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Throughflow and Gravity Modulation Effects on Thermal Convection in a Couple Stress Fluids Saturating a Porous Medium with an Internal Heat Source

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ABSTRACT

In this paper, the effect of throughflow and gravity modulation on thermal convection in a couple stress fluids saturating a porous medium with an internal heating source is investigated. A weakly nonlinear stability analysis is proposed to study the stationary mode of convection. The amplitude of gravity modulation is assumed to be very small and the disturbances are extended in terms of the power series of the amplitude of convection. Using a non-autonomous Ginzburg- Landau amplitude equation, heat transport is evaluated in terms of the Nusselt number. The finite-amplitude of convection has been derived in the third order. The amplitude and frequency of modulation have the effects of increasing or diminishing heat transport. The presence of a couple-stress parameter with internal heat source throughflow and modulation effects has been discussed. The effect of the internal heat source increases or decreases heat transfer in the system. For suitable ranges of Ω the throughflow and internal heating have both destabilizing and stabilizing effects. Finally flow patterns are presented in terms of streamlines and isotherms.

1. Introduction

Natural convection (buoyancy driven convection, where the gravitational force is a major factor) in fluid-saturated porous media is interesting because of its contribution to the numerous practical applications, including the oil recovery process in the petroleum industry, reactor vessel insulation and geothermal energy extraction. There are many examples in nature when the above mentioned uses of porous media are present. In some of these applications control convective instability is to maintain a nonlinear temperature gradient. These methods include volumetric distribution of internal heat source, radioactive heat transfer, proper thermal and rotation modulation, time-dependent heating or cooling at the boundaries, and periodic vibration of the porous material.

A gravitational modulation is essential when the system is under vertical vibrations. In this circumstance, the density gradient is vibrating, and the buoyancy forces that result from the interaction of the gravity field with the density gradient have a complex spatiotemporal structure.

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The time-dependent gravity field has applications in crystal formation, large-scale atmospheric convection, and space laboratory experiments. Nelson [1] has performed a huge amount of theoretical and experimental research on the processing of materials or the physics of fluids in microgravity from inside orbiting spacecraft. According to Wadih and Roux [2] and Wadih *et al.,* [3], the vibrations can either considerably enhancing or delaying the transfer of heat, thus significant affect the convection. The influence of modulated gravity on a convectively stable configuration can significantly influence the stability of a system by increasing or diminishing its susceptibility to convection. The initial studies were conducted by Gershuni *et al.,* [4] and Gresho and Sani [5] where the gravity modulation on the stability of the layer being heated from below is increased by g-jitter. Yang [6] has investigated the instability of a viscoelastic fluid layer heated from below in a modulated gravitational field numerically. It is observed that, modulation has a destabilization effect at lower frequencies and a small stabilization effect at high frequencies that this effect increases as modulation amplitude increases. Additionally, it is also found that for viscoelastic fluid acted on by a gravity modulation as those of Newtonian fluids, modulation has the same effects at both very low and very high frequencies. Malashetty and Padmavathi [7] investigated the onset of convection in both fluid and porous layers using Venezian model. They found that, the low amplitude g-jitter can have a significant effect on the stability of the problem. The viscous flow limit and Darcy limit are obtained as degenerate cases of the Brinkman model. Rees and Pop [8] found the boundary-layer flow generated by a vertical surface with constant temperature fixed in a vibrating porous medium. The g-jitter amplitude is considered to be small when compared to the mean acceleration. Govender [9] investigated a weakly nonlinear analysis in a porous medium under gravity modulation. It is shown that, the vibration in frequency causes the convection of amplitude approaches to zero when the vibration frequency increases which stabilizes the system. The bio convection in a shallow horizontal fluid saturated porous layer that contains a suspension of oxytactic bacteria, such as Bacillusubtilis was investigated by Kuznetsov [10]. It is determined that, analysis of linear stability indicates that g-jitter has a stabilizing effect on the suspension. Some of the important and well documented works on gravity modulation are [11-15].

Throughflow causes the fundamental state temperature to change from linear to nonlinear with layer height, which substantially impacts on the system's stability. When the porous layer's boundaries are of the same types, throughflow is found to be stabilising; however, if they are not, throughflow is found to be destabilising in one particular direction [16-18]. Moreover, it has been observed that the throughflow is unstable even when the boundaries are of the same kind and when there is an extra diffusing component [19]. Nield [20] has investigated how throughflow affects the onset of convective in a horizontally porous medium for a Newtonian fluid. Suma *et al.,* [21] examine the effects of throughflow and a variable gravity field on heat transfer in a porous material. Kiran [22,23] investigates nonlinear throughflow and g-jitter effects on porous media for the stationary and oscillatory modes of convection. Weakly nonlinear system was constructed in order to examine heat and mass transfer across the porous medium, while obtaining the non-autonomous Ginzburg-Landau equation. The same phenomenon for modulation of temperature was investigated by Kiran and Bhadauria [24], who examined three different types of temperature distributions and discovered that when the boundaries are at out of phase modulation the heat transmission is maximum. Darcy convection is investigated for nonlinear stationary mode. Vanishree [25] investigated the effects of throughflow and internal heating onset of convection in a porous medium. Here, throughflow and internal heating effects on onset of convention is mentioned. It is also found that, depending on the direction of the flow and internal heat generation the onset of convection mat stabilise or destabilise the system. Hetsroni *et al.,* [26] have investigated the heat transfer in a screen for the transport of high-energy beams with a porous material made of metal wastes. In the case of internal heating and gravitational modulation, Bhadauria *et al.,* [27] conducted a weakly nonlinear thermal destabilization in a porous media. By using Ginzburg Landau equation to derive amplitude of convection and observed that modulation of gravity used, to alter the heat transfer, whereas internal heating is to increases the heat transfer. Bhadauria *et al.,* [28] and Saravanan [29] were investigated the studies of internal heat generation. Bakar and Roslan [30] have investigated the influences of internal heat generation or absorption parameters in terms of the flow, heat transfer, and Nusselt number. They found that the presence of internal heat generation or absorption has a significant effect on the fluid flow and heat transfer process in the horizontal cavity. Mahat *et al.,* [31] it was found that velocity, temperature, skin friction and heat transfer coefficients of viscoelastic nanofluid depend strongly on viscosity and thermal conductivity together with magnetic field. The critical Rayleigh numbers from the double diffusive binary fluid were obtained by using the Galerkin expansion procedure is studied by Loni *et al.,* [32]. For the different shapes of cavity receiver were studied under the same operating conditions for prediction of the internal heat transfer coefficient correlation for each cavity receiver. Their results reveal that the hemispherical cavity receiver had the highest cavity heat gain, heat transfer coefficient, and Nusselt number values compared to two other cavity receivers. Abidin *et al.,* [33] have investigated the onset of Darcy-Rayleigh convection in a viscoelastic double diffusive binary fluid layer saturated in an anisotropic porous with temperature dependent viscosity. The system is heated from below and cooled from above. It has been observed that, in the literature several models being performed for linear theory with throughflow, but is it scary for nonlinear models with gravity modulation. Because, it is due to Kiran [22,23] and Kiran and Bhadauria [24] studied that, under temperature or gravity modulation, the throughflow effects are considered. They are the first studies to investigate the effects of stationary and oscillatory convection on nonlinear throughflow under modulation. The Peclet number, which evaluates the intensity of throughflow, considers the conduction state temperature to be nonlinear and thus has an effect on the system through related energy and momentum equations. It has been observed that, this nonlinear throughflow has duel effect on heat transport in the system.

