Journal of Advanced Research in Applied Mechanics 112, Issue 1 (2023) 88-101

SEMARAK ILMU PUBLISHING

Journal of Advanced Research in Applied Mechanics

Journal homepage: https://semarakilmu.com.my/journals/index.php/appl_mech/index

ISSN: 2289-7895

Nanoparticle Shape Effects of Aligned Magnetohydrodynamics Mixed Convection Flow of Jeffrey Hybrid Nanofluid over a Stretching Vertical Plate

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ARTICLE INFO

Article history:
Received 13 August 2023
Received in revised form 15 October 2023
Accepted 31 October 2023
Available online 5 January 2024

Keywords:
Jeffrey hybrid nanofluid; mixed convection flow; stretching vertical plate; nanoparticle shape effect; magnetohydrodynamic

ABSTRACT

The nanoparticle shape effects of aligned magnetohydrodynamics (MHD) mixed convection flow of Cu-Al_2O_3/water-EG Jeffrey hybrid nanofluid over a stretching vertical plate are investigated in this study. Five different shapes of nanoparticles which are spherical, cylindrical, blades, bricks, and platelets are considered. The governing equations in the form of Partial Differential Equations (PDEs) had been reduced to nonlinear Ordinary Differential Equations (ODEs) by using similarity transformation. The transformed ODEs are tackled numerically by implementing bvp4c solver in MATLAB towards the dimensionless physical parameters which are aligned angle of magnetic field (α), interaction of magnetic field (M), mixed convection (λ), Deborah number (β), volume fraction of nanoparticles (ϕ), and nanoparticle shape factor (*m*). The effects of nanoparticle shape and other parameters on fluid velocity, temperature, skin friction coefficient, and Nusselt number are illustrated with graphs and tables. This study discovered that blade-shaped nanoparticles have the greatest skin friction coefficient and Nusselt number compared to all different shapes. While λ, and ϕ enhance the skin friction coefficient, α, and M increase the Nusselt number. The parameters α, M, λ, and ϕ reduce the velocity profiles while raising the temperature profiles.

1. Introduction

Nanofluids are a unique type of heat transfer fluid with a very tiny number of nanoparticles stably contained in a carrier liquid [1]. A conductive fluid, such as water or ethylene glycol, is used as the

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https://doi.org/10.37934/aram.112.1.88101

basis fluid in nanofluid. Hybrid nanofluids, which are technologically improved, have better thermal conductivity than regular nanofluids [2]. The combination of exceptional qualities of two different types of dispersed nanoparticles in the base fluid at an affordable price is the most significant hybrid nanofluid exclusivity. The composition of hybrid nanomaterials is carefully chosen to maximize the benefits of nanoparticles while minimizing the drawbacks of using a single material. On hybrid nanofluid, a number of numerical investigations have been conducted. Amira *et al.,* [3] conducted a study on mixed convection flow of a hybrid nanofluid passing a vertical flat plate. Here, the hybrid nanofluid was made by suspending two different nanoparticles which are alumina (Al_2O_3) and copper (Cu) in the base fluid (H_2O). The study revealed that the magnetic parameter rises the fluid velocity while diminishes the fluid temperature.

The Jeffrey model is the simplest fundamental model for non-Newtonian fluid rheological impacts of an elastic fluid [4]. Kumar *et al.,* [5] performed finite element analysis to evaluate the effects of sort number on Jeffrey magnetohydrodynamics (MHD) fluid through a vertical, porous, and moving plate. The mixed convective flow of Jeffrey fluid was investigated by Ijaz and Ayub [6].

Heat transfer is the migration of heat from one surface to another surface with a temperature difference. It flows from higher temperature regions to lower temperature regions. Conduction, radiation, and convection are the three fundamental modes of heat transfer. Temperature changes in various sections of the same body are referred to as conduction. On the other hand, radiation is an electromagnetic radiation that is influenced by the temperature and pace of the internal energy of the highlighted body. Convection flow refers to the transfer of heat from a higher to a lower temperature area in the form of a fluid or gas.

According to Waini *et al.,* [7] convectional heat transfer is a process in which heat is moved from one location to another through the movement of fluids. Forced convection is the process of generating fluid motion from an external source. While, natural or free convection is a process where fluid motion is caused solely by buoyant forces resulting from density fluctuations. Whenever natural and forced convection processes work simultaneously, mixed convection occurs. Mixed convection flows past a stretched surface in a wide range of ways. It plays a significant role in a variety of applications, including the drying of porous solids, solar-oriented beneficiaries exposed to wind currents, atmospheric and oceanic flows, nuclear reactors cooled during an emergency shutdown, heat exchangers placed in a low-speed environment, electronic devices cooled by fans, and so on.

Numerous investigations on the behavior of hybrid nanofluids in mixed convection flow via different types of plates have been conducted. Waini *et al.,* [7] used a hybrid nanofluid to investigate exponentially stretching or shrinking mixed convection flow across a vertical surface. They use water as the base fluid with hybrid nanoparticles of Cu and Al_2O_3. The study proved that nanoparticles increase the heat transfer rate.

