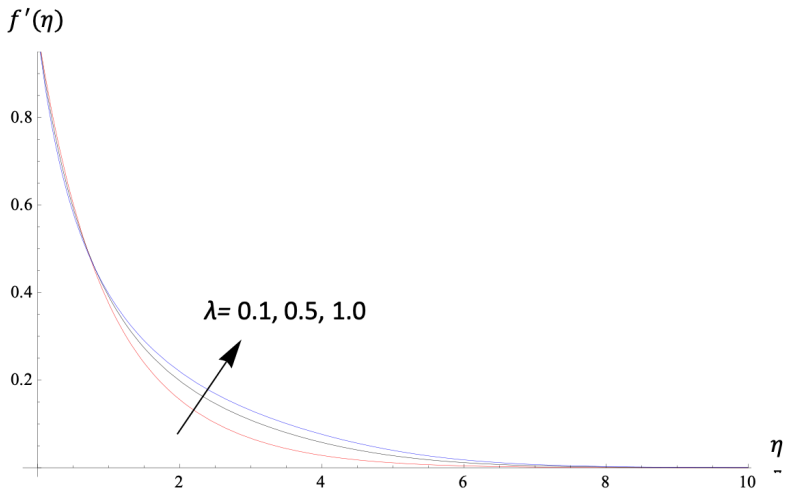

Fig. 3. $f'(\eta)$ against η for numerous magnitudes of λ with n = 1.2 and We = 0.1

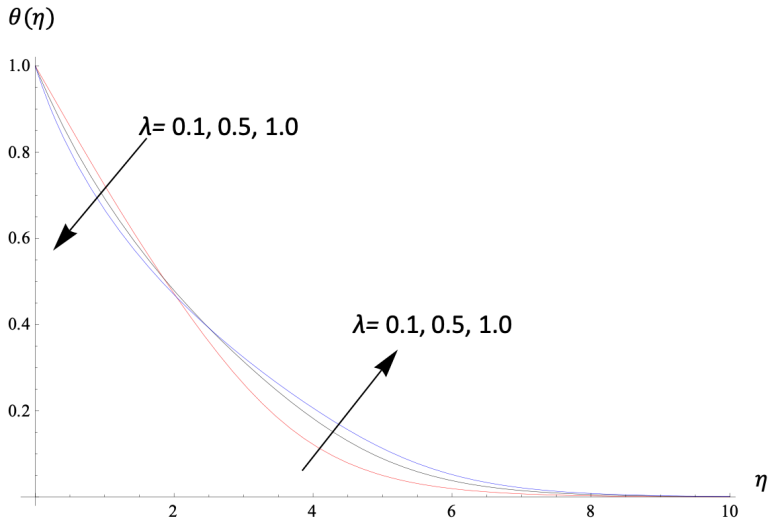

Fig. 4. $\theta(\eta)$ against η for various magnitudes of λ with n = 1.2, We = 0.1, Pr = 0.7, Rd = 1, and ε = Q = 0.2

The space in the fluid is uplifted as the power-law index n has been improved. As a result, the velocity of the liquid is raised as seen in Figure 5. The larger n has made the momentum boundary layer becomes thicker. As n approaches 1, the fluid will behave Newtonian characteristics, leading to a decline in velocity as bigger viscosity is produced. The temperature distribution is diminished regarding the high value of the n, as illustrated in Figure 6.

Figure 8 demonstrates that the velocity profile of $n > 1$ (shear-thickening fluid), is positively proportional to the Weissenberg number We. Contradictory, there is a decrease in the velocity distribution for $n < 1$ (shear − thinning fluid), as illustrated in Figure 7. Physically, We is linearly dependent on Γ (the fluid relaxation time) as given in Equation (17). Accordingly, we assume $\Gamma\dot{\gamma} < 1$, thus the shear rate decreases, which results in $n < 1$ fluid with high viscosity but $n > 1$ fluid with low viscosity. In contrast, the shear-thinning fluid temperature increase for a larger number of the We (see Figure 9). However, a conflicting tendency is illustrated in Figure 10 for the shear-thickening fluid.

Figure 11 interprets that increment enhances the heat transfer in radiation parameters. An alike phenomenon is detected in Figure 12. The heat source parameter Q improves the thermal boundary layer. An improvement in temperature distribution is also achieved by increasing the ε parameter as

established in Figure 13. The thermal conductivity parameter ε increases the α of the fluid. Thus, more thermal is transported from the pipe (tube) to the Carreau fluid and thus induces an increment in kinetic energy of the fluid in particles which enhances the variation of thermal behavior.

