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Comparative Analysis of Blum and P-Y Methods for Design of Flexible Monopile under Lateral Forces

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ABSTRACT

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Berthing operations in maritime transport are highly intricate and of paramount importance, particularly when dealing with hazardous materials. However, the maritime industry has witnessed significant advancements that have greatly enhanced safety and efficiency during these manoeuvres. Integration of advanced technologies, adherence to updated guidelines, and implementation of enhanced safety measures, such as utilizing breasting dolphins equipped with protective fenders, have played a pivotal role in ensuring smooth and secure berthing processes, especially in terminals dealing with hazardous cargo. This study presents a comprehensive comparative analysis of two prominent methods, namely the Blum method and the P-Y method, for designing mooring dolphins. Mooring dolphins are critical structures utilized for safely berthing vessels in offshore terminals, with a particular focus on handling hazardous materials. The Blum method and the P-Y method were both applied in this analysis to examine soil-pile interaction and assess the behaviour and performance of mooring piles. The results of the study demonstrate that both the Blum method and the P-Y method yielded displacements within the allowable limit of 50 centimetres, making them suitable and reliable for the design process. However, it was observed that the P-Y method provided a slightly higher penetration depth, showcasing its ability to exercise improved control over lateral pile head displacement. Additionally, the analysis revealed that the maximum bending moment obtained from the Blum method was approximately 3.6% higher compared to that obtained from the P-Y method.

Keywords:
Monopile design; Mooring dolphin; p-y method, Flexible piles; Deep foundation modelling; Soil-pile interaction

1. Introduction

The berthing operation is a critical and pivotal phase in maritime transport activities, particularly when hazardous materials, such as those found in offshore terminals dedicated to oil and gas handling, are involved. During this crucial process, the vessel approaches the berthing structure, resulting in the transfer of a significant amount of energy. The magnitude of this energy transfer largely depends on the ship's displacement and speed. Once the berthing configuration is achieved, the utmost priority becomes ensuring the safe mooring of the vessel. This is essential to limit any

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unwanted movement caused by external factors such as wind, currents, and tidal forces, as well as to optimize the overall operability of the terminal during the transfer of the tanker's load [1]

Over the years, there has been a growing realization of the need to improve efficiency and increase safety in maritime structures and operations. This realization has prompted the development of updated guidelines for the design of maritime facilities and for ship piloting techniques. As a result, numerous operative instructions and cutting-edge technologies have been devised to ensure increasingly suitable and secure berthing manoeuvres. These advancements have had a profound impact on the safety of personnel, the environment, and the assets involved in the maritime industry based on previous studies [1-9]

Additionally, Piles serve multiple essential functions in various maritime and civil engineering applications. They are commonly utilized as mooring or berthing dolphins in harbours to withstand lateral loads, primarily induced by the impact of ships. Additionally, piles serve as foundational elements for bridges and offshore structures, effectively resisting both lateral and axial loads. With the rapid development of offshore wind farms, a specific type of pile known as monopiles has gained widespread use.

Up until the present, the design of laterally loaded monopiles has been predominantly governed by well-established standards and guidelines, most notably those set forth by the American Petroleum Institute (API) in 2011 and Det Norske Veritas (DNV) in 2014 [10,11] These industry-leading standards recommend the adoption of the P-y model based on the Winkler method [12]. This approach conceptualizes the ground soil as a series of springs that emulate the lateral reactions exerted on the piles along their embedded length [13-16].

A significant advantage of employing the P-y model-based design lies in its ability to accurately capture the non-linear stress-strain response of the soil, a critical consideration in geotechnical engineering [17,18] The origin of the P-y model can be traced back to the groundbreaking work of McClelland and Focht in 1956 [19], and subsequent advancements were made by esteemed researchers such as Reese *et al.,* [20] and Murchison *et al.,* [21], among others.

Among the highly respected design standards and guidelines for laterally loaded piles, particularly in offshore applications, the API and DNV standards hold paramount importance. Often referred to as the API/DNV P-y model, both API and DNV strongly endorse the application of the P-y model, especially for piles in sandy soils, as proposed by Reese *et al.,* [20] and Murchison *et al.,* [21].

