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Enhancing Quantum Information Processing – SU(2) Operator Model Development for Three-Qubit Quantum Systems Entanglement Classification

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ABSTRACT

The study introduces the development of the $SU(2)\times SU(2)\times SU(2)$ operator model within the Local Unitary (LU) protocol for entanglement classification, specifically of the pure three-qubit quantum systems. Addressing the challenge of accurately distinguishing different classes of entanglement, this research aims to enhance the understanding and utilization of entangled quantum states in quantum information processing. A systematic approach was employed in the development of the model, designed to effectively distinguish different classes of entanglement. This study contributes significantly to quantum information processing, by providing valuable insights into the nature of entangled quantum states and enabling researchers to gain a better understanding, utilizing them effectively in various quantum applications. The developed $SU(2)\times SU(2)\times SU(2)$ operator model holds significant potential for the advancements in entanglement classification and broader scope of quantum information processing. The model marks a notable achievement in enabling a more precise and efficient entanglement of entangled quantum systems, which is crucial for advancing various quantum technologies. Future research should extend this model to a higher-qubit and higher-dimensional quantum systems and explore its integration with several other entanglement classification protocols and Lie groups to further advance the field.

1. Introduction

Quantum entanglement is fundamental in quantum mechanics, where it exhibits the correlation relation between particles that cannot be explained by classical physics. This phenomenon has

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captivated the attention of researchers for decades now, particularly in quantum information processing [1,2]. Entanglement plays a crucial role in the development of quantum technologies.

Thus, entanglement classification has its own significant role in empowering the potential for quantum technologies namely quantum computing, quantum cryptography and quantum teleportation as in previous studies [3-13]. These technologies leverage the unique properties of entangled states to perform tasks that are either impossible or inefficient with a classical system.

Entanglement classification process became more complex as the number of qubits grow as indicated in previous studies [14-20]. For instance, while single or two-qubits systems have relatively straightforward classification process, multi-qubit quantum systems such as three-qubit quantum systems or higher, present significant challenges. These complexities arose from the exponentially increasing number of possible entangled states and the intricate relationships within the quantum systems.

There are three main protocols in entanglement classification, that are Local Unitary (LU), Local Operations and Classical Communication (LOCC) and Stochastic Local Operations and Classical Communication (SLOCC). While each has their distinct entanglement classification methods, they are also partly correlated with one another, complimenting the nature of quantum entanglement.

In this study, the multiqubit quantum systems is reviewed, particularly of the three-qubit quantum systems and the LU protocol is utilized. LU protocol in particular, has been deemed as one of the most efficient entanglement classification frameworks, as it preserved the invariance quantum states under local unitary transformation.

The LU protocol's efficiency in distinguishing various entangled states makes it an ideal choice for this research. This protocol has been utilized in number of experiments notably classifying different classes of entanglement as in previous studies [21, 22]. Leveraging the special unitary group, $SU(2)$, the model operator $SU(2)\times SU(2)\times SU(2)$ was developed.

Despite the advancements in quantum information processing, accurately distinguishing different classes of entanglement in three-qubit quantum systems remains a significant challenge. Existing methods lack precision and efficiency, highlighting the need for a robust and systematic approach. This study addresses this gap by introducing a developed operator model within the LU protocol for entanglement classification, which enhances our understanding and utilization of entangled quantum states.

The proposed operator model presents an innovative approach in entanglement classification. It is designed to efficiently classify the entanglement of entangled quantum systems with high accuracy. The developed operator model significantly contributes to quantum information science field, by offering valuable insights into the nature of entangled quantum states and enabling researchers to gain a better understanding of these states, utilizing them effectively in various quantum applications.

For instance, in quantum computing, precise entanglement classification can significantly improve error correction methods and optimize quantum algorithms for various quantum information processing tasks. In another, quantum cryptography, precise entanglement classification can enhance the quantum key distribution protocols, therefore strengthening the security framework.

This main aim of this study is to introduce the developed operator model within the LU protocol for entanglement classification in the three-qubit quantum systems. The paper is organized as follows: Section 2 describes the methodology in detail, explaining the theoretical foundation and practical implementation of the model. Section 3 presents the developed operator model, including its design, functionality and performance evaluation. Finally, section 4 concludes the study, summarizing the key findings and future direction.