MHD is a branch of science that investigates the magnetic characteristics and behavior of electrically conducting fluids [8]. The term MHD is a mix of magneto, hydro, and dynamics. The phrase magneto refers to magnetism, whereas hydro refers to water or fluid, and dynamics refers to fluid movement caused by force. MHD also known as magnetofluid dynamics or hydromagnetic. The concept of MHD is that the magnetic fields influencing the behavior of the fluids and flows. The magnetic field can induce currents into such a non-stationary fluid. MHD is a physical-mathematical framework for electrically conducting fluid flow while accounting for magnetic field dynamics. The MHD for nanofluids is connected to many phenomena, and this theory has been used in different disciplines to address issues.

Since magnetic fields exist naturally everywhere, numerous studies have been conducted in recent years to investigate MHD phenomena. The research aims to explore the MHD mixed convection flow of a hybrid nanofluid past a stretching vertical plate, as encouraged by Rostami *et* *al.,* [9] who developed dual solutions for mixed convective stagnation point flow of an aqueous silica-alumina hybrid nanofluid and Nayan *et al.,* [10] who investigated numerically the aligned MHD flow of hybrid nanofluid across a vertical plate via a porous media.

To the best of the authors' knowledge, mixed convection flow studies have been predominantly conducted on either a fixed plate or moving plate with less consideration on nanoparticle shape effects. Thus, this study was carried out to investigate and identify the nanoparticle shape effects on the steady two-dimensional aligned MHD mixed convection flow of Jeffrey hybrid nanofluid over a stretching vertical plate for various physical parameters, which are aligned angle of magnetic field, interaction of magnetic field, mixed convection, Deborah number, volume fraction of nanoparticles, and nanoparticle shape factor. Using a numerical method, the velocity and temperature profiles, as well as the skin friction coefficient and Nusselt number, are determined.

2. Methodology

This study considers a steady state of two-dimensional laminar mixed convection flow of Jeffrey hybrid nanofluid across stretching vertical plate in the presence of magnetic field and heat transfer with the plane $y=0$. The surface is assumed to stretch with velocity $u_w = ax$, where a is stretching constant. The plate temperature is $T_w = T_\infty + bx$, where T_w is the surface temperature, T_∞ is the ambient fluid temperature and b is constant. A uniform transverse magnetic field of strength B_o is applied parallel to the y-axis. The physical model and coordinate system of this study are depicted in Figure 1.

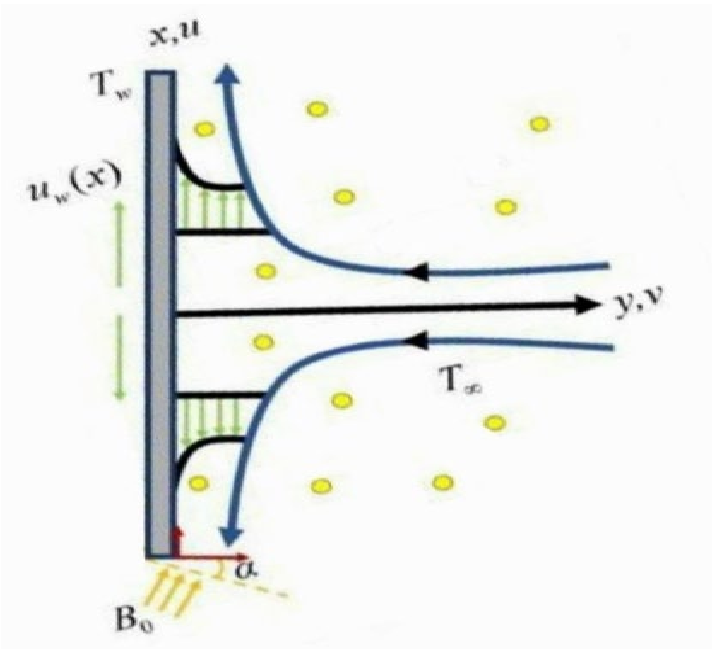


Fig. 1. Physical model and coordinate system [11]

In this study, the continuity, momentum, and energy equations serve as the governing equations for fluid dynamics problems. Together with the Boussinesq and boundary layer approximations, the continuity and energy equations used are as proposed by Ilias *et al.,* [12] while, the equation of momentum used are as presented by Ghalambaz *et al.,* [11] and Shahzad *et al.,* [13]. Therefore, the governing equations of the steady MHD mixed convection boundary layer flow of Jeffrey hybrid nanofluid across a stretching vertical plate can be expressed as follows

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{hnf}}{\rho_{hnf}\left(1+\lambda_1\right)}\left(\frac{\partial^2 u}{\partial y^2}+\lambda_2\left(\begin{array}{l} u\dfrac{\partial^3 u}{\partial x\partial y^2}+ \\ \dfrac{\partial u}{\partial y}\dfrac{\partial^2 u}{\partial x\partial y}+v\dfrac{\partial^3 u}{\partial y^3}-\dfrac{\partial u}{\partial x}\dfrac{\partial^2 u}{\partial y^2}\end{array}\right)\right)$$
$$-\frac{\sigma B_0{}^2}{\rho_{hnf}}\sin^2\alpha\left(u\right)+\frac{\left(\rho\beta\right)_{hnf}}{\rho_{hnf}}g\left(T-T_\infty\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{hnf}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

This study considers the following boundary conditions given by Ahmad & Ishak [14]

As $y=0,\ u\left(x,0\right)=u_w\left(x\right),\ v\left(x,0\right)=0,\ T\left(x,0\right)=T_w.$
As $y\to\infty,\ u\left(x,\infty\right)=0,\ \dfrac{\partial u}{\partial y}\left(x,\infty\right)=0,\ T\left(x,\infty\right)=T_\infty.$ (4)

where u and v indicates the velocity component along the x and y axes, respectively. x and y are the Cartesian coordinate parallel and normal to the surface, respectively. Here μ_{hnf} is dynamic viscosity of the hybrid nanofluid, ρ_{hnf} is density of the hybrid nanofluid, λ_1 is ratio of relaxation to the retardation times, λ_2 is relaxation time, σ is electrical conductivity of the fluid, α is aligned angle of magnetic field, β_{hnf} is thermal expansion coefficient of the hybrid nanofluid and g is the acceleration due to gravity. Furthermore, in Eq. (3), T refers to the fluid temperature in the boundary layer and α_{hnf} denotes to the thermal diffusivity of the hybrid nanofluid.