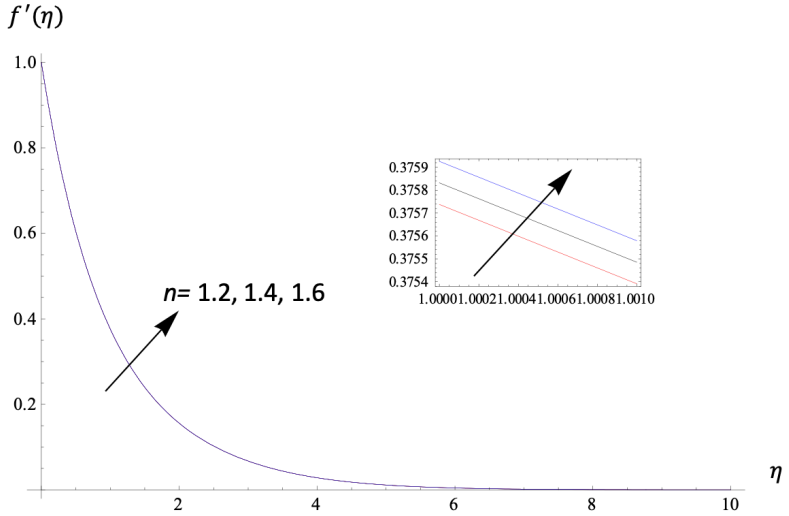

Fig. 5. $f'(\eta)$ against η for numerous magnitudes of n with $We = \lambda = 0.1$

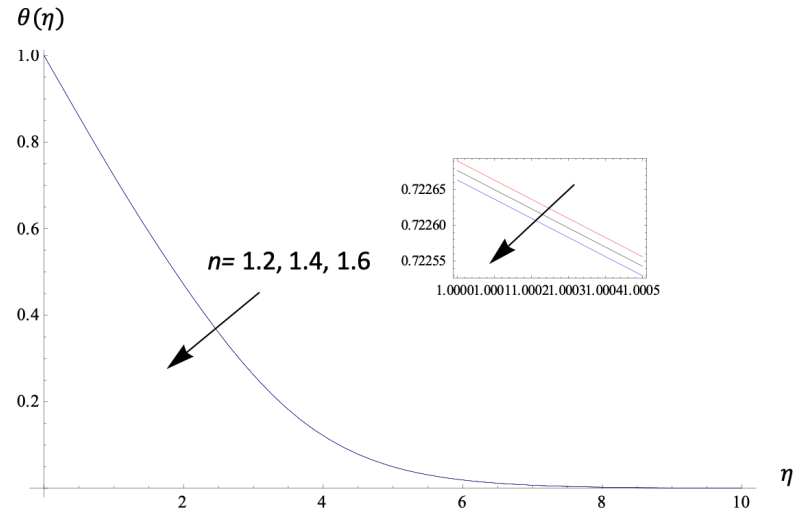

Fig. 6. $\theta(\eta)$ against η for various magnitudes of n with $\lambda = We = 0.1$, $Rd = 1$, $Pr = 0.7$, and $\varepsilon = Q = 0.2$

The behavior of local skin friction and local Nusselt number for a range of pertinent parameters *Pr*, λ, ε, *n We*, *Q,* and *Rd* are directed in Tables 5 and 6, respectively. When n =1.2, the *We* number increases the local skin friction. An opposite trend is noticed when n =0.5 is applied. The curvature and power-law index, respectively, has enhanced the skin friction coefficient, as indicated in Table 5. Furthermore, the local Nusselt number $|\theta'(0)|$ is increased by the value of λ, *Pr, We*, and n, but a decrease happens when n = 0.5 for increasing the number of *We*. A decline is observed in the local Nusselt number $|\theta'(0)|$ when ε, *Rd*, and *Q* is enhanced respectively.