2. Methodology

The development of the model within the LU protocol for entanglement classification involved the utilization of three sets of 2×2 matrix as generators. These generators were employed in the dot product multiplication process, resulting in the formation of the 8×8 matrix. It is then further in the process, sets of parameters selected were implemented to the developed operator model, classifying entanglement. Modelling process of $SU(2)\times SU(2)\times SU(2)$ operator model is illustrated in Figure 1.

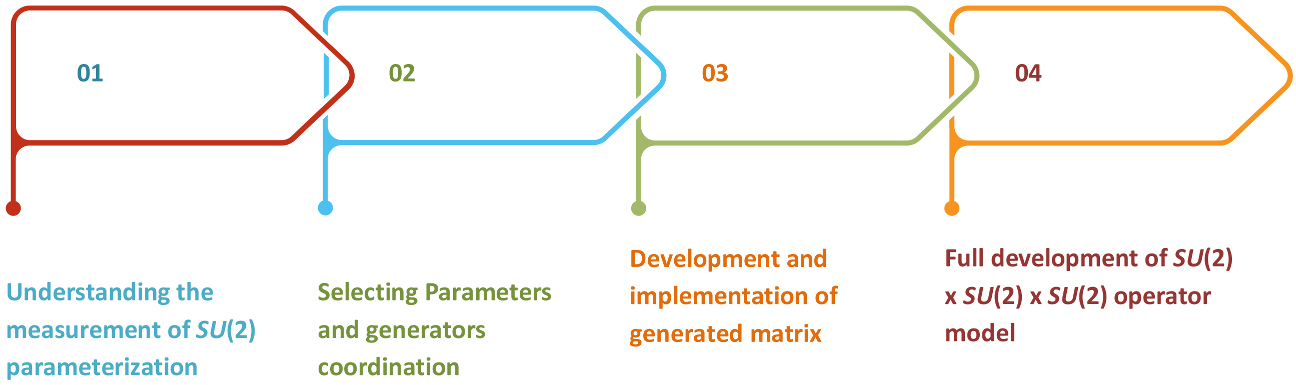

Fig. 1. Modelling process of $SU(2)\times SU(2)\times SU(2)$

The first step started with understanding the parameterization measurement of $SU(2)$. In this stage, the variables of generators and parameters were thoroughly investigated to ensure accurate representation and functionality. Then, the parameter values were determined by coordinating the selected parameter values with generators, ensuring that the model accurately reflects the entanglement characteristics.

At this point, the modelling process enters the development phase. This phase involved the development and implementation of the generated matrix in Mathematica 13.2 software. The choice of software facilitated precise calculations and the ability to handle complex matrix operations efficiently.

Finally, full development of $SU(2)\times SU(2)\times SU(2)$ operator model began, undergoing a three-qubit quantum system creation. This system was achieved through the integration of the developed operator model with initial pure quantum states, allowing for the classification of different entangled states.

The development process of the operator model relied on the series of expansions of exponential, cosine and sine functions. These expansions were critical in representing the nature of entangled quantum states. The operator model of $SU(2)$, which in principle is representing a single-qubit quantum system, were then extended to $SU(2)\times SU(2)\times SU(2)$ operator model, designed to represent a three-qubit quantum system. This extension was crucial in enabling the classification of more complex entangled quantum systems.

The next phase of the development is model validation. This process is vital in assuring that the developed operator model is accurate and reliable. The validation process includes $SU(2)$ and $SU(2)\times SU(2)\times SU(2)$ operator model comparison. The results obtained were compared with manual calculations to verify its accuracy. This step involved cross-referencing the mathematical outputs with the theoretical predictions.

The final step in model validation covered the results obtained in an environment-controlled experiment of the developed $SU(2)\times SU(2)\times SU(2)$ operator model with experimental result from earlier studies. This experimental validation was essential to demonstrate the practical applicability and robustness of the developed operator model in a real quantum information processing task. Figure 2 illustrates the matrix development of $SU(2)\times SU(2)\times SU(2)$ operator model.

