CFD Letters

Journal homepage:
https://semarakilmu.com.my/journals/index.php/CFD_Letters/index
ISSN: 2180-1363

Flow of Viscoelastic Fluid with Microrotation at a Boundary Layer Flow of a Horizontal Circular Cylinder

Laila Amera Aziz[1,*], Abdul Rahman Mohd Kasim[1], Mohd Zuki Salleh[1], Ibrahim Faye[2]

[1] Centre for Mathematical Sciences, Universiti Malaysia Pahang, Lebuhraya Tun Razak, 26300 Gambang, Pahang, Malaysia
[2] Fundamental and Applied Science Department, Universiti Teknologi PETRONAS, 32610 Seri Iskandar, Perak, Malaysia

ARTICLE INFO

Article history:
Received 7 September 2022
Received in revised form 29 September 2022
Accepted 22 October 2022
Available online 31 December 2022

Keywords:
Viscoelastic micropolar; circular cylinder; aligned magnetohydrodynamic

ABSTRACT

In this study, the dynamics of the non-Newtonian viscoelastic fluid with microrotation at a boundary layer of a horizontal circular cylinder is investigated. For this case, Lorentz force is induced by external magnetic field positioned at right angle of the fluid flow, hence the magnetohydrodynamic effect is embedded in the momentum equation of the proposed model in addition to other governing parameters. The constitutive equations are converted to dimensionless form along with the associated boundary conditions before the resulting partial differential equations are solved using finite difference technique in Fortran programming. Results are validated before the velocity and microrotation profiles are examined and the effects of material, viscoelastic and magnetohydrodynamic parameter on the flow is discussed.

1. Introduction

The attribute of a fluid is the classification factor whether it belongs in Newtonian or non-Newtonian family. In simple terms, fluid that flows constantly without resistant at a certain temperature due to unchanged viscosity such as water and gasoline are Newtonian. Meanwhile, fluid which flow comes to a halt over time or gain momentum caused by altered viscosity when force is applied like cosmetics and asphalt belongs to the non-Newtonian family. The non-Newtonian fluid is a more complicated subject of study than Newtonian because it defies the Newton's law of viscosity. However, despite the complication, the study on non-Newtonian fluid is still sought-after due to numerous applications.

A theoretical and experimental study, for example, has been conducted by Mohamed and AbdelGawad [1] where they investigate the suitability of non-Newtonian fluid in medical application, namely for the broken bones bandage. The study accomplished to attest the aptness and capability of non-Newtonian fluid at reducing sudden impact for patient's safety and wellness. The application of non-Newtonian fluid also been discussed by Marinov [2] through modelling the work material as viscoplastic fluid where the result from the investigation can be used to analyze and interpret the

* *Corresponding author.*
E-mail address: laila@ump.edu.my (Laila Amera Aziz)

https://doi.org/10.37934/cfdl.14.12.6674

phenomena in the metal cutting process. Besides that, non-Newtonian fluid is also important in pharmaceutical field. For instance, Thurston and Martin [3] have reviewed the behaviour of a polymeric solution called carhoxymethylcellulose that is commonly used in drug product formulation using the generalized Maxwell model. Different models are used in different studies to represent the flow of different type on fluids depending on the dominant characteristics of the fluid itself.

This study will explore the model for the flow of viscoelastic fluid with microrotation. Viscoelastic fluid is viscous fluid with an extra distinct feature of elasticity. There are varieties of model for viscoelastic fluid documented and the simplest model that could portray the viscous and elastic nature of the fluid is the Maxwell model that has been addressed by Brujan [4]. The extension of the prior model is the Oldroyd-B model which takes into consideration the rate of deformation tensor and can be reduced to Maxwell model when the retardation parameter is set to 0. Aside from these, there is also the Dumbbell model that can be further divided into another two models, namely the elastic dumbbell and rigid dumbbell model. Lastly, there is the BKBZ model which is the integral type of constitutive equations. Among the study that considers these models to represent the flow of viscoelastic fluid for the case of bluff body have been presented by several authors [5-8].