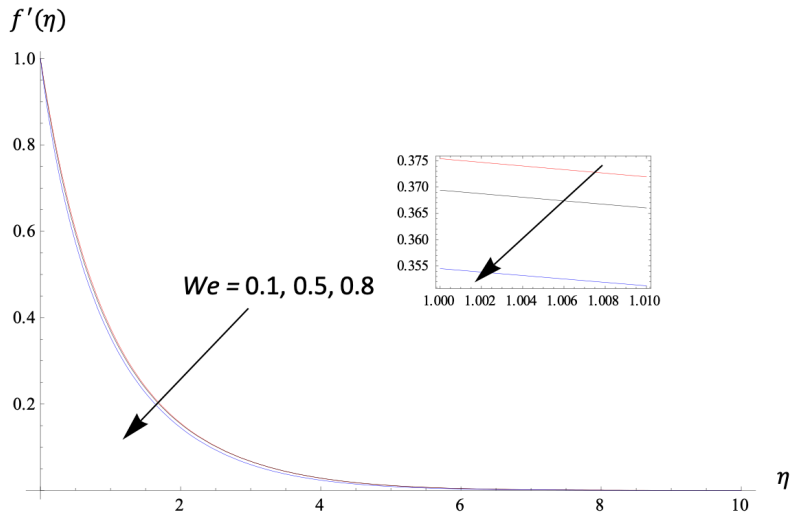

Fig. 7. $f'(\eta)$ against η for various magnitudes of *We* with n = 0.5 and λ =0.1

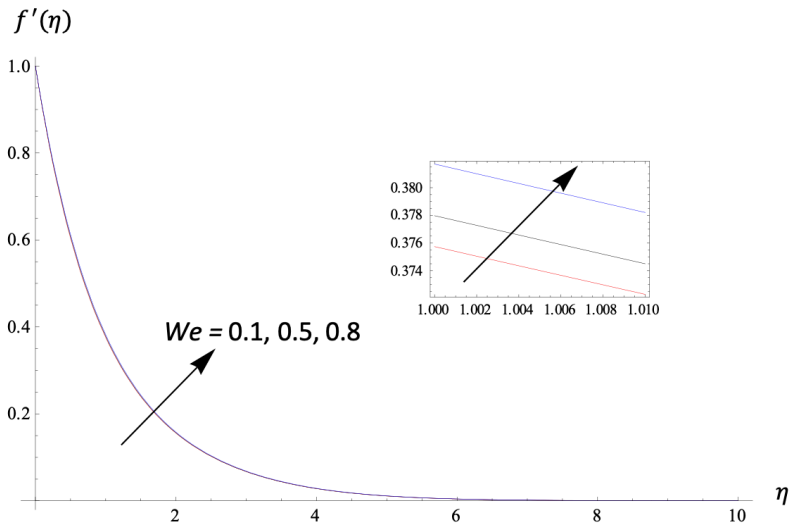

Fig. 8. $f'(\eta)$ against η for several values of *We* with n = 1.2 and λ =0.1

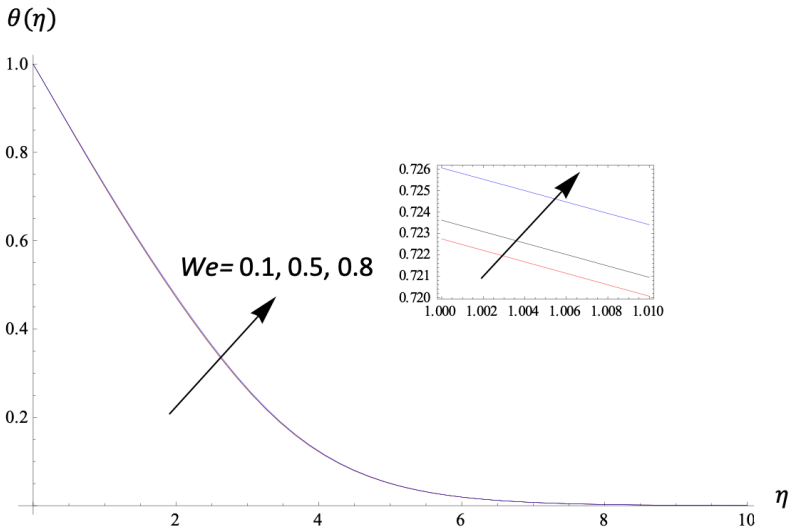

Fig. 9. $\theta(\eta)$ against η for various magnitudes of *We* with n = 0.5, *Pr* = 0.7, Rd = 1, λ = 0.1, and ε = Q = 0.2

