

# Dual Numerical Solutions on Mixed Convection Casson Fluid Flow Due to the Effect of the Rate of Extending and Compressing Sheet – Stability Analysis


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## ARTICLE INFO

## ABSTRACT

### Article history:

Received 21 June 2020

Received in revised form 20 August 2020

Accepted 25 August 2020

Available online 31 August 2020

Numerical model of mixed convection boundary layer flow, heat and mass transfer beyond an extending or compressing sheet are developed by Mathematicians in the field of Applied Mathematics Fluid Dynamics. Most of the numerical results can be used as a comparison with the results by experimental works. In the numerical model of mixed convection with extending and compressing sheet, there is high possibility to achieve dual solutions. One of the solutions is stable, reliable and really occur in actual fluid mechanism. Meanwhile, another solution is unstable, unreliable and rejected. According to this statement, the dual numerical solutions of mixed convection Casson fluid flow, which is subjected to the rate of extending and compressing sheet is reported in this paper. The extending rate is denoted as positive values, otherwise it is declared as compression of the sheet. The usage of similarity variables is to perform the conversion from partial differential equations (PDE) to ordinary differential equations (ODE). The PDE are formed from the equations of momentum, energy and concentration. Finally, ODE are solved numerically by *bvp4c* program in Matlab software. Stability analysis is performed to deal with dual numerical solutions, which select the most stable solution and physically reliable. The numerical results of velocity, temperature and concentration are presented, subjected to the governed parameters in the modelled problem. As a conclusion, the variations of velocity, temperature and concentration against boundary layer thickness are fully affected by compressing and extending parameters.

### Keywords:

Casson fluid; extending/compressing sheet; dual solutions; stability analysis

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## 1. Introduction

Research on non-Newtonian fluid is becoming more important and interesting topics because of its flow characteristics which has variety of industrial applications such as polymer processing, abstraction of crude oil from petroleum products, optical fibers, paper production and hot rolling

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<https://doi.org/10.37934/cfdl.12.8.7684>

[1,2]. Among them, Casson fluid has noticeable features because it will deform when the rate of shear stress is higher than yield stress and in reverse condition it will become a solid. The most familiar Casson fluid are namely as jelly, honey, tomato sauce, concentrated fruit juice and chocolate [3]. As a result, a plenty of numerical fluid model has been developed with casson fluid [1-14], which can be used as a reference by researchers from the same area, but with different method (for example, experimental approach). On the other hand, the usual implementation of fluid flow over stretching/shrinking surface (extending/compressing surface) are as follow: polymer transforming industries, controlling environmental pollution and biological process. Thus, the effect of extending/compressing sheet on casson fluid flow have been explored by many researchers as it is widely using in the sector of science and engineering [2-14]. Mixed convection, which is defined as the combination of forced and natural convection exists when both of these convection mechanisms act together to transfer heat. The numerical results of mixed convection fluid flow bounded by extending/compressing sheet were published, due to their applications in science and engineering [9-12].

In the presence of mixed convection with extending and compressing sheet, there is high possibility to achieve multiple numerical solutions. The occurrence of multiple solutions in the related cases (stretching/shrinking sheet) have been done by some researchers [8-9,14-18]. Thus, to verify the excepted solution stability analysis is used, and the rest is decided as unstable. This investigation can be operated by using the *bvp4c* programme from MATLAB and processed numerically for dual solutions. The solution with initial decay of disturbance labelled as stable solution (first solution). Besides, unstable solution which is mentioned as second solution owns initial growth of disturbance. The application of stability analysis in the related cases (stretching/shrinking sheet) have been done by some researchers [18-21].

The main priority of the study is to examine the dual numerical solutions of mixed convection Casson fluid flow in the presence of extending and compressing sheet. To the best of the researchers' observation, the stability analysis of the current experimentation is not published yet in the literature. This study is chronologically structured as followings: Section 2 is Methodology, Section 3 is Results and Discussion, and final section is Conclusion.