Besides being viscoelastic in nature, the fluid in interest in this study contains microelements that according to Karvelas *et al.,* [9] have varying size and shape, that rotate and move independently of the bulk flow, detached from the flow and rotation of the fluid. Micropolar fluid theory introduced by Eringen [10] is the fittest model to portray the characteristics of these microelements that are suspended in viscous fluid. There are substantial number of literatures on micropolar fluid model but only a handful deliberated on the flow characteristics of micropolar fluid over bluff body. The case of micropolar flow past circular cylinder has been discussed in detail in a number of studies [11-13]. While other studies have highlighted the dynamics of the flow when the geometry involved is sphere [14-16].

However, in this study, an elastic fluid that contains suspended microelements that are in the form of short, rigid, dumb-bell molecule such as blood and liquid crystal elastomers is considered. In order the mimic the characteristics of the fluid to the utmost, both parameters from viscoelastic and micropolar model are linked in a set of constitutive equations for an improvised viscoelastic micropolar model. It is also of interest to see the change in the flow of the fluid when it is exposed to a magnetic field inclined at an angle. The investigation on MHD effect on viscoelastic fluid has been presented by Kasim *et al.,* [17] and Mahat *et al*., [18] while Bhat and Katagi [19] has examined the MHD effect on micropolar fluid flow. Both studies have demonstrated that magnetic effect does exist on the flow. The study of magneto-convective viscoelastic micropolar has been discussed by Aziz *et al.,* [20] but this study will put more focus on the flow itself in terms of the velocity and boundary layer separation without consideration of the heat transfer process.

2. Mathematical Formulation

An incompressible two-dimensional flow of a horizontal circular cylinder with radius a, submerged in a viscoelastic micropolar fluid while imposed to a magnetic field acting at an inclined angle is the subject of this research. This problem is investigated under the assumption that the temperature for the surface of the cylinder remains constant. The illustration of the model is shown in Figure 1 where is the circumference measurement of the cylinder from the lower stagnation point while represents the length perpendicular to the body surface and is the free stream velocity.

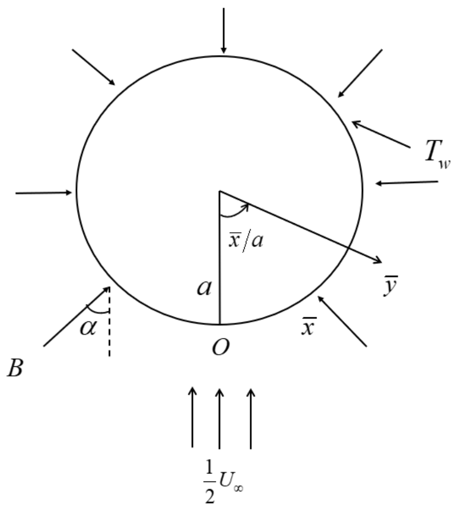

Fig. 1. Schematic diagram for the flow

The problem is governed by a set of equations that each represents the characteristics of the flow as follows:

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0 \tag{1}$$

$$\begin{aligned}\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}=\bar{u}_e\frac{d\bar{u}_e}{d\bar{x}}+\left(\frac{\mu+\kappa}{\rho}\right)\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}+\frac{\kappa}{\rho}\frac{\partial \bar{N}}{\partial \bar{y}}+\frac{k_0}{\rho}\left[\frac{\partial}{\partial \bar{x}}\left(\bar{u}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right)+\bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3}-\frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}}\right]\\ -\frac{\sigma}{\rho}\left(\bar{u}-\bar{u}_e\right)B_0^2\sin^2\alpha_1\end{aligned} \tag{2}$$

$$\rho j\left(\bar{u}\frac{\partial \bar{N}}{\partial \bar{x}}+v\frac{\partial \bar{N}}{\partial \bar{y}}\right)=-\kappa\left(2\bar{N}+\frac{\partial \bar{u}}{\partial \bar{y}}\right)+\gamma\frac{\partial^2 \bar{N}}{\partial \bar{y}^2} \tag{3}$$