The compositions of Jeffrey hybrid nanofluid used in this study are composed of water-EG as the base fluid and Cu-Al_2O_3 as the hybrid nanoparticles. This kind of hybrid nanofluid had also been studied by Ghalambaz *et al.,* [11] for a case of mixed convection flow. The thermophysical properties of nanoparticles and the base fluid are displayed in Table 1.

Table 1
Thermophysical properties of base fluid and nanoparticles [15]

Thermophysical properties	Base fluid	Nanoparticles	
	Water-EG (H_2O+ EG (50%:50%))	Copper (Cu)	Alumina (Al_2O_3)
$C_p\left(J/kgK\right)$	3288	385	765
$\rho\left(kg/m^3\right)$	1056	8933	3970
$k\left(W/mK\right)$	0.425	400	40
$\beta\times10^{-5}\left(mK\right)$	0.00341	1.67	0.85

In the meantime, the density, heat capacity, dynamic viscosity, thermal conductivity, thermal expansion coefficient, and thermal diffusivity of the hybrid nanofluid can be evaluated using the relations introduced in Table 2.

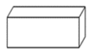
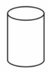
Table 2
Thermophysical relations of hybrid nanofluid [11]

Properties	Hybrid nanofluid
Density	$\rho_{hnf} = (1-\phi_2)\left[(1-\phi_1)\rho_f + \phi_1\rho_{s_1}\right] + \phi_2\rho_{s_2}$
Heat capacity	$\left(\rho C_p\right)_{hnf} = (1-\phi_2)\left[(1-\phi_1)\left(\rho C_p\right)_f + \phi_1\left(\rho C_p\right)_{s_1}\right] + \phi_2\left(\rho C_p\right)_{s_2}$
Viscosity	$\mu_{hnf} = \dfrac{\mu_f}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}}$
Thermal conductivity	$\dfrac{k_{hnf}}{k_{bf}} = \dfrac{k_{s_2} + (m-1)k_{bf} - (m-1)\phi_2\left(k_{bf} - k_{s_2}\right)}{k_{s_2} + (m-1)k_{bf} + \phi_2\left(k_{bf} - k_{s_2}\right)}$ $\dfrac{k_{bf}}{k_f} = \dfrac{k_{s_1} + (m-1)k_f - (m-1)\phi_1\left(k_f - k_{s_1}\right)}{k_{s1} + (m-1)k_f + \phi_1\left(k_f - k_{s1}\right)}$
Thermal expansion coefficient	$\beta_{hnf} = (1-\phi_2)\left[(1-\phi_1)\beta_f + \phi_1(\rho\beta)_{s_1}\right] + \phi_2\beta_{s_2}$ $(\rho\beta)_{hnf} = (1-\phi_2)\left[(1-\phi_1)(\rho\beta)_f + \phi_1(\rho\beta)_{s_1}\right] + \phi_2(\rho\beta)_{s_2}$
Thermal diffusivity	$\alpha_{hnf} = \dfrac{k_{hnf}}{\left(\rho C_p\right)_{hnf}}$

Here, ϕ_1 and ϕ_2 denote the solid volume fractions of two different nanoparticles; ϕ_1 is for Cu and ϕ_2 is for Al_2O_3. ρ_f, ρ_{s_1} and ρ_{s_2} are the density of the base fluid, Cu and Al_2O_3, respectively. $\left(C_p\right)_f$, $\left(C_p\right)_{s_1}$ and $\left(C_p\right)_{s_2}$ are the specific heat at a constant pressure for base fluid, Cu and Al_2O_3, respectively. k_f, k_{s_1} and k_{s_2} are the specific thermal conductivity for base fluid, Cu and Al_2O_3, respectively. β_f, β_{s_1} and β_{s_2} are the specific thermal expansion coefficient for base fluid, Cu and Al_2O_3, respectively. m is the nanoparticle shape factor, and μ_f is the viscosity of the base fluid.

Due to the fact that this study investigates the effects of nanoparticle shape on the behaviour of fluid flow, Table 3 depicts the value of the nanoparticle shape factor, m, for spherical, brick, cylindrical, platelet, and blade shapes.