2. Governing Equation

We consider a non-Newtonian fluid-saturated infinitely extended horizontally porous media bounded within two boundaries that are completely free - free at z = 0 and z = d as heated from the bottom. ΔT is fixed variation in temperature all over the porous media. We have used the reference in Cartesian terms with the origin at the bottom as well as z - axis moving upwards in a vertical direction. Its schematic diagram is shown in Figure 1.

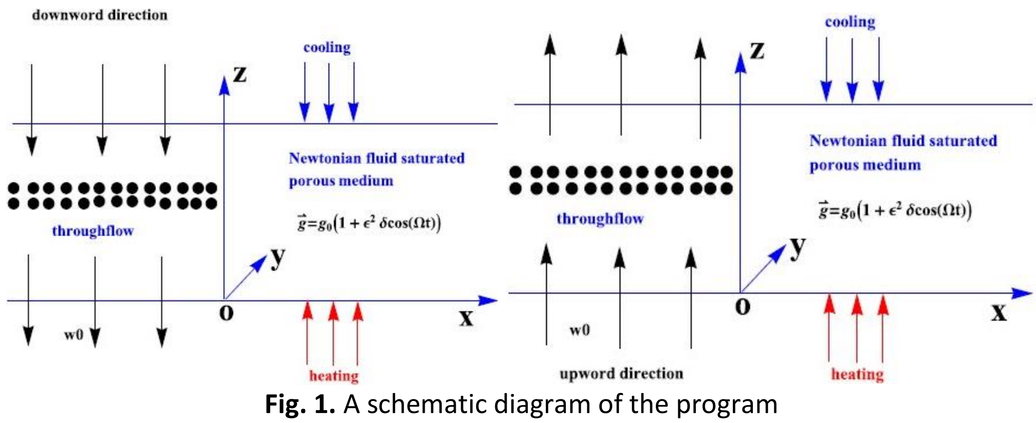

Fig. 1. A schematic diagram of the program

In this paper we consider the throughflow in both vertical and horizontal directions. Furthermore, we consider these assumptions are taken under Darcy Brinkman law and the Oberbeck Boussinesq approximations, the equations which represent the flow model are given by Kiran and Bhadauria [24], and Bhadauria and Kiran [34].

$$\nabla.\vec{q} = 0, \tag{1}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial \vec{q}}{\partial t} + \nabla P = \rho \vec{g} - \frac{\mu}{K}\vec{q} + \frac{\mu_c}{K}\nabla^2 \vec{q}, \tag{2}$$

$$\gamma \frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = k_T \nabla^2 T + Q(T - T_0), \tag{3}$$

$$\rho = \rho_0 \left[1 - \alpha_T (T - T_0)\right], \tag{4}$$

where $\vec{q}$ is velocity, K is permeability, μ is viscosity, P is pressure, μ_c is couple stress fluid, k_T is the coefficient of thermal expansion, ρ is density, T_0 is the temperature with $\rho = \rho_0$ is the standard density and the heat capacity ratio are equal to γ (here γ is taken unity for simplicity). The following are the externally imposed thermal and periodic gravity field:

$$T = T_0 + \Delta T \qquad \text{at} \quad Z = 0, \tag{5}$$

$$T = T_0 \qquad \text{at} \quad Z = d,$$

$$\vec{g} = g_0 (1 + \varepsilon^2 \delta c\,\mathrm{os}(\omega t))\hat{k}, \tag{6}$$

where δ is magnitude of gravity modulation & ω is frequency of modulation.

Therefore, in this stage, the basic state is considered quiescent, with the following quantities:

$$\vec{q} = ((0, 0, w_0(z)), \;\; \rho = \rho_b(z), \;\; P = P_b(z), \;\; T = T_b(z,t) \tag{7}$$

Substituting Eq. (7) into Eq. (1) to Eq. (4), obtain following expressions, they help to define basic state of pressure and temperature:

$$\frac{dp_b}{dz} = -\rho_b g, \tag{8}$$

$$w_0 \frac{dT_b}{dz} = k_T \frac{d^2 T_b}{dz^2} + Q(T - T_0), \tag{9}$$

$$\rho_b = \rho_0 [1 - \alpha_T (T_b - T_0)], \tag{10}$$

The amplitude solution of the Eq. (9) when subjected to thermal boundary condition in Eq. (5) is provided by:

$$T_b = T_0 + \Delta T \frac{\sin\sqrt{R_i}\,(1 - z)}{\sin\sqrt{R_i}}, \tag{11}$$

The perturbations on the basic state with finite amplitude are superposed in the form:

$$\vec{q} = q_b + q',\ \ \rho = \rho_b + \rho',\ \ P = P_b + P',\ \ T = T_b + T'. \tag{12}$$

Since, we introduce two-dimensional convection stream function ψ as $(u', 0, w') = \left(\frac{\partial \psi}{\partial z}, 0, -\frac{\partial \psi}{\partial x} \right)$ which satisfy Eq. (1) and following non-dimensional physical variables are rescaled by:

$$x^* = \frac{x}{d},\ \ y^* = \frac{y}{d},\ \ z^* = \frac{z}{d},\ \ t' = \frac{d^2}{k_T} t^*,\ \ p' = \frac{\mu k_T}{K} p^*,$$
$$q' = \frac{k_T}{d} q^*,\ \ \psi = k_T \psi^*,\ T = \Delta T T^* \text{ and } \Omega^* = \frac{k_T}{d^2}\Omega.$$