The boundary conditions associated to the governing equations are

$$\begin{aligned}&\bar{u}=\bar{v}=0, \quad \bar{N}=-n\frac{\partial \bar{u}}{\partial \bar{y}} \qquad \text{on } \bar{y}=0,\\ &\bar{u}\to\bar{u}_e(x),\ \frac{\partial \bar{u}}{\partial \bar{y}}\to 0,\ \bar{N}\to 0, \quad \text{as } \bar{y}\to\infty\end{aligned} \tag{4}$$

Each parameter that made up the equations represents different physical quantities with different units which justifies the reason why non-dimensionalisation is necessary as the next step. These variables are introduced

$$x=\frac{\bar{x}}{a},\ y=\frac{\mathrm{Re}^{1/2}\,\bar{y}}{a},\ u=\frac{\bar{u}}{U_\infty},\ v=\frac{\mathrm{Re}^{1/2}\,\bar{v}}{U_\infty},\ N=\frac{\mathrm{Re}^{-1/2}\,a\bar{N}}{U_\infty},\ u_e=\frac{\bar{u}_e(\bar{x})}{U_\infty} \tag{5}$$

resulting to the following set of dimensionless equations.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{6}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{du_e}{dx}\left(1+K_1\right)\frac{\partial^2 u}{\partial y^2}+K_1\frac{\partial N}{\partial y}+K\left[\frac{\partial}{\partial x}\left(\bar{u}\frac{\partial^2\bar{u}}{\partial \bar{y}^2}\right)+\bar{v}\frac{\partial^3\bar{u}}{\partial \bar{y}^3}-\frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2\bar{u}}{\partial \bar{x}\partial \bar{y}}\right]-M\left(u-u_e\right)\sin^2\alpha_1 \tag{7}$$

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=-K_1\left(2N+\frac{\partial u}{\partial y}\right)+\left(1+\frac{K_1}{2}\right)\frac{\partial^2 N}{\partial y^2} \tag{8}$$

subjected to the boundary conditions

$$\begin{aligned} &u=v=0,\ N=-n\frac{\partial u}{\partial y} \qquad \text{on } y=0\\ &u\to u_e(x),\ \frac{\partial u}{\partial y}\to 0,\ N\to 0 \quad \text{as } y\to\infty \end{aligned} \tag{9}$$

The main parameters in the constitutive equations, which are the viscoelastic parameter, K, the material parameter, K_1, magnetic parameter, M as well as the microinertia density, j, and the spin gradient viscosity, can be defined as

$$K=\frac{k_0U_\infty}{a\rho\nu},\ K_1=\frac{\kappa}{\mu},\ M=\frac{\sigma B_0^2 a}{\rho U_\infty},\ \ j=\frac{a\nu}{U_\infty},\ \gamma=\left(\mu+\frac{\kappa}{2}\right)j \tag{10}$$

Then, the following set of similarity equations is introduced to simplify the equations by reducing the dependence of some terms to a single variable instead of two. The variables that follow are assumed to solve Eq. (6) to Eq. (8) subjected to the boundary conditions in Eq. (9).

$$\psi=xf(x,y),\ N=xg(x,y) \tag{11}$$

where the stream function, is defined as

$$u=\frac{\partial\psi}{\partial y},\ \ v=-\frac{\partial\psi}{\partial x} \tag{12}$$

As a result from the transformation, these equations are obtained.