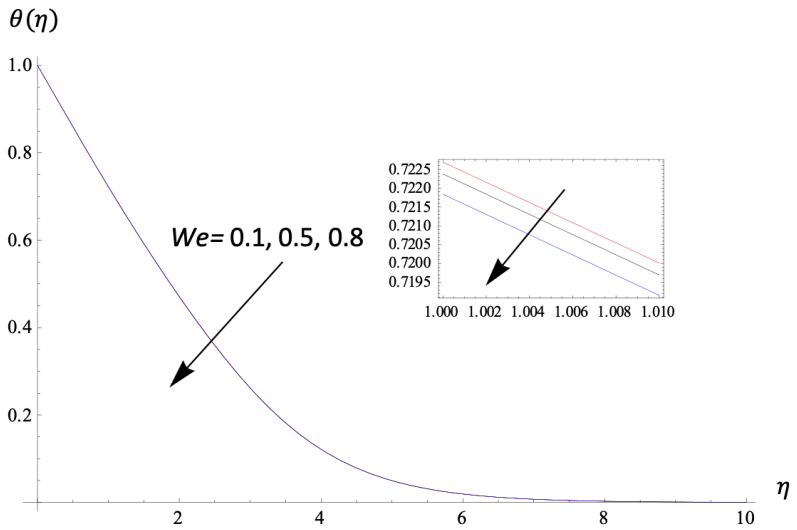

Fig. 10. $\theta(\eta)$ against η for various magnitudes of *We* with n = 1.2, Pr = 0.7, λ =0.1, Rd = 1, and ε = Q = 0.2

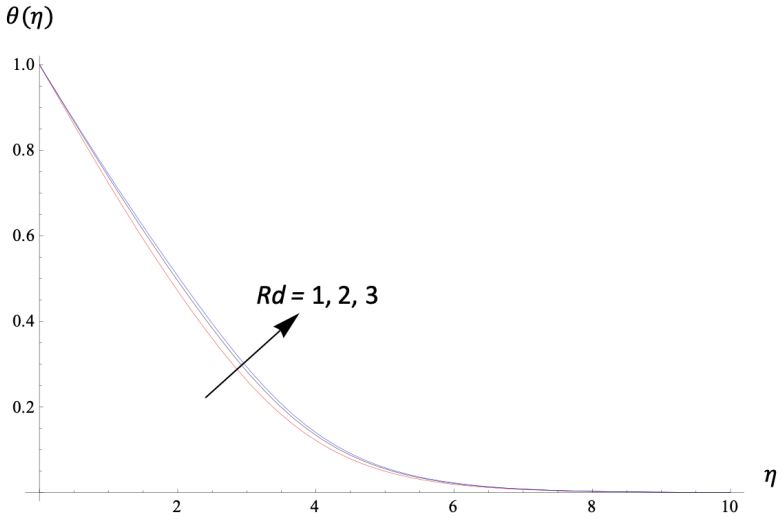

Fig. 11. $\theta(\eta)$ against η for different magnitudes of *Rd* with n = 1.2, ε = Q = 0.2, Pr = 0.7, and λ = *We* = 0.1

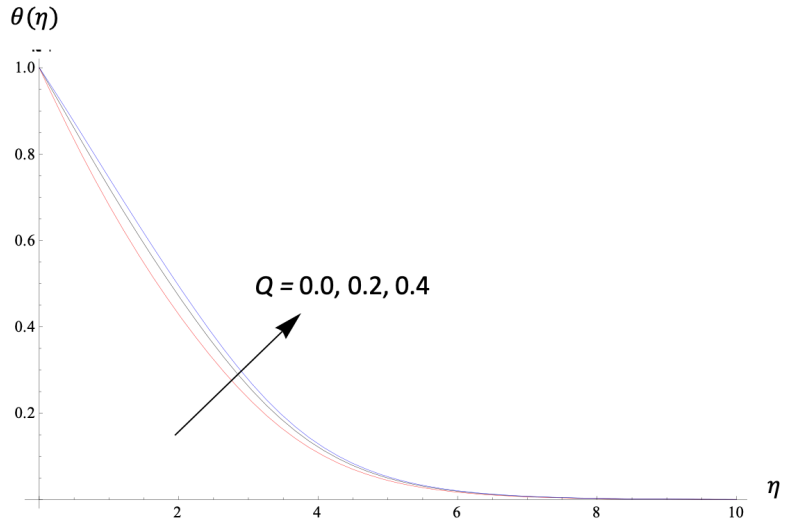

Fig. 12. $\theta(\eta)$ against η for different magnitudes of Q with λ = 0.1, Rd = 1, *We* = 0.1, Pr = 0.7, n = 1.2, and ε = 0.2.