Substituting Eq. (12) into the Eq. (1) to Eq. (4), the resulting non-dimensionlized governing system (dropping its asterisk *) by using the dimensionless variables stated above and eliminating the pressure gradient term:

$$\nabla^2 \psi + \frac{1}{\Pr_D} \frac{\partial}{\partial t} (\nabla^2 \psi) = C \nabla^4 \psi - Ra g_m \frac{\partial T}{\partial x}, \tag{13}$$
$$\frac{\partial T}{\partial t} - \frac{\partial \psi}{\partial x} \frac{\partial T_b}{\partial z} - (\nabla^2 + Ri - Pe \frac{\partial}{\partial z}) T = \frac{\partial (\psi, T)}{\partial (x, z)}. \tag{14}$$

where $Pe = \frac{w_0 d^2}{k_T}$ is Peclet number, $\Pr_D = \frac{\phi v d^2}{K k_T}$ is Darcy Prandtl number, $Ra = \frac{\alpha_T g \Delta T d K}{v k_T}$ is internal Rayleigh number, $g_m = (1 + \varepsilon^2 \delta \cos(\omega t))\vec{k}$, and $C = \frac{\mu_c}{\mu d^2}$, is the couple stress parameter, $Ri = \frac{Q d^2}{K_T}$ is internal heat source. The Eq. (14) shows that the factor $\frac{\partial T_b}{\partial z}$, has been given in Eq. (11), the basic state solutions have an effect on the stability problem. The basic state temperature which seen in Eq. (14) is obtained from the Eq. (9) numerically and it is given by:

$$Pe \frac{dT_b}{dz} = \frac{d^2 T_b}{dz^2} + R_i T_b \tag{15}$$

Considering small change of time and re-scaling it as $\tau = \varepsilon^2 t$, the convection in a stationary mode is to be discussed. The linear and non-linear system of Eqs. (13) – (14) may be represented in the matrix form as follows

$$\begin{bmatrix} \nabla^2 - C\nabla^4 & Ra g_m \frac{\partial}{\partial x} \\ -\frac{\partial T_b}{\partial z} \frac{\partial}{\partial x} & -(\nabla^2 + Ri - Pe \frac{\partial}{\partial z}) \end{bmatrix} \begin{bmatrix} \psi \\ T \end{bmatrix} = \begin{bmatrix} -\frac{\varepsilon^2}{\Pr_D} \frac{\partial}{\partial \tau} \nabla^2 \\ -\frac{\varepsilon^2}{\Pr_D} \frac{\partial T}{\partial \tau} + \frac{\partial (\psi, V)}{\partial (x, z)} \end{bmatrix} \tag{16}$$

The above system will be solved by considering stress free and isothermal boundary conditions as given below (Bhadauria & Kiran [35], Kiran [36, 37]).

$$\psi = T = 0 \quad \text{on} \quad Z = 0 \text{ and } Z = 1 \tag{17}$$

3. Heat Transport and Stationary Instability

In order to derive the solution and to resolve nonlinearity the following asymptotic solutions are given in the above Eq. (15) [14,35,36]:

$$\left.\begin{aligned} Ra &= R_0 + \varepsilon^2 R_2 + ... \\ \psi &= \varepsilon\psi_1 + \varepsilon^2\psi_2 + \varepsilon^3\psi_3 + ... \\ T &= \varepsilon T_1 + \varepsilon^2 T_2 + \varepsilon^3 T_3 + ... \\ \delta &= \delta_0 + \varepsilon\delta_1 + \varepsilon^2\delta_2 + \varepsilon^3\delta_3 + ... \end{aligned}\right\} \tag{18}$$

In the absence of gravitational modulations, R_0 would be the critical Rayleigh number where convection starts. The statement δ is suitable with a basic state solution such that if δ_0 disappears at the lower order (following Govender [9], Bhadauria, and Kiran [35]). Further, in addition, δ_1 vanishes, the equations that were derived in order $\varepsilon \; and \; \varepsilon^2$ shows that the solution has a singularity. These findings (Bhadauria and Kiran [35]) show that gravity modulation effect must be provided at an early stage $\delta = \varepsilon^2\delta_2$, which enable consistency. Furthermore, system will be studied for different orders of ε.

3.1 First Order System

The system uses the following format at the lowest level:

$$\begin{bmatrix} \nabla^2 - C\nabla^4 & Rag_m \dfrac{\partial}{\partial x} \\ -\dfrac{\partial T_b}{\partial z}\dfrac{\partial}{\partial x} & -(\nabla^2 + R_i - Pe\dfrac{\partial}{\partial z}) \end{bmatrix} \begin{bmatrix} \psi_1 \\ T_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{19}$$

Lowest-order solution accoriding to initial conditions, Eq. (16) evaluated as follows:

$$\psi_1 = A\sin(k_c x)\sin(\pi z), \tag{20}$$

$$T_1 = -\frac{2k_c I_1 A}{(\delta^2 - R_i)}\cos(k_c x)\sin(\pi z), \tag{21}$$

where $\delta^2 = k_c{}^2 + \pi^2$ and $I_1 = \int_0^1 \frac{dT_b}{dz}\sin^2(\pi z)dz$. Onset of stationary convection is quantitatively determined by using value of critical Rayleigh number with the related wave number and expressions are given by:

$$R_0 = \frac{(\delta^2 + \delta^4 C)(R_i - \delta^2)}{2{k_c}^2 I_1}, \tag{22}$$

3.2 System of Second Order

Now, the system adopts the following form:

$$\begin{bmatrix} \nabla^2 - C\nabla^4 & R_0 \dfrac{\partial}{\partial x} \\ -\dfrac{\partial T_b}{\partial z}\dfrac{\partial}{\partial x} & -(\nabla^2 + R_i - Pe\dfrac{\partial}{\partial z}) \end{bmatrix} \begin{bmatrix} \psi_2 \\ T_2 \end{bmatrix} = \begin{bmatrix} R_{21} \\ R_{22} \end{bmatrix} \tag{23}$$

Where

$$R_{21} = 0, \tag{24}$$

$$R_{22} = \frac{\partial \psi_1}{\partial x}\frac{\partial T_1}{\partial z} - \frac{\partial \psi_1}{\partial z}\frac{\partial T_1}{\partial x}. \tag{25}$$

The solutions of second-order subjected to initial conditions as in Eq. (16) are given by:

$$\psi_2 = 0, \tag{26}$$

$$T_2 = -\frac{2{k_c}^2 I_1}{(4\pi^2 - R_i)(\delta^2 - R_i)} A^2 \sin(2\pi z) \tag{27}$$

For convection in a stationary mode, the horizontally averaged Nusselt number Nu is calculated as follows:

$$Nu = 1 + \left[\frac{k_c}{2\pi} \int_0^{\frac{2\pi}{k_c}} \frac{\partial T_2}{\partial z} \partial x \right] \div \left[\frac{k_c}{2\pi} \int_0^{\frac{2\pi}{k_c}} \frac{\partial T_b}{\partial z} \partial x \right]$$

$$Nu = 1 + \frac{{k_c}^2 I_1 \sin\sqrt{R_i}}{(4\pi^2 - R^2{}_i)(\delta^2 - R_i)} A^2. \tag{28}$$

In the situation of a porous media which is isotropic in the absence of fluid flow, the following conclusions are found in Eq. (21), Eq. (22) and Eq. (28) are presented by Bhadauria *et al.,* [12,13], and Lapwood [41].