$$\begin{aligned} &\left(1+K_1\right)\frac{\partial^3 f}{\partial y^3}+f\frac{\partial^2 f}{\partial y^2}-\left(\frac{\partial f}{\partial y}\right)^2+\frac{\sin x\cos x}{x}+K_1\frac{\partial g}{\partial y}-M\left(\frac{\partial f}{\partial y}-\frac{\sin x}{x}\right)\sin^2\alpha\\ &+K\left\{2\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}-f\frac{\partial^4 f}{\partial y^4}-\left(\frac{\partial^2 f}{\partial y^2}\right)^2+x\left(\frac{\partial^2 f}{\partial x\partial y}\frac{\partial^3 f}{\partial y^3}-\frac{\partial f}{\partial x}\frac{\partial^4 f}{\partial y^4}+\frac{\partial f}{\partial y}\frac{\partial^4 f}{\partial x\partial y^3}-\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial x\partial y^2}\right)\right\}\\ &=x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}-\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right) \end{aligned} \tag{13}$$

$$\left(1+\frac{K_1}{2}\right)\frac{\partial^2 g}{\partial y^2}+f\frac{\partial g}{\partial y}-g\frac{\partial f}{\partial y}-K_1\left(2g+\frac{\partial^2 f}{\partial y^2}\right)=x\left(\frac{\partial f}{\partial y}\frac{\partial g}{\partial x}-\frac{\partial f}{\partial x}\frac{\partial g}{\partial y}\right) \tag{14}$$

bounded by the transformed boundary conditions

$$\begin{aligned} &f=\frac{\partial f}{\partial y}=0, \quad g=-\frac{1}{2}\frac{\partial^2 f}{\partial y^2} \qquad \text{at } y=0 \\ &\frac{\partial f}{\partial y}=\frac{\sin x}{x}, \quad \frac{\partial^2 f}{\partial y^2}=0, \quad g=0 \quad \text{as } y\to\infty \end{aligned} \tag{15}$$

The value $n=1/2$ is chosen for this problem where it indicates weak concentration in microelements and rotation is plausible, while the anti-symmetric part of the stress tensor is vanishing [21]. At the lower stagnation point of the cylinder, the constitutive equations of the viscoelastic micropolar flow are reduced to a set of differential equation given by

$$\left(1+K_1\right)f'''+ff''-f'^2+1+K_1g'-M\left(f'-1\right)\sin^2\alpha+K\left(2f'f'''-f\,f^{iv}-f''^2\right)=0 \tag{16}$$

$$\left(1+\frac{K_1}{2}\right)g''+fg'-f'g-K_1\left(2g+f''\right)=0 \tag{17}$$

corresponds to the boundary conditions

$$\begin{aligned} &f(0)=f'(0)=0, \quad g(0)=-\frac{1}{2}f''(0) \quad \text{at } y=0 \\ &f'(\infty)=1, \quad f''(\infty)=0, \quad g(\infty)=0 \qquad \text{as } y\to\infty \end{aligned} \tag{18}$$

where the derivatives are with respect to *y* variable.

3. Results and Discussion

The numerical solutions for the non-Newtonian flow model are attained by using the Keller-box method in Fortran programming. For verification purpose, the results obtained are compared to the exact solution from Ariel [22] and numerical solution from another limiting case of viscoelastic model by Anwar *et al.,* [23]. The comparison of the resulting skin friction values for a variety of viscoelastic parameter values to the mentioned literatures is presented in Table 1. Considering the commendable degree of similarity between the current result and the exact solution, it is decent to proclaim that the current model is trustworthy.

Table 1
Values of $f''(0)$ at different values of *K* when *M*=K_1=0

K	Exact solution (Ariel [22])	Present	Viscoelastic model (Anwar *et al.,* [23])
0	1.232588	1.232657	-
0.01	-	1.221447	1.222693
0.05	1.179830	1.179893	-
0.1	1.134114	1.134172	1.135982
0.2	1.058131	1.058180	1.045412
0.3	0.996844	0.996886	0.960922
0.4	0.945869	0.945907	0.882512
0.5	0.902500	0.902535	0.810182
0.6	-	0.864985	0.743933
0.7	-	0.832019	0.683763
0.8	-	0.802749	0.629673
0.9	-	0.776511	0.581664
1	0.752766	0.752803	-
100	0.099515	0.100783	-
500	0.044677	0.045487	-
1000	0.031607	0.032229	-

Figure 2, Figure 3 and Figure 4 show the velocity profiles of the flow when the different values of viscoelastic, material and magnetic parameters are considered. The result reveals that a rise in the values of viscoelastic and material parameter lead to reduction in the velocity profile while the elevation of magnetic parameter enhanced the velocity of the flow. From the result, it appears that the MHD effects may boost the velocity of viscoelastic micropolar fluid and since only positive values are recorded, it means that there is no reversal flows for the selected values of magnetic parameter. From Figure 7, it is evident that the Lorentz force formed when the electrically conducting fluid moves through the induced magnetic field affect the skin friction, which in turns delay the undesirable boundary layer separation as the magnetic parameter increases.