Table 3
The nanoparticles shape factors (m) [2]

Nanoparticles shape	Shape structure	Shape factor (m)
Spherical		3
Brick		3.7
Cylindrical		4.9
Platelet		5.7
Blade		8.6

Observed that Eq. (1)-(3) are nonlinear partial differential equations with a large number of dependent and independent variables. It is also presented in dimensional forms that are challenging to solve directly. Therefore, the following similarity variables are introduced to solve the governing equations in Eq. (1)-(3) as in the study conducted by Ilias *et al.,* [12]

$$\eta = \sqrt{\frac{u_w}{\nu_f x}}y = \sqrt{\frac{ax}{\nu_f x}}y = \left(\frac{a}{\nu_f}\right)^{\frac{1}{2}} y,\ \psi = \sqrt{\nu_f x(ax)}f(\eta) = \left(a\nu_f\right)^{\frac{1}{2}} xf(\eta),\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}. \tag{5}$$

where η refers to the similarity variable, ψ is the stream function, $f(\eta)$ is dimensionless stream function, and $\theta(\eta)$ denotes dimensionless temperature function. The continuity equation in Eq. (1) is satisfied by the stream function ψ such that

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x}. \tag{6}$$

Upon substitution of Eqns. (5)-(6) into momentum equation, Eq. (2), energy equation, Eq. (3) and boundary conditions, Eq. (4), the transformed non-dimensional ordinary differential equations can be written as follows

$$\begin{aligned} &f'''(\eta) + \beta\left(\left(f''(\eta)\right)^2 - f(\eta)f^{iv}(\eta)\right) - A_1(1+\lambda_1)Mf'(\eta)\sin^2\alpha + \\ &A_1A_3(1+\lambda_1)\lambda\theta(\eta) + A_1A_2(1+\lambda_1)\left[f(\eta)f''(\eta) - \left(f'(\eta)\right)^2\right] = 0 \end{aligned} \tag{7}$$

$$\left(\frac{k_{hnf}}{k_f}\right)\theta''(\eta) - A_4 \Pr\left[\theta(\eta)f'(\eta) - \theta'(\eta)f(\eta)\right] = 0 \tag{8}$$

while the transformed boundary conditions are:

$$\begin{aligned} &f(0) = 0,\ f'(0) = 1,\ \theta(0) = 1 \text{ at } \eta = 0 \\ &f'(\eta) = 0,\ f''(\eta) = 0,\ \theta(\eta) = 0 \text{ as } \eta \to \infty \end{aligned} \tag{9}$$

where

$$A_1 = (1-\phi_1)^{2.5}(1-\phi_2)^{2.5},$$

$$A_2 = (1-\phi_2)\left[(1-\phi_1)+\phi_1\frac{\rho_{s_1}}{\rho_f}\right]+\phi_2\frac{\rho_{s_2}}{\rho_f},$$

$$A_3 = (1-\phi_2)\left[(1-\phi_1)+\phi_1\frac{(\rho\beta)_{s_1}}{(\rho\beta)_f}\right]+\phi_2\frac{(\rho\beta)_{s_2}}{(\rho\beta)_f},$$

$$A_4 = \left[(1-\phi_2)\left[(1-\phi_1)+\phi_1\frac{(\rho C_p)_{s_1}}{(\rho C_p)_f}\right]+\phi_2\frac{(\rho C_p)_{s_2}}{(\rho C_p)_f}\right]$$

The parameters involved in the non-dimensional equations are Deborah number, $\beta = a\lambda_2$, magnetic parameter, $M = \frac{\sigma B_0^{\,2}}{a\rho_f}$, and Prandtl number $\left(\Pr = \frac{\mu_f (C_p)_f}{k_f}\right)$. $\lambda = \frac{Gr_x}{(\mathrm{Re}_x)^2}$ is mixed convection parameter with Grashof number, $Gr_x = \frac{g\beta_f (T_w - T_\infty)x^3}{\nu_f^{\,2}}$ and Reynolds number, $\mathrm{Re}_x = \frac{u_w x}{\nu_f}$.

The pertinent physical quantities are the skin friction coefficient, C_f [16] and local Nusselt number, Nu_x [2] which given by:

$$C_f = \frac{\tau_w}{\rho_f u_w^{\,2}}, \quad Nu_x = \frac{x q_w}{k_f (T_w - T_\infty)} \tag{10}$$

where τ_w is the wall shear stress and q_w is the wall heat flux which are defined by:

$$\tau_w = \frac{\mu_{hnf}}{(1+\lambda_1)}\left(\frac{\partial u}{\partial y}+\lambda_2\left[u\frac{\partial^2 u}{\partial y \partial x}+v\frac{\partial^2 u}{\partial y^2}\right]\right)_{y=0}, \quad q_w = -k_{hnf}\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{11}$$

Using the similarity variables in Eq. (5) and definition in Eq. (11), Eq. (10) can be reduced as:

$$\frac{1}{2}\sqrt{\mathrm{Re}_x}C_f = \frac{1}{(1+\lambda_1)(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}}\left(f''(0)+\beta\left[f'(0)f''(0)-f(0)f'''(0)\right]\right),$$
$$\frac{Nu_x}{\sqrt{\mathrm{Re}_x}} = -\frac{k_{hnf}}{k_f}\theta'(0) \tag{12}$$

3. Results

This study investigated the aligned magnetohydrodynamics mixed convection flow of Jeffrey hybrid nanofluid over a stretching vertical plate using numerical methods. The bvp4c solver in MATLAB is used to solve the system of ordinary differential equations given by Eqns. (7)-(8) with boundary conditions given by Eq. (9) with shooting method approach. Bvp4c is a numerical method for solving a set of ordinary differential equations [17]. It accepts directly problems with undetermined parameters because it was a developed instrument to address the boundary value problems.