3.3 System of Third Order

Now for this point system takes the form as:

$$\begin{bmatrix} \nabla^2 - C\nabla^4 & R_0 \dfrac{\partial}{\partial x} \\ -\dfrac{\partial T_b}{\partial z}\dfrac{\partial}{\partial x} & -(\nabla^2 + R_i - Pe\dfrac{\partial}{\partial z}) \end{bmatrix} \begin{bmatrix} \psi_3 \\ T_3 \end{bmatrix} = \begin{bmatrix} R_{31} \\ R_{32} \end{bmatrix} \tag{29}$$

Here, terms of RHS are given by:

$$R_{31} = -\frac{1}{\Pr_D}\frac{\partial \nabla^2 \psi_1}{\partial \tau} - R_0 \delta_2 \cos(\omega\tau)\frac{\partial T_1}{\partial x} - R_2 \frac{\partial T_1}{\partial x}, \tag{30}$$

$$R_{32} = \frac{\partial T_2}{\partial z}\frac{\partial \psi_1}{\partial x} - \frac{\partial T_1}{\partial \tau}, \tag{31}$$

Now, putting first-order and second-order solutions into the following Eqs. (30) – (31) and easily we get the expressions for R_{31} and R_{32}. Under solvability condition, we get Ginzburg-Landau equation for existence of third order system. The Ginzburg-Landau expression is given by:

$$Q_1 \frac{dA(\tau)}{d\tau} - Q_2 A(\tau) + Q_3 A^3(\tau) = 0, \tag{32}$$

where Q_1, Q_2, Q_3 are coefficients:

$$Q_1(\tau) = \left(\frac{\delta^2 - \delta^4 C}{\Pr_D} + \frac{2R_0 k_c{}^2 I_1}{(\delta^2 - R_i)^2} \right),$$
$$Q_2(\tau) = \left(\frac{2k_c{}^2 I_1}{(\delta^2 - R_i)}\left[R_2 + R_0\delta_2 \cos(\Omega\tau)\right] \right),$$
$$Q_3 = \frac{R_0 \pi^2 k_c{}^4 I_1}{(\delta^2 - R_i)^2 (4\pi^2 - R_i)}$$

The Eq. (32) is known as Bernoulli equation, because of its non-autonomous structure, finding an analytical solution is very difficult in the presence of modulation. As a result, it was numerically solved by using Mathematica 12.0 built-in function ND Solution, when necessary initial condition at $A_0 = a_0$ where a_0 is defined as present initial convection magnitude. Its analytic solution of Eq. (32) for such an un-modulated case is as follows:

$$A(\tau) = \frac{1}{\sqrt{\left(\dfrac{Q_3}{Q_2} + C_1 e^{\left[-\frac{2Q_2}{Q_1}\tau\right]} \right)}}, \tag{33}$$

where Q_1, Q_3 as same in Eq. (32),

$Q_2 = \left(\dfrac{2k_c{}^2 I_1 R_0}{(\delta^2 - R_i)} \right)$ and C_1 is an integration constant which appears in Eq. (33), which can be obtained by adopting appropriate boundary conditions.

4. Results and Discussions

In this paper, we have investigated effects of throughflow and gravity modification on convective instability in a porous media saturated with a coupled stress fluid in the presence of internal heat source and sink. To study the effects of gravity modulation and vertical throughflow on coupled stress fluid and heat transport, an analysis of nonlinear phenomena of stability was being used. There is only minimal amount of amplitude gravity modulation has been considered for Benard - Darcy convection where effects of gravity modulation had been assumed for third order O (ε^2). The objective for weakly nonlinear theory is to investigate transmission of heat in ways that a linear study could not. In order to study transfer of heat in porous media external regulations are important. The purpose of this study is to consider such a gravity modulation and vertical throughflow. Because here we examining small amplitude on transport of heat, then the value of δ_2 may be very small, around 0.1. Furthermore, because low - frequency has the maximum effect on which onset of convection and heat transfer, the amplitude of gravitational modulation is considered to be minimal. The problem's purpose is to take gravity modulation and vertical throughflow into consideration for either increasing or decreasing heat transfer, where the Darcy- Brinkmen model is considered in a momentum equation since the porous medium is assumed to closely packed. Another important subject is the internal heat source or sink, which is explained through the energy equation. This concept is most important where the system provides its own internal heat generation. Due its dominant nature, the moderate values of R_i are considered.

The values of Pr_D may consider (modern porous medium applications, such as mushy layer in solidification of binary alloys and fractured porous medium) around 1, and also for low porosity medium, the large values considered for Pr_D . The values of δ and Ω are treated to be small, for small values of frequency and amplitudes, the transfer of heat can reach the maximum. The numeric findings for Nu from Eq. (27) in respect to amplitude Eq. (31), and the results are shown in Figures 2-4. The effects of every parameter on heat transfer are shown in Figures 2-4 where Nusselt number Nu versus Ω is graphically presented. From the figures it is found that the values of Nu (Nusselt number) oscillates maximum for low frequencies and further for the values of Ω there is no oscillations can be observed, which shows that, the system can convect more quickly due to the low modulated frequencies. Therefore, we get nature of oscillatory figures is because of modulation only.