This study addresses a research gap by conducting a comprehensive comparative analysis of the P-Y and Blum methods for designing monopile mooring dolphins in oil terminals. The significance of this study lies in its contribution to the understanding of the strengths and limitations of these methods, offering insights to designers and engineers for achieving optimal design outcomes. The primary objective is to evaluate the accuracy, efficiency, and practicality of both methods in predicting the structural behaviour of mooring dolphins under varying loading conditions. By comparing these methods, the study aims to provide valuable recommendations and insights to enhance the design process for monopile mooring dolphins in the context of oil terminals.

2. P-Y Curve Method

The p-y curve method is a valuable technique that establishes a correlation between the nonlinear behaviour of soil resistance (p) and the lateral deflection of the pile (y). The p-y curve itself is a graphical representation of this relationship at a specific depth, as depicted in Figure 1 by Haiderali *et al.,* [22]. To develop and validate these p-y curves, extensive field tests are conducted on fully instrumented piles.

In the 1950s, significant advancements enabled full-scale testing for laterally loaded piles. Two key developments played a crucial role: the availability of digital computers to solve relevant equations and the capability to remotely read strain gauges, facilitating the acquisition of critical soil response data, as highlighted by Reese *et al.,* [23] The p-y curve method inherently considers the nonlinearity of the pile-soil system. However, one limitation is that each p-y curve is unique to a specific combination of soil and pile properties, as noted by Horvath *et al.,* [24].

Despite this limitation, the p-y curve method gained popularity due to its reasonably accurate results. Consequently, API (American Petroleum Institute) [10] endorsed its use in engineering practice in 1987 and 2007. Additionally, FHWA (Federal Highway Administration) [25] recommended the utilization of software such as LPILE or FBPIER, which applies the p-y curve method, for analysing laterally loaded piles, as suggested by Favaretti *et al.,* [26].

The original concept of the p-y curve method was proposed by McClelland *et al.,* [27]. They employed the finite difference method to solve beam bending moment equations, considering applied nonlinear loads versus deflection curves to model soil response. Their comprehensive work involved both full-scale testing on a 60 cm (24 in) steel pipe pile and laboratory tests on undisturbed clay samples.

The lateral resistance of soil near the surface plays a critical role in pile design, and it is essential to consider the potential impact of scour on this resistance. The ultimate lateral bearing capacity for sand has been observed to vary, depending on the depth. At shallow depths, the ultimate bearing capacity is determined by Eq. (1), while at deeper depths, it is determined by Eq. (2). To ensure conservatism, the equation yielding the smallest value of p_u should be used as the ultimate bearing capacity for a given depth. However, it is important to note that these equations may not be conservative enough for layered soil conditions, particularly when the sand is overlain by soft clay [10].

The equations for ultimate lateral resistance are given by Eq. (1) and Eq. (2):

$$p_{us} = (C_1 z + C_2 D)\gamma' z \tag{1}$$

$$p_{ud} = C_3 D \gamma' z \tag{2}$$

Where p_u is the ultimate resistance (s=shallow, d=deep), γ' is the submerged soil unit weight; z is the depth below the original seafloor, $\phi'$is the angle of internal friction of sand, D is the pile outside diameter and $C_1, C_{2,}\ C_3$ are the coefficients determined as function of ϕ' shown in Figure 1.