It can be observed from Figure 3 that higher number of macro particles in the fluid hinder the fluid movement, causing the velocity to reduce while Figure 6 suggests that high concentrated mass in the flow downscale the effect of skin friction on the cylinder surface. As for the viscoelasticity effect, being highly viscoelastic can retard the flow of fluid as well as procrastinate the separation. Similar result has been documented by Jones and Lewis [24] when they investigate the separation phenomenon for the flow past a cylinder. The study concludes that viscoelasticity causes the point of separation to move towards the front stagnation point.

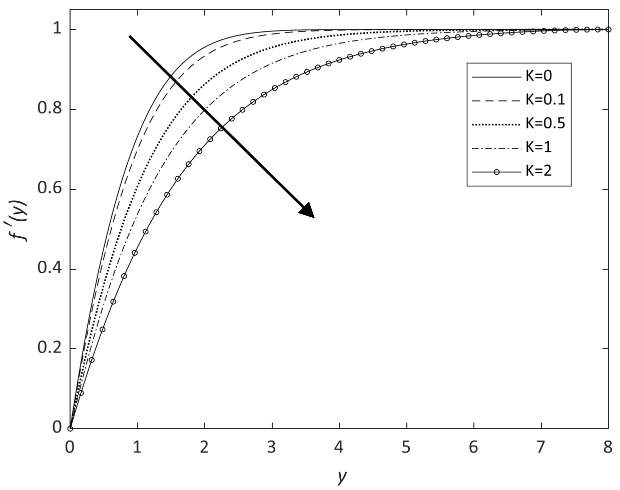

Fig. 2. Velocity profile at various *K* values

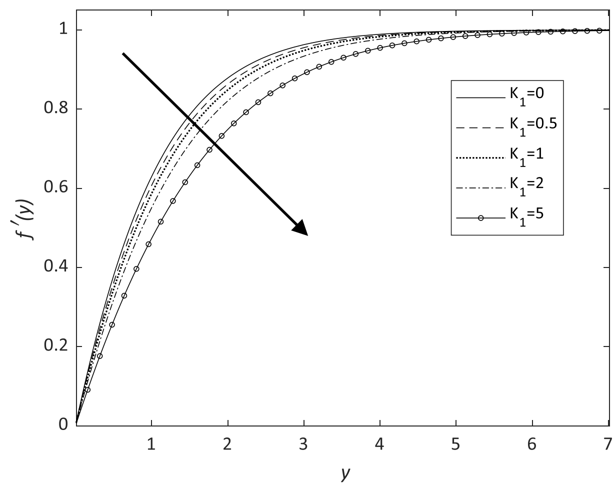

Fig. 3. Velocity profile at various K_1 values

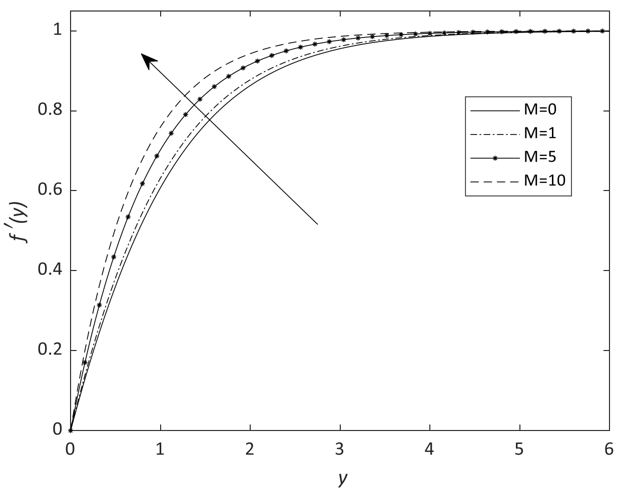

Fig. 4. Velocity profile at various *M* values