Now, let us look about the effects of gravity modulation: in Figures 2 the corresponding results of internal Rayleigh number and Peclet number are presented. The effect of Pe with heat transport is studied for the circumstances from upward and downwards directed flows. The upward throughflow (also known as pro-gravity) (Pe > 0), has destabilizing effect which is given in Figure 2a, whereas downward throughflow (also known as anti-gravity) (Pe < 0), has stabilizing effect which is presented in Figure 2b. The related results for linear theory are due to Vanishree [26], this is because of temperature dependence of viscosity. When Pe varies in either direction the nonlinear temperature distribution significantly matters at conduction state and there is significant or drastic variation in the energy supply to the disturbance. The corresponding results are also obtained by Nield [21] in the case of fluid layer for small amount of throughflow. The present results are computable with the results obtained by Shivakumara *et al.,* [18] and Suma *et al.,* [22]. Shivakumara *et al.,* [19] pointed that; the destabilization effect may be due to the distortion of basic state temperature distribution from linear to nonlinear throughflow. The corresponding results of throughflow under modulation the reader may look at the articles [25, 26]. It is noted that, for nonlinear theories Pe has duel effect [25] but, for linear case Pe only stabilizing effect [26], the reason could be guessed as the finite amplitude interaction in the fluid flow through coupled formation of momentum and energy equations.

The effects of the internal Rayleigh number R_i, on heat transfer is given in Figure 2c for both the cases of heat source and heat sink, Figure 2c for heat source and Figure 2d for heat sink. For many chemical and industrial experiments, the system may produce or absorb energy throughout its operation, which is why internal heat occurs. In such a circumstance, one needs to learn how to control these oscillations in the system. As a result, it can be seen from the graphs that the heat transmission in the system is enhanced or decreased by positive and negative values of the heat source or heat sink, respectively. To understand more about internal heat generation, we can see [3, 13, 26]. In Astrophysics provides an example of internally driven convection for uniformly heated medium. In the cores of stars heat is produced by thermonuclear reactions. In such situation the heating rate is very sensitive to temperature, and this sensitivity creates steep thermal gradients that drive powerful convection in the cores (Kippenhahn *et al.,* [41]). Clearly, this and many other instances (related space science) of internally driven convection contain more complications. It is observed that, for low porosity medium, large values [29] considered for Pr_D. The Nusselt number Nu increases with Pr_D showing heat transfer increases (see Figure 3a). The results could be gained for lower values of Prandtl Darcy number. The related article which presents the results corresponding to heat transfer in a porous medium under modulation may be observed in the following studies [12, 13, 35].

Further, it is observed that, large values of Pr_D are considered, for low porosity medium. The Nusselt number Nu increases with value of Pr_D, which shows that heat transfer increases. The outcomes could be improved for smaller values of Prandtl-Darcy numbers. The following research [3, 12, 13], contains the relevant article that offers the results corresponding to heat transport in the porous media under modulation effect.

According to Figures 2 - 4, the effect of frequency of modulation on heat transport diminishes heat transfer where it can be observed that as increase in Ω decreases the magnitude of Nu. The gravitational modulation on convective instability completely vanishes at high rates of frequencies. The above findings are agree quite well for temperature modulation with the linear theory of Venezian [38], which shows that in the critical value of Rayleigh number due to thermal modulation becomes almost zero at higher frequency. The more related research results for gravity modulation on weakly nonlinear studies we can see [7, 11, 23, 29]. In general the modulation frequency compresses the wavelengths and reduces the amplitude of Nu, because of this reason heat transfer decreases. Furthermore, Figure 3b shows that the effect of modulation amplitude and it is observed that heat transfer increases as δ increases. Therefore, amplitude of modulation is to increase the heat transfer in the system. In Figure 4, the comparison between modulated and unmodulated case are presented. It is shown that gravitationally modulated system flows transport less heat transfer than unmodulated systems the corresponding finding obtained from the studies of [39, 40]. Therefore, it should be noticed that Eq. (32) is the analytical solution for unmodulated case.