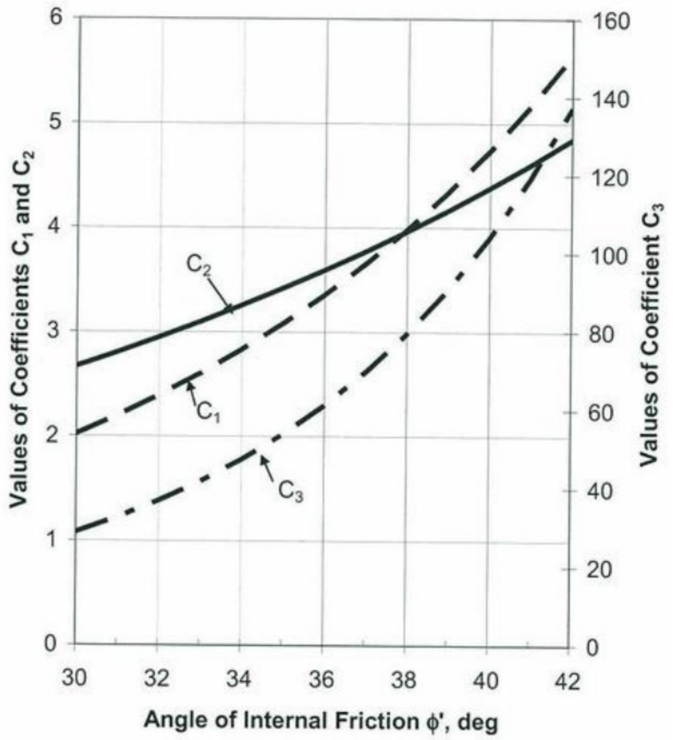

Fig. 1. Coefficients $C_1, C_{2,} C_3$, as functions of ϕ', [10]

The lateral soil resistance-deflection (p-y) relationship for sand exhibits a non-linear behaviour. In situations where more detailed information is not readily accessible, it is possible to approximate this relationship at a specific depth, z, by employing Eq. (3).

$$p = A \times p_u \tanh\left(\frac{k \times z}{A \times p_u} y\right) \tag{3}$$

where A is the factor to account for cyclic or static load condition, k is the rate of increase with depth of initial modulus of subgrade reaction shown in Table 1, y is the lateral deflection at depth z.

Table 1
Rate of increase with depth of initial modulus of subgrade reaction [10]

ϕ'	k	
	MN/m3	lb/in3
25°	5.4	20
30°	11	40
35°	22	80
40°	45	165

3. Blum Method

The method first introduced by Blum [28], which has been in use for over 80 years, remains widely applicable today. Its simplicity and swift convergence to solutions have positioned it as the preferred alternative to costly and computationally complex methods. Blum's method assumes that the soil's response is entirely independent of the pile displacement. In this method, the anchor at the end of the pile is considered to have zero resistance, while shear force exists. The maximum soil resistance response is determined by the Rankine horizontal earth pressure coefficient, k [29].

In the Blum method the location of the maximum bending moment below the seabed is determined by using Eq. (4), considering the maximum allowable bending moment:

$$M_{max} = \frac{k_p \cdot \gamma'}{24} X_m^2 (3X_m^2 + X_m(4h + 8b) + 12hb) \tag{4}$$

where X_m is the distance from the location of the maximum bending moment occurrence to the seabed, b is the width of the mooring dolphin, h is the free length of the pile between the point of load entry and the seabed level and γ' is the effective unit weight of the submerged soil.

4. Study Area

Bandar Anzali is situated in the northwestern region of Iran, within the province of Gilan. Located approximately 40 kilometres northwest of the city of Rasht, it is positioned in the central part of Anzali County, between the Caspian Sea and Anzali Wetland, see Figure 2. It serves as the largest and most active port on the southern shores of the Caspian Sea, equipped with modern facilities for efficient loading and unloading operations. Leveraging its strategic geographical location, Bandar Anzali plays a vital role in facilitating the exchange and facilitation of commercial goods, particularly in sectors such as fuel, iron and steel, wood, and grains within the region.

In terms of tonnage for loading and unloading, the port holds the third position, following Imam Khomeini Port and Shahid Rajaee Port. Additionally, its favourable location allows for close proximity to essential ports in Russia, such as Astrakhan and Lagan, Turkmenbashi Port in Turkmenistan, Aktau in Kazakhstan, and Baku in Azerbaijan, enabling effective connections with regional markets surrounding the Caspian Sea.