## 2. Methodology

### 2.1 Mathematical Formulation

Consider the two-dimensional incompressible, viscous and electrically conducting magneto-hydrodynamics Casson fluid over a permeable extending and compressing surface. The horizontal  $x$ -axis is a shrinking/stretching sheet and transverse magnetic field is placed at vertical  $y$ -axis. The parameter of Casson fluid  $\omega$  in Eq. (2) has been derived by the previous report [10]. The mathematical formulae are governed by the following equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \left( 1 + \frac{1}{\omega} \right) \frac{\partial^2 u}{\partial y^2} + g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty) + \frac{\sigma B_0^2}{\rho} u \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{DK_T}{C_S C_P} \frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \frac{DK_T}{T_m} \frac{\partial^2 T}{\partial y^2} \tag{4}$$

where  $u$  and  $v$  are the components of velocity in the  $x$  and  $y$ -directions,  $\nu = \frac{\mu}{\rho}$  is the kinematic viscosity,  $\mu$  denotes the viscosity,  $\rho$  is the fluid density,  $g$  is the gravitational acceleration,  $\beta_T$  is the coefficient of thermal expansion,  $\beta_C$  is the coefficient of solutal expansions,  $T$  is the temperature of the fluid,  $C$  is concentration of the fluid,  $\sigma$  is the electrical conductivity,  $B_0$  is the constant strength of magnetic field,  $\alpha$  is the thermal diffusivity,  $D$  is the solutal diffusivity of the medium,  $K_T$  is the thermal diffusion ratio,  $C_S$  is the concentration susceptibility,  $C_p$  is the specific heat at constant pressure and  $T_m$  is the mean fluid temperature. The subscripts  $w$  and  $\infty$  at parameters  $T$  and  $C$  represent the situation at the wall and at the outer region of the boundary layer. The appropriate boundary conditions are

$$\begin{aligned}
 u = u_w(x) &= \lambda U_0 \exp(x/L), v = v_w(x), T_w(x) = T_\infty + T_0 \exp(x/2L), \\
 C_w(x) &= C_\infty + C_0 \exp(x/2L), \quad \text{at } y = 0 \quad (5) \\
 u \rightarrow 0, T &\rightarrow T_\infty, C \rightarrow C_\infty \quad \text{as } y \rightarrow \infty
 \end{aligned}$$

where  $\lambda < 0$  is the compressing parameter whereas positive  $\lambda$  represent the extending parameter. Besides, the wall mass suction velocity is represented by  $v_w(x) < 0$ . The term  $\exp(x/2L)$  in temperature  $T_w$  and concentration distribution  $C_w$  are taken from the previous study [18,22-24] in the problem for two-dimensional fluid flow with the presence of Soret Dufour effects and when concentration equation is included in governing equations. The assumption of temperature and concentration functions are to make sure the numerical results will satisfy final boundary condition after substituting similarity variables. The stream function is stated as

$$\psi(x, y) = (2\nu L U_0)^{\frac{1}{2}} \exp(x/2L) f(\eta), u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \quad (6)$$

Introducing new similarity variables

$$\begin{aligned}
 \theta(\eta) &= \frac{T - T_\infty}{T_w - T_\infty}, \varphi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \eta = y \left( \frac{U_0}{2\nu L} \right)^{1/2} \exp(x/2L), \\
 u &= U_0 \exp(x/L) f'(\eta), \quad (7) \\
 v &= -\left( \frac{\nu U_0}{2L} \right)^{1/2} \exp(x/2L) [f(\eta) + \eta f'(\eta)]
 \end{aligned}$$

where prime indicates the differentiation with respect to  $\eta$ . Substituting Eq. (7) into Eqs. (2 – 5), transformed ordinary differential equations occur in the following form

$$\left( 1 + \frac{1}{\omega} \right) f'''' + f f'' - 2(f')^2 + 2Ri \left[ \exp\left( \frac{-3X}{2} \right) \right] (\theta + N\varphi) - 2H[\exp(-X)] f' = 0 \quad (8)$$

$$\frac{1}{Pr} \theta'' + f \theta' - f' \theta + Db \varphi'' = 0 \quad (9)$$

$$\frac{1}{Sc} \varphi'' + f \varphi' - f' \varphi + Sr \theta'' = 0 \quad (10)$$

The new boundary conditions are

$$f'(\eta) = \lambda, f(\eta) = S, \theta(\eta) = 1, \varphi(\eta) = 1 \text{ at } \eta = 0 \quad (11)$$

$$f'(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0, \varphi(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty$$

The parameters involved in this problem are mixed convection parameter  $Ri = Gr/Re^2$ , magnetic field parameter  $H = 2\sigma LB_0^2/\rho U_0$ , thermal Grashof number  $Gr = g\beta_T(T_0 - T_\infty)L^3/\nu^2$ , Reynolds number  $Re = U_0L/\nu$ , dimensionless coordinate along the plate parameter  $X = x/L$ , length of the extended/compressed sheet  $L$ , buoyancy ratio  $N = \beta_C(C_0 - C_\infty)/\beta_T(T_0 - T_\infty)$ , Prandtl number  $Pr = \nu/\alpha$ , Schmidt number  $Sc = \nu/D$ , Soret number  $Sr = DK_T(T_0 - T_\infty)/T_m\nu(C_0 - C_\infty)$ , Dufour number  $Db = DK_T(C_0 - C_\infty)/C_S C_P \nu(T_0 - T_\infty)$ , and suction parameter is defined as  $S = (v_w(x)/\exp(x/2L)) \times \sqrt{2L/\nu U_0} > 0$ . The opposing flow is when  $Ri < 0$ . Otherwise, the positive  $Ri$  indicates the case of aiding flow.