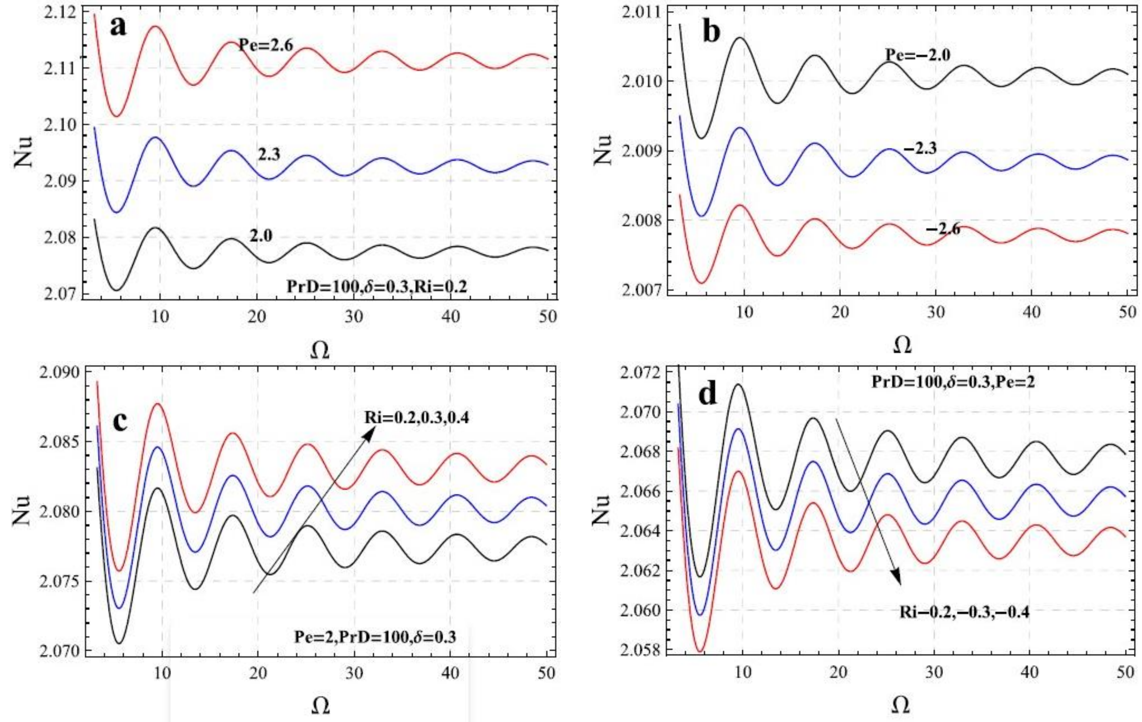

Fig. 2. Effect of Pe and R_i on heat transport

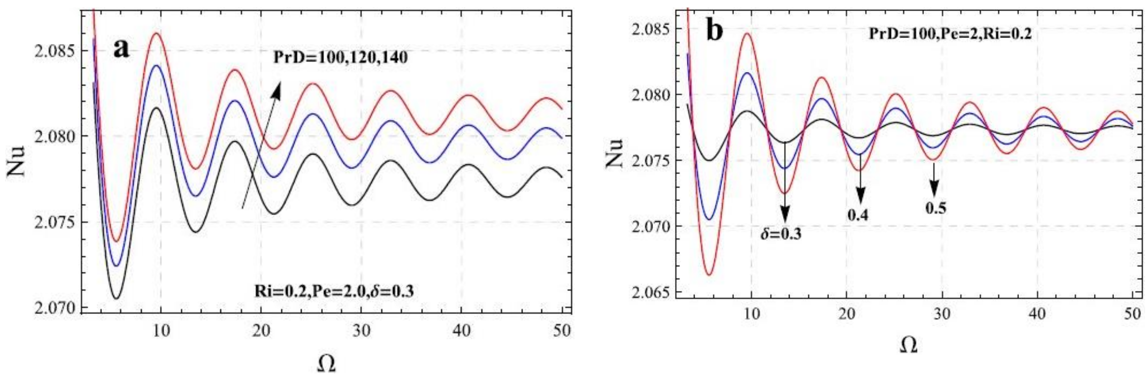

Fig. 3. Ect of Pr_D and δ on heat transport

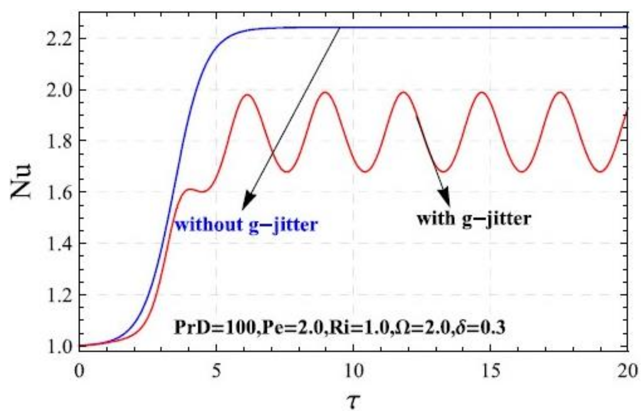

Fig. 4. Comparison between modulated and unmodulated case

Finally, the nature of stream lines and corresponding isotherms presented in Figure 5 and Figure 6. Figure 5 shows the variation of stream lines and isotherms at different instants of times, respectively. Figure 5 shows the magnitudes of stream lines increase as time increases. Also, initially the isotherms are flat and parallel, thus heat transport is due to conduction only. However, as time increases, isotherms form contours, showing convective regime is taking place, after reaching certain instant there is no change in the magnitude of stream lines and isotherms, thus showing the steady state. In particular Figure 6 is drawn to see the effect of throughflow on isotherms. While strengthening the throughflow there forms a boundary layer at the bottom plate, then the significant results may observe at corresponding isotherms, where isotherms are missing at the bottom plate due to throughflow disturbances. The reason would be, when there is flow at the bottom plate there may not be stream line flow due to heavy boundary layer and hence the nature of the Figure 6.