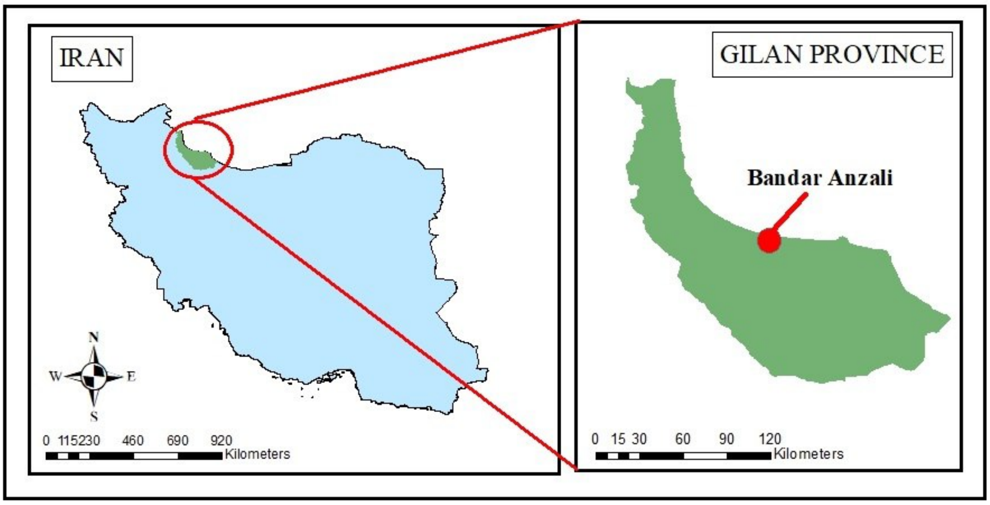

Fig. 2. Location of study site in Bandar-e-Anzali Port, north of Iran

In the development plan for Bandar Anzali, considering the classification of different types of exchanged goods at the various stages of future development and their projected volumes, specific quays have been designed for accommodating container ships, general cargo, and petroleum products. To handle oil tankers, three dolphin-type quays have been designated for mooring, as indicated in Figure 3.

Fig. 3. Dolphin-type quays location

5. Specifications of Mooring Vessels

The design of mooring dolphins necessitates careful consideration of the diverse loads and forces that these structures may experience during berthing and mooring operations. Table 2 provides essential data, including Gross Register Tonnage (GRT), Deadweight Tonnage (DWT), Displacement Weight (DT), Length Overall ($L_{(\mathrm{OA})}$), Breadth (B), and Draft (D_f), all of which play a crucial role in determining the design loads [30].

Table 2
Specifications of the design vessels (largest and smallest vessels)

Type		GRT	DWT	DT	$L_{(\mathrm{OA})}$	B	D_f
		[tons]	[tons]	[tons]	[m]	[m]	[m]
Design Vessel (Largest)	Oil Tanker	8300	12000	16500	147	19	8.6
Design Vessel (smallest)	Oil Tanker	2765	5000	7383	102	16.8	6.4

6. Geotechnical Characterization of Seabed

One of the most significant factors directly influencing the design of the mooring dolphin is the geotechnical conditions of the seabed in the specific area of interest. In this study, the geotechnical conditions of the subsurface layers were investigated at 15 marine boreholes with depths of 20, 25, and 35 meters. Based on the log data from these boreholes, the stratification of the seabed in the study area was found to be relatively uniform, mainly consisting of densely to very densely compacted fine to medium sand with a unified classification of SM and SM-SP. Field tests, including Standard Penetration Tests (SPT), were conducted at intervals of 1.5 to 3.0 meters. The graph depicting the variations of the SPT N-value corrected at different depths for the studied boreholes is presented in Figure 4.