## 2.2 Stability Analysis

The initial step of stability analysis is to write down the governing equations in the state of unsteady. Then, the dimensionless variable  $\tau$  is introduced [25] in similarity variables.

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \left(1 + \frac{1}{\omega}\right) \frac{\partial^2 u}{\partial y^2} + g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty) + \frac{\sigma B_0^2}{\rho} u \quad (12)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{DK_T}{C_S C_P} \frac{\partial^2 C}{\partial y^2} \quad (13)$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \frac{DK_T}{T_m} \frac{\partial^2 T}{\partial y^2} \quad (14)$$

New similarity variables are

$$\theta(\eta, \tau) = \frac{T - T_\infty}{T_w - T_\infty}, \varphi(\eta, \tau) = \frac{C - C_\infty}{C_w - C_\infty}, \eta = y \left(\frac{U_0}{2\nu L}\right)^{1/2} \exp(x/2L),$$

$$u = U_0 \exp(x/L) \frac{\partial}{\partial \eta} f(\eta, \tau), \quad \tau = \frac{U_0}{2L} t \exp(x/L) \quad (15)$$

$$v = -\sqrt{2\nu L U_0} \left[ \frac{\eta}{2L} \exp(x/2L) \frac{\partial}{\partial \eta} f(\eta, \tau) + \frac{U_0}{2L^2} t \exp(3x/L) \frac{\partial}{\partial \tau} f(\eta, \tau) + \frac{1}{2L} \exp(x/2L) f(\eta, \tau) \right]$$

The analysis introduced by previous investigators in stability field [25] are stated as below

$$f(\eta, \tau) = f_0(\eta) + F(\eta, \tau) \exp(-\gamma\tau)$$

$$\theta(\eta, \tau) = \theta_0(\eta) + G(\eta, \tau) \exp(-\gamma\tau) \quad (16)$$

$$\varphi(\eta, \tau) = \varphi_0(\eta) + H(\eta, \tau) \exp(-\gamma\tau)$$

where  $\gamma$  denotes an unknown eigenvalue while  $F(\eta, \tau)$ ,  $G(\eta, \tau)$  and  $H(\eta, \tau)$  are small relative to  $f_0(\eta)$ ,  $\theta_0(\eta)$  and  $\varphi_0(\eta)$ . Eq. (16) is substituted into governing equations, Eqs. (12 -14) and boundary conditions Eq. (11) and set  $\tau = 0$ . As a result, the following equations are obtained

$$\left(1 + \frac{1}{\omega}\right) F_0''' + f_0 F_0'' + f_0'' F_0 - 4f_0' F_0' + 2Ri \left[ \exp\left(\frac{-3X}{2}\right) \right] (G_0 + NH_0) + [\gamma - 2H\{\exp(-X)\}] F_0' = 0 \quad (17)$$

$$\frac{1}{Pr} G_0'' + f_0 G_0' + \theta_0' F_0 - \theta_0 F_0' - G_0 f_0' + Db H_0'' + \gamma G_0' = 0 \quad (18)$$

$$\frac{1}{Sc} H_0'' + f_0 H_0' + \varphi_0' F_0 - \varphi_0 F_0' - H_0 f_0' + Sr G_0'' + \gamma H_0' = 0 \quad (19)$$

with the boundary conditions

$$F_0'(\eta) = 0, F_0(\eta) = 0, G_0(\eta) = 0, H_0(\eta) = 0 \quad \text{at } \eta = 0$$

$$F_0'(\eta) \rightarrow 0, G_0(\eta) \rightarrow 0, H_0(\eta) \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \quad (20)$$

For the present problem, the first boundary condition as  $\eta \rightarrow \infty$  is relaxed. The relaxation technique of boundary condition has been described by the previous investigators [26]. Finally, the Eqs. (17 – 19) along with the new boundary condition  $F_0''(0) = 1$  is solved in Matlab software.