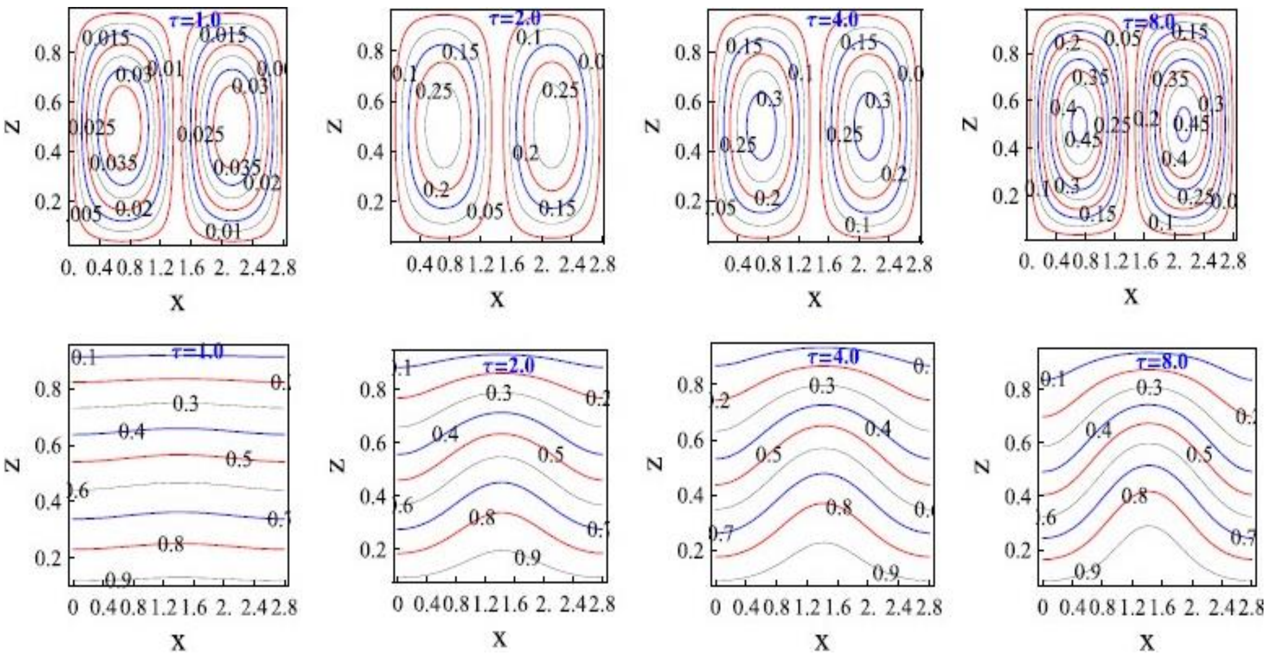

Fig. 5. Streamlines and Isotherms for Pe = 0.3, R_i = 0, Pr_D = 100, $\in$ = 0.5

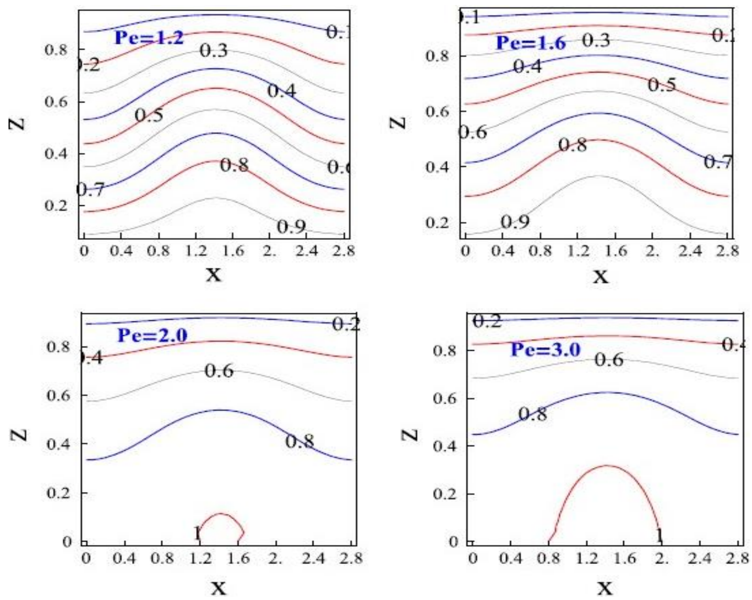

Fig. 6. Isotherms for τ = 4.0, R_i = 1.0, Pr_D = 100, $\in$ = 0.5

5. Conclusions

The effects of gravity modulation and vertical throughflow and with an internal heating on Benard-Dacry convection is investigated with a weakly nonlinear theory. Internal heating is applied to the porous media. For weakly nonlinear theory, the Ginzburg-Landau model is used. On previous analysis the following conclusions are made:

a) The effects of Pr_D is to increases the transport of heat for smaller values of Ω and decreases for large values of Ω.
b) Heat transmission is increased by antigravity flow $(Pe > 0)$, while it is decreased by pro-gravity flow $(Pe < 0)$.
c) With the increasing value of δ, rate of heat transfer increases.
d) The amplitude of convection diminishes with an increase in gravity modulation frequency Ω., and for larger values of Ω, the influence of the g-jitter is negligible.
e) G-jitter exhibits its maximum stabilizing or destabilizing influence at low frequencies, whereas high frequencies stabilize the system.
f) Anti-gravity flow $(Pe > 0)$ and Heat source $(R_i > 0)$ have destabilizing effect, whereas pro-gravity flow $(Pe < 0)$ and heat sink $(R_i < 0)$ have stabilizing effect on the system.
g) The magnitude of streamlines expands with time and reaches a maximum for increasing values of the time.
h) Isotherms are flat initially due to the conduction state before taking on a contour to show the convection regime.

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Reference
[1] Nelson, Emily S. *An examination of anticipation of g-jitter on space station and its effects on materials processes*. NASA Technical Memorandum 103775, 1994.
[2] Wadih, M., and B. Roux. "Natural convection in a long vertical cylinder under gravity modulation." *Journal of Fluid Mechanics* 193 (1988): 391-415. https://doi.org/10.1017/S0022112088002198
[3] Wadih, M., N. Zahibo, and B. Roux. "Effect of gravity jitter on natural convection in a vertical cylinder." *Low Gravity Fluid Dynamics and Transport Phenomena*, edited by J. N. Koster and R. L. Sani (AIAA, New York, 1990) (1990): 309-354. https://doi.org/10.2514/5.9781600866036.0309.0352
[4] Gershuni, G. Z., E. M. Zhukhovitskii, and Iu S. Iurkov. "On convective stability in the presence of periodically varying parameter: PMM vol. 34, n≗ 3, 1970, pp. 470-480." *Journal of Applied Mathematics and Mechanics* 34, no. 3 (1970): 442-452. https://doi.org/10.1016/0021-8928(70)90090-0
[5] Gresho, P. M., and R. L. Sani. "The effects of gravity modulation on the stability of a heated fluid layer." *Journal of Fluid Mechanics* 40, no. 4 (1970): 783-806. https://doi.org/10.1017/S0022112070000447
[6] Yang, Wen-Mei. "Stability of viscoelastic fluids in a modulated gravitational field." *International Journal of Heat and Mass Transfer* 40, no. 6 (1997): 1401-1410. https://doi.org/10.1016/S0017-9310(96)00194-9
[7] Malashetty, M. S., and V. Padmavathi. "Effect of gravity modulation on the onset of convection in a fluid and porous layer." *International Journal of Engineering Science* 35, no. 9 (1997): 829-840. https://doi.org/10.1016/S0020-7225(97)80002-X
[8] Rees, D. Andrew S., and I. Pop. "The effect of g-jitter on vertical free convection boundary-layer flow in porous media." *International Communications in Heat and Mass Transfer* 27, no. 3 (2000): 415-424. https://doi.org/10.1016/S0735-1933(00)00122-6

[9] Govender, S. "Stability of convection in a gravity modulated porous layer heated from below." *Transport in Porous Media* 57, no. 1 (2004): 113-123. https://doi.org/10.1023/B:TIPM.0000032739.39927.af

[10] Kuznetsov, A. V. "Linear stability analysis of the effect of vertical vibration on bioconvection in a horizontal porous layer of finite depth." *Journal of Porous Media* 9, no. 6 (2006): 597-608. https://doi.org/10.1615/JPorMedia.v9.i6.80

[11] Bhadauria, B. S., and P. Kiran. "Weak nonlinear double diffusive magneto-convection in a Newtonian liquid under gravity modulation." *Journal of Applied Fluid Mechanics* 8, no. 4 (2015): 735-746. https://doi.org/10.18869/acadpub.jafm.67.223.22740

[12] Bhadauria, B. S., P. G. Siddheshwar, Jogendra Kumar, and Om P. Suthar. "Weakly nonlinear stability analysis of temperature/gravity-modulated stationary rayleigh-bénard convection in a rotating porous medium." *Transport in Porous Media* 92, no. 3 (2012): 633-647. https://doi.org/10.1007/s11242-011-9925-4

[13] Bhadauria, B. S., Ishak Hashim, and P. G. Siddheshwar. "Study of heat transport in a porous medium under G-jitter and internal heating effects." *Transport in Porous Media* 96, no. 1 (2013): 21-37. https://doi.org/10.1007/s11242-012-0071-4

[14] Bhadauria, B. S., and Palle Kiran. "Nonlinear thermal Darcy convection in a nanofluid saturated porous medium under gravity modulation." *Advanced Science Letters* 20, no. 5-6 (2014): 903-910. https://doi.org/10.1166/asl.2014.5466

[15] Kiran, Palle. "Nonlinear thermal convection in a viscoelastic nanofluid saturated porous medium under gravity modulation." *Ain Shams Engineering Journal* 7, no. 2 (2016): 639-651. https://doi.org/10.1016/j.asej.2015.06.005

[16] Homsy, George M., and Albert E. Sherwood. "Convective instabilities in porous media with through flow." *AIChE Journal* 22, no. 1 (1976): 168-174. https://doi.org/10.1002/aic.690220121

[17] Jones, M. C., and J. M. Persichetti. "Convective instability in packed beds with throughflow." *AIChE Journal* 32, no. 9 (1986): 1555-1557. https://doi.org/10.1002/aic.690320916