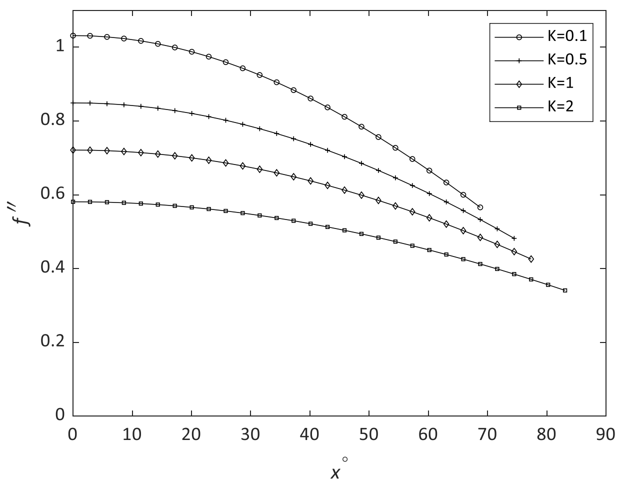

Fig. 5. Skin friction coefficient at various *K* values

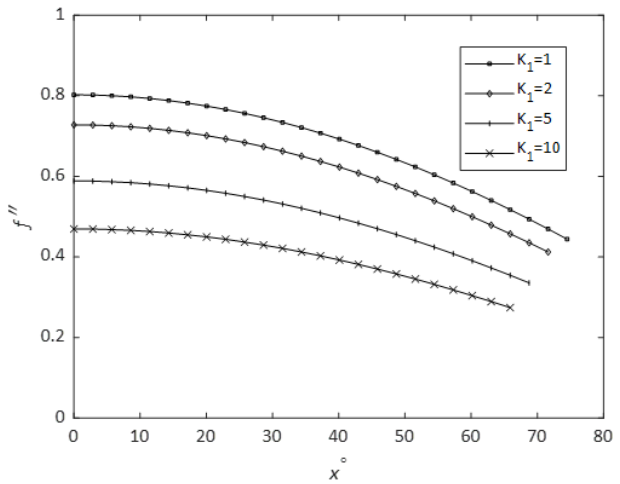

Fig. 6. Skin friction coefficient at various K_1 values

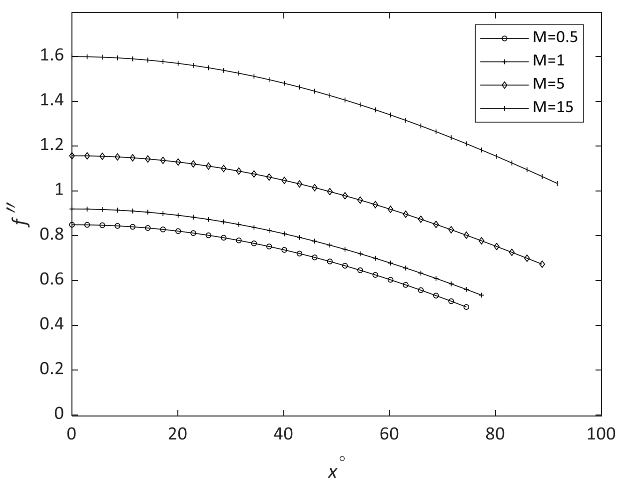

Fig. 7. Skin friction coefficient at various *M* values

4. Conclusions

The problem of boundary layer flow of viscoelastic micropolar fluid with aligned MHD effect is investigated. Holding other parameters constant, the results obtained suggested that:

I. higher viscoelastic parameter will result in higher velocity and decrease the skin friction coefficient.
II. rise in the value of magnetic parameter boost the velocity as well as the skin friction coefficient.
III. when the material parameter increases, both velocity and skin friction coefficient decrease.
IV. boundary layer separation is delayed for greater values of magnetic and viscoelastic parameter, but lower value of material parameter.