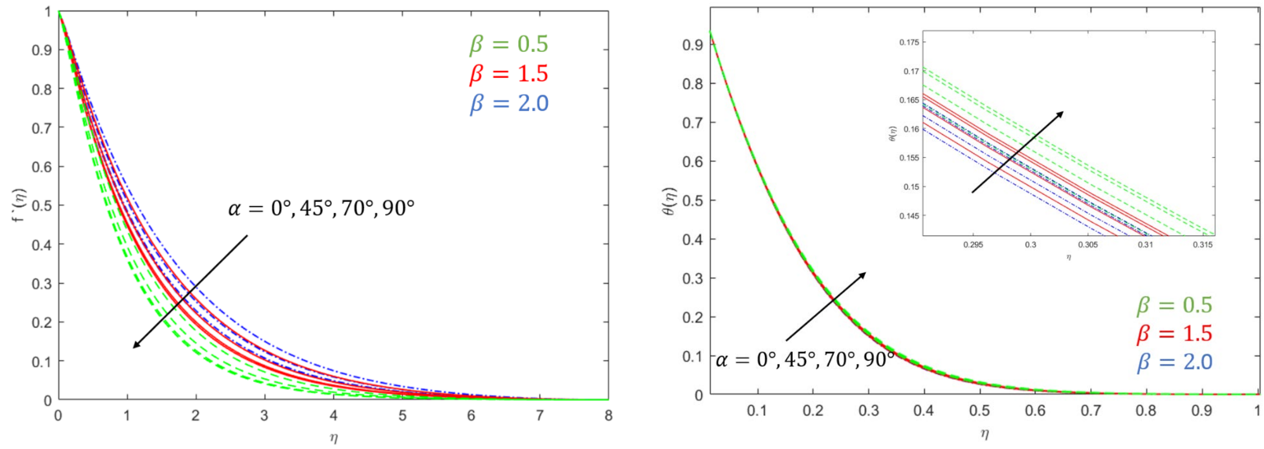
In order to check the validity of the present code, a comparison of the results for the skin friction coefficient, $C_f \operatorname{Re}_x^{1/2}$ obtained from this study was made with previous studies for the case of $\gamma = M = \lambda = 0$. The absence of the buoyancy term, $\lambda\theta$ in Eq. (7) is considered. The comparison is presented in Table 4. Evidently, the present values $C_f \operatorname{Re}_x^{1/2}$ are in excellent agreement.

Table 4
Comparison results for the values of $C_f \operatorname{Re}_x^{1/2}$ for different values of a/c when the buoyancy force term $\lambda\theta$ is absent

a/c	[18]	[19]	[14]	Present
0.1	-0.9694	-0.9694	-0.9694	-0.9694
0.2	-0.9181	-0.9181	-0.9181	-0.9181
0.5	-0.6673	-0.6673	-0.6673	-0.6673
2	2.0175	2.0176	2.0176	2.0176
3	4.7293	4.7296	4.7294	4.7293

The effect of dimensionless parameters on velocity and temperature profiles with its physical interpretation on skin friction coefficient and Nusselt number are presented using graphs and tables. In this study, the Prandtl number of Water-EG was determined to be 29.86. The volume fraction of the solid nanoparticle, ϕ is taken in the range of $0 \leq \phi \leq 0.2$. $\phi = 0$ represents the pure base fluid Water-EG. Unless otherwise specified, we fit the non-dimensional values as follows for numerical computation, $\alpha = 90^\circ$, $M = 1$, $\lambda = 0.02$, $\phi_1 = 0.1$, and $\phi_2 = 0.1$.

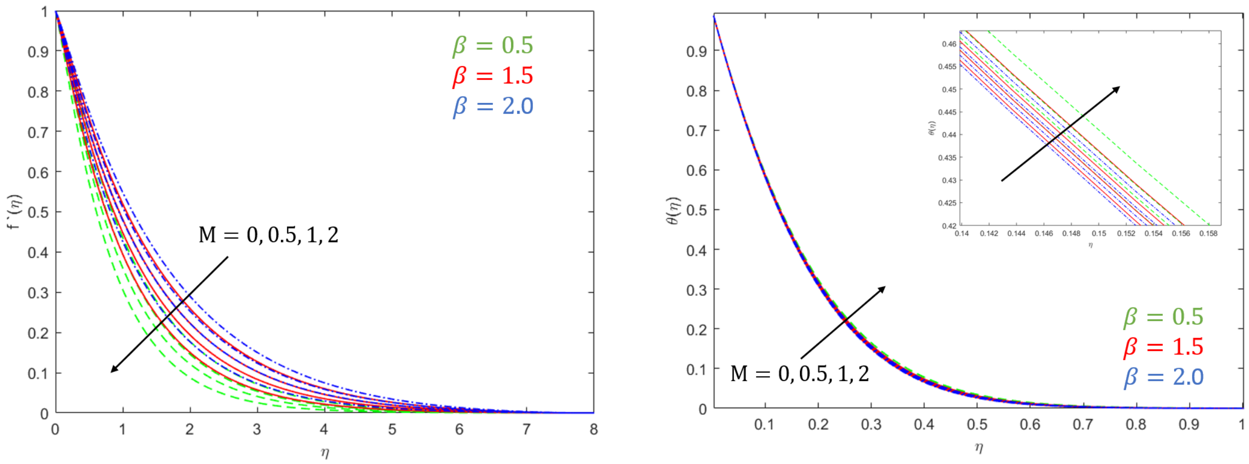
Figures 2 through 5 illustrate how velocity and temperature profiles vary as α, M, λ, and ϕ are varied on spherical shape of Cu-Al_2O_3/Water-EG Jeffrey hybrid nanofluid with diverse values of β. Table 5 illustrates the physical interpretation of variation in skin friction coefficient and Nusselt number for Cu-Al_2O_3/Water-EG Jeffrey hybrid nanofluid with varying dimensionless parameters. Table 6 displays the effect of dimensionless parameters on the skin friction coefficient and Nusselt number, along with their physical interpretations for nanoparticle shape.