[18] Shivakumara, I. S., and S. Sureshkumar. "Convective instabilities in a viscoelastic-fluid-saturated porous medium with throughflow." *Journal of Geophysics and Engineering* 4, no. 1 (2007): 104. https://doi.org/10.1088/1742-2132/4/1/012

[19] Khalili, Arzhang, and I. S. Shivakumara. "Onset of convection in a porous layer with net through-flow and internal heat generation." *Physics of Fluids* 10, no. 1 (1998): 315-317. https://doi.org/10.1063/1.869540

[20] Nield, D. A. "Convective instability in porous media with throughflow." *AIChE Journal* 33, no. 7 (1987): 1222-1224. https://doi.org/10.1002/aic.690330719

[21] Suma, S. P., Y. H. Gangadharaiah, and R. Indira. "Effect of throughflow and variable gravity field on thermal convection in a porous layer." *International Journal of Engineering Science and Technology* 3, no. 10 (2011): 7657-7668.

[22] Kiran, P. "Throughflow and g-jitter effects on binary fluid saturated porous medium." *Applied Mathematics and Mechanics* 36, no. 10 (2015): 1285-1304. https://doi.org/10.1007/s10483-015-1984-9

[23] Kiran, P. "Throughflow and gravity modulation effects on heat transport in a porous medium." *Journal of Applied Fluid Mechanics* 9, no. 3 (2016): 1105-1113. https://doi.org/10.18869/acadpub.jafm.68.228.24682

[24] Kiran, Palle, and B. S. Bhadauria. "Nonlinear throughflow effects on thermally modulated porous medium." *Ain Shams Engineering Journal* 7, no. 1 (2016): 473-482. https://doi.org/10.1016/j.asej.2015.03.010

[25] Vanishree, R. K. "Effects of through-flow and internal heat generation on a thermo convective instability in an anisotropic porous medium." *Journal of Applied Fluid Mechanics* 7, no. 4 (2014): 581-590. https://doi.org/10.36884/jafm.7.04.19410

[26] Hetsroni, G., M. Gurevich, and R. Rozenblit. "Natural convection in metal foam strips with internal heat generation." *Experimental Thermal and Fluid Science* 32, no. 8 (2008): 1740-1747. https://doi.org/10.1016/j.expthermflusci.2008.06.011

[27] Bhadauria, B. S., Ishak Hashim, and P. G. Siddheshwar. "Study of heat transport in a porous medium under G-jitter and internal heating effects." *Transport in Porous Media* 96, no. 1 (2013): 21-37. https://doi.org/10.1007/s11242-012-0071-4

[28] Bhadauria, B. S., Anoj Kumar, Jogendra Kumar, Nirmal C. Sacheti, and Pallath Chandran. "Natural convection in a rotating anisotropic porous layer with internal heat generation." *Transport in Porous Media* 90, no. 2 (2011): 687-705. https://doi.org/10.1007/s11242-011-9811-0

[29] Saravanan, S. "Thermal non-equilibrium porous convection with heat generation and density maximum." *Transport in Porous Media* 76, no. 1 (2009): 35-43. https://doi.org/10.1007/s11242-008-9232-x

[30] Bakar, Norhaliza Abu, and Rozaini Roslan. "Mixed Convection in a Lid-Driven Horizontal Cavity in the Presence of Internal Heat Generation or Absorption." *Journal of Advanced Research in Numerical Heat Transfer* 3, no. 1 (2020): 1-11.

[31] Mahat, Rahimah, Muhammad Saqib, Imran Ulah, Sharidan Shafie, and Sharena Mohamad Isa. "MHD Mixed Convection of Viscoelastic Nanofluid Flow due to Constant Heat Flux." *Journal of Advanced Research in Numerical Heat Transfer* 9, no. 1 (2022): 19-25.

[32] Loni, Reyhaneh, Gholamhassan Najafi, Rizalman Mamat, Mohd Fairusham Ghazali, and Nor Azwadi Che Sidik. "Nusselt Number Prediction for Oil and Water in Solar Tubular Cavity Receivers." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 97, no. 2 (2022): 157-174. https://doi.org/10.37934/arfmts.97.2.157174

[33] Abidin, Nurul Hafizah Zainal, Nor Fadzillah Mohd Mokhtar, Izzati Khalidah Khalid, and Siti Nur Aisyah Azeman. "Oscillatory Mode of Darcy-Rayleigh Convection in a Viscoelastic Double Diffusive Binary Fluid Layer Saturated Anisotropic Porous Layer." *Journal of Advanced Research in Numerical Heat Transfer* 10, no. 1 (2022): 8-19.

[34] Bhadauria, B. S., and Palle Kiran. "Heat transport in an anisotropic porous medium saturated with variable viscosity liquid under temperature modulation." *Transport in Porous Media* 100, no. 2 (2013): 279-295. https://doi.org/10.1007/s11242-013-0216-0

[35] Kiran, Palle. "Throughflow and non-uniform heating effects on double diffusive oscillatory convection in a porous medium." *Ain Shams Engineering Journal* 7, no. 1 (2016): 453-462. https://doi.org/10.1016/j.asej.2015.04.003

[36] Kiran, Palle. "Weak nonlinear oscillatory convection in a nonuniform heating porous medium with throughflow." *International Journal of Engineering and Mathematical Modelling* 2, no. 3 (2015): 63-78.

[37] Venezian, Giulio. "Effect of modulation on the onset of thermal convection." *Journal of Fluid Mechanics* 35, no. 2 (1969): 243-254. https://doi.org/10.1017/S0022112069001091

[38] Davis, Stephen H. "The stability of time-periodic flows." *Annual Review of Fluid Mechanics* 8, no. 1 (1976): 57-74. https://doi.org/10.1146/annurev.fl.08.010176.000421

[39] Srivastava, Alok, B. S. Bhadauria, and I. Hashim. "Effect of internal heating on double diffusive convection in a couple stress fluid saturated anisotropic porous medium." *Advances in Materials Science and Applications* 3, no. 1 (2014): 24-45. https://doi.org/10.5963/AMSA0301004

[40] Kippenhahn, Rudolf, and Alfred Weigert. *Stellar structure and evolution*. Berlin: Springer-Verlag, 1994.

[41] Lapwood, E. R. "Convection of a fluid in a porous medium." In *Mathematical Proceedings of the Cambridge Philosophical Society*, vol. 44, no. 4, pp. 508-521. Cambridge University Press, 1948. https://doi.org/10.1017/S030500410002452X