Acknowledgement
This research was funded by a grant from Universiti Malaysia Pahang (RDU 223201).

References

[1] Mohamed, Ahmed Bahgat, and Ahmed Farouk AbdelGawad. "Experimental Study of Non-Newtonian Fluid Behavior by Utilizing Drop Test for Medical Applications." In *Proceedings of Fourteenth International Conference of Fluid Dynamics (ICFD14)*. Cairo, Egypt, 2021.
[2] Marinov, Valery. "Application of Non-Newtonian Fluid Mechanics in Modeling of the Metal Cutting Process: An Overview." In *Proceedings of the Sixth CIRP International Workshop on Modeling of Machining Operations*. Hamilton, Ontario, Canada, May, pp. 19-20. 2003.
[3] Thurston, George B., and Alfred Martin. "Rheology of pharmaceutical systems: oscillatory and steady shear of non-Newtonian viscoelastic liquids." *Journal of Pharmaceutical Sciences* 67, no. 11 (1978): 1499-1506. https://doi.org/10.1002/jps.2600671103
[4] Brujan, Emil. *Cavitation in Non-Newtonian fluids: with biomedical and bioengineering applications*. Springer Science & Business Media, 2011. https://doi.org/10.1007/978-3-642-15343-3
[5] Arif, Muhammad, Poom Kumam, Dolat Khan, and Wiboonsak Watthayu. "Thermal performance of GO-MoS_2/engine oil as Maxwell hybrid nanofluid flow with heat transfer in oscillating vertical cylinder." *Case Studies in Thermal Engineering* 27 (2021): 101290. https://doi.org/10.1016/j.csite.2021.101290
[6] Abbasi, A., F. Mabood, W. Farooq, and Z. Hussain. "Non-orthogonal stagnation point flow of Maxwell nano-material over a stretching cylinder." *International Communications in Heat and Mass Transfer* 120 (2021): 105043. https://doi.org/10.1016/j.icheatmasstransfer.2020.105043
[7] Abro, K. A., and A. Abdon. "A computational technique for thermal analysis in coaxial cylinder of one-dimensional flow of fractional Oldroyd-B nanofluid." *International Journal of Ambient Energy* (2021): 1-9. https://doi.org/10.1080/01430750.2021.1939157
[8] Davoodi, M., K. Zografos, P. J. Oliveira, and R. J. Poole. "On the similarities between the simplified Phan-Thien-Tanner model and the finitely extensible nonlinear elastic dumbbell (Peterlin closure) model in simple and complex flows." *Physics of Fluids* 34, no. 3 (2022): 033110. https://doi.org/10.1063/5.0083717
[9] Karvelas, Evangelos, Giorgos Sofiadis, Thanasis Papathanasiou, and Ioannis Sarris. "Effect of micropolar fluid properties on the blood flow in a human carotid model." *Fluids* 5, no. 3 (2020): 125. https://doi.org/10.3390/fluids5030125
[10] Eringen, A. Cemal. "Theory of micropolar fluids." *Journal of Mathematics and Mechanics* (1966): 1-18. https://doi.org/10.1512/iumj.1967.16.16001
[11] Nazar, Roslinda, Norsarahaida Amin, and Ioan Pop. "Mixed convection boundary-layer flow from a horizontal circular cylinder in micropolar fluids: case of constant wall temperature." *International Journal of Numerical Methods for Heat & Fluid Flow* 13, no. 1 (2003): 86-109. https://doi.org/10.1108/09615530310456778
[12] Alwawi, Firas, Ibrahim M. Sulaiman, Mohammed Z. Swalmeh, and Nusayba Yaseen. "Energy transport boosters of magneto micropolar fluid flowing past a cylinder: A case of laminar combined convection." *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science* (2022): 09544062221111055. https://doi.org/10.1177/09544062221111055