(a) (b)

Fig. 2. Effects of α on (a) velocity profiles and (b) temperature profiles

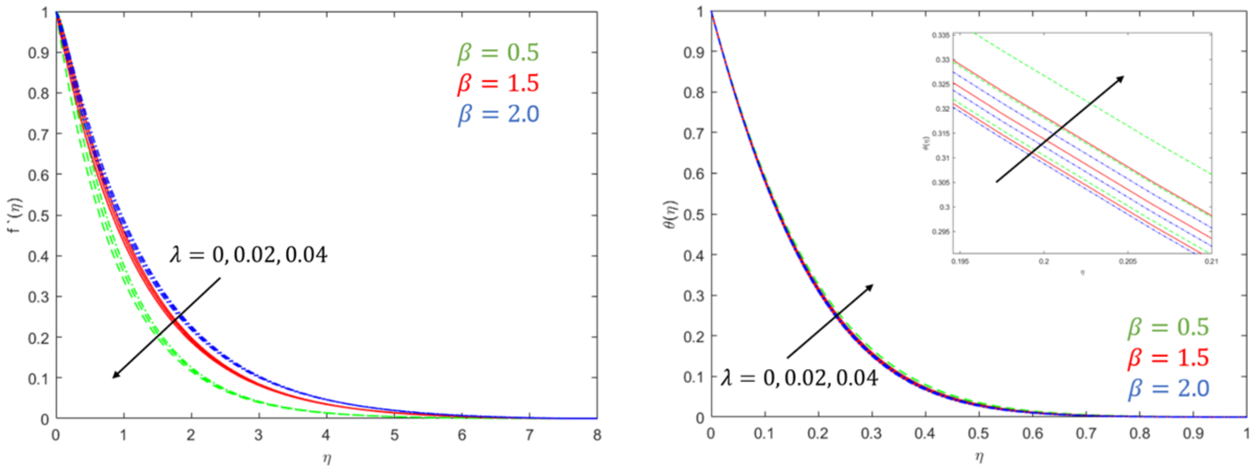
Figure 2(a) presents the effect of the aligned angle of magnetic field, α on the velocity profile. If α increases, the velocity profile will decrease. The velocity profile has reduced the momentum boundary layer thickness. The velocity profile is plotted for different values for the β. There is a growth in the velocity profile with an increment β because it is directly related to the stretching vertical plate. An augmentation in β leads to the acceleration of the nanofluid motion in the momentum boundary layer. Additionally, an increment in the nanofluid motion results in a decrease in the momentum boundary layer thickness. Figure 2(b) presents the effect of α on the temperature profile. This study found that α improved the temperature profile. When α increases, the temperature profile increase. When the temperature profile increases, it also increases the thermal boundary layer thickness.



(a) (b)

Fig. 3. Effects of M on (a) velocity profiles and (b) temperature profiles

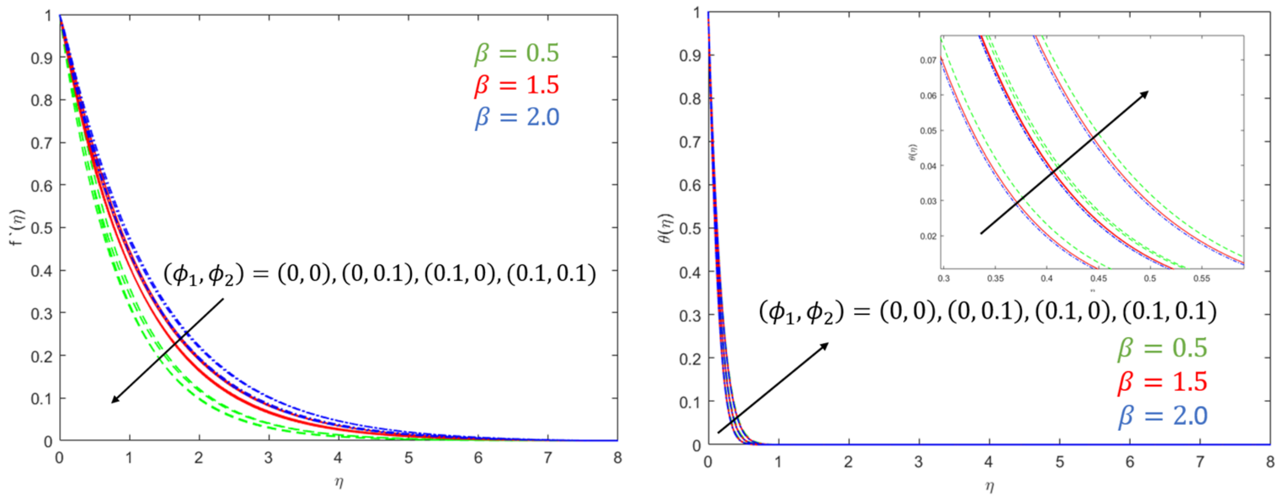
The interaction of the magnetic parameter, M on the velocity profile, $f'(\eta)$ is shown in Figure 3(a). If M increases, the velocity profile will decrease. The velocity profile has reduced the momentum boundary layer thickness. The interaction of the magnetic parameter, M on the temperature profile, $\theta(\eta)$ is shown in Figure 3(b). If M increases, the temperature profile will increase. When the temperature profile increases, it also increases the thermal boundary layer thickness.