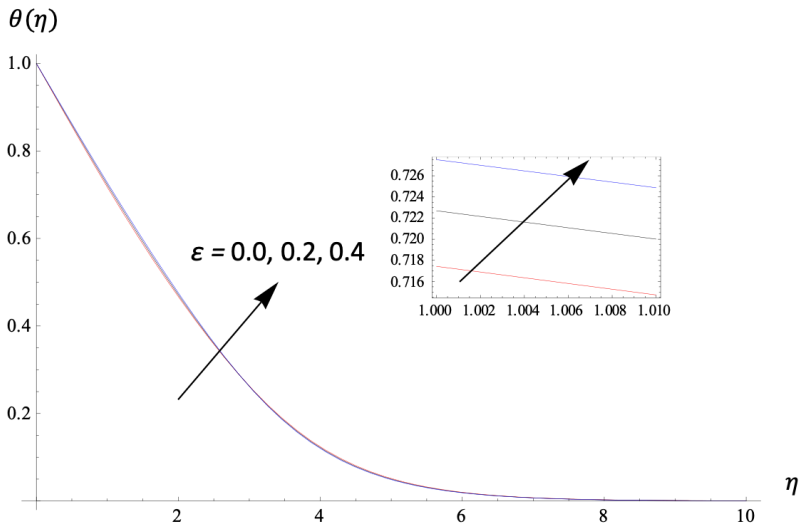

Fig. 13. $\theta(\eta)$ against η for different magnitudes of ε with n = 1.2, ε = 0.2, Pr = 0.7, Rd = 1, and λ = We = 0.1

Table 5
The results of the $C_f\sqrt{Re_x}$ for a range of values of *We*, *n*, and λ

We	*n*	λ	*Cf/Re*
0.1	0.5	0.1	-1.03890
0.5	-	-	-1.01732
0.8	-	-	-0.956351
0.1	1.2	0.1	-1.04012
0.5	-	-	-1.04782
0.8	-	-	-1.05928
0.1	1.4	0.1	-1.04045
-	1.6	-	-1.04078
0.1	1.2	0.5	-1.19653
-	-	1.0	-1.38680

Table 6
The results of the $Nu/\sqrt{Re_x}$ for a range of values of *Rd*, *Pr*, ε, *Q*, *We*, *n*, and λ

Rd	*Pr*	ε	*Q*	λ	*We*	*n*	*Nu/Re*
1	0.7	0.2	0.2	0.1	0.1	1.2	-0.278256
2	-	-	-	-	-	-	-0.266297
3	-	-	-	-	-	-	-0.261270
1	1	0.2	0.2	0.1	0.1	1.2	-0.288260
-	2	-	-	-	-	-	-0.338026
1	0.7	0.0	0.2	0.1	0.1	1.2	-0.285406
-	-	0.4	-	-	-	-	-0.272009
1	0.7	0.2	0.0	0.1	0.1	1.2	-0.346583
-	-	-	0.4	-	-	-	-0.241521
1	0.7	0.2	0.2	0.5	0.1	1.2	-0.387327
-	-	-	-	1.0	-	-	-0.506564
1	0.7	0.2	0.2	0.1	0.5	1.2	-0.278624
-	-	-	-	-	0.8	-	-0.279249
1	0.1	0.2	0.2	0.1	0.1	1.4	-0.278271
-	-	-	-	-	-	1.6	-0.278287
-	-	-	-	-	0.1	0.5	-0.278201
-	-	-	-	-	0.5	-	-0.277169
-	-	-	-	-	0.8	-	-0.274279

4. Conclusions

The thermal and Carreau fluid characteristic passing a horizontal stretching cylinder have been concerned, and the influence of thermal radiation and heat source are included. Furthermore, to be more realistic, the thermal conductivity of the Carreau fluid is taken to be dependent on the temperature of the fluid. The similarity conversion technique is utilized to convert the momentum and energy equations into nonlinear ordinary differential equations. Then, the homotopy analysis scheme approximates the analytical solutions of the similarity-transformed governing equations. The method is verified by comparing the current results with the existing solutions for a limiting case in the literature. The method is found appropriate for solving the proposed problem. The effects of the Weissenberg number, heat generation parameter, radiative number, curvature number, and temperature-dependent thermal conductivity on the fluid movement and thermal distribution are discussed. From this investigation, we can conclude:

- The HAM is appropriate to be used to solve the developed highly non-linear ordinary governing equations.
- *Q, λ, ε,* and *Rd* accelerate the thermal distribution. In contrast, the *n* and *Pr* reduce the temperature.
- A conflict behavior is identified on the temperature profiles due to the Weissenberg number. The temperature shows an increasing function at $n = 0.5$ when *We* is increased but an opposite tendency is detected when $n = 1.2$.
- For the velocity profiles, the magnitude is enhanced when *We* is amplified for $n = 1.2$. Contradicting, the momentum thickness is reduced when $n = 0.5$ and *We* is enhanced.
- The velocity profile is improved by the λ and *n.*

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