### 3. Results and Discussion

Numerical computations are generated by using bvp4c MATLAB programming method and the resulting data are presented graphically by velocity profiles, temperature profiles and concentration profiles for different values of extending and compressing parameter. Negative  $\lambda$  denotes the shrinking parameter, where as positive one for stretching parameter. The calculation is performed by using the fixed values of the following parameters unless otherwise mentioned:  $\omega = 1800$ ,  $Pr = 1$ ,  $Sc = 0.5$ ,  $Ri = -0.01$ ,  $N = 0.5$ ,  $Db = 0.1$ ,  $Sr = 1.8$ ,  $S = 3.1$ ,  $X = -0.5$ ,  $H = 0.02$ . Since we got dual solutions, first solution is identified as solid line and the dashed line indicates the second solution.

The authentication of numerical results occurred from MATLAB programming is performed. The current numerical data is compared with the result by previous investigators [22] for the case of exponentially extending sheet ( $\lambda > 0$ ) and for various values of Prandtl number  $Pr$ . This table is tabulated for the following fixed values:  $Ri = 1.0$ ,  $Sr = 2.0$ ,  $Db = 0.03$ ,  $X = 0.5$ ,  $\omega = 100000000$ ,  $Sc = 0.22$ ,  $N = 0.5$  and  $H = S = 0$ . The comparison values of skin friction coefficient, local Nusselt number and local Sherwood number are presented in Table 1. The formulation of skin friction coefficient, local Nusselt number and local Sherwood number are reported as  $C_f \sqrt{2Re_x} \exp\left(\frac{-3X}{2}\right)$ ,  $Nu_x \sqrt{2/Re_x} \exp\left(\frac{-X}{2}\right)$  and  $Sh_x \sqrt{2/Re_x} \exp\left(\frac{-X}{2}\right)$  respectively [22]. As a conclusion, these values are in good agreement compared with previous researchers.

The function of stability analysis is to select the solution which is stable and physically reliable. The declaration of the stable solution is obtained when the values of smallest eigenvalue  $\gamma$  is positive, otherwise it is unstable and not physically occur in actual state in fluid. Table 2 represents the stability analysis for several values of compressing parameter at  $S = 1.7$  and  $1.72$ . For the smallest

eigenvalue,  $\gamma$  first solution are positive whereas opposite for second solution. Positive value represents the acceptance of first solution and the second solution is unstable and rejected. Moreover, the stable solution always following the conditions at the boundary Eq. (11), with minimal existence of minimum or maximum peaks (see Figures 1-3). Therefore, all of the results in this articles (only for the first solution) are stable, accepted and really occur in the actual mechanism in fluid flow.

From the first solution in Figure 1, the velocity value for the compressing sheet is negative at the wall and the value is increasing until it reaches zero velocity for large  $\eta$ . However, for the case of extending sheet the velocity value for the first solution is positive for the small  $\eta$ , and it decreases until velocity is zero. Negative velocity when  $\lambda < 0$  indicates the opposite direction of instantaneous fluid flow, compared to the case of fluid flow affected by extending sheet  $\lambda > 0$ . Moreover, the highest velocity is obtained at the sheet ( $\eta = 0$ ) for the extending sheet (highest  $\lambda$ ). However, minimum peak arises for the second solution in velocity profile. For the temperature and concentration profiles (Figure 2 and 3) these values go down with the raising of  $\eta$  and  $\lambda$ . The effect of  $\lambda$  on temperature and concentration profiles are significant at the point close to the sheet (small  $\eta$ ). The occurrence of maximum peak is observed at concentration profile, near to the sheet. Therefore, the concentration level is the highest at that point.

**Table 1**

The comparison with the previous researchers [22] for a) skin friction coefficient, b) local Nusselt number and c) local Sherwood number

$\lambda$	$Pr$	a)		b)		c)	
		[22]	Present	[22]	Present	[22]	Present
1	1.0	-0.63185	-0.63185	1.11260	1.11260	0.07884	0.07884
	3.0	-0.70730	-0.70730	2.03606	2.03605	-0.13554	-0.31553
	5.0	-0.73931	-0.73931	2.70661	2.70657	-0.60416	-0.60414
1.02	1.0	-	-0.52867	-	1.09382	-	0.08111
	3.0	-	-0.60440	-	1.99450	-	-0.30392
	5.0	-	-0.63670	-	2.64803	-	-0.58536

**Table 2**

The smallest eigenvalues for various values of compressing parameter

$S$	$\lambda$	Solutions	
		First solution	Second solution
1.70	-0.4000	0.19877	-0.20665
	-0.4002	0.19785	-0.20566
	-0.4004	0.19693	-0.20466
1.72	-0.4000	0.26728	-0.28206
	-0.4002	0.26661	-0.28131
	-0.4004	0.26594	-0.28055