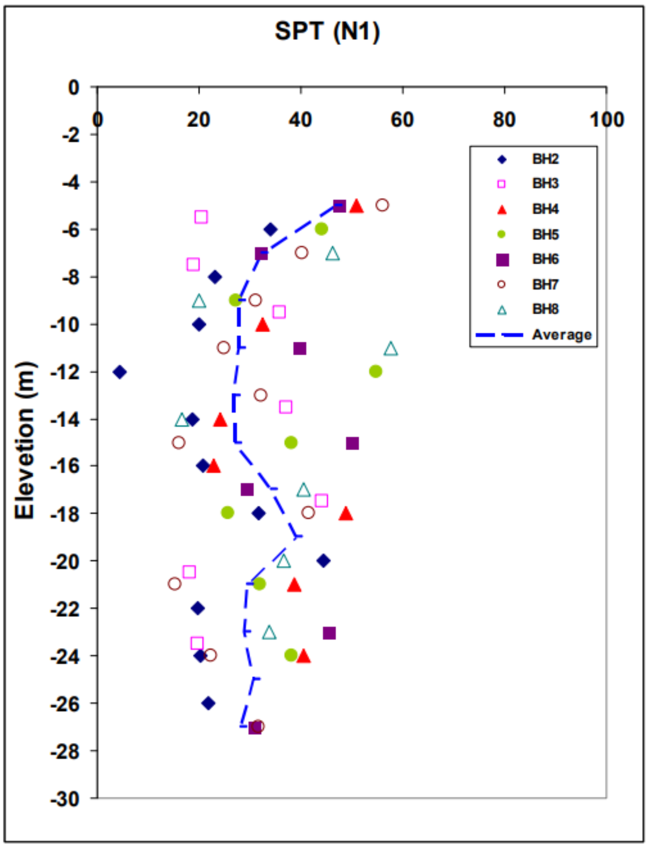

Fig. 4. SPT N-value corrected at different depths

According to the graph, the corrected SPT N-value with respect to overburden pressure (N1) in the sandy layers (SM) ranges from 15 to 57, with an average value of 33, indicating densely to very densely compacted conditions for the sandy layers. Based on the obtained data, the geotechnical parameters considered for the design of the mooring dolphin are presented in Table 3.

Table 3
Geotechnical parameters of the seabed soil layer

Geotechnical Parameters	Qualitative/Quantitative Description
Soil Type	Sand
Layer Thickness (m)	20-35
Soil Classification	SM or SP-SM
Soil Compaction	Medium to Dense
Dry Unit Weight (t/m3)	1.7
Effective Friction Angle (φ) °	30
Effective Cohesion (t/m2)	0

7. Soil–Pile Interaction Analysis

The allowable stress on the mooring dolphin system has been determined based on the API standard. According to this standard, in the design of mooring dolphins, the maximum mooring force should be less than the allowable lateral stress of the dolphin, and the resulting displacement due to this force should be less than 50 centimetres. Additionally, the applied force on the mooring system is determined based on the specifications of the design vessel, as indicated in Table 4. For vessels with a weight of 8300 tons, this value is equal to 70 tons [31].

Table 4
Standard Values of Tractive Force by Ships [31]

GT of ship (ton)	Tractive force acting on a bollard (kN)	Tractive force acting on a mooring post (kN)
200<GT<500	150	150
500<GT<1000	250	250
1000<GT<2000	250	350
2000<GT<3000	350	350
3000<GT<5000	350	500
5000<GT<10000	500	700
10000<GT<20000	700	1000
20000<GT<50000	1000	1500

The analysis of the flexible mooring dolphin aims to determine the force-displacement curve assuming a known Tractive force. The interaction between the dolphin, modelled as a single pile, and the soil is analysed using both the Blum and P-Y methods for critical loading combinations. The specifications of the modelled pile are provided in Table 5.

Table 5
Pile dimensions

Pile Length (m)	Pile diameter (m)	Pile thickness (mm)
20	1.7	16

In order to obtain the optimal penetration depth in the P-Y method, the penetration depth is first calculated using the Blum method. Then, springs are placed below the pile to exceed the penetration depth obtained from the Blum method by at least 1.5 times. The analysis is performed in each step while gradually removing the springs from the bottom. This process continues until the analytical model exhibits significant deformations or equilibrium conditions are not met. As a result, the minimum required depth to achieve equilibrium and control lateral deformation of the pile is determined. Additionally, this process ensures non-linear analysis for the penetration depth obtained from the Blum method and provides the force-displacement diagram and moment diagram with depth. The arrangement of the springs is shown in Figure 5.