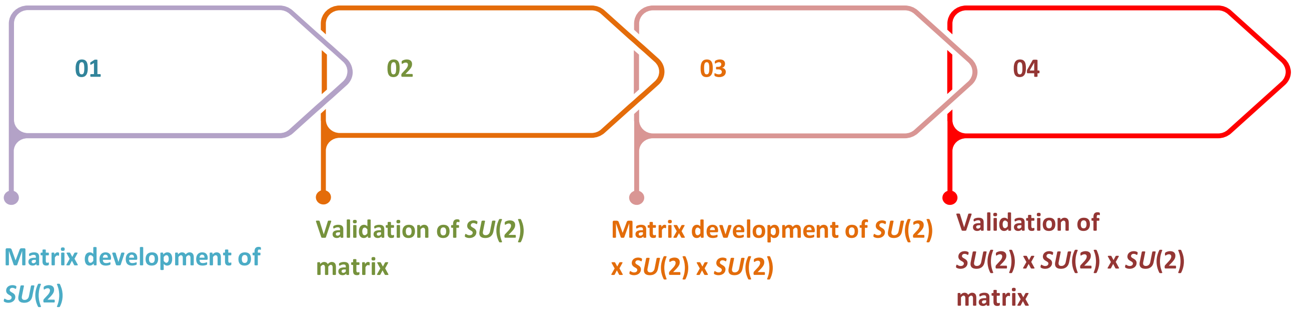

Fig. 2. $SU(2)\times SU(2)\times SU(2)$ operator model matrix development

3. Results

Figure 3 illustrates the successfully developed representation of the tensor product multiplication of $SU(2)\times SU(2)\times SU(2)$ operator model. This process resulted in a comprehensive 8×8 composite matrix derived by the $SU(2)$ group generators as presented in Eq. (1).

$$SU(2)=e^{i\sigma_3\beta_1}e^{i\sigma_2\beta_2}e^{i\sigma_3\beta_3} \tag{1}$$

with $\sigma_1=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$, $\sigma_2=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$ and $\sigma_3=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$ are the individual operator generators representing a single-qubit quantum system. Parameters β_1, β_2, β_3 are the angle of qubit rotation within the range of $0\le\beta_1,\beta_3\le\pi$ and $0\le\beta_2\le\frac{\pi}{2}$ in a *Hilbert* space. Eq. (2) illustrates the full expansion of the $SU(2)\times SU(2)\times SU(2)$ operator model.

The expanded $SU(2)\times SU(2)\times SU(2)$ operator model as illustrated in Figure 3, showcases the intricate structure of a three-qubit quantum system. The exponential, cosine and sine functions are vital to the development of the operator model, ensuring that the quantum states are represented accurately.