(a) (b)

Fig. 4. Effects of λ on (a) velocity profiles and (b) temperature profiles

Figure 4(a) presents the velocity profile when Deborah number, $\beta = 0.5,\ 1.5,\ 2.0$ for a few values of the mixed convection parameter λ respectively. It is well known that $\lambda = 0$ is corresponds to pure forced convection, and the presence of thermal buoyancy ($\lambda \neq 0$) will lead to stronger buoyancy force, which includes more flow along the surface. If λ increases, the velocity profile will decrease. The velocity profile has reduced the momentum boundary layer thickness. Figure 4(b) present the graph of the temperature profile. If λ increases, the temperature profile will increase. When the temperature profile increases, it also increases the thermal boundary layer thickness.



(a) (b)

Fig. 5. Effects of ϕ on (a) velocity profiles and (b) temperature profiles

Figure 5(a) observed that an increment in the nanoparticles volume fraction ϕ_1 and ϕ_2 decreases the velocity profile. If ϕ_1 and ϕ_2 increases, the velocity profiles will decrease. The velocity profile has reduced the momentum boundary layer thickness. Figure 5(b) present the graph of the temperature profile. If ϕ_1 and ϕ_2 increases, the temperature profiles will increase. When the temperature profile increases, it also increases the thermal boundary layer thickness.

Effect of dimensionless parameters on skin friction coefficient and Nusselt number on spherical shape of Cu-Al_2O_3/Water-EG Jeffrey Hybrid nanofluid with diverse values of Deborah number, β are shown in Table 5.

Table 5
Variation in skin friction coefficient and Nusselt number of Cu-Al_2O_3/Water-EG Jeffrey hybrid nanofluid with diverse values of Deborah number, β

α	M	λ	ϕ_1	ϕ_2	Skin friction coefficient (Nusselt number)		
					$\beta = 0.5$	$\beta = 1.5$	$\beta = 2$
0°	1	0.02	0.1	0.1	-0.444299 (-8.771940)	-0.740811 (-8.801654)	-0.870645 (-8.814249)
45°					-0.575070 (-8.729126)	-0.908942 (-8.768202)	-1.054386 (-8.783697)
70°					-0.666989 (-8.698951)	-1.027135 (-8.744607)	-1.183677 (-8.762121)
90°					-0.693870 (-8.690117)	-1.061688 (-8.737705)	-1.221481 (-8.755803)
90°	0	0.02	0.1	0.1	-0.444299 (-8.771940)	-0.740811 (-8.801654)	-0.870645 (-8.814249)
	0.5				-0.575070 (-8.729126)	-0.908942 (-8.768202)	-1.054386 (-8.783697)
	1				-0.693870 (-8.690117)	-1.061688 (-8.737705)	-1.221481 (-8.755803)
	2				-0.905546 (-8.620386)	-1.333507 (-8.683250)	-1.518877 (-8.705981)
90°	1	0	0.1	0.1	-1.334846 (-8.564247)	-1.723285 (-8.668727)	-1.887748 (-8.699847)
		0.02			-0.693870 (-8.690117)	-1.061688 (-8.737705)	-1.221481 (-8.755803)
		0.04			-0.073877 (-8.803439)	-0.412490 (-8.802370)	-0.565511 (-8.808836)
90°	1	0.02	0	0	-1.729307 (-6.577013)	-2.233093 (-6.640299)	-2.446394 (-6.659175)
			0	0.1	-1.306474 (-7.509644)	-1.731292 (-7.577695)	-1.911857 (-7.598766)
			0.1	0	-0.906782 (-7.622231)	-1.356132 (-7.665795)	-1.550595 (-7.681833)
			0.1	0.1	-0.693870 (-8.690117)	-1.061688 (-8.737705)	-1.221481 (-8.755803)

According to Table 5, the aligned angle of the magnetic field, the interaction of magnetic parameters, and Deborah number were found to reduce the skin friction coefficient. Increases in the aligned angle of magnetic field, interaction of magnetic parameter, and Deborah number cause a reduction in the skin friction. It occurs differently with mixed convection parameter and volume fraction of nanoparticles. The skin friction coefficient increases as the mixed convection parameter and volume fraction of nanoparticles increase.

This study discovered a positive correlation between the parameters; aligned angle of the magnetic field and interaction of magnetic parameter with the Nusselt number. When the aligned angle of magnetic field and interaction of magnetic parameter increase, the Nusselt number also increases. However, as mixed convection parameter, volume fraction of nanoparticles and Deborah number increase, the Nusselt number decrease.