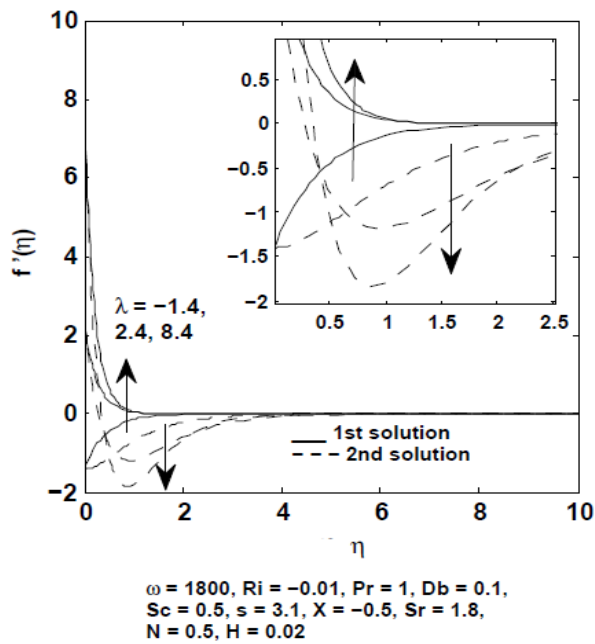


Fig. 1. Velocity profile for different values of  $\lambda$

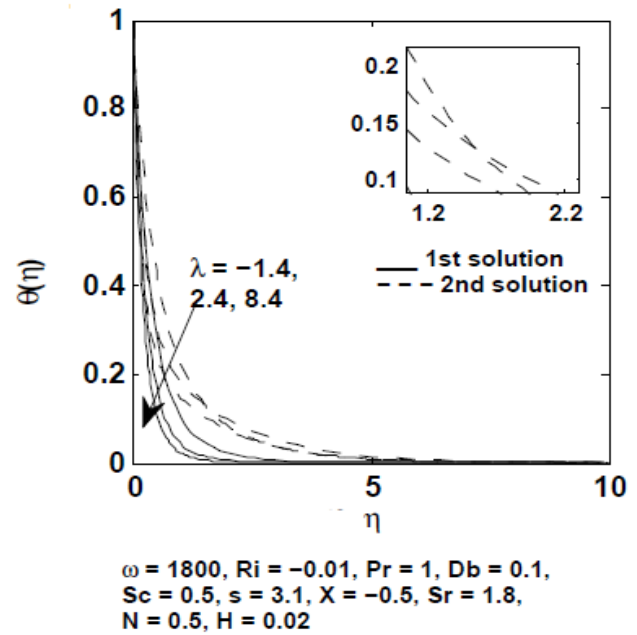


Fig. 2. Temperature profile for different values of  $\lambda$

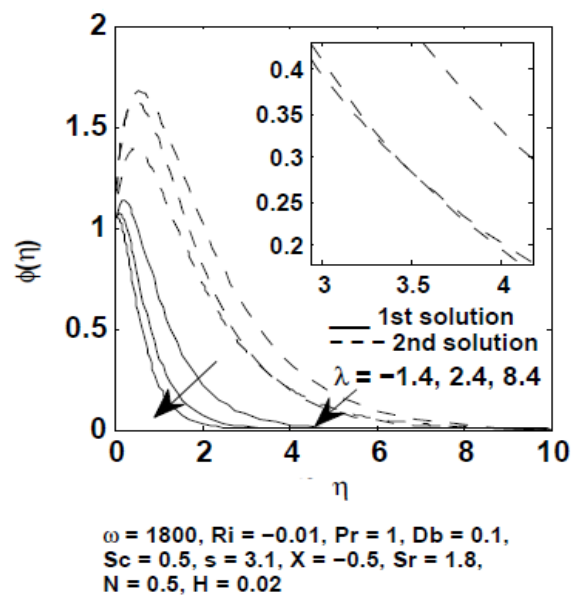


Fig. 3. Concentration profile for different values of  $\lambda$  *Fiiger*

#### 4. Conclusions

The dual numerical solutions of mixed convection Casson fluid flow, which is subjected to the rate of extending and compressing sheet are investigated. Observing the findings from figures and tables following conclusions are noted

- i. Results comparing with the previous work it is clear that Matlab programming are acceptable to obtain the numerical calculation.
- ii. Velocity is negative near the wall for shrinking sheet, whereas it is positive for extending case and coincides as it goes far from the wall.



- iii. Temperature and concentration profiles, both are declined with the increment of  $\lambda$  and  $\eta$  for both solutions and if the distance from the wall increased ( $\eta$  become larger), effects of temperature and concentration becomes negligible.

### Acknowledgement

This research was funded by a grant from Universiti Putra Malaysia (Project code: GP-IPM/2018/9596900).

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