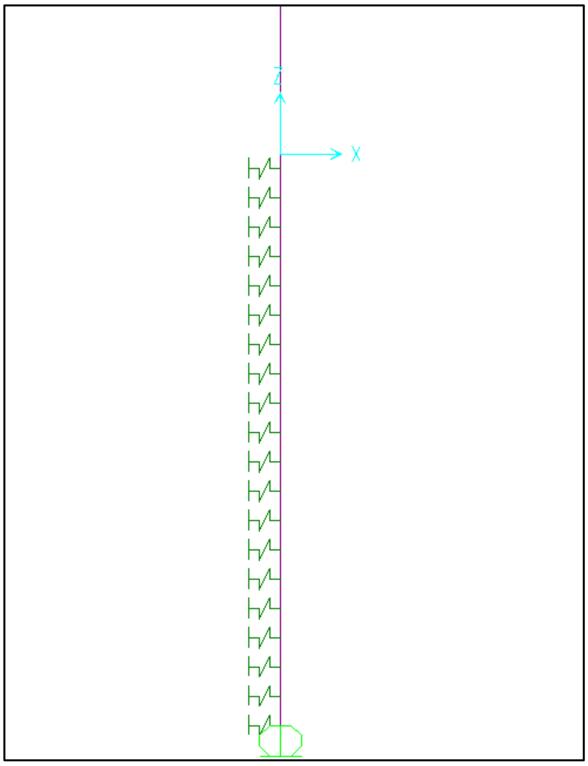

Fig. 5. Pile Model, Maximum Penetration Depth (20 meters), and Placement of Springs

8. Results

The pile is analysed using the Blum method, and the P-Y curves are used to control the pile behaviour. The pile is modelled in the SAP2000 software, and the surrounding soil is simulated using non-linear springs. This approach allows us to calculate the energy absorption capacity and the elastic and permanent displacements of the pile. The first spring is considered at a depth of half a meter below the ground level, and the subsequent springs are placed at one-meter intervals. Accordingly, the relationship between P and Y is calculated and applied to each level. Thus, for each specified depth, a corresponding P-Y curve is obtained, as illustrated in Figure 6.