Table 6
Effect of dimensionless parameters on skin friction coefficient and Nusselt number with its physical interpretation for nanoparticle shape

α	M	λ	ϕ_1	ϕ_2	Skin Friction Coefficient (Nusselt number)				
					Spherical $(m=3)$	Bricks $(m=3.7)$	Cylindrical $(m=4.9)$	Platelets $(m=5.7)$	Blades $(m=8.6)$
0°	1	0.02	0.1	0.1	-0.740811 (-8.801654)	-0.706678 (-8.333977)	-0.650213 (-7.652911)	-0.614049 (-7.267632)	-0.493113 (-6.200194)
45°					-0.908942 (-8.768202)	-0.874828 (-8.300624)	-0.818411 (-7.619737)	-0.782292 (-7.234584)	-0.661602 (-6.167621)
70°					-1.027135 (-8.744607)	-0.993064 (-8.277091)	-0.936733 (-7.596317)	-0.900683 (-7.211243)	-0.780303 (-6.144584)
90°					-1.061688 (-8.737705)	-1.027632 (-8.270206)	-0.971335 (-7.589463)	-0.935310 (-7.204411)	-0.815040 (-6.137836)
90°	0	0.02	0.1	0.1	-0.740811 (-8.801654)	-0.706678 (-8.333977)	-0.650213 (-7.652911)	-0.614049 (-7.267632)	-0.493113 (-6.200194)
	0.5				-0.908942 (-8.768202)	-0.874828 (-8.300624)	-0.818411 (-7.619737)	-0.782292 (-7.234584)	-0.661602 (-6.167621)
	1				-1.061688 (-8.737705)	-1.027632 (-8.270206)	-0.971335 (-7.589463)	-0.935310 (-7.204411)	-0.815040 (-6.137836)
	2				-1.333507 (-8.683250)	-1.299637 (-8.215871)	-1.243699 (-7.535349)	-1.207942 (-7.150454)	-1.088817 (-6.084493)
90°	1	0	0.1	0.1	-1.723285 (-8.668727)	-1.723285 (-8.197314)	-1.723285 (-7.510029)	-1.723285 (-7.120745)	-1.723285 (-6.039808)
		0.02			-1.061688 (-8.737705)	-1.027632 (-8.270206)	-0.971335 (-7.589463)	-0.935310 (-7.204411)	-0.815040 (-6.137836)
		0.04			-0.412490 (-8.802370)	-0.346351 (-8.338063)	-0.237421 (-7.662492)	-0.167987 (-7.280675)	0.062243 (-6.224414)
90°	1	0.02	0	0	-2.233093 (-6.640299)	-2.233093 (-6.640299)	-2.233093 (-6.640299)	-2.233093 (-6.640299)	-2.233093 (-6.640299)
			0	0.1	-1.731292 (-7.577695)	-1.726929 (-7.373138)	-1.719878 (-7.063315)	-1.715447 (-6.880631)	-1.700899 (-6.337674)
			0.1	0	-1.356132 (-7.665795)	-1.336462 (-7.447587)	-1.304200 (-7.113228)	-1.283618 (-6.913822)	-1.214276 (-6.310642)
			0.1	0.1	-1.061688 (-8.737705)	-1.027632 (-8.270206)	-0.971335 (-7.589463)	-0.935310 (-7.204411)	-0.815040 (-6.137836)

The effect of dimensionless parameters on the skin friction coefficient and Nusselt number, along with their physical interpretation for nanoparticle shape, is presented in Table 6. It was discovered that the values of the skin friction coefficient of spherical, brick-shaped, cylindrical, platelet-shaped, and blade-shaped nanoparticles diminish as the inclination angle of the magnetic field and interaction of magnetic parameter increase. The values of the skin friction coefficient of spherical, brick-shaped, cylindrical, platelet-shaped, and blade-shaped nanoparticles increase as the mixed convection parameter and volume fraction of nanoparticles increase.

The Nusselt number of all nanoparticle shapes increases as the inclination angle of the magnetic field and interaction of magnetic parameter increase whereas it decreases as mixed convection parameter increases. The Nusselt number decreases as the volume fraction of spherical, brick-shaped, cylindrical, and platelet-shaped nanoparticles increases. Nonetheless, the Nusselt number increases as the volume fraction of blade-shaped nanoparticles increases.

4. Conclusions

Due to the problem of the nanoparticle shape effects of aligned MHD mixed convection flow of Jeffrey hybrid nanofluid over a stretching vertical plate, Cu-Al2O3/Water-EG Jeffrey hybrid nanofluid has been analysed in this study. The dimensionless equations in ODE are obtained by reducing the governing equations consisting of continuity, momentum, and energy equations using the similarity transformation method. These equations were coded and solved using the bvp4c solver in MATLAB software. The results were presented graphically for velocity and temperature profiles, while they were tabulated for the skin friction coefficient and Nusselt number in relation to six distinct physical condition parameters. In addition, tabulated data is also used to illustrate the effect of dimensionless parameters on the skin friction coefficient and Nusselt number for five different nanoparticle shapes, including spherical, cylindrical, blades, bricks, and platelets. The results can be summed up as follows:

i. The velocity profile and the momentum boundary layer thickness decrease as the physical parameters α, M, λ, and ϕ increase.
ii. The temperature profile and the thermal boundary layer thickness increase as the physical parameters α, M, λ, and ϕ increase.
iii. Skin friction coefficient increases as the physical parameters λ and ϕ increase.
iv. Nusselt number increase as the physical parameters α and M increase.
v. As β increase, the velocity profile increases while the temperature profile, skin friction coefficient, and Nusselt number decrease.
vi. Blade-shape nanoparticles has the greatest value of skin friction coefficient and Nusselt number.

Acknowledgement
The authors extend their appreciation to Universiti Teknologi MARA for funding this work through Geran Penyelidikan MyRA (GPM) under grant number 600-RMC 5/3/GPM (041/2022).

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