[13] Khan, Aamir Abbas, Nadeem Abbas, S. Nadeem, Qiu-Hong Shi, M. Y. Malik, Muhammad Ashraf, and Shafiq Hussain. "Non-Newtonian based micropolar fluid flow over nonlinear starching cylinder under Soret and Dufour numbers effects." *International Communications in Heat and Mass Transfer* 127 (2021): 105571. https://doi.org/10.1016/j.icheatmasstransfer.2021.105571

[14] Alkasasbeh, Hamzeh Taha. "Numerical solution of micropolar Casson fluid behaviour on steady MHD natural convective flow about a solid sphere." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 50, no. 1 (2018): 55-66.

[15] Aparna, Podila, Podila Padmaja, Nalimela Pothanna, and Josyula Venkata Ramana Murthy. "Uniform Flow of Viscous Fluid Past a Porous Sphere Saturated with Micro Polar Fluid." *Biointerface Research in Applied Chemistry* 13, no. 1 (2022): 1-12. https://doi.org/10.33263/BRIAC131.069

[16] Madasu, Krishna Prasad, and Tina Bucha. "Influence of MHD on micropolar fluid flow past a sphere implanted in porous media." *Indian Journal of Physics* 95, no. 6 (2021): 1175-1183. https://doi.org/10.1007/s12648-020-01759-7

[17] Kasim, A. R. M., N. F. Mohammad, I. Anwar, and S. Sharidan. "Mhd Effect on Convective Boundary Layer Flow of a Viscoelastic Fluid Embedded in Porous Medium with Newtonian Heating." *Recent Advances in Mathematics* 4 (2013): 182-189.

[18] Mahat, Rahimah, Muhammad Saqib, Imran Ulah, Sharidan Shafie, and Sharena Mohamad Isa. "MHD Mixed Convection of Viscoelastic Nanofluid Flow due to Constant Heat Flux." *Journal of Advanced Research in Numerical Heat Transfer* 9, no. 1 (2022): 19-25.

[19] Bhat, Ashwini, and Nagaraj N. Katagi. "Magnetohydrodynamic flow of micropolar fluid and heat transfer between a porous and a non-porous disk." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 75, no. 2 (2020): 59-78. https://doi.org/10.37934/arfmts.75.2.5978

[20] Aziz, Laila Amera, Abdul Rahman Mohd Kasim, Mohd Zuki Salleh, Sharidan Shafie, and Wan Nur Syahidah Wan Yusoff. "Boundary layer flow of mixed convection viscoelastic micropolar fluid over a horizontal circular cylinder with aligned magnetohydrodynamic effect." *Malaysian Journal of Fundamental and Applied Sciences* 13, no. 4 (2017): 567-571. https://doi.org/10.11113/mjfas.v0n0.590

[21] Agarwal, Vandana, Bhupander Singh, Amrita Kumari, Wasim Jamshed, Kottakkaran Sooppy Nisar, Abdulrazak H. Almaliki, and H. Y. Zahran. "Steady Magnetohydrodynamic Micropolar Fluid Flow and Heat and Mass Transfer in Permeable Channel with Thermal Radiation." *Coatings* 12, no. 1 (2021): 11. https://doi.org/10.3390/coatings12010011

[22] Ariel, P. D. "On extra boundary condition in the stagnation point flow of a second grade fluid." *International Journal of Engineering Science* 40, no. 2 (2002): 145-162. https://doi.org/10.1016/S0020-7225(01)00031-3

[23] Anwar, Ilyana, Norsarahaida Amin, and Ioan Pop. "Mixed convection boundary layer flow of a viscoelastic fluid over a horizontal circular cylinder." *International Journal of Non-Linear Mechanics* 43, no. 9 (2008): 814-821. https://doi.org/10.1016/j.ijnonlinmec.2008.04.008

[24] Jones, J. R., and M. K. Lewis. "On the streamline motion of elastico-viscous liquids in rotating channels." *Rheologica Acta* 7, no. 4 (1968): 307-316. https://doi.org/10.1007/BF01984843