$$SU(2) \otimes SU(2) \otimes SU(2) =$$

$$\begin{pmatrix}
\cos[\beta 2]^3 (\cos[\beta 1] + i \sin[\beta 1])^3 (\cos[\beta 3] + i \sin[\beta 3])^3 & \dots\dots\dots\dots\dots & (\cos[\beta 1] + i \sin[\beta 1])^3 \sin[\beta 2]^3 (\cos[\beta 3] - i \sin[\beta 3])^3 \\
-\cos[\beta 2]^2 (\cos[\beta 1] - i \sin[\beta 1])(\cos[\beta 1] + i \sin[\beta 1])^2 \sin[\beta 2](\cos[\beta 3] + i \sin[\beta 3])^3 & \dots\dots\dots\dots\dots\dots & \cos[\beta 2](\cos[\beta 1] - i \sin[\beta 1])(\cos[\beta 1] + i \sin[\beta 1])^2 \sin[\beta 2]^2 (\cos[\beta 3] - i \sin[\beta 3])^3 \\
-\cos[\beta 2]^2 (\cos[\beta 1] - i \sin[\beta 1])(\cos[\beta 1] + i \sin[\beta 1])^2 \sin[\beta 2](\cos[\beta 3] + i \sin[\beta 3])^3 & \dots\dots\dots\dots\dots\dots & \cos[\beta 2](\cos[\beta 1] - i \sin[\beta 1])(\cos[\beta 1] + i \sin[\beta 1])^2 \sin[\beta 2]^2 (\cos[\beta 3] - i \sin[\beta 3])^3 \\
\cos[\beta 2](\cos[\beta 1] - i \sin[\beta 1])^2 (\cos[\beta 1] + i \sin[\beta 1]) \sin[\beta 2]^2 (\cos[\beta 3] + i \sin[\beta 3])^3 & \dots\dots\dots\dots\dots\dots & \cos[\beta 2]^2 (\cos[\beta 1] - i \sin[\beta 1])^2 (\cos[\beta 1] + i \sin[\beta 1]) \sin[\beta 2](\cos[\beta 3] - i \sin[\beta 3])^3 \\
-\cos[\beta 2]^2 (\cos[\beta 1] - i \sin[\beta 1])(\cos[\beta 1] + i \sin[\beta 1])^2 \sin[\beta 2](\cos[\beta 3] + i \sin[\beta 3])^3 & \dots\dots\dots\dots\dots\dots & \cos[\beta 2](\cos[\beta 1] - i \sin[\beta 1])(\cos[\beta 1] + i \sin[\beta 1])^2 \sin[\beta 2]^2 (\cos[\beta 3] - i \sin[\beta 3])^3 \\
\cos[\beta 2](\cos[\beta 1] - i \sin[\beta 1])^2 (\cos[\beta 1] + i \sin[\beta 1]) \sin[\beta 2]^2 (\cos[\beta 3] + i \sin[\beta 3])^3 & \dots\dots\dots\dots\dots\dots & \cos[\beta 2]^2 (\cos[\beta 1] - i \sin[\beta 1])^2 (\cos[\beta 1] + i \sin[\beta 1]) \sin[\beta 2](\cos[\beta 3] - i \sin[\beta 3])^3 \\
\cos[\beta 2](\cos[\beta 1] - i \sin[\beta 1])^2 (\cos[\beta 1] + i \sin[\beta 1]) \sin[\beta 2]^2 (\cos[\beta 3] + i \sin[\beta 3])^3 & \dots\dots\dots\dots\dots\dots & \cos[\beta 2]^2 (\cos[\beta 1] - i \sin[\beta 1])^2 (\cos[\beta 1] + i \sin[\beta 1]) \sin[\beta 2](\cos[\beta 3] - i \sin[\beta 3])^3 \\
-(\cos[\beta 1] - i \sin[\beta 1])^3 \sin[\beta 2]^3 (\cos[\beta 3] + i \sin[\beta 3])^3 & \dots\dots\dots\dots\dots & \cos[\beta 2]^3 (\cos[\beta 1] - i \sin[\beta 1])^3 (\cos[\beta 3] - i \sin[\beta 3])^3
\end{pmatrix} \quad (2)$$

Fig. 3. The successfully developed representation of the tensor product multiplication of operator model

To validate the accuracy and reliability of the developed model, a series of comparative analyses was done:

i. Manual calculations: The results obtained from the developed operator model were carefully compared with manual calculations. This step ensured that an accurate mathematical formulations and parameterizations, providing a strong theoretical foundation for the model.
ii. Controlled experiments: The developed operator model was tested in a controlled experiment environment. The results obtained were compared side by side with experimental results conducted by Mohd *et al.,* [20]. The comparison of the operator model developed, demonstrated high fidelity in entanglement classification, aligned closely to the established experimental results.

The successful development and validation of the $SU(2)\times SU(2)\times SU(2)$ operator model marks a significant contribution to the field of entanglement classification and quantum information science. This model provides a robust framework for the classification of three-qubit entangled quantum systems under the LU protocol. It offers several advantages over conventional methods including enhanced efficiency, scalability, and accuracy.

4. Conclusion

This study introduces a novel $SU(2)\times SU(2)\times SU(2)$ operator model systematically designed and developed to classify entanglement within the three-qubit pure quantum systems under the LU protocol. The operator model developed represents a significant leap in the field of entanglement classification and quantum information science, providing a deeper understanding of entanglement classification.

The developed operator model not only sheds light on the nature of entanglement, but also empowers researchers to effectively harness the potential of these concepts in various quantum applications. By accelerating the entanglement classification, the developed operator model significantly enhances quantum information processing.

The $SU(2)\times SU(2)\times SU(2)$ operator model marks a notable achievement in entanglement classification, enabling a more precise and efficient classification of entangled quantum systems, which crucial for the advancement of several quantum information science fields namely, quantum computing, quantum cryptography, and quantum teleportation.

Future research should extends to the expansion of this developed operator model to higher-qubit and higher-dimensional quantum systems and explores its integration with other entanglement classification protocols and Lie groups to further advance the field. Such research direction is a catalyst to a more transformative innovations in quantum technologies, paving the way for enhanced technological capabilities and novel quantum-enabled applications.

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