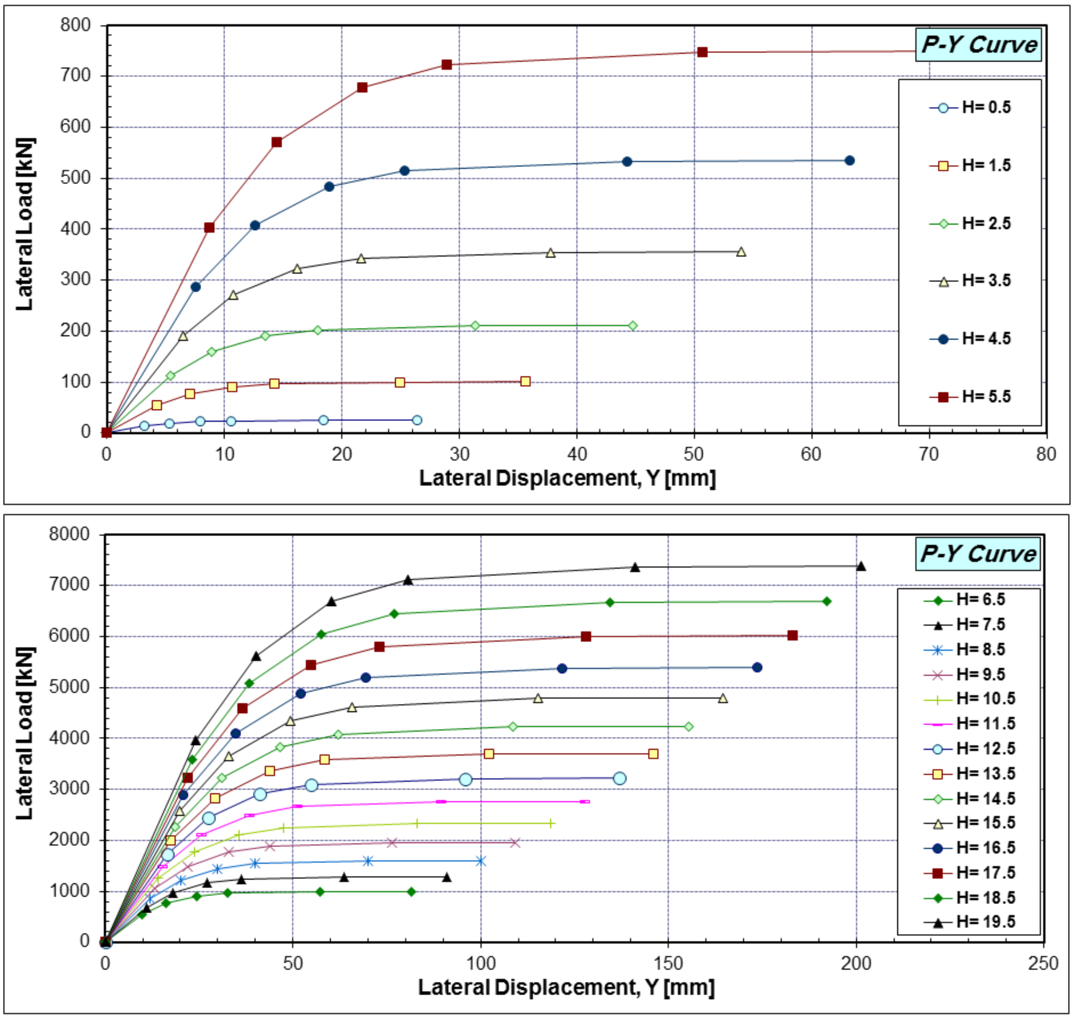

Fig. 6. Pile P-Y curve for different depths

Figure 7 illustrates the variations in the pile penetration depth with the displacement of the pile head. As observed, the graph remains relatively constant from approximately 11 to 12 meters below the ground level. However, if the depth is placed below this range, the displacement of the pile head undergoes sudden changes. Therefore, based on the p-y method, the pile penetration depth is determined to be approximately 12 meters below the ground level, which is slightly higher than the depth obtained from the Blum method. Consequently, a safe pile penetration depth of around 12 meters is selected, where the pile head displacement remains within acceptable limits and is well controlled.

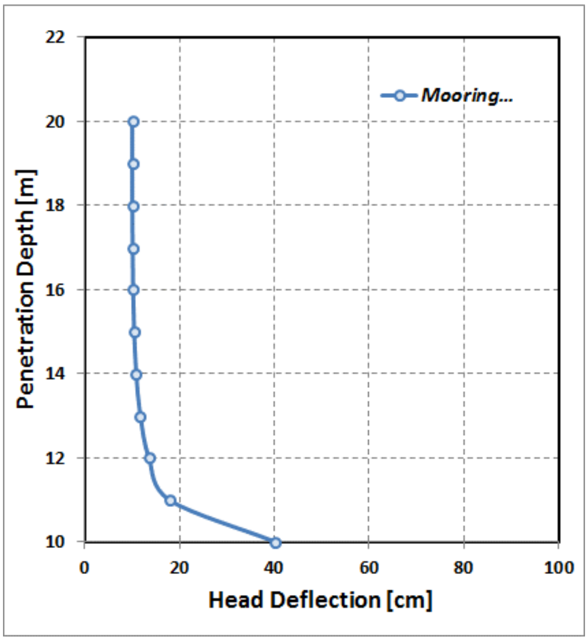

Fig. 7. Variations in Pile Head Displacement with Penetration Depth in the P-Y Method

Additionally, the bending moment and shear forces distribution along the length of the mooring pile and the mooring pile's bending moment diagram are presented in Figures 8 and 9, respectively.

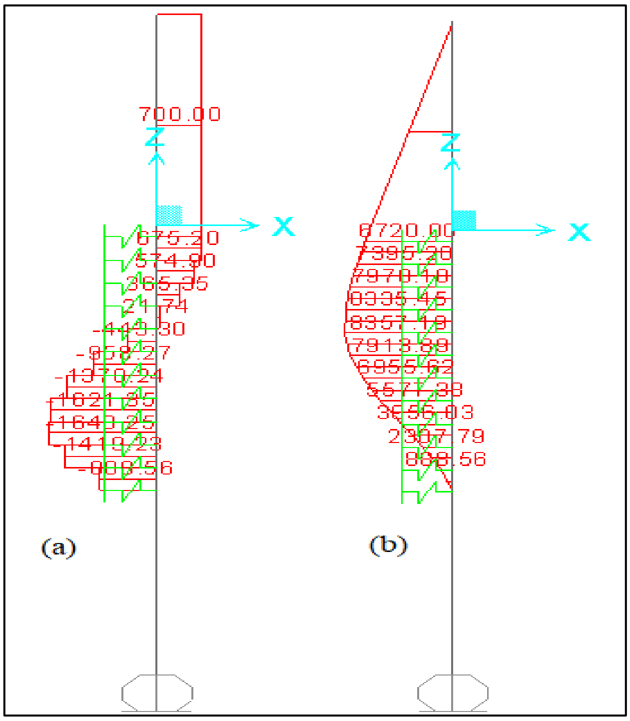

Fig. 8. (a) shear force (kN) and (b) bending moment $(kN.m)$ Curves of mooring pile with 1.7-meter Diameter and 12-meter Penetration Depth under 70-ton Load using P-Y Method

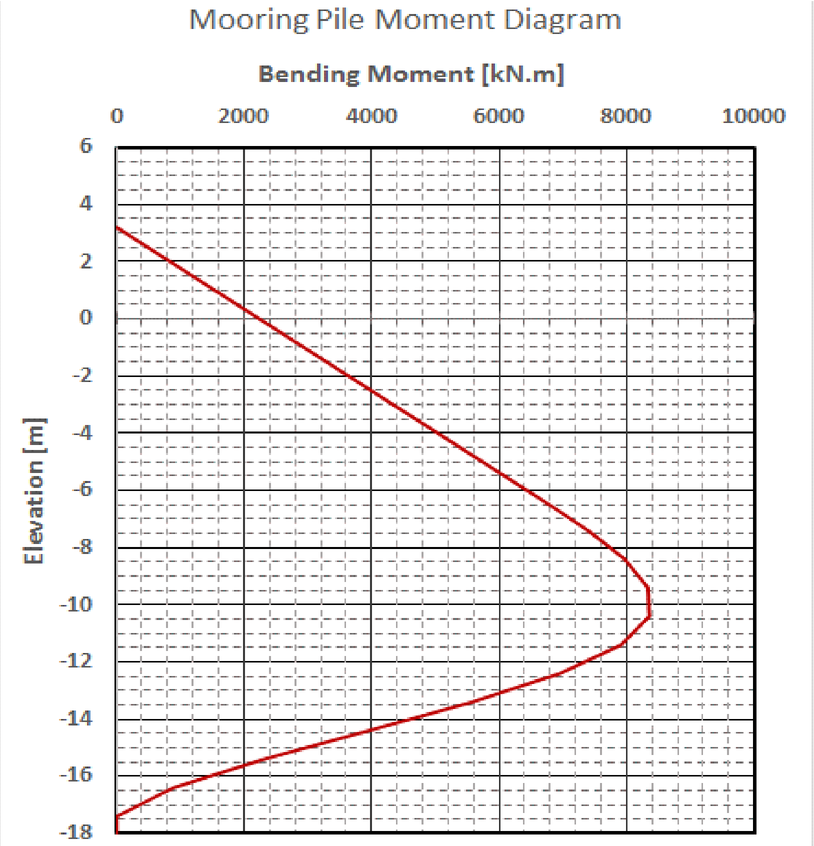

Fig. 9. Bending moment variation with mooring pile depth

9. Conclusions

The analysis of mooring piles' performance in oil terminals holds significant importance in understanding their behaviour during vessel berthing. In this study, we utilized the P-Y method, a well-established approach for analysing deep foundations under lateral loads, to investigate the lateral deformation of mooring piles in the terminal. By considering non-linear force-displacement curves, we assessed the soil-pile interaction and compared the results with those obtained from the Blum method.

The findings from the mooring pile analysis revealed that the p-y method yielded a maximum lateral displacement of 13.5 centimetres and a penetration depth of 12 meters, while the Blum method produced values of 7.4 centimetres and 10.45 meters, respectively. It is important to note that both methods demonstrated displacements within the allowable limit of 50 centimetres.

Moreover, the distance from the point of maximum bending moment occurrence to the seabed was determined to be 4.5 meters using the P-Y method and 3.9 meters with the Blum method.

Additionally, the maximum bending moment imposed on the mooring pile, as obtained from the Blum method, measured 8660 $kN.m$, which was approximately 3.6% higher than the maximum bending moment determined by the p-y method. These comprehensive results offer valuable insights into the behaviour and performance of mooring piles, aiding in the design and optimization of oil terminal structures.

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